

Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.1-Quadratic/1.2.1.6-
 $g+h-x^m-a+b-x+c-x^2-p-d+e-x+f-x^2-q$

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3.97	$\int \frac{x}{\sqrt{a+bx+cx^2}(d-fx^2)} dx$	685
3.98	$\int \frac{1}{\sqrt{a+bx+cx^2}(d-fx^2)} dx$	690
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3.101	$\int \frac{1}{x^3\sqrt{a+bx+cx^2}(d-fx^2)} dx$	707
3.102	$\int \frac{x^4}{(a+bx+cx^2)^{3/2}(d-fx^2)} dx$	712
3.103	$\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d-fx^2)} dx$	719
3.104	$\int \frac{x^2}{(a+bx+cx^2)^{3/2}(d-fx^2)} dx$	725
3.105	$\int \frac{x}{(a+bx+cx^2)^{3/2}(d-fx^2)} dx$	730
3.106	$\int \frac{1}{(a+bx+cx^2)^{3/2}(d-fx^2)} dx$	735
3.107	$\int \frac{1}{x(a+bx+cx^2)^{3/2}(d-fx^2)} dx$	740
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3.111	$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2} dx$	762

3.112	$\int \frac{\sqrt{a+bx+cx^2}}{x(d+ex+fx^2)} dx$	767
3.113	$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex+fx^2)} dx$	773
3.114	$\int \frac{x^3}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx$	779
3.115	$\int \frac{x^2}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx$	785
3.116	$\int \frac{x}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx$	791
3.117	$\int \frac{1}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx$	801
3.118	$\int \frac{1}{x\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx$	811
3.119	$\int \frac{1}{x^2\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx$	816
3.120	$\int \frac{1}{x^3\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx$	821
3.121	$\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)} dx$	828
3.122	$\int \frac{x^2}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)} dx$	834
3.123	$\int \frac{x}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)} dx$	839
3.124	$\int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)} dx$	844
3.125	$\int \frac{1}{x(a+bx+cx^2)^{3/2}(d+ex+fx^2)} dx$	851
3.126	$\int \frac{x^4}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx$	859
3.127	$\int \frac{x^3}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx$	866
3.128	$\int \frac{x^2}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx$	873
3.129	$\int \frac{x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx$	879
3.130	$\int \frac{1}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx$	883
3.131	$\int \frac{1}{x\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx$	888
3.132	$\int \frac{1}{x^2\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx$	894
3.133	$\int (2+3x)^2 (30+31x-12x^2)^2 \sqrt{6+17x+12x^2} dx$	901
3.134	$\int (2+3x) (30+31x-12x^2) \sqrt{6+17x+12x^2} dx$	906
3.135	$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)(30+31x-12x^2)} dx$	910

3.136	$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^2(30+31x-12x^2)^2} dx$	914
3.137	$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^3(30+31x-12x^2)^3} dx$	919
3.138	$\int (-3+2x)(-3x+x^2)^{2/3} dx$	925
3.139	$\int ((-3+x)x)^{2/3}(-3+2x) dx$	928
3.140	$\int \frac{x(9-9x+2x^2)}{\sqrt[3]{-3x+x^2}} dx$	931
3.141	$\int \frac{x(9-9x+2x^2)}{\sqrt[3]{(-3+x)x}} dx$	934
3.142	$\int \frac{g+hx}{\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}(g^2+3h^2x^2)} dx$	937
3.143	$\int \frac{g+hx}{\sqrt[3]{\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2} \left(\frac{f\left(b^2-\frac{-c^2g^2+bcgh+2b^2h^2}{3h^2}\right)}{c^2} + \frac{bfx}{c} + fx^2 \right)} dx$	941

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Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [143]. This is test number [37].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric₂F₁ functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (143)	% 0.00 (0)
Mathematica	% 99.30 (142)	% 0.70 (1)
Maple	% 98.60 (141)	% 1.40 (2)
Maxima	% 10.49 (15)	% 89.51 (128)
Fricas	% 46.15 (66)	% 53.85 (77)
Sympy	% 7.69 (11)	% 92.31 (132)
Giac	% 34.97 (50)	% 65.03 (93)
Mupad	% 13.29 (19)	% 86.71 (124)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

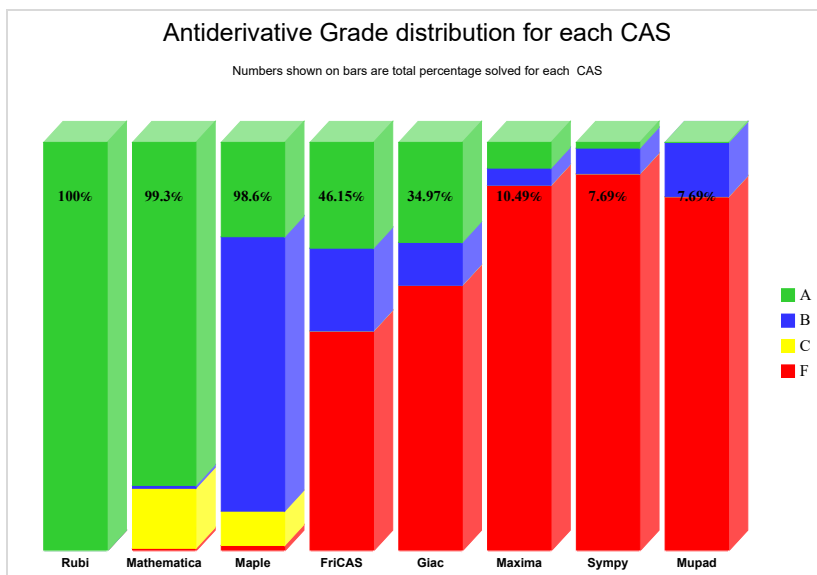
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

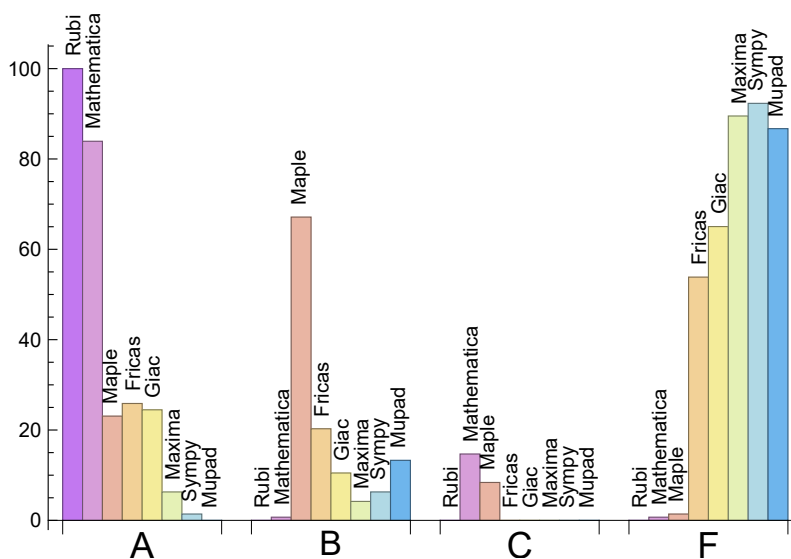
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	83.92	0.70	14.69	0.70
Maple	23.08	67.13	8.39	1.40
Maxima	6.29	4.20	0.00	89.51
Fricas	25.87	20.28	0.00	53.85
Sympy	1.40	6.29	0.00	92.31
Giac	24.48	10.49	0.00	65.03
Mupad	0.00	13.29	0.00	86.71

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure **F**.

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	1	100.00 %	0.00 %	0.00 %
Maple	2	100.00 %	0.00 %	0.00 %
Maxima	128	52.34 %	0.00 %	47.66 %
Fricas	77	1.30 %	98.70 %	0.00 %
Sympy	132	87.88 %	12.12 %	0.00 %
Giac	93	22.58 %	18.28 %	59.14 %
Mupad	124	99.19 %	0.81 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

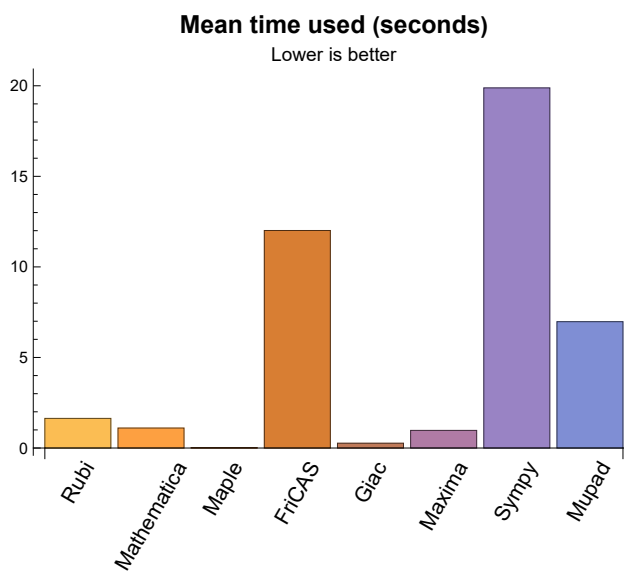
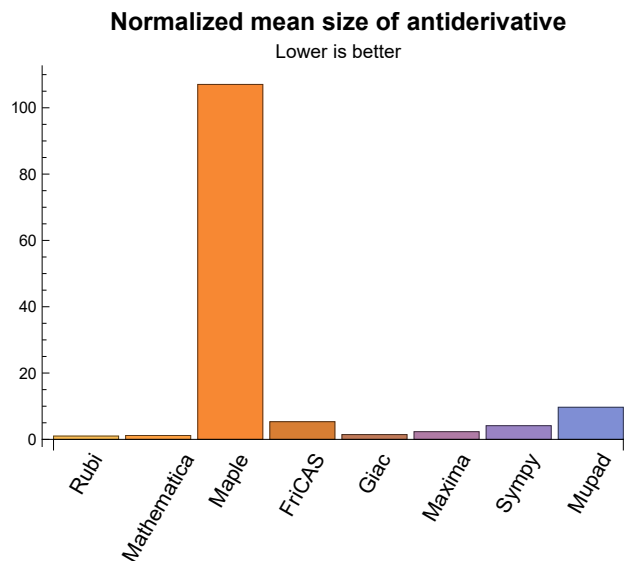
1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	1.64	333.90	1.00	302.00	1.00
Mathematica	1.10	347.54	1.17	290.50	0.97
Maple	0.03	52015.57	107.04	1172.00	3.85
Maxima	0.97	388.27	2.32	220.00	1.11
Fricas	12.01	1356.59	5.33	337.00	2.16
Sympy	19.88	971.36	4.13	680.00	4.09
Giac	0.27	259.92	1.44	146.00	1.21
Mupad	6.98	7822.16	9.68	187.00	1.25

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.



1.4 list of integrals that has no closed form antiderivative

{

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {25, 28, 142}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at <https://>

ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount = 1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

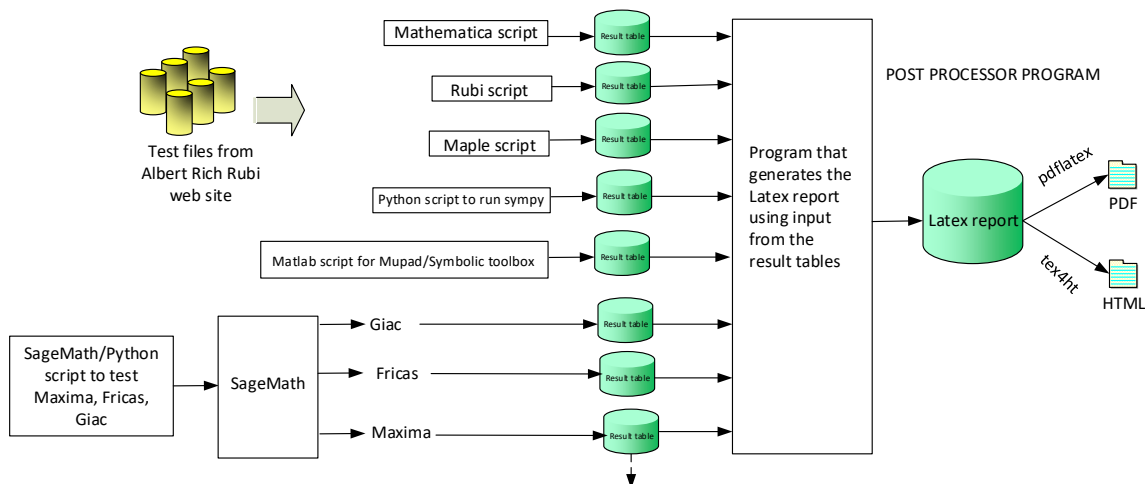
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand, the_variable)
```

Which gives $\sin(x)^2/2$

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
 2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
 3. integer. Leaf size of result.
 4. integer. Leaf size of the optimal antiderivative.
 5. number. CPU time used to solve this integral. 0 if failed.
 6. string. The integral in Latex format
 7. string. The input used in CAS own syntax.
 8. string. The result (antiderivative) produced by CAS in Latex format
 9. string. The optimal antiderivative in Latex format.
 10. integer. 0 or 1. Indicates if problem has known antiderivative or not
 11. String. The result (antiderivative) in CAS own syntax.
 12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
- The following field present only in Rubi and Mathematica Tables*
13. integer. 1 if result was verified or 0 if not verified.
- The following fields present only in Rubi Tables*
14. integer. Number of rules used.
 15. integer. Integrand leaf size.
 16. real number. Ratio of field 14 over field 15
 17. integer. 1 if result was verified or 0 if not verified.
 18. String of form "{n,n,...}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 36, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 133, 134, 135, 136, 137, 138, 139, 140, 141 }

B grade: { 35 }

C grade: { 11, 12, 31, 32, 33, 34, 37, 38, 62, 63, 75, 76, 93, 126, 127, 128, 129, 130, 131, 132, 142 }

F grade: { 143 }

2.1.3 Maple

A grade: { 1, 2, 3, 10, 22, 28, 31, 32, 33, 34, 36, 38, 39, 92, 94, 95, 96, 99, 100, 101, 126, 127, 128, 129, 130, 131, 132, 133, 134, 138, 139, 140, 141 }

B grade: { 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 29, 30, 35, 37, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 97, 98, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 135, 136, 137 }

C grade: { 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51 }

F grade: { 142, 143 }

2.1.4 Maxima

A grade: { 1, 2, 3, 10, 92, 133, 134, 138, 139 }

B grade: { 25, 26, 27, 28, 29, 30 }

C grade: { }

F grade: { 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 135, 136, 137, 140, 141, 142, 143 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 10, 13, 14, 33, 36, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 92, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141 }

B grade: { 7, 11, 17, 18, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 37, 55, 66, 67, 81, 82, 93, 97, 98, 99, 116, 117, 135 }

C grade: { }

F grade: { 4, 5, 6, 8, 9, 12, 15, 16, 19, 20, 21, 35, 39, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 83, 84, 85, 86, 87, 88, 89, 90, 91, 94, 95, 96, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 118, 119, 120, 121, 122, 123, 124, 125, 142, 143 }

2.1.6 Sympy

A grade: { 33, 139 }

B grade: { 1, 2, 3, 13, 14, 17, 18, 31, 138 }

C grade: { }

F grade: { 4, 5, 6, 7, 8, 9, 10, 11, 12, 15, 16, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 140, 141, 142, 143 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 10, 13, 14, 15, 17, 18, 28, 29, 30, 36, 40, 41, 42, 43, 44, 45, 46, 47, 48, 92, 126, 127, 129, 131, 133, 134, 137, 138, 139, 140, 141 }

B grade: { 5, 11, 16, 31, 32, 33, 34, 37, 38, 51, 128, 130, 132, 135, 136 }

C grade: { }

F grade: { 6, 7, 8, 9, 12, 19, 20, 21, 22, 23, 24, 25, 26, 27, 35, 39, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 142, 143 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 13, 14, 16, 17, 18, 31, 33, 36, 133, 134, 138, 139, 140, 141 }

C grade: { }

F grade: { 6, 7, 8, 9, 10, 11, 12, 15, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 34, 35, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 135, 136, 137, 142, 143 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	86	133	84	200	333	87	97
normalized size	1	1.00	0.91	1.41	0.89	2.13	3.54	0.93	1.03
time (sec)	N/A	0.113	0.072	0.031	0.968	0.860	1.854	0.151	3.436
Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	204	373	220	500	933	263	253
normalized size	1	1.00	0.89	1.64	0.96	2.19	4.09	1.15	1.11
time (sec)	N/A	0.330	0.204	0.012	0.979	0.688	7.873	0.159	0.249
Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	441	441	422	822	471	1014	1962	623	552
normalized size	1	1.00	0.96	1.86	1.07	2.30	4.45	1.41	1.25
time (sec)	N/A	0.621	0.459	0.008	0.989	0.710	23.623	0.164	3.792

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	212	745	0	0	0	266	3888
normalized size	1	1.00	0.77	2.72	0.00	0.00	0.00	0.97	14.19
time (sec)	N/A	0.282	0.368	0.016	0.000	0.000	0.000	0.168	38.323

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	596	596	523	9311	0	0	0	1313	23006
normalized size	1	1.00	0.88	15.62	0.00	0.00	0.00	2.20	38.60
time (sec)	N/A	1.772	1.842	0.055	0.000	0.000	0.000	0.205	7.534

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	322	3358	0	0	0	0	-1
normalized size	1	1.00	0.97	10.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.601	0.801	0.085	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	249	714	0	6113	0	0	-1
normalized size	1	1.00	1.00	2.87	0.00	24.55	0.00	0.00	-0.00
time (sec)	N/A	0.204	0.240	0.024	0.000	147.137	0.000	0.000	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	380	440	2758	0	0	0	0	-1
normalized size	1	1.00	1.15	7.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.798	0.650	0.026	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	797	796	674	6422	0	0	0	0	-1
normalized size	1	1.00	0.85	8.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.870	4.060	0.031	0.000	0.000	0.000	0.000	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	49	46	65	46	0	48	-1
normalized size	1	1.00	1.04	0.98	1.38	0.98	0.00	1.02	-0.02
time (sec)	N/A	0.034	0.008	0.018	0.973	1.120	0.000	0.246	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	78	637	0	758	0	457	-1
normalized size	1	1.00	0.67	5.44	0.00	6.48	0.00	3.91	-0.01
time (sec)	N/A	0.169	0.033	0.176	0.000	1.005	0.000	0.376	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	484	484	136	6871419	0	0	0	0	-1
normalized size	1	1.00	0.28	14197.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	23.581	0.083	0.515	0.000	0.000	0.000	0.000	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	182	175	510	0	583	1260	191	273
normalized size	1	0.99	0.95	2.77	0.00	3.17	6.85	1.04	1.48
time (sec)	N/A	0.346	0.211	0.008	0.000	1.387	16.651	0.169	3.852

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	542	542	535	1672	0	1837	4663	738	893
normalized size	1	1.00	0.99	3.08	0.00	3.39	8.60	1.36	1.65
time (sec)	N/A	1.102	0.604	0.012	0.000	1.957	145.638	0.246	4.854

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	406	398	267	1698	0	0	0	416	-1
normalized size	1	0.98	0.66	4.18	0.00	0.00	0.00	1.02	-0.00
time (sec)	N/A	0.478	0.462	0.013	0.000	0.000	0.000	0.169	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1075	1067	952	51470	0	0	0	3226	118429
normalized size	1	0.99	0.89	47.88	0.00	0.00	0.00	3.00	110.17
time (sec)	N/A	4.177	6.700	0.054	0.000	0.000	0.000	0.451	30.314

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	131	340	0	1150	709	219	395
normalized size	1	1.00	0.94	2.43	0.00	8.21	5.06	1.56	2.82
time (sec)	N/A	0.131	0.143	0.006	0.000	1.419	2.437	0.172	0.422

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	131	340	0	1130	680	207	375
normalized size	1	1.00	0.94	2.43	0.00	8.07	4.86	1.48	2.68
time (sec)	N/A	0.104	0.027	0.006	0.000	0.886	2.259	0.170	4.020

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	617	615	517	16209	0	0	0	0	-1
normalized size	1	1.00	0.84	26.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	8.998	2.086	0.051	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1092	1092	1627	59465	0	0	0	0	-1
normalized size	1	1.00	1.49	54.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	18.867	6.564	0.032	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	393	2269	0	0	0	0	-1
normalized size	1	1.00	0.94	5.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.702	4.175	0.044	0.000	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	780	780	254	784	0	6861	0	0	-1
normalized size	1	1.00	0.33	1.01	0.00	8.80	0.00	0.00	-0.00
time (sec)	N/A	5.162	0.409	0.039	0.000	134.783	0.000	0.000	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	283	1771	0	8977	0	0	-1
normalized size	1	1.00	0.94	5.86	0.00	29.73	0.00	0.00	-0.00
time (sec)	N/A	0.843	0.430	0.033	0.000	124.907	0.000	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	154	608	0	1515	0	0	-1
normalized size	1	1.00	1.52	6.02	0.00	15.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.174	0.021	0.000	1.199	0.000	0.000	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	140	324	361	322	0	0	-1
normalized size	1	1.00	1.01	2.33	2.60	2.32	0.00	0.00	-0.01
time (sec)	N/A	0.220	0.301	0.068	1.074	0.928	0.000	0.000	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	167	760	678	344	0	0	-1
normalized size	1	1.00	1.01	4.58	4.08	2.07	0.00	0.00	-0.01
time (sec)	N/A	0.221	0.378	0.027	1.112	0.899	0.000	0.000	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	185	1560	1276	439	0	0	-1
normalized size	1	1.00	0.96	8.08	6.61	2.27	0.00	0.00	-0.01
time (sec)	N/A	0.266	0.596	0.020	1.263	0.990	0.000	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	148	186	363	245	0	93	-1
normalized size	1	1.00	0.98	1.23	2.40	1.62	0.00	0.62	-0.01
time (sec)	N/A	0.229	0.356	0.051	1.069	1.228	0.000	0.477	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	172	466	668	365	0	112	-1
normalized size	1	1.00	0.99	2.68	3.84	2.10	0.00	0.64	-0.01
time (sec)	N/A	0.255	0.570	0.019	1.099	0.974	0.000	0.499	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	190	878	1276	435	0	121	-1
normalized size	1	1.00	0.96	4.46	6.48	2.21	0.00	0.61	-0.01
time (sec)	N/A	0.303	0.710	0.017	1.241	1.018	0.000	0.541	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	79	14	0	49	36	31	13
normalized size	1	1.00	5.27	0.93	0.00	3.27	2.40	2.07	0.87
time (sec)	N/A	0.017	0.045	0.020	0.000	0.870	6.999	0.347	3.763

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	101	40	0	106	0	108	-1
normalized size	1	1.00	2.30	0.91	0.00	2.41	0.00	2.45	-0.02
time (sec)	N/A	0.052	0.055	0.012	0.000	0.992	0.000	0.374	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	90	20	0	34	68	39	19
normalized size	1	1.00	3.75	0.83	0.00	1.42	2.83	1.62	0.79
time (sec)	N/A	0.019	0.063	0.018	0.000	1.097	6.813	0.355	3.781

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	114	45	0	307	0	133	-1
normalized size	1	1.00	2.04	0.80	0.00	5.48	0.00	2.38	-0.02
time (sec)	N/A	0.050	0.061	0.012	0.000	1.098	0.000	0.319	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	767	3606	0	0	0	0	-1
normalized size	1	1.00	3.08	14.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.910	1.489	0.053	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	46	48	0	85	0	81	49
normalized size	1	1.00	0.96	1.00	0.00	1.77	0.00	1.69	1.02
time (sec)	N/A	0.050	0.185	0.005	0.000	1.973	0.000	0.266	3.753

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	165	94	0	56	0	98	-1
normalized size	1	1.00	9.71	5.53	0.00	3.29	0.00	5.76	-0.06
time (sec)	N/A	0.022	0.288	0.017	0.000	1.344	0.000	0.231	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	150	123	0	132	0	163	-1
normalized size	1	1.00	1.74	1.43	0.00	1.53	0.00	1.90	-0.01
time (sec)	N/A	0.185	0.109	0.013	0.000	1.022	0.000	0.275	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	108	121	0	0	0	0	-1
normalized size	1	1.00	0.79	0.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.091	0.027	0.000	119.134	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	129	103	0	175	0	117	-1
normalized size	1	1.00	0.61	0.49	0.00	0.83	0.00	0.55	-0.00
time (sec)	N/A	0.116	0.151	0.048	0.000	0.937	0.000	0.253	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	117	83	0	157	0	98	-1
normalized size	1	1.00	0.73	0.52	0.00	0.98	0.00	0.61	-0.01
time (sec)	N/A	0.067	0.116	0.010	0.000	1.000	0.000	0.213	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	85	65	0	128	0	79	-1
normalized size	1	1.00	0.57	0.44	0.00	0.86	0.00	0.53	-0.01
time (sec)	N/A	0.057	0.063	0.008	0.000	0.785	0.000	0.271	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	139	94	0	341	0	102	-1
normalized size	1	1.00	0.87	0.59	0.00	2.13	0.00	0.64	-0.01
time (sec)	N/A	0.119	0.143	0.009	0.000	0.993	0.000	0.247	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	118	118	0	333	0	126	-1
normalized size	1	1.00	0.76	0.76	0.00	2.13	0.00	0.81	-0.01
time (sec)	N/A	0.113	0.281	0.013	0.000	1.047	0.000	0.260	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	126	141	0	377	0	199	-1
normalized size	1	1.00	0.78	0.88	0.00	2.34	0.00	1.24	-0.01
time (sec)	N/A	0.116	0.116	0.015	0.000	0.758	0.000	0.275	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	198	530	0	517	0	368	-1
normalized size	1	1.00	0.62	1.67	0.00	1.63	0.00	1.16	-0.00
time (sec)	N/A	0.333	0.275	0.018	0.000	1.033	0.000	0.311	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	147	381	0	391	0	268	-1
normalized size	1	1.00	0.65	1.68	0.00	1.72	0.00	1.18	-0.00
time (sec)	N/A	0.127	0.131	0.013	0.000	0.902	0.000	0.283	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	134	257	0	287	0	185	-1
normalized size	1	1.00	0.68	1.30	0.00	1.45	0.00	0.93	-0.01
time (sec)	N/A	0.101	0.127	0.011	0.000	0.935	0.000	0.302	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	149	214	0	651	0	0	-1
normalized size	1	1.00	0.71	1.01	0.00	3.09	0.00	0.00	-0.00
time (sec)	N/A	0.221	0.198	0.011	0.000	2.544	0.000	0.000	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	155	249	0	647	0	0	-1
normalized size	1	1.00	0.77	1.23	0.00	3.20	0.00	0.00	-0.00
time (sec)	N/A	0.194	0.189	0.013	0.000	1.402	0.000	0.000	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	161	358	0	693	0	450	-1
normalized size	1	1.00	0.75	1.67	0.00	3.22	0.00	2.09	-0.00
time (sec)	N/A	0.183	0.218	0.016	0.000	1.813	0.000	0.413	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	452	452	516	7739	0	0	0	0	-1
normalized size	1	1.00	1.14	17.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.967	2.500	0.039	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	395	422	5581	0	0	0	0	-1
normalized size	1	1.00	1.07	14.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.931	1.524	0.015	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	282	3249	0	0	0	0	-1
normalized size	1	1.00	0.95	10.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.376	0.363	0.013	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	314	3544	0	2266	0	0	-1
normalized size	1	1.00	0.88	9.90	0.00	6.33	0.00	0.00	-0.00
time (sec)	N/A	1.314	0.651	0.019	0.000	83.145	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	382	569	3703	0	0	0	0	-1
normalized size	1	1.00	1.49	9.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.417	3.033	0.025	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	507	507	642	3993	0	0	0	0	-1
normalized size	1	1.00	1.27	7.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.880	2.607	0.019	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	795	795	793	19148	0	0	0	0	-1
normalized size	1	1.00	1.00	24.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.264	3.449	0.030	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	553	553	755	14709	0	0	0	0	-1
normalized size	1	1.00	1.37	26.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.431	2.011	0.015	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	484	482	603	8954	0	0	0	0	-1
normalized size	1	1.00	1.25	18.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.240	1.073	0.014	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	496	496	746	9728	0	0	0	0	-1
normalized size	1	1.00	1.50	19.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.569	1.575	0.018	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	604	604	885	9912	0	0	0	0	-1
normalized size	1	1.00	1.47	16.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.809	4.159	0.020	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	668	668	904	10298	0	0	0	0	-1
normalized size	1	1.00	1.35	15.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.465	3.223	0.019	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	380	378	2397	0	0	0	0	-1
normalized size	1	1.00	0.99	6.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.170	1.333	0.029	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	334	1796	0	0	0	0	-1
normalized size	1	1.00	0.97	5.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.541	0.701	0.015	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	275	1172	0	5085	0	0	-1
normalized size	1	1.00	0.94	3.99	0.00	17.30	0.00	0.00	-0.00
time (sec)	N/A	0.235	0.320	0.013	0.000	3.458	0.000	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	247	589	0	5073	0	0	-1
normalized size	1	1.00	0.93	2.21	0.00	19.07	0.00	0.00	-0.00
time (sec)	N/A	0.151	0.291	0.013	0.000	3.446	0.000	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	319	681	0	0	0	0	-1
normalized size	1	1.00	0.97	2.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.825	0.742	0.017	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	367	356	736	0	0	0	0	-1
normalized size	1	1.00	0.97	2.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.199	0.838	0.018	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	457	457	460	911	0	0	0	0	-1
normalized size	1	1.00	1.01	1.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.863	1.569	0.018	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	499	499	577	6124	0	0	0	0	-1
normalized size	1	1.00	1.16	12.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.107	2.696	0.032	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	509	4752	0	0	0	0	-1
normalized size	1	1.00	1.24	11.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.709	2.073	0.019	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	457	3000	0	0	0	0	-1
normalized size	1	1.00	1.11	7.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.826	0.806	0.015	0.000	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	320	1713	0	0	0	0	-1
normalized size	1	1.00	0.77	4.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.617	2.061	0.015	0.000	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	526	526	497	1945	0	0	0	0	-1
normalized size	1	1.00	0.94	3.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.183	3.497	0.017	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	618	618	557	2046	0	0	0	0	-1
normalized size	1	1.00	0.90	3.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.280	3.604	0.019	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	392	392	327	1817	0	0	0	0	-1
normalized size	1	1.00	0.83	4.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.952	0.889	0.018	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	302	1810	0	0	0	0	-1
normalized size	1	1.00	0.96	5.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.489	0.485	0.017	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	272	1667	0	0	0	0	-1
normalized size	1	1.00	0.96	5.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.295	0.294	0.014	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	253	1669	0	0	0	0	-1
normalized size	1	1.00	0.95	6.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.226	0.166	0.014	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	255	1764	0	1253	0	0	-1
normalized size	1	1.00	0.96	6.61	0.00	4.69	0.00	0.00	-0.00
time (sec)	N/A	0.778	0.295	0.018	0.000	26.876	0.000	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	275	1819	0	1094	0	0	-1
normalized size	1	1.00	0.96	6.36	0.00	3.83	0.00	0.00	-0.00
time (sec)	N/A	0.706	0.431	0.020	0.000	38.759	0.000	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	316	1953	0	0	0	0	-1
normalized size	1	1.00	0.90	5.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.879	0.522	0.017	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	501	501	447	4884	0	0	0	0	-1
normalized size	1	1.00	0.89	9.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.412	1.493	0.032	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	417	417	395	4900	0	0	0	0	-1
normalized size	1	1.00	0.95	11.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.015	1.017	0.018	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	330	4567	0	0	0	0	-1
normalized size	1	1.00	0.95	13.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.522	0.805	0.015	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	298	4574	0	0	0	0	-1
normalized size	1	1.00	0.95	14.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.517	0.812	0.015	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	469	469	755	4765	0	0	0	0	-1
normalized size	1	1.00	1.61	10.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.274	0.505	0.018	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	463	765	4799	0	0	0	0	-1
normalized size	1	1.00	1.65	10.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.201	0.638	0.018	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	614	614	303	5056	0	0	0	0	-1
normalized size	1	1.00	0.49	8.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.436	0.970	0.020	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	181	1346	0	0	0	0	-1
normalized size	1	1.00	0.96	7.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.285	0.583	0.018	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	102	83	70	0	73	-1
normalized size	1	1.00	1.00	1.36	1.11	0.93	0.00	0.97	-0.01
time (sec)	N/A	0.058	0.038	0.015	0.975	1.386	0.000	0.195	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	120	789	0	777	0	0	-1
normalized size	1	1.00	0.92	6.07	0.00	5.98	0.00	0.00	-0.01
time (sec)	N/A	0.160	0.161	0.137	0.000	1.281	0.000	0.000	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	300	516	0	0	0	0	-1
normalized size	1	1.00	0.81	1.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.809	1.709	0.027	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	325	410	0	0	0	0	-1
normalized size	1	1.00	1.13	1.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.633	1.158	0.018	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	250	399	0	0	0	0	-1
normalized size	1	1.00	0.94	1.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.216	0.425	0.015	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	211	354	0	2753	0	0	-1
normalized size	1	1.00	0.96	1.61	0.00	12.51	0.00	0.00	-0.00
time (sec)	N/A	0.130	0.173	0.015	0.000	2.398	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	209	358	0	2641	0	0	-1
normalized size	1	1.00	0.95	1.63	0.00	12.00	0.00	0.00	-0.00
time (sec)	N/A	0.118	0.096	0.015	0.000	2.147	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	252	391	0	5995	0	0	-1
normalized size	1	1.00	0.94	1.46	0.00	22.45	0.00	0.00	-0.00
time (sec)	N/A	0.663	0.443	0.016	0.000	149.604	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	325	427	0	0	0	0	-1
normalized size	1	1.00	1.12	1.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.657	1.033	0.017	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	314	519	0	0	0	0	-1
normalized size	1	1.00	0.84	1.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.734	2.009	0.022	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	466	466	562	1648	0	0	0	0	-1
normalized size	1	1.00	1.21	3.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.347	1.405	0.022	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	414	1480	0	0	0	0	-1
normalized size	1	1.00	1.21	4.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.041	1.197	0.019	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	352	1427	0	0	0	0	-1
normalized size	1	1.00	1.19	4.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.454	0.402	0.019	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	356	1360	0	0	0	0	-1
normalized size	1	1.00	1.19	4.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.400	0.353	0.016	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	360	1376	0	0	0	0	-1
normalized size	1	1.00	1.16	4.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.411	0.399	0.016	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	436	1518	0	0	0	0	-1
normalized size	1	1.00	1.11	3.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.182	0.866	0.019	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	454	454	488	1656	0	0	0	0	-1
normalized size	1	1.00	1.07	3.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.193	1.193	0.019	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	761	761	552	14815	0	0	0	0	-1
normalized size	1	1.00	0.73	19.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.135	2.312	0.019	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	549	549	496	10138	0	0	0	0	-1
normalized size	1	1.00	0.90	18.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	7.028	1.776	0.015	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	431	431	417	6019	0	0	0	0	-1
normalized size	1	1.00	0.97	13.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.650	0.738	0.015	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	523	521	454	6460	0	0	0	0	-1
normalized size	1	1.00	0.87	12.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.701	1.309	0.019	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	736	736	520	6765	0	0	0	0	-1
normalized size	1	1.00	0.71	9.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.476	1.683	0.019	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	545	545	550	3131	0	0	0	0	-1
normalized size	1	1.00	1.01	5.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.722	2.201	0.032	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	463	468	2321	0	0	0	0	-1
normalized size	1	1.00	1.01	5.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.436	0.947	0.017	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	407	1516	0	11311	0	0	-1
normalized size	1	1.00	1.01	3.77	0.00	28.14	0.00	0.00	-0.00
time (sec)	N/A	0.963	0.974	0.014	0.000	11.374	0.000	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	376	761	0	11287	0	0	-1
normalized size	1	1.00	1.01	2.03	0.00	30.18	0.00	0.00	-0.00
time (sec)	N/A	0.314	0.789	0.014	0.000	10.296	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	451	451	450	859	0	0	0	0	-1
normalized size	1	1.00	1.00	1.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.632	2.414	0.019	0.000	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	543	543	533	983	0	0	0	0	-1
normalized size	1	1.00	0.98	1.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.592	1.408	0.017	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	679	679	669	1296	0	0	0	0	-1
normalized size	1	1.00	0.99	1.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	11.226	2.110	0.020	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	779	779	1066	14651	0	0	0	0	-1
normalized size	1	1.00	1.37	18.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	14.170	2.624	0.035	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	609	609	1097	11341	0	0	0	0	-1
normalized size	1	1.00	1.80	18.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	5.842	6.585	0.021	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	609	609	770	7163	0	0	0	0	-1
normalized size	1	1.00	1.26	11.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	5.643	5.684	0.017	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	666	666	700	4099	0	0	0	0	-1
normalized size	1	1.00	1.05	6.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.746	5.042	0.018	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	816	814	1121	4594	0	0	0	0	-1
normalized size	1	1.00	1.37	5.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	15.916	6.560	0.036	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	210	159	0	178	0	188	-1
normalized size	1	1.00	1.50	1.14	0.00	1.27	0.00	1.34	-0.01
time (sec)	N/A	0.501	0.527	0.019	0.000	0.986	0.000	0.203	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	192	144	0	175	0	185	-1
normalized size	1	1.00	1.67	1.25	0.00	1.52	0.00	1.61	-0.01
time (sec)	N/A	0.420	0.438	0.015	0.000	1.504	0.000	0.247	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	159	130	0	161	0	171	-1
normalized size	1	1.00	1.62	1.33	0.00	1.64	0.00	1.74	-0.01
time (sec)	N/A	0.198	0.187	0.009	0.000	0.924	0.000	0.226	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	69	174	92	0	50	0	68	-1
normalized size	1	1.01	2.56	1.35	0.00	0.74	0.00	1.00	-0.01
time (sec)	N/A	0.062	0.168	0.011	0.000	0.876	0.000	0.185	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	150	121	0	132	0	165	-1
normalized size	1	1.00	1.58	1.27	0.00	1.39	0.00	1.74	-0.01
time (sec)	N/A	0.111	0.100	0.011	0.000	0.952	0.000	0.188	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	200	152	0	170	0	199	-1
normalized size	1	1.00	1.54	1.17	0.00	1.31	0.00	1.53	-0.01
time (sec)	N/A	0.417	0.437	0.019	0.000	0.665	0.000	0.201	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	225	169	0	194	0	269	-1
normalized size	1	1.00	1.49	1.12	0.00	1.28	0.00	1.78	-0.01
time (sec)	N/A	0.454	0.405	0.020	0.000	0.731	0.000	0.483	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	87	147	155	88	0	85	187
normalized size	1	1.00	0.58	0.99	1.04	0.59	0.00	0.57	1.26
time (sec)	N/A	0.093	0.191	0.019	1.001	0.582	0.000	0.297	5.085

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	72	96	104	73	0	70	136
normalized size	1	1.00	0.70	0.93	1.01	0.71	0.00	0.68	1.32
time (sec)	N/A	0.042	0.033	0.006	0.981	0.587	0.000	0.204	4.690

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	37	163	0	53	0	63	-1
normalized size	1	1.00	1.32	5.82	0.00	1.89	0.00	2.25	-0.04
time (sec)	N/A	0.048	0.098	0.016	0.000	0.700	0.000	0.272	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	114	245	0	126	0	159	-1
normalized size	1	1.00	1.36	2.92	0.00	1.50	0.00	1.89	-0.01
time (sec)	N/A	0.078	0.238	0.023	0.000	0.721	0.000	0.272	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	131	306	0	186	0	232	-1
normalized size	1	1.00	0.94	2.20	0.00	1.34	0.00	1.67	-0.01
time (sec)	N/A	0.117	0.367	0.021	0.000	0.638	0.000	0.260	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	13	16	11	11	31	11	15
normalized size	1	1.00	0.87	1.07	0.73	0.73	2.07	0.73	1.00
time (sec)	N/A	0.004	0.007	0.004	0.434	0.608	0.311	0.153	3.723

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	13	14	9	11	10	11	13
normalized size	1	1.00	0.81	0.88	0.56	0.69	0.62	0.69	0.81
time (sec)	N/A	0.006	0.003	0.003	0.428	0.561	4.252	0.150	3.664

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	13	20	0	11	0	11	15
normalized size	1	1.00	0.87	1.33	0.00	0.73	0.00	0.73	1.00
time (sec)	N/A	0.028	0.006	0.004	0.000	0.411	0.000	0.160	3.675

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	13	18	0	11	0	11	13
normalized size	1	1.00	0.87	1.20	0.00	0.73	0.00	0.73	0.87
time (sec)	N/A	0.058	0.004	0.004	0.000	0.424	0.000	0.201	3.607

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	268	0	0	0	0	0	-1
normalized size	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.093	0.560	0.135	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	488	488	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.360	0.535	0.449	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [63] had the largest ratio of [.5556]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	1.00	25	0.160
2	A	5	4	1.00	27	0.148
3	A	5	4	1.00	27	0.148
4	A	8	8	1.00	27	0.296
5	A	9	9	1.00	27	0.333
6	A	9	6	1.00	30	0.200
7	A	5	3	1.00	30	0.100
8	A	6	4	1.00	30	0.133
9	A	7	5	1.00	30	0.167
10	A	5	4	1.00	23	0.174
11	A	5	4	1.00	23	0.174
12	A	5	4	1.00	30	0.133
13	A	6	5	0.99	28	0.179
14	A	6	5	1.00	30	0.167
15	A	9	5	0.98	30	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
16	A	10	6	0.99	30	0.200
17	A	5	5	1.00	34	0.147
18	A	5	5	1.00	34	0.147
19	A	9	6	1.00	32	0.188
20	A	10	7	1.00	32	0.219
21	A	5	3	1.00	32	0.094
22	A	5	3	1.00	29	0.103
23	A	5	3	1.00	29	0.103
24	A	6	6	1.00	26	0.231
25	A	5	4	1.00	30	0.133
26	A	7	6	1.00	30	0.200
27	A	7	6	1.00	30	0.200
28	A	5	3	1.00	30	0.100
29	A	6	4	1.00	30	0.133
30	A	7	5	1.00	30	0.167
31	A	2	2	1.00	26	0.077
32	A	5	5	1.00	26	0.192
33	A	2	2	1.00	24	0.083
34	A	5	5	1.00	20	0.250
35	A	6	5	1.00	36	0.139
36	A	2	2	1.00	36	0.056
37	A	2	2	1.00	32	0.062
38	A	13	9	1.00	32	0.281
39	A	5	5	1.00	38	0.132
40	A	6	6	1.00	35	0.171
41	A	5	5	1.00	33	0.152

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
42	A	5	5	1.00	32	0.156
43	A	8	8	1.00	35	0.229
44	A	8	8	1.00	35	0.229
45	A	8	8	1.00	35	0.229
46	A	6	6	1.00	38	0.158
47	A	5	5	1.00	36	0.139
48	A	5	5	1.00	35	0.143
49	A	7	6	1.00	38	0.158
50	A	7	6	1.00	38	0.158
51	A	7	6	1.00	38	0.158
52	A	9	6	1.00	27	0.222
53	A	9	6	1.00	25	0.240
54	A	8	5	1.00	24	0.208
55	A	12	9	1.00	27	0.333
56	A	18	12	1.00	27	0.444
57	A	22	13	1.00	27	0.482
58	A	10	7	1.00	27	0.259
59	A	10	7	1.00	25	0.280
60	A	9	6	1.00	24	0.250
61	A	17	11	1.00	27	0.407
62	A	21	14	1.00	27	0.518
63	A	26	15	1.00	27	0.556
64	A	10	6	1.00	27	0.222
65	A	8	5	1.00	27	0.185
66	A	5	3	1.00	25	0.120
67	A	5	3	1.00	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
68	A	10	7	1.00	27	0.259
69	A	11	8	1.00	27	0.296
70	A	15	9	1.00	27	0.333
71	A	10	7	1.00	27	0.259
72	A	6	4	1.00	27	0.148
73	A	6	4	1.00	25	0.160
74	A	6	4	1.00	24	0.167
75	A	12	9	1.00	27	0.333
76	A	14	11	1.00	27	0.407
77	A	15	9	1.00	28	0.321
78	A	9	6	1.00	28	0.214
79	A	9	6	1.00	26	0.231
80	A	8	5	1.00	25	0.200
81	A	17	9	1.00	28	0.321
82	A	16	8	1.00	28	0.286
83	A	20	10	1.00	28	0.357
84	A	17	10	1.00	28	0.357
85	A	10	7	1.00	28	0.250
86	A	10	7	1.00	26	0.269
87	A	9	6	1.00	25	0.240
88	A	19	11	1.00	28	0.393
89	A	18	10	1.00	28	0.357
90	A	26	13	1.00	28	0.464
91	A	9	6	1.00	24	0.250
92	A	8	6	1.00	22	0.273
93	A	10	9	1.00	17	0.529

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
94	A	13	7	1.00	28	0.250
95	A	10	6	1.00	28	0.214
96	A	8	5	1.00	28	0.179
97	A	5	3	1.00	26	0.115
98	A	5	3	1.00	25	0.120
99	A	9	4	1.00	28	0.143
100	A	10	5	1.00	28	0.179
101	A	13	6	1.00	28	0.214
102	A	13	9	1.00	28	0.321
103	A	9	6	1.00	28	0.214
104	A	6	4	1.00	28	0.143
105	A	6	4	1.00	26	0.154
106	A	6	4	1.00	25	0.160
107	A	12	7	1.00	28	0.250
108	A	12	7	1.00	28	0.250
109	A	9	6	1.00	30	0.200
110	A	9	6	1.00	28	0.214
111	A	8	5	1.00	27	0.185
112	A	17	9	1.00	30	0.300
113	A	23	10	1.00	30	0.333
114	A	12	6	1.00	30	0.200
115	A	8	5	1.00	30	0.167
116	A	5	3	1.00	28	0.107
117	A	5	3	1.00	27	0.111
118	A	9	4	1.00	30	0.133
119	A	12	5	1.00	30	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
120	A	16	7	1.00	30	0.233
121	A	10	7	1.00	30	0.233
122	A	6	4	1.00	30	0.133
123	A	6	4	1.00	28	0.143
124	A	6	4	1.00	27	0.148
125	A	12	7	1.00	30	0.233
126	A	24	14	1.00	30	0.467
127	A	20	13	1.00	30	0.433
128	A	16	12	1.00	30	0.400
129	A	6	4	1.01	28	0.143
130	A	10	8	1.00	27	0.296
131	A	17	11	1.00	30	0.367
132	A	20	12	1.00	30	0.400
133	A	8	6	1.00	34	0.176
134	A	6	5	1.00	30	0.167
135	A	3	3	1.00	34	0.088
136	A	5	5	1.00	34	0.147
137	A	7	7	1.00	34	0.206
138	A	1	1	1.00	17	0.059
139	A	1	1	1.00	15	0.067
140	A	2	2	1.00	23	0.087
141	A	3	3	1.00	21	0.143
142	A	2	2	1.00	40	0.050
143	A	2	2	1.00	104	0.019

Chapter 3

Listing of integrals

$$3.1 \quad \int \frac{(A+Bx)(a+bx+cx^2)}{d+fx^2} dx$$

Optimal. Leaf size=94

$$-\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(-aAf + Acd + bBd)}{\sqrt{d}f^{3/2}} - \frac{\log(d + fx^2)(-aBf - Abf + Bcd)}{2f^2} + \frac{x(Ac + bB)}{f} + \frac{Bcx^2}{2f}$$

[Out] (A*c+B*b)*x/f+1/2*B*c*x^2/f-1/2*(-A*b*f-B*a*f+B*c*d)*ln(f*x^2+d)/f^2-(-A*a*f+A*c*d+B*b*d)*arctan(x*f^(1/2)/d^(1/2))/f^(3/2)/d^(1/2)

Rubi [A] time = 0.11, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1629, 635, 205, 260}

$$-\frac{\log(d + fx^2)(-aBf - Abf + Bcd)}{2f^2} - \frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(-aAf + Acd + bBd)}{\sqrt{d}f^{3/2}} + \frac{x(Ac + bB)}{f} + \frac{Bcx^2}{2f}$$

Antiderivative was successfully verified.

[In] Int[((A + B*x)*(a + b*x + c*x^2))/(d + f*x^2), x]

[Out] ((b*B + A*c)*x)/f + (B*c*x^2)/(2*f) - ((b*B*d + A*c*d - a*A*f)*ArcTan[(Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]*f^(3/2)) - ((B*c*d - A*b*f - a*B*f)*Log[d + f*x^2])/(2*f^2)

Rule 205

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]`

Rule 260

`Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]`

Rule 635

`Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]`

Rule 1629

`Int[(Pq_)*((d_) + (e_.)*(x_)^(m_.))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]`

Rubi steps

$$\begin{aligned} \int \frac{(A + Bx)(a + bx + cx^2)}{d + fx^2} dx &= \int \left(\frac{bB + Ac}{f} + \frac{Bcx}{f} - \frac{bBd + Acd - aAf + (Bcd - Abf - aBf)x}{f(d + fx^2)} \right) dx \\ &= \frac{(bB + Ac)x}{f} + \frac{Bcx^2}{2f} - \frac{\int \frac{bBd + Acd - aAf + (Bcd - Abf - aBf)x}{d + fx^2} dx}{f} \\ &= \frac{(bB + Ac)x}{f} + \frac{Bcx^2}{2f} - \frac{(bBd + Acd - aAf) \int \frac{1}{d + fx^2} dx}{f} - \frac{(Bcd - Abf - aBf) \int \frac{x}{d + fx^2} dx}{f} \\ &= \frac{(bB + Ac)x}{f} + \frac{Bcx^2}{2f} - \frac{(bBd + Acd - aAf) \tan^{-1} \left(\frac{\sqrt{f}x}{\sqrt{d}} \right)}{\sqrt{d} f^{3/2}} - \frac{(Bcd - Abf - aBf) \ln |d + fx^2|}{2f^2} \end{aligned}$$

Mathematica [A] time = 0.07, size = 86, normalized size = 0.91

$$\frac{\log(d + fx^2)(aBf + Abf - Bcd) - \frac{2\sqrt{f} \tan^{-1} \left(\frac{\sqrt{f}x}{\sqrt{d}} \right) (-aAf + Acd + bBd)}{\sqrt{d}} + fx(2Ac + 2bB + Bcx)}{2f^2}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*x)*(a + b*x + c*x^2))/(d + f*x^2), x]

[Out] (f*x*(2*b*B + 2*A*c + B*c*x) - (2*Sqrt[f]*(b*B*d + A*c*d - a*A*f)*ArcTan[(Sqrt[f]*x)/Sqrt[d]])/Sqrt[d] + (- (B*c*d) + A*b*f + a*B*f)*Log[d + f*x^2]/(2*f^2)

fricas [A] time = 0.86, size = 200, normalized size = 2.13

$$\left[\frac{Bcdfx^2 + 2(Bb + Ac)dfx - (Aaf - (Bb + Ac)d)\sqrt{-df} \log\left(\frac{fx^2 - 2\sqrt{-df}x - d}{fx^2 + d}\right) - (Bcd^2 - (Ba + Ab)df) \log(fx^2 + d)}{2df^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+d), x, algorithm="fricas")

[Out] [1/2*(B*c*d*f*x^2 + 2*(B*b + A*c)*d*f*x - (A*a*f - (B*b + A*c)*d)*sqrt(-d*f)*log((f*x^2 - 2*sqrt(-d*f)*x - d)/(f*x^2 + d)) - (B*c*d^2 - (B*a + A*b)*d*f)*log(f*x^2 + d))/(d*f^2), 1/2*(B*c*d*f*x^2 + 2*(B*b + A*c)*d*f*x + 2*(A*a*f - (B*b + A*c)*d)*sqrt(d*f)*arctan(sqrt(d*f)*x/d) - (B*c*d^2 - (B*a + A*b)*d*f)*log(f*x^2 + d))/(d*f^2)]

giac [A] time = 0.15, size = 87, normalized size = 0.93

$$\frac{(Bbd + Acd - Aaf) \arctan\left(\frac{fx}{\sqrt{df}}\right) - (Bcd - Baf - Abf) \log(fx^2 + d)}{\sqrt{df} f} + \frac{Bcfx^2 + 2Bbfx + 2Acfx}{2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+d), x, algorithm="giac")

[Out] -(B*b*d + A*c*d - A*a*f)*arctan(f*x/sqrt(d*f))/(sqrt(d*f)*f) - 1/2*(B*c*d - B*a*f - A*b*f)*log(f*x^2 + d)/f^2 + 1/2*(B*c*f*x^2 + 2*B*b*f*x + 2*A*c*f*x)/f^2

maple [A] time = 0.03, size = 133, normalized size = 1.41

$$\frac{Aa \arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}} - \frac{Acd \arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df} f} - \frac{Bbd \arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df} f} + \frac{Bcx^2}{2f} + \frac{Ab \ln(fx^2 + d)}{2f} + \frac{Acx}{f} + \frac{Ba \ln(fx^2 + d)}{2f} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x+A)*(c*x^2+b*x+a)/(f*x^2+d), x)

[Out] $\frac{1}{2}Bcx^2/f + 1/fA^*cx + 1/fb^*B^*x + 1/2/f*\ln(f*x^2+d)*A^*b + 1/2/f*\ln(f*x^2+d)*B^*a - 1/2/f^2*\ln(f*x^2+d)*B^*c*d + 1/(d*f)^{(1/2)}*\arctan(x*f/(d*f)^{(1/2)})*A^*a - 1/f/(d*f)^{(1/2)}*\arctan(x*f/(d*f)^{(1/2)})*A^*c*d - 1/f/(d*f)^{(1/2)}*\arctan(x*f/(d*f)^{(1/2)})*B^*b*d$

maxima [A] time = 0.97, size = 84, normalized size = 0.89

$$\frac{(Aaf - (Bb + Ac)d) \arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}f} + \frac{Bcx^2 + 2(Bb + Ac)x}{2f} - \frac{(Bcd - (Ba + Ab)f) \log(fx^2 + d)}{2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="maxima")

[Out] $(A^*a*f - (B^*b + A^*c)*d)*\arctan(f*x/\text{sqrt}(d*f))/(\text{sqrt}(d*f)*f) + 1/2*(B^*c*x^2 + 2*(B^*b + A^*c)*x)/f - 1/2*(B^*c*d - (B^*a + A^*b)*f)*\log(f*x^2 + d)/f^2$

mupad [B] time = 3.44, size = 97, normalized size = 1.03

$$\frac{x(Ac + Bb)}{f} - \frac{\text{atan}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(Acd - Aaf + Bbd)}{\sqrt{d}f^{3/2}} + \frac{Bcx^2}{2f} + \frac{\ln(fx^2 + d)(4Abdf^3 + 4Badf^3 - 4Bcd^2f^2)}{8df^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x)*(a + b*x + c*x^2))/(d + f*x^2),x)

[Out] $(x*(A^*c + B^*b))/f - (\text{atan}((f^{(1/2)}*x)/d^{(1/2)})*(A^*c*d - A^*a*f + B^*b*d))/(d^{(1/2)}*f^{(3/2)}) + (B^*c*x^2)/(2*f) + (\log(d + f*x^2)*(4*A^*b*d*f^3 + 4*B^*a*d*f^3 - 4*B^*c*d^2*f^2))/(8*d*f^4)$

sympy [B] time = 1.85, size = 333, normalized size = 3.54

$$\frac{Bcx^2}{2f} + x\left(\frac{Ac}{f} + \frac{Bb}{f}\right) + \left(\frac{Abf + Baf - Bcd}{2f^2} - \frac{\sqrt{-df^5}(Aaf - Acd - Bbd)}{2df^4}\right) \log\left(x + \frac{-Abdf - Badf + Bcd^2 + 2df^2}{Aaf^2 - \dots}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x**2+b*x+a)/(f*x**2+d),x)

[Out] $B^*c*x**2/(2*f) + x*(A^*c/f + B^*b/f) + ((A^*b*f + B^*a*f - B^*c*d)/(2*f**2) - \text{sqrt}(-d*f**5)*(A^*a*f - A^*c*d - B^*b*d)/(2*d*f**4))*\log(x + (-A^*b*d*f - B^*a*d*f + B^*c*d**2 + 2*d*f**2*(A^*b*f + B^*a*f - B^*c*d)/(2*f**2) - \text{sqrt}(-d*f**5)*(A$

$$\frac{(a^2f - acd - b^2d)/(2d^2f^4)}{(a^2f^2 - acd^2f - b^2d^2f)} + \left(\frac{(abf + B^2a^2f - B^2c^2d)/(2f^2) + \sqrt{-df^5}(a^2f - acd - b^2d)/(2d^2f^4)}{\log(x + (-ab^2d^2f - B^2a^2d^2f + B^2c^2d^2 + 2d^2f^2((abf + B^2a^2f - B^2c^2d)/(2f^2) + \sqrt{-df^5}(a^2f - acd - b^2d)/(2d^2f^4))))/(a^2f^2 - acd^2f - b^2d^2f)} \right)$$

$$3.2 \quad \int \frac{(A+Bx)(a+bx+cx^2)^2}{d+fx^2} dx$$

Optimal. Leaf size=228

$$\frac{\log(d+fx^2)(2Abf(cd-af)-B(-f(b^2d-a^2f)-2acdf+c^2d^2))}{2f^3} + \frac{x^2(2Abcf-B(-2acf+b^2(-f)+c^2d))}{2f^2}$$

[Out] (A*b^2*f-A*c*(-2*a*f+c*d)-b*B*(-2*a*f+2*c*d))*x/f^2+1/2*(2*A*b*c*f-B*(-2*a*c*f-b^2*f+c^2*d))*x^2/f^2+1/3*c*(A*c+2*B*b)*x^3/f+1/4*B*c^2*x^4/f-1/2*(2*A*b*f*(-a*f+c*d)-B*(c^2*d^2-2*a*c*d*f-f*(-a^2*f+b^2*d)))*ln(f*x^2+d)/f^3-(A*b^2*d*f-2*b*B*d*(-a*f+c*d)-A*(-a*f+c*d)^2)*arctan(x*f^(1/2)/d^(1/2))/f^(5/2)/d^(1/2)

Rubi [A] time = 0.33, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1012, 635, 205, 260}

$$\frac{\log(d+fx^2)(2Abf(cd-af)-B(-f(b^2d-a^2f)-2acdf+c^2d^2))}{2f^3} + \frac{x^2(2Abcf-B(-2acf+b^2(-f)+c^2d))}{2f^2}$$

Antiderivative was successfully verified.

[In] Int[((A + B*x)*(a + b*x + c*x^2)^2)/(d + f*x^2), x]

[Out] ((A*b^2*f - A*c*(c*d - 2*a*f) - b*B*(2*c*d - 2*a*f))*x)/f^2 + ((2*A*b*c*f - B*(c^2*d - b^2*f - 2*a*c*f))*x^2)/(2*f^2) + (c*(2*b*B + A*c)*x^3)/(3*f) + (B*c^2*x^4)/(4*f) - ((A*b^2*d*f - 2*b*B*d*(c*d - a*f) - A*(c*d - a*f)^2)*ArcTan[(Sqrt[f]*x)/Sqrt[d]]/(Sqrt[d]*f^(5/2)) - ((2*A*b*f*(c*d - a*f) - B*(c^2*d^2 - 2*a*c*d*f - f*(b^2*d - a^2*f)))*Log[d + f*x^2])/(2*f^3)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635


```
Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1012

```
Int[((g_) + (h_)*(x_))*((a_) + (c_)*(x_)^2)^(p_)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Int[ExpandIntegrand[(a + c*x^2)^p*(d + e*x + f*x^2)^q*(g + h*x), x], x] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && IntegersQ[p, q] && (GtQ[p, 0] || GtQ[q, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(A + Bx)(a + bx + cx^2)^2}{d + fx^2} dx &= \int \left(\frac{Ab^2f - Ac(cd - 2af) - bB(2cd - 2af)}{f^2} + \frac{(2Abcf - B(c^2d - b^2f - 2acf))}{f^2} \right) dx \\ &= \frac{(Ab^2f - Ac(cd - 2af) - bB(2cd - 2af))x}{f^2} + \frac{(2Abcf - B(c^2d - b^2f - 2acf))}{2f^2} \\ &= \frac{(Ab^2f - Ac(cd - 2af) - bB(2cd - 2af))x}{f^2} + \frac{(2Abcf - B(c^2d - b^2f - 2acf))}{2f^2} \\ &= \frac{(Ab^2f - Ac(cd - 2af) - bB(2cd - 2af))x}{f^2} + \frac{(2Abcf - B(c^2d - b^2f - 2acf))}{2f^2} \end{aligned}$$

Mathematica [A] time = 0.20, size = 204, normalized size = 0.89

$$\frac{6 \log(d + fx^2) (B(a^2f^2 - 2acdf + b^2(-d)f + c^2d^2) + 2Abf(af - cd)) + fx(4Ac(6af - 3cd + cfx^2) + 4bB(6af - 3cd + cfx^2))}{12f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x + c*x^2)^2)/(d + f*x^2), x]
```

```
[Out] (((-(A*b^2*d*f) + 2*b*B*d*(c*d - a*f) + A*(c*d - a*f)^2)*ArcTan[(Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]*f^(5/2)) + (f*x*(12*A*b*c*f*x + 6*b^2*f*(2*A + B*x) + 3*B*c*x*(-2*c*d + 4*a*f + c*f*x^2) + 4*A*c*(-3*c*d + 6*a*f + c*f*x^2) + 4*b*B*(-6*c*d + 6*a*f + 2*c*f*x^2)) + 6*(2*A*b*f*(-(c*d) + a*f) + B*(c^2*d^2 - b^2*d*f - 2*a*c*d*f + a^2*f^2))*Log[d + f*x^2])/(12*f^3)
```

fricas [A] time = 0.69, size = 500, normalized size = 2.19

$$\frac{3 B c^2 d f^2 x^4 + 4 (2 B b c + A c^2) d f^2 x^3 - 6 (B c^2 d^2 f - (B b^2 + 2 (B a + A b) c) d f^2) x^2 - 6 (A a^2 f^2 + (2 B b c + A c^2) d^2 -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="fricas")

[Out] [1/12*(3*B*c^2*d*f^2*x^4 + 4*(2*B*b*c + A*c^2)*d*f^2*x^3 - 6*(B*c^2*d^2*f - (B*b^2 + 2*(B*a + A*b)*c)*d*f^2)*x^2 - 6*(A*a^2*f^2 + (2*B*b*c + A*c^2)*d^2 - (2*B*a*b + A*b^2 + 2*A*a*c)*d*f)*sqrt(-d*f)*log((f*x^2 - 2*sqrt(-d*f)*x - d)/(f*x^2 + d)) - 12*((2*B*b*c + A*c^2)*d^2*f - (2*B*a*b + A*b^2 + 2*A*a*c)*d*f^2)*x + 6*(B*c^2*d^3 - (B*b^2 + 2*(B*a + A*b)*c)*d^2*f + (B*a^2 + 2*A*a*b)*d*f^2)*log(f*x^2 + d))/(d*f^3), 1/12*(3*B*c^2*d*f^2*x^4 + 4*(2*B*b*c + A*c^2)*d*f^2*x^3 - 6*(B*c^2*d^2*f - (B*b^2 + 2*(B*a + A*b)*c)*d*f^2)*x^2 + 12*(A*a^2*f^2 + (2*B*b*c + A*c^2)*d^2 - (2*B*a*b + A*b^2 + 2*A*a*c)*d*f)*sqrt(d*f)*arctan(sqrt(d*f)*x/d) - 12*((2*B*b*c + A*c^2)*d^2*f - (2*B*a*b + A*b^2 + 2*A*a*c)*d*f^2)*x + 6*(B*c^2*d^3 - (B*b^2 + 2*(B*a + A*b)*c)*d^2*f + (B*a^2 + 2*A*a*b)*d*f^2)*log(f*x^2 + d))/(d*f^3)]

giac [A] time = 0.16, size = 263, normalized size = 1.15

$$\frac{(2 B b c d^2 + A c^2 d^2 - 2 B a b d f - A b^2 d f - 2 A a c d f + A a^2 f^2) \arctan\left(\frac{f x}{\sqrt{d f}}\right) + (B c^2 d^2 - B b^2 d f - 2 B a c d f - 2 A b c d f - 2 A a^2 f^2) \arctan\left(\frac{f x}{\sqrt{d f}}\right)}{\sqrt{d f} f^2} + \frac{(B c^2 d^2 - B b^2 d f - 2 B a c d f - 2 A b c d f - 2 A a^2 f^2) \arctan\left(\frac{f x}{\sqrt{d f}}\right)}{2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="giac")

[Out] (2*B*b*c*d^2 + A*c^2*d^2 - 2*B*a*b*d*f - A*b^2*d*f - 2*A*a*c*d*f + A*a^2*f^2)*arctan(f*x/sqrt(d*f))/(sqrt(d*f)*f^2) + 1/2*(B*c^2*d^2 - B*b^2*d*f - 2*B*a*c*d*f - 2*A*b*c*d*f + B*a^2*f^2 + 2*A*a*b*f^2)*log(f*x^2 + d)/f^3 + 1/12*(3*B*c^2*f^3*x^4 + 8*B*b*c*f^3*x^3 + 4*A*c^2*f^3*x^3 - 6*B*c^2*d*f^2*x^2 + 6*B*b^2*f^3*x^2 + 12*B*a*c*f^3*x^2 + 12*A*b*c*f^3*x^2 - 24*B*b*c*d*f^2*x - 12*A*c^2*d*f^2*x + 24*B*a*b*f^3*x + 12*A*b^2*f^3*x + 24*A*a*c*f^3*x)/f^4

maple [A] time = 0.01, size = 373, normalized size = 1.64

$$\frac{B c^2 x^4}{4 f} + \frac{A c^2 x^3}{3 f} + \frac{2 B b c x^3}{3 f} + \frac{A a^2 \arctan\left(\frac{f x}{\sqrt{d f}}\right)}{\sqrt{d f}} - \frac{2 A a c d \arctan\left(\frac{f x}{\sqrt{d f}}\right)}{\sqrt{d f} f} - \frac{A b^2 d \arctan\left(\frac{f x}{\sqrt{d f}}\right)}{\sqrt{d f} f} + \frac{A b c x^2}{f} + \frac{A c^2 d^2 \arctan\left(\frac{f x}{\sqrt{d f}}\right)}{\sqrt{d f}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+d),x)`

[Out] $\frac{1}{4}B^2c^2x^4/f + \frac{1}{3}fA^2x^3c^2 + \frac{2}{3}fB^2x^3bc + \frac{1}{f}A^2x^2b^2c + \frac{1}{f}B^2x^2a^2c + \frac{1}{2}fB^2x^2b^2 - \frac{1}{2}f^2B^2x^2c^2d + \frac{2}{f}A^2ac^2x + \frac{1}{f}A^2b^2x - \frac{1}{f^2}A^2c^2dx + \frac{2}{f}B^2a^2bx - \frac{2}{f^2}B^2b^2cdx + \frac{1}{f} \ln(fx^2+d)A^2ab - \frac{1}{f^2} \ln(fx^2+d)A^2b^2cd + \frac{1}{2}f \ln(fx^2+d)B^2a^2 - \frac{1}{f^2} \ln(fx^2+d)B^2a^2cd - \frac{1}{2}f^2 \ln(fx^2+d)B^2b^2d + \frac{1}{2}f^3 \ln(fx^2+d)B^2c^2d^2 + \frac{1}{(df)^{1/2}} \arctan\left(\frac{1}{(df)^{1/2}}fx\right)A^2a^2 - \frac{2}{f} \arctan\left(\frac{1}{(df)^{1/2}}fx\right)A^2ac^2d - \frac{1}{f} \arctan\left(\frac{1}{(df)^{1/2}}fx\right)A^2b^2d + \frac{1}{f^2} \arctan\left(\frac{1}{(df)^{1/2}}fx\right)A^2c^2d^2 - \frac{2}{f} \arctan\left(\frac{1}{(df)^{1/2}}fx\right)B^2a^2bd + \frac{2}{f^2} \arctan\left(\frac{1}{(df)^{1/2}}fx\right)B^2b^2cd^2$

maxima [A] time = 0.98, size = 220, normalized size = 0.96

$$\frac{(Aa^2f^2 + (2Bbc + Ac^2)d^2 - (2Bab + Ab^2 + 2Aac)df) \arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}f^2} + \frac{3Bc^2fx^4 + 4(2Bbc + Ac^2)fx^3 - 6(Bc^2d^2 + 2Bab + Ab^2 + 2Aac)fx^2 - 12((2B^2bc + A^2c^2)d - (2B^2a^2b + A^2b^2 + 2A^2ac)fx) + (B^2a^2 + 2A^2ab)fx^2 + \log(fx^2 + d)}{f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="maxima")`

[Out] $(A^2a^2f^2 + (2B^2bc + A^2c^2)d^2 - (2B^2ab + A^2b^2 + 2A^2ac)df) \arctan\left(\frac{fx}{\sqrt{df}}\right) / (\sqrt{df}f^2) + \frac{1}{12} \left(3B^2c^2fx^4 + 4(2B^2bc + A^2c^2)fx^3 - 6(B^2c^2d - (B^2b^2 + 2(B^2a + A^2b)c)fx) + 12((2B^2bc + A^2c^2)d - (2B^2a^2b + A^2b^2 + 2A^2ac)fx) + (B^2a^2 + 2A^2ab)fx^2 + \log(fx^2 + d) \right) / f^3$

mupad [B] time = 0.25, size = 253, normalized size = 1.11

$$x \left(\frac{Ab^2 + 2Bab + 2Aac}{f} - \frac{d(Ac^2 + 2Bbc)}{f^2} \right) + x^2 \left(\frac{Bb^2 + 2Ac b + 2Bac}{2f} - \frac{Bc^2d}{2f^2} \right) + \frac{x^3(Ac^2 + 2Bbc)}{3f} + \frac{Bc^2d^2}{4f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x)*(a + b*x + c*x^2)^2)/(d + f*x^2),x)`

[Out] $x \left(\frac{A^2b^2 + 2A^2ac + 2B^2a^2b}{f} - \frac{d(A^2c^2 + 2B^2bc)}{f^2} \right) + x^2 \left(\frac{B^2b^2 + 2A^2b^2c + 2B^2a^2c}{2f} - \frac{B^2c^2d}{2f^2} \right) + \frac{x^3(A^2c^2 + 2B^2bc)}{3f} + \frac{B^2c^2x^4}{4f} + \frac{\operatorname{atan}\left(\frac{f^{1/2}x}{d^{1/2}}\right) \left(A^2a^2f^2 + A^2c^2d^2 + 2B^2b^2cd^2 - A^2b^2d^2f - 2A^2ac^2d^2f - 2B^2a^2b^2d^2f \right)}{d^{1/2}f^{5/2}} + \frac{\log(d + fx^2) \left(4B^2a^2d^2f^5 - 4B^2b^2d^2f^4 + 4B^2c^2d^3f^3 + 8A^2a^2bd^2f^5 - 8A^2b^2cd^2f^4 - 8B^2a^2cd^2f^4 \right)}{8d^2f^6}$

sympy [B] time = 7.87, size = 933, normalized size = 4.09

$$\frac{Bc^2x^4}{4f} + x^3 \left(\frac{Ac^2}{3f} + \frac{2Bbc}{3f} \right) + x^2 \left(\frac{Abc}{f} + \frac{Bac}{f} + \frac{Bb^2}{2f} - \frac{Bc^2d}{2f^2} \right) + x \left(\frac{2Aac}{f} + \frac{Ab^2}{f} - \frac{Ac^2d}{f^2} + \frac{2Bab}{f} - \frac{2Bbcd}{f^2} \right) + \left(\frac{2Aabf}{f^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x**2+b*x+a)**2/(f*x**2+d), x)

[Out] B*c**2*x**4/(4*f) + x**3*(A*c**2/(3*f) + 2*B*b*c/(3*f)) + x**2*(A*b*c/f + B*a*c/f + B*b**2/(2*f) - B*c**2*d/(2*f**2)) + x*(2*A*a*c/f + A*b**2/f - A*c**2*d/f**2 + 2*B*a*b/f - 2*B*b*c*d/f**2) + ((2*A*a*b*f**2 - 2*A*b*c*d*f + B*a**2*f**2 - 2*B*a*c*d*f - B*b**2*d*f + B*c**2*d**2)/(2*f**3) - sqrt(-d*f**7)*(A*a**2*f**2 - 2*A*a*c*d*f - A*b**2*d*f + A*c**2*d**2 - 2*B*a*b*d*f + 2*B*b*c*d**2)/(2*d*f**6))*log(x + (-2*A*a*b*d*f**2 + 2*A*b*c*d**2*f - B*a**2*d*f**2 + 2*B*a*c*d**2*f + B*b**2*d**2*f - B*c**2*d**3 + 2*d*f**3*((2*A*a*b*f**2 - 2*A*b*c*d*f + B*a**2*f**2 - 2*B*a*c*d*f - B*b**2*d*f + B*c**2*d**2)/(2*f**3) - sqrt(-d*f**7)*(A*a**2*f**2 - 2*A*a*c*d*f - A*b**2*d*f + A*c**2*d**2 - 2*B*a*b*d*f + 2*B*b*c*d**2)/(2*d*f**6))))/(A*a**2*f**3 - 2*A*a*c*d*f**2 - A*b**2*d*f**2 + A*c**2*d**2*f - 2*B*a*b*d*f**2 + 2*B*b*c*d**2*f)) + ((2*A*a*b*f**2 - 2*A*b*c*d*f + B*a**2*f**2 - 2*B*a*c*d*f - B*b**2*d*f + B*c**2*d**2)/(2*f**3) + sqrt(-d*f**7)*(A*a**2*f**2 - 2*A*a*c*d*f - A*b**2*d*f + A*c**2*d**2 - 2*B*a*b*d*f + 2*B*b*c*d**2)/(2*d*f**6))*log(x + (-2*A*a*b*d*f**2 + 2*A*b*c*d**2*f - B*a**2*d*f**2 + 2*B*a*c*d**2*f + B*b**2*d**2*f - B*c**2*d**3 + 2*d*f**3*((2*A*a*b*f**2 - 2*A*b*c*d*f + B*a**2*f**2 - 2*B*a*c*d*f - B*b**2*d*f + B*c**2*d**2)/(2*f**3) + sqrt(-d*f**7)*(A*a**2*f**2 - 2*A*a*c*d*f - A*b**2*d*f + A*c**2*d**2 - 2*B*a*b*d*f + 2*B*b*c*d**2)/(2*d*f**6))))/(A*a**2*f**3 - 2*A*a*c*d*f**2 - A*b**2*d*f**2 + A*c**2*d**2*f - 2*B*a*b*d*f**2 + 2*B*b*c*d**2*f))

$$3.3 \quad \int \frac{(A+Bx)(a+bx+cx^2)^3}{d+fx^2} dx$$

Optimal. Leaf size=441

$$\frac{\log(d+fx^2) \left(Abf(-f(b^2d-3a^2f)-6acdf+3c^2d^2) - B(cd-af)(-f(3b^2d-a^2f)-2acdf+c^2d^2) \right)}{2f^4} x^2 (A$$

[Out] $-(b^3B*d*f+3A*b^2*f*(-a*f+c*d)-3*b*B*(-a*f+c*d)^2-A*c*(3*a^2*f^2-3*a*c*d*f+c^2*d^2))*x/f^3-1/2*(A*b*f*(-6*a*c*f-b^2*f+3*c^2*d)-B*(c^3*d^2-3*a*c^2*d*f+3*a*b^2*f^2-3*c*f*(-a^2*f+b^2*d)))*x^2/f^3+1/3*(b^3*B*f+3A*b^2*c*f-A*c^2*(-3*a*f+c*d)-3*b*B*c*(-2*a*f+c*d))*x^3/f^2+1/4*c*(3A*b*c*f-B*(-3*a*c*f-3*b^2*f+c^2*d))*x^4/f^2+1/5*c^2*(A*c+3B*b)*x^5/f+1/6*B*c^3*x^6/f+1/2*(A*b*f*(3*c^2*d^2-6*a*c*d*f-f*(-3*a^2*f+b^2*d))-B*(-a*f+c*d)*(c^2*d^2-2*a*c*d*f-f*(-a^2*f+3*b^2*d)))*ln(f*x^2+d)/f^4+(b^3*B*d^2*f+3A*b^2*d*f*(-a*f+c*d)-3*b*B*d*(-a*f+c*d)^2-A*(-a*f+c*d)^3)*arctan(x*f^(1/2)/d^(1/2))/f^(7/2)/d^(1/2)$

Rubi [A] time = 0.62, antiderivative size = 441, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1012, 635, 205, 260}

$$\frac{x^2 \left(Abf(-6acf+b^2(-f)+3c^2d) - B(-3cf(b^2d-a^2f)+3ab^2f^2-3ac^2df+c^3d^2) \right)}{2f^3} + \frac{\log(d+fx^2) \left(Abf(-$$

Antiderivative was successfully verified.

[In] Int[((A + B*x)*(a + b*x + c*x^2)^3)/(d + f*x^2), x]

[Out] $-\left(\left(\left(b^3B*d*f+3A*b^2*f*(c*d-a*f)-3*b*B*(c*d-a*f)^2-A*c*(c^2*d^2-3*a*c*d*f+3*a^2*f^2)\right)*x\right)/f^3-\left(\left(A*b*f*(3*c^2*d-b^2*f-6*a*c*f)-B*(c^3*d^2-3*a*c^2*d*f+3*a*b^2*f^2-3*c*f*(b^2*d-a^2*f)\right)\right)*x^2\right)/(2*f^3)+\left(\left(b^3*B*f+3A*b^2*c*f-A*c^2*(c*d-3*a*f)-3*b*B*c*(c*d-2*a*f)\right)\right)*x^3/(3*f^2)+\left(c*(3A*b*c*f-B*(c^2*d-3*b^2*f-3*a*c*f))\right)*x^4/(4*f^2)+\left(c^2*(3*b*B+A*c)\right)*x^5/(5*f)+\left(B*c^3*x^6\right)/(6*f)+\left(\left(b^3*B*d^2*f+3A*b^2*d*f*(c*d-a*f)-3*b*B*d*(c*d-a*f)^2-A*(c*d-a*f)^3\right)\right)*ArcTan[\left(\frac{\sqrt{f*x}}{\sqrt{d}}\right)]/\left(\sqrt{d}*f^{7/2}\right)+\left(\left(A*b*f*(3*c^2*d^2-6*a*c*d*f-f*(b^2*d-3*a^2*f))-B*(c*d-a*f)*(c^2*d^2-2*a*c*d*f-f*(3*b^2*d-a^2*f))\right)\right)*Log[d+f*x^2]/(2*f^4)$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1012

```
Int[((g_) + (h_)*(x_))*((a_) + (c_)*(x_)^2)^(p_)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Int[ExpandIntegrand[(a + c*x^2)^p*(d + e*x + f*x^2)^q*(g + h*x), x], x] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && IntegersQ[p, q] && (GtQ[p, 0] || GtQ[q, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(A + Bx)(a + bx + cx^2)^3}{d + fx^2} dx &= \int \left(-\frac{b^3Bdf + 3Ab^2f(cd - af) - 3bB(cd - af)^2 - Ac(c^2d^2 - 3acdf + 3a^2f^2)}{f^3} \right. \\ &= -\frac{(b^3Bdf + 3Ab^2f(cd - af) - 3bB(cd - af)^2 - Ac(c^2d^2 - 3acdf + 3a^2f^2))x}{f^3} \\ &= -\frac{(b^3Bdf + 3Ab^2f(cd - af) - 3bB(cd - af)^2 - Ac(c^2d^2 - 3acdf + 3a^2f^2))x}{f^3} \\ &= -\frac{(b^3Bdf + 3Ab^2f(cd - af) - 3bB(cd - af)^2 - Ac(c^2d^2 - 3acdf + 3a^2f^2))x}{f^3} \end{aligned}$$

Mathematica [A] time = 0.46, size = 422, normalized size = 0.96

$$\frac{fx(3b(4B(15a^2f^2 + 10acf(fx^2 - 3d) + c^2(15d^2 - 5dfx^2 + 3f^2x^4)) + 15Acfx(4af - 2cd + cfx^2)) + c(4A(45$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x + c*x^2)^3)/(d + f*x^2), x]
```

```
[Out] ((b^3*B*d^2*f + 3*A*b^2*d*f*(c*d - a*f) - 3*b*B*d*(c*d - a*f)^2 - A*(c*d - a*f)^3)*ArcTan[(Sqrt[f]*x)/Sqrt[d]]/(Sqrt[d]*f^(7/2)) + (f*x*(10*b^3*f*(-6*B*d + 3*A*f*x + 2*B*f*x^2) + 15*b^2*f*(3*B*x*(-2*c*d + 2*a*f + c*f*x^2) + 4*A*(-3*c*d + 3*a*f + c*f*x^2)) + 3*b*(15*A*c*f*x*(-2*c*d + 4*a*f + c*f*x^2) + 4*B*(15*a^2*f^2 + 10*a*c*f*(-3*d + f*x^2) + c^2*(15*d^2 - 5*d*f*x^2 + 3*f^2*x^4))) + c*(5*B*x*(18*a^2*f^2 + 9*a*c*f*(-2*d + f*x^2) + c^2*(6*d^2 - 3*d*f*x^2 + 2*f^2*x^4)) + 4*A*(45*a^2*f^2 + 15*a*c*f*(-3*d + f*x^2) + c^2*(15*d^2 - 5*d*f*x^2 + 3*f^2*x^4)))) - 30*(A*b*f*(-3*c^2*d^2 + b^2*d*f + 6*a*c*d*f - 3*a^2*f^2) + B*(c*d - a*f)*(c^2*d^2 - 3*b^2*d*f - 2*a*c*d*f + a^2*f^2))*Log[d + f*x^2])/(60*f^4)
```

fricas [A] time = 0.71, size = 1014, normalized size = 2.30

$$\frac{10Bc^3df^3x^6 + 12(3Bbc^2 + Ac^3)df^3x^5 - 15(Bc^3d^2f^2 - 3(Bb^2c + (Ba + Ab)c^2)df^3)x^4 - 20((3Bbc^2 + Ac^3)d^2f^2 - (Bb^3 + 3Aa^2c^2 + 3(2Bab + Ab^2)c)d^2f^3)x^3 + 30(Bc^3d^3f - 3(Bb^2c + (Ba + Ab)c^2)d^2f^2 + (3Bab^2 + Ab^3 + 3(Ba^2 + 2Aab)c)d^2f^3)x^2 - 30(Aa^3f^3 - (3Bb^2c + Ac^3)d^3 + (Bb^3 + 3Aa^2c^2 + 3(2Bab + Ab^2)c)d^2f - 3(Ba^2b + Aab^2 + Aa^2c)d^2f^2)*\sqrt{-d*f})\log((f*x^2 - 2*\sqrt{-d*f})x - d)/(f*x^2 + d)) + 60((3Bb^2c + Ac^3)d^3f - (Bb^3 + 3Aa^2c^2 + 3(2Bab + Ab^2)c)d^2f^2 + 3(Ba^2b + Aab^2 + Aa^2c)d^2f^3)x - 30(Bc^3d^4 - 3(Bb^2c + (Ba + Ab)c^2)d^3f + (3Bab^2 + Ab^3 + 3(Ba^2 + 2Aab)c)d^2f^2 - (Ba^3 + 3Aa^2b)d^2f^3)*\log(f*x^2 + d))/(d*f^4), 1/60*(10Bc^3df^3x^6 + 12(3Bb^2c + Ac^3)df^3x^5 - 15(Bc^3d^2f^2 - 3(Bb^2c + (Ba + Ab)c^2)df^3)x^4 - 20((3Bb^2c + Ac^3)d^2f^2 - (Bb^3 + 3Aa^2c^2 + 3(2Bab + Ab^2)c)d^2f^3)x^3 + 30(Bc^3d^3f - 3(Bb^2c + (Ba + Ab)c^2)d^2f^2 + (3Bab^2 + Ab^3 + 3(Ba^2 + 2Aab)c)d^2f^3)x^2 + 60(Aa^3f^3 - (3Bb^2c + Ac^3)d^3 + (Bb^3 + 3Aa^2c^2 + 3(2Bab + Ab^2)c)d^2f - 3(Ba^2b + Aab^2 + Aa^2c)d^2f^2)*\sqrt{d*f})\arctan(\sqrt{d*f}*x/d) + 60((3Bb^2c + Ac^3)d^3f - (Bb^3 + 3Aa^2c^2 + 3(2Bab + Ab^2)c)d^2f^2 + 3(Ba^2b + Aab^2 + Aa^2c)d^2f^3)x - 30(Bc^3d^4 - 3(Bb^2c + (Ba + Ab)c^2)d^3f + (3Bab^2 + Ab^3 + 3(Ba^2 + 2Aab)c)d^2f^2 - (Ba^3 + 3Aa^2b)d^2f^3)*\log(f*x^2 + d))/(d*f^4)]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^3/(f*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/60*(10*B*c^3*d*f^3*x^6 + 12*(3*B*b*c^2 + A*c^3)*d*f^3*x^5 - 15*(B*c^3*d^2*f^2 - 3*(B*b^2*c + (B*a + A*b)*c^2)*d*f^3)*x^4 - 20*((3*B*b*c^2 + A*c^3)*d^2*f^2 - (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d*f^3)*x^3 + 30*(B*c^3*d^3*f - 3*(B*b^2*c + (B*a + A*b)*c^2)*d^2*f^2 + (3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*d*f^3)*x^2 - 30*(A*a^3*f^3 - (3*B*b*c^2 + A*c^3)*d^3 + (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d^2*f - 3*(B*a^2*b + A*a*b^2 + A*a^2*c)*d*f^2)*sqrt(-d*f)*log((f*x^2 - 2*sqrt(-d*f)*x - d)/(f*x^2 + d)) + 60*((3*B*b*c^2 + A*c^3)*d^3*f - (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d^2*f^2 + 3*(B*a^2*b + A*a*b^2 + A*a^2*c)*d*f^3)*x - 30*(B*c^3*d^4 - 3*(B*b^2*c + (B*a + A*b)*c^2)*d^3*f + (3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*d^2*f^2 - (B*a^3 + 3*A*a^2*b)*d*f^3)*log(f*x^2 + d))/(d*f^4), 1/60*(10*B*c^3*d*f^3*x^6 + 12*(3*B*b*c^2 + A*c^3)*d*f^3*x^5 - 15*(B*c^3*d^2*f^2 - 3*(B*b^2*c + (B*a + A*b)*c^2)*d*f^3)*x^4 - 20*((3*B*b*c^2 + A*c^3)*d^2*f^2 - (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d*f^3)*x^3 + 30*(B*c^3*d^3*f - 3*(B*b^2*c + (B*a + A*b)*c^2)*d^2*f^2 + (3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*d*f^3)*x^2 + 60*(A*a^3*f^3 - (3*B*b*c^2 + A*c^3)*d^3 + (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d^2*f - 3*(B*a^2*b + A*a*b^2 + A*a^2*c)*d*f^2)*sqrt(d*f)*arctan(sqrt(d*f)*x/d) + 60*((3*B*b*c^2 + A*c^3)*d^3*f - (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d^2*f^2 + 3*(B*a^2*b + A*a*b^2 + A*a^2*c)*d*f^3)*x - 30*(B*c^3*d^4 - 3*(B*b^2*c + (B*a + A*b)*c^2)*d^3*f + (3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*d^2*f^2 - (B*a^3 + 3*A*a^2*b)*d*f^3)*log(f*x^2 + d))/(d*f^4)]
```

giac [A] time = 0.16, size = 623, normalized size = 1.41

$$\frac{(3 Bbc^2d^3 + Ac^3d^3 - Bb^3d^2f - 6 Babcd^2f - 3 Ab^2cd^2f - 3 Aac^2d^2f + 3 Ba^2bdf^2 + 3 Aab^2df^2 + 3 Aa^2cdf^2 - A}{\sqrt{df} f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)^3/(f*x^2+d),x, algorithm="giac")

[Out] $-(3*B*b*c^2*d^3 + A*c^3*d^3 - B*b^3*d^2*f - 6*B*a*b*c*d^2*f - 3*A*b^2*c*d^2*f - 3*A*a*c^2*d^2*f + 3*B*a^2*b*d*f^2 + 3*A*a*b^2*d*f^2 + 3*A*a^2*c*d*f^2 - A*a^3*f^3)*\arctan(f*x/\sqrt{d*f})/(\sqrt{d*f}*f^3) - 1/2*(B*c^3*d^3 - 3*B*b^2*c*d^2*f - 3*B*a*c^2*d^2*f - 3*A*b*c^2*d^2*f + 3*B*a*b^2*d*f^2 + A*b^3*d*f^2 + 3*B*a^2*c*d*f^2 + 6*A*a*b*c*d*f^2 - B*a^3*f^3 - 3*A*a^2*b*f^3)*\log(f*x^2 + d)/f^4 + 1/60*(10*B*c^3*f^5*x^6 + 36*B*b*c^2*f^5*x^5 + 12*A*c^3*f^5*x^5 - 15*B*c^3*d*f^4*x^4 + 45*B*b^2*c*f^5*x^4 + 45*B*a*c^2*f^5*x^4 + 45*A*b*c^2*f^5*x^4 - 60*B*b*c^2*d*f^4*x^3 - 20*A*c^3*d*f^4*x^3 + 20*B*b^3*f^5*x^3 + 120*B*a*b*c*f^5*x^3 + 60*A*b^2*c*f^5*x^3 + 60*A*a*c^2*f^5*x^3 + 30*B*c^3*d^2*f^3*x^2 - 90*B*b^2*c*d*f^4*x^2 - 90*B*a*c^2*d*f^4*x^2 - 90*A*b*c^2*d*f^4*x^2 + 90*B*a*b^2*f^5*x^2 + 30*A*b^3*f^5*x^2 + 90*B*a^2*c*f^5*x^2 + 180*A*a*b*c*f^5*x^2 + 180*B*b*c^2*d^2*f^3*x + 60*A*c^3*d^2*f^3*x - 60*B*b^3*d*f^4*x - 360*B*a*b*c*d*f^4*x - 180*A*b^2*c*d*f^4*x - 180*A*a*c^2*d*f^4*x + 180*B*a^2*b*f^5*x + 180*A*a*b^2*f^5*x + 180*A*a^2*c*f^5*x)/f^6$

maple [A] time = 0.01, size = 822, normalized size = 1.86

$$\frac{Bc^3x^6}{6f} + \frac{Ac^3x^5}{5f} + \frac{3Bbc^2x^5}{5f} + \frac{3Abc^2x^4}{4f} + \frac{3Bac^2x^4}{4f} + \frac{3Bb^2cx^4}{4f} - \frac{Bc^3dx^4}{4f^2} + \frac{Aac^2x^3}{f} + \frac{Ab^2cx^3}{f} - \frac{Ac^3dx^3}{3f^2} + \frac{2Babcx^3}{f} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x+A)*(c*x^2+b*x+a)^3/(f*x^2+d),x)

[Out] $6/f^2/(d*f)^{(1/2)}*\arctan(1/(d*f)^{(1/2)}*f*x)*B*a*b*c*d^2-1/f^2*b^3*B*d*x+3/f*A*a*b^2*x+1/6*B*c^3*x^6/f-3/2/f^2*\ln(f*x^2+d)*B*a^2*c*d-3/2/f^2*\ln(f*x^2+d)*B*a*b^2*d+3/2/f^3*\ln(f*x^2+d)*B*a*c^2*d^2+3/2/f^3*\ln(f*x^2+d)*B*b^2*c*d^2-1/f^3/(d*f)^{(1/2)}*\arctan(1/(d*f)^{(1/2)}*f*x)*A*c^3*d^3+1/f^2/(d*f)^{(1/2)}*\arctan(1/(d*f)^{(1/2)}*f*x)*b^3*B*d^2+3/f^3*B*b*c^2*d^2*x+2/f*B*x^3*a*b*c-1/f^2*B*x^3*b*c^2*d+3/f*A*x^2*a*b*c-3/2/f^2*A*x^2*b*c^2*d-3/2/f^2*B*x^2*a*c^2*d-3/2/f^2*B*x^2*b^2*c*d-3/f^2*A*a*c^2*d*x-3/f^2*A*b^2*c*d*x+3/2/f^3*\ln(f*x^2+d)*A*b*c^2*d^2+1/2/f*\ln(f*x^2+d)*B*a^3+1/(d*f)^{(1/2)}*\arctan(1/(d*f)^{(1/2)}*f*x)*A*a^3+1/3/f*B*x^3*b^3+1/2/f*A*x^2*b^3+1/5/f*A*x^5*c^3-3/f^2*\ln(f*x^2+d)*A*a*b*c*d-3/f/(d*f)^{(1/2)}*\arctan(1/(d*f)^{(1/2)}*f*x)*A*a^2*c*d-3/f/(d*f)^{(1/2)}*\arctan(1/(d*f)^{(1/2)}*f*x)*A*a*b^2*d+3/f^2/(d*f)^{(1/2)}*\arctan(1/(d*f)^{(1/2)}*f*x)$

/2)*f*x)*A*a*c^2*d^2+3/f^2/(d*f)^(1/2)*arctan(1/(d*f)^(1/2)*f*x)*A*b^2*c*d^2-3/f/(d*f)^(1/2)*arctan(1/(d*f)^(1/2)*f*x)*B*a^2*b*d-3/f^3/(d*f)^(1/2)*arctan(1/(d*f)^(1/2)*f*x)*B*b*c^2*d^3-6/f^2*B*a*b*c*d*x+3/5/f*B*x^5*b*c^2+3/4/f*A*x^4*b*c^2+1/f*A*x^3*a*c^2+3/2/f*ln(f*x^2+d)*A*a^2*b-1/2/f^2*ln(f*x^2+d)*A*b^3*d-1/2/f^4*ln(f*x^2+d)*B*c^3*d^3+1/f*A*x^3*b^2*c+1/2/f^3*B*x^2*c^3*d^2+1/f^3*A*c^3*d^2*x+3/f*B*a^2*b*x-1/3/f^2*A*x^3*c^3*d+3/2/f*B*x^2*a^2*c+3/2/f*B*x^2*a*b^2+3/f*A*a^2*c*x+3/4/f*B*x^4*a*c^2+3/4/f*B*x^4*b^2*c-1/4/f^2*B*x^4*c^3*d

maxima [A] time = 0.99, size = 471, normalized size = 1.07

$$\frac{(Aa^3f^3 - (3Bbc^2 + Ac^3)d^3 + (Bb^3 + 3Aac^2 + 3(2Bab + Ab^2)c)d^2f - 3(Ba^2b + Aab^2 + Aa^2c)df^2) \arctan\left(\frac{f}{\sqrt{df}}\right)}{\sqrt{df} f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)^3/(f*x^2+d),x, algorithm="maxima")

[Out] (A*a^3*f^3 - (3*B*b*c^2 + A*c^3)*d^3 + (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d^2*f - 3*(B*a^2*b + A*a*b^2 + A*a^2*c)*d*f^2)*arctan(f*x/sqrt(d*f))/(sqrt(d*f)*f^3) + 1/60*(10*B*c^3*f^2*x^6 + 12*(3*B*b*c^2 + A*c^3)*f^2*x^5 - 15*(B*c^3*d*f - 3*(B*b^2*c + (B*a + A*b)*c^2)*f^2)*x^4 - 20*((3*B*b*c^2 + A*c^3)*d*f - (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*f^2)*x^3 + 30*(B*c^3*d^2 - 3*(B*b^2*c + (B*a + A*b)*c^2)*d*f + (3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*f^2)*x^2 + 60*((3*B*b*c^2 + A*c^3)*d^2 - (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d*f + 3*(B*a^2*b + A*a*b^2 + A*a^2*c)*f^2)*x)/f^3 - 1/2*(B*c^3*d^3 - 3*(B*b^2*c + (B*a + A*b)*c^2)*d^2*f + (3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*d*f^2 - (B*a^3 + 3*A*a^2*b)*f^3)*log(f*x^2 + d)/f^4

mupad [B] time = 3.79, size = 552, normalized size = 1.25

$$x^2 \left(\frac{3Bca^2 + 3Bab^2 + 6Acab + Ab^3}{2f} - \frac{d \left(\frac{3Bb^2c + 3Abc^2 + 3Bac^2}{f} - \frac{Bc^3d}{f^2} \right)}{2f} \right) + x \left(\frac{3Ba^2b + 3Aca^2 + 3Aab^2}{f} - \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x)*(a + b*x + c*x^2)^3)/(d + f*x^2),x)

[Out] x^2*((A*b^3 + 3*B*a*b^2 + 3*B*a^2*c + 6*A*a*b*c)/(2*f) - (d*((3*A*b*c^2 + 3*B*a*c^2 + 3*B*b^2*c)/f - (B*c^3*d)/f^2))/(2*f)) + x*((3*A*a*b^2 + 3*A*a^2*c + 3*B*a^2*b)/f - (d*((B*b^3 + 3*A*a*c^2 + 3*A*b^2*c + 6*B*a*b*c)/f - (d*(A*c^3 + 3*B*b*c^2))/f^2))/f) + x^3*((B*b^3 + 3*A*a*c^2 + 3*A*b^2*c + 6*B*a*

$$\begin{aligned}
& b*c)/(3*f) - (d*(A*c^3 + 3*B*b*c^2))/(3*f^2)) + x^4*((3*A*b*c^2 + 3*B*a*c^2 \\
& + 3*B*b^2*c)/(4*f) - (B*c^3*d)/(4*f^2)) + (x^5*(A*c^3 + 3*B*b*c^2))/(5*f) \\
& + (B*c^3*x^6)/(6*f) + (\log(d + f*x^2)*(4*B*a^3*d*f^7 - 4*A*b^3*d^2*f^6 - 4* \\
& B*c^3*d^4*f^4 - 12*B*a*b^2*d^2*f^6 + 12*A*b*c^2*d^3*f^5 + 12*B*a*c^2*d^3*f^ \\
& 5 - 12*B*a^2*c*d^2*f^6 + 12*B*b^2*c*d^3*f^5 + 12*A*a^2*b*d*f^7 - 24*A*a*b*c \\
& *d^2*f^6))/(8*d*f^8) + (\operatorname{atan}((f^{1/2})*x)/d^{1/2})*(A*a^3*f^3 - A*c^3*d^3 - \\
& 3*B*b*c^2*d^3 + B*b^3*d^2*f - 3*A*a*b^2*d*f^2 + 3*A*a*c^2*d^2*f - 3*A*a^2*c \\
& *d*f^2 - 3*B*a^2*b*d*f^2 + 3*A*b^2*c*d^2*f + 6*B*a*b*c*d^2*f))/(d^{1/2}*f^{(\\
& 7/2)})
\end{aligned}$$

sympy [B] time = 23.62, size = 1962, normalized size = 4.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x**2+b*x+a)**3/(f*x**2+d), x)

[Out] B*c**3*x**6/(6*f) + x**5*(A*c**3/(5*f) + 3*B*b*c**2/(5*f)) + x**4*(3*A*b*c*
*2/(4*f) + 3*B*a*c**2/(4*f) + 3*B*b**2*c/(4*f) - B*c**3*d/(4*f**2)) + x**3*
(A*a*c**2/f + A*b**2*c/f - A*c**3*d/(3*f**2) + 2*B*a*b*c/f + B*b**3/(3*f) -
B*b*c**2*d/f**2) + x**2*(3*A*a*b*c/f + A*b**3/(2*f) - 3*A*b*c**2*d/(2*f**2)
) + 3*B*a**2*c/(2*f) + 3*B*a*b**2/(2*f) - 3*B*a*c**2*d/(2*f**2) - 3*B*b**2*
c*d/(2*f**2) + B*c**3*d**2/(2*f**3)) + x*(3*A*a**2*c/f + 3*A*a*b**2/f - 3*A
*a*c**2*d/f**2 - 3*A*b**2*c*d/f**2 + A*c**3*d**2/f**3 + 3*B*a**2*b/f - 6*B*
a*b*c*d/f**2 - B*b**3*d/f**2 + 3*B*b*c**2*d**2/f**3) + ((3*A*a**2*b*f**3 -
6*A*a*b*c*d*f**2 - A*b**3*d*f**2 + 3*A*b*c**2*d**2*f + B*a**3*f**3 - 3*B*a*
*2*c*d*f**2 - 3*B*a*b**2*d*f**2 + 3*B*a*c**2*d**2*f + 3*B*b**2*c*d**2*f - B
*c**3*d**3)/(2*f**4) - sqrt(-d*f**9)*(A*a**3*f**3 - 3*A*a**2*c*d*f**2 - 3*A
*a*b**2*d*f**2 + 3*A*a*c**2*d**2*f + 3*A*b**2*c*d**2*f - A*c**3*d**3 - 3*B*
a**2*b*d*f**2 + 6*B*a*b*c*d**2*f + B*b**3*d**2*f - 3*B*b*c**2*d**3)/(2*d*f*
*8))*log(x + (-3*A*a**2*b*d*f**3 + 6*A*a*b*c*d**2*f**2 + A*b**3*d**2*f**2 -
3*A*b*c**2*d**3*f - B*a**3*d*f**3 + 3*B*a**2*c*d**2*f**2 + 3*B*a*b**2*d**2
*f**2 - 3*B*a*c**2*d**3*f - 3*B*b**2*c*d**3*f + B*c**3*d**4 + 2*d*f**4*((3*
A*a**2*b*f**3 - 6*A*a*b*c*d*f**2 - A*b**3*d*f**2 + 3*A*b*c**2*d**2*f + B*a*
*3*f**3 - 3*B*a**2*c*d*f**2 - 3*B*a*b**2*d*f**2 + 3*B*a*c**2*d**2*f + 3*B*b
2*c*d2*f - B*c**3*d**3)/(2*f**4) - sqrt(-d*f**9)*(A*a**3*f**3 - 3*A*a**
2*c*d*f**2 - 3*A*a*b**2*d*f**2 + 3*A*a*c**2*d**2*f + 3*A*b**2*c*d**2*f - A*
c**3*d**3 - 3*B*a**2*b*d*f**2 + 6*B*a*b*c*d**2*f + B*b**3*d**2*f - 3*B*b*c*
*2*d**3)/(2*d*f**8)))/(A*a**3*f**4 - 3*A*a**2*c*d*f**3 - 3*A*a*b**2*d*f**3
+ 3*A*a*c**2*d**2*f**2 + 3*A*b**2*c*d**2*f**2 - A*c**3*d**3*f - 3*B*a**2*b*
d*f**3 + 6*B*a*b*c*d**2*f**2 + B*b**3*d**2*f**2 - 3*B*b*c**2*d**3*f)) + ((3
*A*a**2*b*f**3 - 6*A*a*b*c*d*f**2 - A*b**3*d*f**2 + 3*A*b*c**2*d**2*f + B*a
3*f3 - 3*B*a**2*c*d*f**2 - 3*B*a*b**2*d*f**2 + 3*B*a*c**2*d**2*f + 3*B*
b**2*c*d**2*f - B*c**3*d**3)/(2*f**4) + sqrt(-d*f**9)*(A*a**3*f**3 - 3*A*a*
*2*c*d*f**2 - 3*A*a*b**2*d*f**2 + 3*A*a*c**2*d**2*f + 3*A*b**2*c*d**2*f - A

$$\begin{aligned}
& *c^{**3}d^{**3} - 3*B*a^{**2}b*d*f^{**2} + 6*B*a*b*c*d^{**2}f + B*b^{**3}d^{**2}f - 3*B*b*c \\
& **2*d^{**3})/(2*d*f^{**8}))*\log(x + (-3*A*a^{**2}b*d*f^{**3} + 6*A*a*b*c*d^{**2}f^{**2} + A \\
& *b^{**3}d^{**2}f^{**2} - 3*A*b*c^{**2}d^{**3}f - B*a^{**3}d*f^{**3} + 3*B*a^{**2}c*d^{**2}f^{**2} \\
& + 3*B*a*b^{**2}d^{**2}f^{**2} - 3*B*a*c^{**2}d^{**3}f - 3*B*b^{**2}c*d^{**3}f + B*c^{**3}d^{** \\
& 4 + 2*d*f^{**4}*((3*A*a^{**2}b*f^{**3} - 6*A*a*b*c*d*f^{**2} - A*b^{**3}d*f^{**2} + 3*A*b*c \\
& **2*d^{**2}f + B*a^{**3}f^{**3} - 3*B*a^{**2}c*d*f^{**2} - 3*B*a*b^{**2}d*f^{**2} + 3*B*a*c \\
& *2*d^{**2}f + 3*B*b^{**2}c*d^{**2}f - B*c^{**3}d^{**3})/(2*f^{**4}) + \text{sqrt}(-d*f^{**9})*(A*a \\
& *3*f^{**3} - 3*A*a^{**2}c*d*f^{**2} - 3*A*a*b^{**2}d*f^{**2} + 3*A*a*c^{**2}d^{**2}f + 3*A*b \\
& **2*c*d^{**2}f - A*c^{**3}d^{**3} - 3*B*a^{**2}b*d*f^{**2} + 6*B*a*b*c*d^{**2}f + B*b^{**3} \\
& d^{**2}f - 3*B*b*c^{**2}d^{**3})/(2*d*f^{**8}))/((A*a^{**3}f^{**4} - 3*A*a^{**2}c*d*f^{**3} - 3 \\
& *A*a*b^{**2}d*f^{**3} + 3*A*a*c^{**2}d^{**2}f^{**2} + 3*A*b^{**2}c*d^{**2}f^{**2} - A*c^{**3}d^{** \\
& 3*f - 3*B*a^{**2}b*d*f^{**3} + 6*B*a*b*c*d^{**2}f^{**2} + B*b^{**3}d^{**2}f^{**2} - 3*B*b*c \\
& *2*d^{**3}f))
\end{aligned}$$

$$3.4 \quad \int \frac{A+Bx}{(a+bx+cx^2)(d+fx^2)} dx$$

Optimal. Leaf size=274

$$\frac{\log(a+bx+cx^2)(-aBf+Abf+Bcd)}{2(f(a^2f+b^2d)-2acdf+c^2d^2)} - \frac{\log(d+fx^2)(-aBf+Abf+Bcd)}{2(f(a^2f+b^2d)-2acdf+c^2d^2)} + \frac{\sqrt{f} \tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(aAf-Acd+bf)}{\sqrt{d}(f(a^2f+b^2d)-2acdf+c^2d^2)}$$

[Out] 1/2*(A*b*f-B*a*f+B*c*d)*ln(c*x^2+b*x+a)/(c^2*d^2-2*a*c*d*f+f*(a^2*f+b^2*d))-1/2*(A*b*f-B*a*f+B*c*d)*ln(f*x^2+d)/(c^2*d^2-2*a*c*d*f+f*(a^2*f+b^2*d))-(A*b^2*f+2*A*c*(-a*f+c*d)-b*B*(a*f+c*d))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(c^2*d^2-2*a*c*d*f+f*(a^2*f+b^2*d))/(-4*a*c+b^2)^(1/2)+(A*a*f-A*c*d+B*b*d)*arctan(x*f^(1/2)/d^(1/2))*f^(1/2)/(c^2*d^2-2*a*c*d*f+f*(a^2*f+b^2*d))/d^(1/2)

Rubi [A] time = 0.28, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {1023, 634, 618, 206, 628, 635, 205, 260}

$$\frac{\log(a+bx+cx^2)(-aBf+Abf+Bcd)}{2(f(a^2f+b^2d)-2acdf+c^2d^2)} - \frac{\log(d+fx^2)(-aBf+Abf+Bcd)}{2(f(a^2f+b^2d)-2acdf+c^2d^2)} + \frac{\sqrt{f} \tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(aAf-Acd+bf)}{\sqrt{d}(f(a^2f+b^2d)-2acdf+c^2d^2)}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x)/((a + b*x + c*x^2)*(d + f*x^2)), x]

[Out] (Sqrt[f]*(b*B*d - A*c*d + a*A*f)*ArcTan[(Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))) - ((A*b^2*f + 2*A*c*(c*d - a*f) - b*B*(c*d + a*f))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))) + ((B*c*d + A*b*f - a*B*f)*Log[a + b*x + c*x^2])/(2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))) - ((B*c*d + A*b*f - a*B*f)*Log[d + f*x^2])/(2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f)))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 260

$\text{Int}[(x_)^m / ((a_) + (b_)*(x_)^n), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^n, x]] / (b*n), x] \text{ ; FreeQ}[\{a, b, m, n\}, x] \ \&\& \ \text{EqQ}[m, n - 1]$

Rule 618

$\text{Int}[(a_) + (b_)*(x_) + (c_)*(x_)^2]^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] \text{ ; FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 628

$\text{Int}[(d_) + (e_)*(x_)] / ((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]) / b, x] \text{ ; FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rule 634

$\text{Int}[(d_) + (e_)*(x_)] / ((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] \rightarrow \text{Dist}[(2*c*d - b*e) / (2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e / (2*c), \text{Int}[(b + 2*c*x) / (a + b*x + c*x^2), x], x] \text{ ; FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4*a*c]$

Rule 635

$\text{Int}[(d_) + (e_)*(x_)] / ((a_) + (c_)*(x_)^2), x_Symbol] \rightarrow \text{Dist}[d, \text{Int}[1/(a + c*x^2), x], x] + \text{Dist}[e, \text{Int}[x/(a + c*x^2), x], x] \text{ ; FreeQ}[\{a, c, d, e\}, x] \ \&\& \ !\text{NiceSqrtQ}[-(a*c)]$

Rule 1023

$\text{Int}[(g_) + (h_)*(x_)] / (((a_) + (b_)*(x_) + (c_)*(x_)^2)*((d_) + (f_)*(x_)^2)), x_Symbol] \rightarrow \text{With}[\{q = \text{Simplify}[c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2]\}, \text{Dist}[1/q, \text{Int}[\text{Simp}[g*c^2*d + g*b^2*f - a*b*h*f - a*g*c*f + c*(h*c*d + g*b*f - a*h*f)*x, x] / (a + b*x + c*x^2), x], x] + \text{Dist}[1/q, \text{Int}[\text{Simp}[b*h*d*f - g*c*d*f + a*g*f^2 - f*(h*c*d + g*b*f - a*h*f)*x, x] / (d + f*x^2), x], x] \text{ ; NeQ}[q, 0] \text{ ; FreeQ}[\{a, b, c, d, f, g, h\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx}{(a + bx + cx^2)(d + fx^2)} dx &= \int \frac{-abBf + A(c^2d + b^2f - acf) + c(Bcd + Abf - aBf)x}{a + bx + cx^2} dx + \int \frac{f(bBd - Acd + aAf) - f(Bcd + Abf - aBf)x}{d + fx^2} dx \\
&= \frac{(f(bBd - Acd + aAf)) \int \frac{1}{d + fx^2} dx}{c^2d^2 - 2acdf + f(b^2d + a^2f)} + \frac{(Bcd + Abf - aBf) \int \frac{b + 2cx}{a + bx + cx^2} dx}{2(c^2d^2 - 2acdf + f(b^2d + a^2f))} - \frac{(f(Bcd + Abf - aBf)x)}{c^2d^2 - 2acdf + f(b^2d + a^2f)} \\
&= \frac{\sqrt{f}(bBd - Acd + aAf) \tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)}{\sqrt{d}(c^2d^2 - 2acdf + f(b^2d + a^2f))} + \frac{(Bcd + Abf - aBf) \log(a + bx + cx^2)}{2(c^2d^2 - 2acdf + f(b^2d + a^2f))} \\
&= \frac{\sqrt{f}(bBd - Acd + aAf) \tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)}{\sqrt{d}(c^2d^2 - 2acdf + f(b^2d + a^2f))} - \frac{(Ab^2f + 2Ac(cd - af) - bB(cd + af)) \tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)}{\sqrt{b^2 - 4ac}(c^2d^2 - 2acdf + f(b^2d + a^2f))}
\end{aligned}$$

Mathematica [A] time = 0.37, size = 212, normalized size = 0.77

$$\frac{\sqrt{d} \left(\sqrt{4ac - b^2} (-aBf + Abf + Bcd) (\log(a + x(b + cx)) - \log(d + fx^2)) + 2 \tan^{-1} \left(\frac{b + 2cx}{\sqrt{4ac - b^2}} \right) (2Ac(cd - af) - bB(cd + af)) \right)}{2\sqrt{d} \sqrt{4ac - b^2} (f(a^2f + b^2d) - 2acdf + c^2d^2)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x)/((a + b*x + c*x^2)*(d + f*x^2)), x]

[Out] (2*Sqrt[-b^2 + 4*a*c]*Sqrt[f]*(b*B*d - A*c*d + a*A*f)*ArcTan[(Sqrt[f]*x)/Sqrt[d]] + Sqrt[d]*(2*(A*b^2*f + 2*A*c*(c*d - a*f) - b*B*(c*d + a*f))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a*c]*(B*c*d + A*b*f - a*B*f)*(-Log[d + f*x^2] + Log[a + x*(b + c*x)])))/(2*Sqrt[-b^2 + 4*a*c]*Sqrt[d]*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f)))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d), x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.17, size = 266, normalized size = 0.97

$$\frac{(Bcd - Baf + Abf) \log(cx^2 + bx + a)}{2(c^2d^2 + b^2df - 2acdf + a^2f^2)} - \frac{(Bcd - Baf + Abf) \log(fx^2 + d)}{2(c^2d^2 + b^2df - 2acdf + a^2f^2)} + \frac{(Bbdf - Acdf + Aaf^2) \arctan\left(\frac{fx}{\sqrt{df}}\right)}{(c^2d^2 + b^2df - 2acdf + a^2f^2)\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="giac")

[Out] $\frac{1}{2}*(B*c*d - B*a*f + A*b*f)*\log(c*x^2 + b*x + a)/(c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2) - \frac{1}{2}*(B*c*d - B*a*f + A*b*f)*\log(f*x^2 + d)/(c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2) + (B*b*d*f - A*c*d*f + A*a*f^2)*\arctan(f*x/\sqrt{d*f})/((c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2)*\sqrt{d*f}) - (B*b*c*d - 2*A*c^2*d + B*a*b*f - A*b^2*f + 2*A*a*c*f)*\arctan((2*c*x + b)/\sqrt{-b^2 + 4*a*c})/((c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2)*\sqrt{-b^2 + 4*a*c})$

maple [B] time = 0.02, size = 745, normalized size = 2.72

$$\frac{2Aac f \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(a^2 f^2 - 2acdf + b^2 df + c^2 d^2) \sqrt{4ac - b^2}} + \frac{Aa f^2 \arctan\left(\frac{fx}{\sqrt{df}}\right)}{(a^2 f^2 - 2acdf + b^2 df + c^2 d^2) \sqrt{df}} + \frac{A b^2 f \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(a^2 f^2 - 2acdf + b^2 df + c^2 d^2) \sqrt{4ac - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d),x)

[Out] $-1/2*f/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)*\ln(f*x^2+d)*A*b+1/2*f/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)*\ln(f*x^2+d)*B*a-1/2/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)*\ln(f*x^2+d)*B*c*d+f^2/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)/(d*f)^{(1/2)}*\arctan(1/(d*f)^{(1/2)}*f*x)*A*a-f/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)/(d*f)^{(1/2)}*\arctan(1/(d*f)^{(1/2)}*f*x)*A*c*d+f/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)/(d*f)^{(1/2)}*\arctan(1/(d*f)^{(1/2)}*f*x)*B*b*d+1/2/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)*\ln(c*x^2+b*x+a)*A*b*f-1/2/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)*\ln(c*x^2+b*x+a)*B*a*f+1/2/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)*c*\ln(c*x^2+b*x+a)*B*d-2/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*A*a*c*f+1/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*A*b^2*f+2/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*A*c^2*d-1/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*B*a*b*f-1/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*B*b*c*d$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* h

elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details) Is $4*a*c-b^2$ positive or negative?

mupad [B] time = 38.32, size = 3888, normalized size = 14.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x)/((d + f*x^2)*(a + b*x + c*x^2)),x)`

[Out]
$$\begin{aligned} & (\log(B^3*c^2*f^2*x + (((B*c*d^2)/2 + (A*b*d*f)/2 - (B*a*d*f)/2 - (A*a*f*(-d*f)^{(1/2)}))/2 + (A*c*d*(-d*f)^{(1/2)}))/2 - (B*b*d*(-d*f)^{(1/2)}))/2 * (((B*c*d^2)/2 + (A*b*d*f)/2 - (B*a*d*f)/2 - (A*a*f*(-d*f)^{(1/2)}))/2 + (A*c*d*(-d*f)^{(1/2)}))/2 - (B*b*d*(-d*f)^{(1/2)}))/2 * (4*A*a^2*c^2*f^4 + 4*A*c^4*d^2*f^2 - c*f^2 * x*(3*A*b^3*f^2 + 4*B*c^3*d^2 - B*a*b^2*f^2 + 4*B*a^2*c*f^2 - 12*A*a*b*c*f^2 + 12*A*b*c^2*d*f - 8*B*a*c^2*d*f - 3*B*b^2*c*d*f) - 3*A*b^2*c^2*d*f^3 - 4*B*b*c^3*d^2*f^2 - A*a*b^2*c*f^4 - 8*A*a*c^3*d*f^3 + B*b^3*c*d*f^3 + (2*c*f^2*((B*c*d^2)/2 + (A*b*d*f)/2 - (B*a*d*f)/2 - (A*a*f*(-d*f)^{(1/2)}))/2 + (A*c*d*(-d*f)^{(1/2)}))/2 - (B*b*d*(-d*f)^{(1/2)}))/2 * (2*b*c^3*d^3 + 4*c^4*d^3*x - a^2*b^2*f^3*x + 4*a*b^3*d*f^2 - 2*b^3*c*d^2*f + 4*a^3*c*f^3*x + 3*b^4*d*f^2*x + 12*a*b*c^2*d^2*f - 14*a^2*b*c*d*f^2 - 4*a*c^3*d^2*f*x - 4*a^2*c^2*d*f^2*x + 3*b^2*c^2*d^2*f*x - 10*a*b^2*c*d*f^2*x)) / (d*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f) + 4*B*a*b*c^2*d*f^3) / (d*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f) - c*f^2*x*(2*B^2*c^2*d - 4*A^2*c^2*f - B^2*b^2*f + 2*B^2*a*c*f + 2*A*B*b*c*f) + A^2*b*c^2*f^3 + B^2*b*c^2*d*f^2 - 4*A*B*a*c^2*f^3 + A*B*b^2*c*f^3 - 4*A*B*c^3*d*f^2) / (d*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f) + A*B^2*c^2*f^2)*(f*((B*a*d)/2 - (A*b*d)/2 + (A*a*(-d*f)^{(1/2)}))/2 - (B*c*d^2)/2 - (A*c*d*(-d*f)^{(1/2)}))/2 + (B*b*d*(-d*f)^{(1/2)}))/2) / (c^2*d^3 + a^2*d*f^2 + b^2*d^2*f - 2*a*c*d^2*f) - (\log(B^3*c^2*f^2*x + (((B*c*d^2)/2 + (A*b*d*f)/2 - (B*a*d*f)/2 + (A*a*f*(-d*f)^{(1/2)}))/2 - (B*a*d*f)/2 + (A*a*f*(-d*f)^{(1/2)}))/2 - (A*c*d*(-d*f)^{(1/2)}))/2 + (B*b*d*(-d*f)^{(1/2)}))/2 * (((B*c*d^2)/2 + (A*b*d*f)/2 - (B*a*d*f)/2 + (A*a*f*(-d*f)^{(1/2)}))/2 - (A*c*d*(-d*f)^{(1/2)}))/2 + (B*b*d*(-d*f)^{(1/2)}))/2 * (4*A*a^2*c^2*f^4 + 4*A*c^4*d^2*f^2 - c*f^2*x*(3*A*b^3*f^2 + 4*B*c^3*d^2 - B*a*b^2*f^2 + 4*B*a^2*c*f^2 - 12*A*a*b*c*f^2 + 12*A*b*c^2*d*f - 8*B*a*c^2*d*f - 3*B*b^2*c*d*f) - 3*A*b^2*c^2*d*f^3 - 4*B*b*c^3*d^2*f^2 - A*a*b^2*c*f^4 - 8*A*a*c^3*d*f^3 + B*b^3*c*d*f^3 + (2*c*f^2*((B*c*d^2)/2 + (A*b*d*f)/2 - (B*a*d*f)/2 + (A*a*f*(-d*f)^{(1/2)}))/2 - (A*c*d*(-d*f)^{(1/2)}))/2 + (B*b*d*(-d*f)^{(1/2)}))/2 * (2*b*c^3*d^3 + 4*c^4*d^3*x - a^2*b^2*f^3*x + 4*a*b^3*d*f^2 - 2*b^3*c*d^2*f + 4*a^3*c*f^3*x + 3*b^4*d*f^2*x + 12*a*b*c^2*d^2*f - 14*a^2*b*c*d*f^2 - 4*a*c^3*d^2*f*x - 4*a^2*c^2*d*f^2*x + 3*b^2*c^2*d^2*f*x - 10*a*b^2*c*d*f^2*x)) / (d*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f) + 4*B*a*b*c^2*d*f^3) / (d*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f) - c*f^2*x*(2*B^2*c^2*d - 4*A^2*c^2*f - B^2*b^2*f + 2*B^2*a*c*f + 2*A*B*b*c*f) + A^2*b*c^2*f^3 + B^2*b*c^2*d*f^2 - 4*A*B*a*c^2*f^3 + A*B*b^2*c*f^3 - 4*A*B*c^3*d*f^2) / (d*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f) + A*B^2*c^2*f^2)*(f*((A*b*d)/2 - (B*a*d)/2 + (A$$

$$\begin{aligned}
& a*(-d*f)^{(1/2)}/2 + (B*c*d^2)/2 - (A*c*d*(-d*f)^{(1/2)}/2 + (B*b*d*(-d*f)^{(1/2)}/2))/ (c^2*d^3 + a^2*d*f^2 + b^2*d^2*f - 2*a*c*d^2*f) - (\log(B^3*c^2*f^2*x + A*B^2*c^2*f^2 - (((A*f*(b^2 - 4*a*c))^{(3/2)} + 2*A*b*f*(4*a*c - b^2) - 2*B*a*f*(4*a*c - b^2) + 2*B*c*d*(4*a*c - b^2) + 4*A*c^2*d*(b^2 - 4*a*c))^{(1/2)} + A*b^2*f*(b^2 - 4*a*c)^{(1/2)} - 2*B*a*b*f*(b^2 - 4*a*c)^{(1/2)} - 2*B*b*c*d*(b^2 - 4*a*c)^{(1/2)})*(c*f^2*x*(3*A*b^3*f^2 + 4*B*c^3*d^2 - B*a*b^2*f^2 + 4*B*a^2*c*f^2 - 12*A*a*b*c*f^2 + 12*A*b*c^2*d*f - 8*B*a*c^2*d*f - 3*B*b^2*c*d*f) - 4*A*c^4*d^2*f^2 - 4*A*a^2*c^2*f^4 + 3*A*b^2*c^2*d*f^3 + 4*B*b*c^3*d^2*f^2 + A*a*b^2*c*f^4 + 8*A*a*c^3*d*f^3 - B*b^3*c*d*f^3 - 4*B*a*b*c^2*d*f^3 + (c*f^2*(A*f*(b^2 - 4*a*c))^{(3/2)} + 2*A*b*f*(4*a*c - b^2) - 2*B*a*f*(4*a*c - b^2) + 2*B*c*d*(4*a*c - b^2) + 4*A*c^2*d*(b^2 - 4*a*c))^{(1/2)} + A*b^2*f*(b^2 - 4*a*c)^{(1/2)} - 2*B*a*b*f*(b^2 - 4*a*c)^{(1/2)} - 2*B*b*c*d*(b^2 - 4*a*c)^{(1/2)})*(2*b*c^3*d^3 + 4*c^4*d^3*x - a^2*b^2*f^3*x + 4*a*b^3*d*f^2 - 2*b^3*c*d^2*f + 4*a^3*c*f^3*x + 3*b^4*d*f^2*x + 12*a*b*c^2*d^2*f - 14*a^2*b*c*d*f^2 - 4*a*c^3*d^2*f*x - 4*a^2*c^2*d*f^2*x + 3*b^2*c^2*d^2*f*x - 10*a*b^2*c*d*f^2*x))/(2*(4*a*c - b^2)*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f)))/ (4*(4*a*c - b^2)*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f)) - c*f^2*x*(2*B^2*c^2*d - 4*A^2*c^2*f - B^2*b^2*f + 2*B^2*a*c*f + 2*A*B*b*c*f) + A^2*b*c^2*f^3 + B^2*b*c^2*d*f^2 - 4*A*B*a*c^2*f^3 + A*B*b^2*c*f^3 - 4*A*B*c^3*d*f^2)*(A*f*(b^2 - 4*a*c))^{(3/2)} + 2*A*b*f*(4*a*c - b^2) - 2*B*a*f*(4*a*c - b^2) + 2*B*c*d*(4*a*c - b^2) + 4*A*c^2*d*(b^2 - 4*a*c)^{(1/2)} + A*b^2*f*(b^2 - 4*a*c)^{(1/2)} - 2*B*a*b*f*(b^2 - 4*a*c)^{(1/2)} - 2*B*b*c*d*(b^2 - 4*a*c)^{(1/2)))/ (4*(4*a*c - b^2)*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f)))*(A*f*(b^2 - 4*a*c))^{(3/2)} + 2*A*b*f*(4*a*c - b^2) - 2*B*a*f*(4*a*c - b^2) + 2*B*c*d*(4*a*c - b^2) + 4*A*c^2*d*(b^2 - 4*a*c)^{(1/2)} + A*b^2*f*(b^2 - 4*a*c)^{(1/2)} - 2*B*a*b*f*(b^2 - 4*a*c)^{(1/2)} - 2*B*b*c*d*(b^2 - 4*a*c)^{(1/2)))/ (b^2*(4*a^2*f^2 + 4*c^2*d^2 + 4*b^2*d*f - 24*a*c*d*f) - 4*a*c*(4*a^2*f^2 + 4*c^2*d^2 - 8*a*c*d*f)) + (\log(B^3*c^2*f^2*x + A*B^2*c^2*f^2 + (((A*f*(b^2 - 4*a*c))^{(3/2)} - 2*A*b*f*(4*a*c - b^2) + 2*B*a*f*(4*a*c - b^2) - 2*B*c*d*(4*a*c - b^2) + 4*A*c^2*d*(b^2 - 4*a*c))^{(1/2)} + A*b^2*f*(b^2 - 4*a*c)^{(1/2)} - 2*B*a*b*f*(b^2 - 4*a*c)^{(1/2)} - 2*B*b*c*d*(b^2 - 4*a*c)^{(1/2)})*(4*A*a^2*c^2*f^4 + 4*A*c^4*d^2*f^2 - c*f^2*x*(3*A*b^3*f^2 + 4*B*c^3*d^2 - B*a*b^2*f^2 + 4*B*a^2*c*f^2 - 12*A*a*b*c*f^2 + 12*A*b*c^2*d*f - 8*B*a*c^2*d*f - 3*B*b^2*c*d*f) - 3*A*b^2*c^2*d*f^3 - 4*B*b*c^3*d^2*f^2 - A*a*b^2*c*f^4 - 8*A*a*c^3*d*f^3 + B*b^3*c*d*f^3 + 4*B*a*b*c^2*d*f^3 + (c*f^2*(A*f*(b^2 - 4*a*c))^{(3/2)} - 2*A*b*f*(4*a*c - b^2) + 2*B*a*f*(4*a*c - b^2) - 2*B*c*d*(4*a*c - b^2) + 4*A*c^2*d*(b^2 - 4*a*c))^{(1/2)} + A*b^2*f*(b^2 - 4*a*c)^{(1/2)} - 2*B*a*b*f*(b^2 - 4*a*c)^{(1/2)} - 2*B*b*c*d*(b^2 - 4*a*c)^{(1/2)})*(2*b*c^3*d^3 + 4*c^4*d^3*x - a^2*b^2*f^3*x + 4*a*b^3*d*f^2 - 2*b^3*c*d^2*f + 4*a^3*c*f^3*x + 3*b^4*d*f^2*x + 12*a*b*c^2*d^2*f - 14*a^2*b*c*d*f^2 - 4*a*c^3*d^2*f*x - 4*a^2*c^2*d*f^2*x + 3*b^2*c^2*d^2*f*x - 10*a*b^2*c*d*f^2*x))/(2*(4*a*c - b^2)*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f)))/ (4*(4*a*c - b^2)*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f)) - c*f^2*x*(2*B^2*c^2*d - 4*A^2*c^2*f - B^2*b^2*f + 2*B^2*a*c*f + 2*A*B*b*c*f) + A^2*b*c^2*f^3 + B^2*b*c^2*d*f^2 - 4*A*B*a*c^2*f^3 + A*B*b^2*c*f^3 - 4*A*B*c^3*d*f^2)*(A*f*(b^2 - 4*a*c))^{(3/2)} - 2*A*b*f*(4*a*c - b^2) +
\end{aligned}$$

$$\frac{2*B*a*f*(4*a*c - b^2) - 2*B*c*d*(4*a*c - b^2) + 4*A*c^2*d*(b^2 - 4*a*c)^{(1/2)} + A*b^2*f*(b^2 - 4*a*c)^{(1/2)} - 2*B*a*b*f*(b^2 - 4*a*c)^{(1/2)} - 2*B*b*c*d*(b^2 - 4*a*c)^{(1/2))}{(4*(4*a*c - b^2)*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f))}*(A*f*(b^2 - 4*a*c)^{(3/2)} - 2*A*b*f*(4*a*c - b^2) + 2*B*a*f*(4*a*c - b^2) - 2*B*c*d*(4*a*c - b^2) + 4*A*c^2*d*(b^2 - 4*a*c)^{(1/2)} + A*b^2*f*(b^2 - 4*a*c)^{(1/2)} - 2*B*a*b*f*(b^2 - 4*a*c)^{(1/2)} - 2*B*b*c*d*(b^2 - 4*a*c)^{(1/2)))/(b^2*(4*a^2*f^2 + 4*c^2*d^2 + 4*b^2*d*f - 24*a*c*d*f) - 4*a*c*(4*a^2*f^2 + 4*c^2*d^2 - 8*a*c*d*f))$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x**2+b*x+a)/(f*x**2+d),x)

[Out] Timed out

$$3.5 \quad \int \frac{A+Bx}{(a+bx+cx^2)^2(d+fx^2)} dx$$

Optimal. Leaf size=596

$$\frac{f^{3/2} \tan^{-1}\left(\frac{\sqrt{fx}}{\sqrt{d}}\right) \left(-A(cd-af)^2 + 2bBd(cd-af) + Ab^2df\right) f \log(a+bx+cx^2) \left(B(-f(b^2d-a^2f) - 2acdf + 2b^2d^2) - 2acdf + c^2d^2\right)}{\sqrt{d} \left(f(a^2f + b^2d) - 2acdf + c^2d^2\right)^2}$$

[Out] (A*b*c*(a*f+c*d)-(A*b-B*a)*(-2*a*c*f+b^2*f+2*c^2*d)-c*(A*b^2*f+2*A*c*(-a*f+c*d)-b*B*(a*f+c*d))*x)/(-4*a*c+b^2)/(b^2*d*f+(-a*f+c*d)^2)/(c*x^2+b*x+a)-(b^5*B*d*f^2-2*A*b^4*f^2*(-a*f+c*d)-4*A*c^2*(-3*a*f+c*d)*(-a*f+c*d)^2+b^3*B*f*(-a^2*f^2-4*a*c*d*f+5*c^2*d^2)-4*A*b^2*c*f*(3*a^2*f^2-3*a*c*d*f+2*c^2*d^2)+2*b*B*c*(3*a^3*f^3+3*a^2*c*d*f^2-7*a*c^2*d^2*f+c^3*d^3))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/(c^2*d^2-2*a*c*d*f+f*(a^2*f+b^2*d))^2-1/2*f*(2*A*b*f*(-a*f+c*d)+B*(c^2*d^2-2*a*c*d*f-f*(-a^2*f+b^2*d)))*ln(c*x^2+b*x+a)/(c^2*d^2-2*a*c*d*f+f*(a^2*f+b^2*d))^2+1/2*f*(2*A*b*f*(-a*f+c*d)+B*(c^2*d^2-2*a*c*d*f-f*(-a^2*f+b^2*d)))*ln(f*x^2+d)/(c^2*d^2-2*a*c*d*f+f*(a^2*f+b^2*d))^2-f^(3/2)*(A*b^2*d*f+2*b*B*d*(-a*f+c*d)-A*(-a*f+c*d)^2)*arctan(x*f^(1/2)/d^(1/2))/(c^2*d^2-2*a*c*d*f+f*(a^2*f+b^2*d))^2/d^(1/2)

Rubi [A] time = 1.77, antiderivative size = 596, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1018, 1074, 634, 618, 206, 628, 635, 205, 260}

$$\frac{f^{3/2} \tan^{-1}\left(\frac{\sqrt{fx}}{\sqrt{d}}\right) \left(-A(cd-af)^2 + 2bBd(cd-af) + Ab^2df\right) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right) \left(-4Ab^2cf(3a^2f^2 - 3acdf + 2b^2d^2) - 2acdf + c^2d^2\right)}{\sqrt{d} \left(f(a^2f + b^2d) - 2acdf + c^2d^2\right)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x)/((a + b*x + c*x^2)^2*(d + f*x^2)),x]

[Out] (A*b*c*(c*d + a*f) - (A*b - a*B)*(2*c^2*d + b^2*f - 2*a*c*f) - c*(A*b^2*f + 2*A*c*(c*d - a*f) - b*B*(c*d + a*f))*x)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(a + b*x + c*x^2)) - (f^(3/2)*(A*b^2*d*f + 2*b*B*d*(c*d - a*f) - A*(c*d - a*f)^2)*ArcTan[(Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))^2) - ((b^5*B*d*f^2 - 2*A*b^4*f^2*(c*d - a*f) - 4*A*c^2*(c*d - 3*a*f)*(c*d - a*f)^2 + b^3*B*f*(5*c^2*d^2 - 4*a*c*d*f - a^2*f^2) - 4*A*b^2*c*f*(2*c^2*d^2 - 3*a*c*d*f + 3*a^2*f^2) + 2*b*B*c*(c^3*d^3 - 7*a*c^2*d^2*f + 3*a^2*c*d*f^2 + 3*a^3*f^3))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))^2) - (f*(2*A*b*f*(c*d - a*f) + B*(c^2*d^2 - 2*a*c*d*f - f*(b^2*d - a^2*f)))*Log[a + b*x + c*x^2])/(2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))^2) + (f*(2*A*b*f*(

$$c*d - a*f) + B*(c^2*d^2 - 2*a*c*d*f - f*(b^2*d - a^2*f))*\text{Log}[d + f*x^2]/(2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))^2)$$

Rule 205

$$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] \text{ /; FreeQ}\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b]$$

Rule 206

$$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] \text{ /; FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$$

Rule 260

$$\text{Int}[(x_)^{(m_)} / ((a_ + (b_)*(x_)^{(n_)}), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^n, x]] / (b*n), x] \text{ /; FreeQ}\{a, b, m, n\}, x] \ \&\& \ \text{EqQ}[m, n - 1]$$

Rule 618

$$\text{Int}[(a_ + (b_)*(x_) + (c_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] \text{ /; FreeQ}\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$$

Rule 628

$$\text{Int}[(d_ + (e_)*(x_)) / ((a_ + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] \text{ /; FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$$

Rule 634

$$\text{Int}[(d_ + (e_)*(x_)) / ((a_ + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] \rightarrow \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] \text{ /; FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4*a*c]$$

Rule 635

$$\text{Int}[(d_ + (e_)*(x_)) / ((a_ + (c_)*(x_)^2), x_Symbol] \rightarrow \text{Dist}[d, \text{Int}[1/(a + c*x^2), x], x] + \text{Dist}[e, \text{Int}[x/(a + c*x^2), x], x] \text{ /; FreeQ}\{a, c, d, e\}, x] \ \&\& \ !\text{NiceSqrtQ}[-(a*c)]$$

Rule 1018

```

Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1)*((g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p + q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] && !(IntegerQ[p] && ILtQ[q, -1])

```

Rule 1074

```

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.)*(x_)^2)), x_Symbol] := With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2}, Dist[1/q, Int[(A*c^2*d - a*c*C*d + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f + c*(B*c*d - b*C*d + A*b*f - a*B*f)*x)/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[(c*C*d^2 + b*B*d*f - A*c*d*f - a*C*d*f + a*A*f^2 - f*(B*c*d - b*C*d + A*b*f - a*B*f)*x)/(d + f*x^2), x], x] /; NeQ[q, 0] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx}{(a + bx + cx^2)^2 (d + fx^2)} dx &= \frac{Abc(cd + af) - (Ab - aB)(2c^2d + b^2f - 2acf) - c(Ab^2f + 2Ac(cd - af) - bcd)}{(b^2 - 4ac)(b^2df + (cd - af)^2)(a + bx + cx^2)} \\
&= \frac{Abc(cd + af) - (Ab - aB)(2c^2d + b^2f - 2acf) - c(Ab^2f + 2Ac(cd - af) - bcd)}{(b^2 - 4ac)(b^2df + (cd - af)^2)(a + bx + cx^2)} \\
&= \frac{Abc(cd + af) - (Ab - aB)(2c^2d + b^2f - 2acf) - c(Ab^2f + 2Ac(cd - af) - bcd)}{(b^2 - 4ac)(b^2df + (cd - af)^2)(a + bx + cx^2)} \\
&= \frac{Abc(cd + af) - (Ab - aB)(2c^2d + b^2f - 2acf) - c(Ab^2f + 2Ac(cd - af) - bcd)}{(b^2 - 4ac)(b^2df + (cd - af)^2)(a + bx + cx^2)} \\
&= \frac{Abc(cd + af) - (Ab - aB)(2c^2d + b^2f - 2acf) - c(Ab^2f + 2Ac(cd - af) - bcd)}{(b^2 - 4ac)(b^2df + (cd - af)^2)(a + bx + cx^2)}
\end{aligned}$$

Mathematica [A] time = 1.84, size = 523, normalized size = 0.88

$$f \log(d + fx^2) \left(B(f(a^2f - b^2d) - 2acdf + c^2d^2) + 2Abf(cd - af) \right) + f \log(a + x(b + cx)) \left(B(f(b^2d - a^2f) + 2$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x)/((a + b*x + c*x^2)^2*(d + f*x^2)),x]

[Out]
$$\begin{aligned}
&((-2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))*A*(b^3*f + b*c*(c*d - 3*a*f) \\
&+ b^2*c*f*x + 2*c^2*(c*d - a*f)*x) + B*(2*a^2*c*f - b*c^2*d*x - a*(2*c^2*d \\
&+ b^2*f + b*c*f*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*f^(3/2)*(-(A \\
&*b^2*d*f) + A*(c*d - a*f)^2 + 2*b*B*d*(-(c*d) + a*f))*ArcTan[(Sqrt[f]*x)/Sqrt[d]])/Sqrt[d] - (2*(b^5*B*d*f^2 - 4*A*c^2*(c*d - 3*a*f)*(c*d - a*f)^2 + 2 \\
&*A*b^4*f^2*(-(c*d) + a*f) - b^3*B*f*(-5*c^2*d^2 + 4*a*c*d*f + a^2*f^2) - 4* \\
&A*b^2*c*f*(2*c^2*d^2 - 3*a*c*d*f + 3*a^2*f^2) + 2*b*B*c*(c^3*d^3 - 7*a*c^2*d^2*f + 3*a^2*c*d*f^2 + 3*a^3*f^3))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]]) \\
&/(-b^2 + 4*a*c)^(3/2) + f*(2*A*b*f*(c*d - a*f) + B*(c^2*d^2 - 2*a*c*d*f + f \\
&*(-(b^2*d) + a^2*f)))*Log[d + f*x^2] + f*(2*A*b*f*(-(c*d) + a*f) + B*(-(c^2 \\
&*d^2) + 2*a*c*d*f + f*(b^2*d - a^2*f)))*Log[a + x*(b + c*x)]/(2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))^2)
\end{aligned}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [B] time = 0.21, size = 1313, normalized size = 2.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="giac")

[Out]
$$-1/2*(B*c^2*d^2*f - B*b^2*d*f^2 - 2*B*a*c*d*f^2 + 2*A*b*c*d*f^2 + B*a^2*f^3 - 2*A*a*b*f^3)*\log(c*x^2 + b*x + a)/(c^4*d^4 + 2*b^2*c^2*d^3*f - 4*a*c^3*d^3*f + b^4*d^2*f^2 - 4*a*b^2*c*d^2*f^2 + 6*a^2*c^2*d^2*f^2 + 2*a^2*b^2*d*f^3 - 4*a^3*c*d*f^3 + a^4*f^4) + 1/2*(B*c^2*d^2*f - B*b^2*d*f^2 - 2*B*a*c*d*f^2 + 2*A*b*c*d*f^2 + B*a^2*f^3 - 2*A*a*b*f^3)*\log(f*x^2 + d)/(c^4*d^4 + 2*b^2*c^2*d^3*f - 4*a*c^3*d^3*f + b^4*d^2*f^2 - 4*a*b^2*c*d^2*f^2 + 6*a^2*c^2*d^2*f^2 + 2*a^2*b^2*d*f^3 - 4*a^3*c*d*f^3 + a^4*f^4) - (2*B*b*c*d^2*f^2 - A*c^2*d^2*f^2 - 2*B*a*b*d*f^3 + A*b^2*d*f^3 + 2*A*a*c*d*f^3 - A*a^2*f^4)*\arctan(f*x/\sqrt{d*f})/((c^4*d^4 + 2*b^2*c^2*d^3*f - 4*a*c^3*d^3*f + b^4*d^2*f^2 - 4*a*b^2*c*d^2*f^2 + 6*a^2*c^2*d^2*f^2 + 2*a^2*b^2*d*f^3 - 4*a^3*c*d*f^3 + a^4*f^4)*\sqrt{d*f}) + (2*B*b*c^4*d^3 - 4*A*c^5*d^3 + 5*B*b^3*c^2*d^2*f - 14*B*a*b*c^3*d^2*f - 8*A*b^2*c^3*d^2*f + 20*A*a*c^4*d^2*f + B*b^5*d*f^2 - 4*B*a*b^3*c*d*f^2 - 2*A*b^4*c*d*f^2 + 6*B*a^2*b*c^2*d*f^2 + 12*A*a*b^2*c^2*d*f^2 - 28*A*a^2*c^3*d*f^2 - B*a^2*b^3*f^3 + 2*A*a*b^4*f^3 + 6*B*a^3*b*c*f^3 - 12*A*a^2*b^2*c*f^3 + 12*A*a^3*c^2*f^3)*\arctan((2*c*x + b)/\sqrt{-b^2 + 4*a*c})/((b^2*c^4*d^4 - 4*a*c^5*d^4 + 2*b^4*c^2*d^3*f - 12*a*b^2*c^3*d^3*f + 16*a^2*c^4*d^3*f + b^6*d^2*f^2 - 8*a*b^4*c*d^2*f^2 + 22*a^2*b^2*c^2*d^2*f^2 - 24*a^3*c^3*d^2*f^2 + 2*a^2*b^4*d*f^3 - 12*a^3*b^2*c*d*f^3 + 16*a^4*c^2*d*f^3 + a^4*b^2*f^4 - 4*a^5*c*f^4)*\sqrt{-b^2 + 4*a*c}) + (2*B*a*c^4*d^3 - A*b*c^4*d^3 + 3*B*a*b^2*c^2*d^2*f - 2*A*b^3*c^2*d^2*f - 6*B*a^2*c^3*d^2*f + 5*A*a*b*c^3*d^2*f + B*a*b^4*d*f^2 - A*b^5*d*f^2 - 4*B*a^2*b^2*c*d*f^2 + 5*A*a*b^3*c*d*f^2 + 6*B*a^3*c^2*d*f^2 - 7*A*a^2*b*c^2*d*f^2 + B*a^3*b^2*f^3 - A*a^2*b^3*f^3 - 2*B*a^4*c*f^3 + 3*A*a^3*b*c*f^3 + (B*b*c^4*d^3 - 2*A*c^5*d^3 + B*b^3*c^2*d^2*f - B*a*b*c^3*d^2*f - 3*A*b^2*c^3*d^2*f + 6*A*a*c^4*d^2*f + B*a*b^3*c*d*f^2 - A*b^4*c*d*f^2 - B*a^2*b*c^2*d*f^2 + 4*A*a*b^2*c^2*d*f^2 - 6*A*a^2*c^3*d*f^2 + B*a^3*b*c*f^3 - A*a^2*b^2*c*f^3 + 2*A*a^3*c^2*f^3)*x)/((c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2)^2*(c*x^2 + b*x + a)*(b^2 - 4*a*c))$$

maple [B] time = 0.06, size = 9311, normalized size = 15.62

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+d),x)$

[Out] result too large to display

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+d),x, \text{algorithm}=\text{"maxima"})$

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 7.53, size = 23006, normalized size = 38.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((A + B*x)/((d + f*x^2)*(a + b*x + c*x^2)^2),x)$

[Out]
$$\begin{aligned} & ((A*b^3*f + A*b*c^2*d - 2*B*a*c^2*d - B*a*b^2*f + 2*B*a^2*c*f - 3*A*a*b*c*f) / ((4*a*c - b^2)*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f)) - (x*(2*A*a*c^2*f - 2*A*c^3*d + B*b*c^2*d - A*b^2*c*f + B*a*b*c*f)) / ((4*a*c - b^2)*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f))) / (a + b*x + c*x^2) + \text{symsum}(\log((x*(4*A^3*b^3*c^4*f^6 + 16*B^3*a^3*c^4*f^6 - 3*B^3*a^2*b^2*c^3*f^6 + B^3*b^2*c^5*d^2*f^4 - 16*A^3*a*b*c^5*f^6 + 20*A^2*B*a^2*c^5*f^6 - 3*A^2*B*b^4*c^3*f^6 + 4*A^2*B*c^7*d^2*f^4 - 16*B^3*a^2*c^5*d*f^5 + 6*B^3*a*b^2*c^4*d*f^5 - 24*A^2*B*a*c^6*d*f^5 + 6*A*B^2*a*b^3*c^3*f^6 - 28*A*B^2*a^2*b*c^4*f^6 + 8*A^2*B*a*b^2*c^4*f^6 - 4*A*B^2*b*c^6*d^2*f^4 - 6*A*B^2*b^3*c^4*d*f^5 + 8*A^2*B*b^2*c^5*d*f^5 + 16*A*B^2*a*b*c^5*d*f^5)) / (16*a^2*c^6*d^4 + a^4*b^4*f^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 + 2*b^6*c^2*d^3*f + 96*a^4*c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3) - \text{root}(2560*a^3*b^2*c^9*d^8*f*z^4 - 1152*a^2*b^4*c^8*d^8*f*z^4 + 384*a^5*b^8*c^5*d^7*f^2*z^4 + \end{aligned}$$

$$\begin{aligned}
& 288*a^3*b^{10}*c*d^4*f^5*z^4 + 288*a*b^{10}*c^3*d^6*f^3*z^4 + 224*a^7*b^6*c*d^2 \\
& *f^7*z^4 - 192*a^{10}*b^2*c^2*d*f^8*z^4 + 224*a*b^6*c^7*d^8*f*z^4 + 80*a*b^{12} \\
& *c*d^5*f^4*z^4 + 48*a^9*b^4*c*d*f^8*z^4 - 33920*a^6*b^2*c^6*d^5*f^4*z^4 + 2 \\
& 7936*a^5*b^4*c^5*d^5*f^4*z^4 + 26112*a^7*b^2*c^5*d^4*f^5*z^4 + 26112*a^5*b^ \\
& 2*c^7*d^6*f^3*z^4 - 20352*a^6*b^4*c^4*d^4*f^5*z^4 - 20352*a^4*b^4*c^6*d^6*f \\
& ^3*z^4 - 13080*a^4*b^6*c^4*d^5*f^4*z^4 - 11520*a^8*b^2*c^4*d^3*f^6*z^4 - 11 \\
& 520*a^4*b^2*c^8*d^7*f^2*z^4 + 8736*a^5*b^6*c^3*d^4*f^5*z^4 + 8736*a^3*b^6*c \\
& ^5*d^6*f^3*z^4 + 7488*a^7*b^4*c^3*d^3*f^6*z^4 + 7488*a^3*b^4*c^7*d^7*f^2*z^ \\
& 4 + 3840*a^3*b^8*c^3*d^5*f^4*z^4 + 2560*a^9*b^2*c^3*d^2*f^7*z^4 - 2416*a^6* \\
& b^6*c^2*d^3*f^6*z^4 - 2416*a^2*b^6*c^6*d^7*f^2*z^4 - 2160*a^4*b^8*c^2*d^4*f \\
& ^5*z^4 - 2160*a^2*b^8*c^4*d^6*f^3*z^4 - 1152*a^8*b^4*c^2*d^2*f^7*z^4 - 720* \\
& a^2*b^{10}*c^2*d^5*f^4*z^4 - 16*b^8*c^6*d^8*f*z^4 - 2048*a^4*c^{10}*d^8*f*z^4 + \\
& 256*a^{11}*c^3*d*f^8*z^4 - 4*a^8*b^6*d*f^8*z^4 + 48*a*b^4*c^9*d^9*z^4 - 24*b \\
& ^{10}*c^4*d^7*f^2*z^4 - 16*b^{12}*c^2*d^6*f^3*z^4 + 17920*a^7*c^7*d^5*f^4*z^4 - \\
& 14336*a^8*c^6*d^4*f^5*z^4 - 14336*a^6*c^8*d^6*f^3*z^4 + 7168*a^9*c^5*d^3*f \\
& ^6*z^4 + 7168*a^5*c^9*d^7*f^2*z^4 - 2048*a^{10}*c^4*d^2*f^7*z^4 - 24*a^4*b^{10} \\
& *d^3*f^6*z^4 - 16*a^6*b^8*d^2*f^7*z^4 - 16*a^2*b^{12}*d^4*f^5*z^4 - 192*a^2*b \\
& ^2*c^{10}*d^9*z^4 - 4*b^{14}*d^5*f^4*z^4 - 4*b^6*c^8*d^9*z^4 + 256*a^3*c^{11}*d^9 \\
& *z^4 + 912*A*B*a^6*b*c^3*d*f^6*z^2 + 192*A*B*a^4*b^5*c*d*f^6*z^2 + 920*A*B* \\
& a^4*b^3*c^3*d^2*f^5*z^2 - 480*A*B*a^2*b^5*c^3*d^3*f^4*z^2 - 336*A*B*a^2*b^3 \\
& *c^5*d^4*f^3*z^2 - 272*A*B*a^3*b^3*c^4*d^3*f^4*z^2 + 240*A*B*a^3*b^5*c^2*d^ \\
& 2*f^5*z^2 + 192*A*B*a*b*c^8*d^6*f*z^2 - 2496*A*B*a^5*b*c^4*d^2*f^5*z^2 + 18 \\
& 72*A*B*a^4*b*c^5*d^3*f^4*z^2 - 744*A*B*a^5*b^3*c^2*d*f^6*z^2 - 720*A*B*a^2* \\
& b*c^7*d^5*f^2*z^2 + 504*A*B*a*b^3*c^6*d^5*f^2*z^2 + 256*A*B*a^3*b*c^6*d^4*f \\
& ^3*z^2 + 168*A*B*a*b^7*c^2*d^3*f^4*z^2 - 144*A*B*a^2*b^7*c*d^2*f^5*z^2 + 14 \\
& 4*A*B*a*b^5*c^4*d^4*f^3*z^2 - 56*B^2*a*b^2*c^7*d^6*f*z^2 - 36*B^2*a^5*b^4*c \\
& *d*f^6*z^2 - 16*B^2*a*b^8*c*d^3*f^4*z^2 - 164*A^2*a^3*b^6*c*d*f^6*z^2 - 16* \\
& A^2*a*b^8*c*d^2*f^5*z^2 - 96*A*B*b^5*c^5*d^5*f^2*z^2 - 24*A*B*b^7*c^3*d^4*f \\
& ^3*z^2 - 580*B^2*a^4*b^2*c^4*d^3*f^4*z^2 + 536*B^2*a^3*b^4*c^3*d^3*f^4*z^2 \\
& - 348*B^2*a^4*b^4*c^2*d^2*f^5*z^2 + 316*B^2*a^2*b^2*c^6*d^5*f^2*z^2 + 200*B \\
& ^2*a^5*b^2*c^3*d^2*f^5*z^2 - 120*B^2*a^2*b^4*c^4*d^4*f^3*z^2 - 66*B^2*a^2*b \\
& ^6*c^2*d^3*f^4*z^2 - 16*B^2*a^3*b^2*c^5*d^4*f^3*z^2 + 1952*A^2*a^4*b^2*c^4* \\
& d^2*f^5*z^2 - 1792*A^2*a^3*b^2*c^5*d^3*f^4*z^2 - 1272*A^2*a^3*b^4*c^3*d^2*f \\
& ^5*z^2 + 976*A^2*a^2*b^2*c^6*d^4*f^3*z^2 + 960*A^2*a^2*b^4*c^4*d^3*f^4*z^2 \\
& + 282*A^2*a^2*b^6*c^2*d^2*f^5*z^2 - 72*A*B*b^3*c^7*d^6*f*z^2 - 16*A*B*b^9*c \\
& *d^3*f^4*z^2 - 16*A*B*a^3*b^7*d*f^6*z^2 + 16*A*B*a*b^9*d^2*f^5*z^2 - 180*B^ \\
& 2*a*b^4*c^5*d^5*f^2*z^2 + 132*B^2*a^6*b^2*c^2*d*f^6*z^2 + 108*B^2*a^3*b^6*c \\
& *d^2*f^5*z^2 + 20*B^2*a*b^6*c^3*d^4*f^3*z^2 - 736*A^2*a^5*b^2*c^3*d*f^6*z^2 \\
& + 624*A^2*a^4*b^4*c^2*d*f^6*z^2 - 416*A^2*a*b^2*c^7*d^5*f^2*z^2 - 276*A^2* \\
& a*b^4*c^5*d^4*f^3*z^2 - 196*A^2*a*b^6*c^3*d^3*f^4*z^2 + 31*B^2*b^6*c^4*d^5* \\
& f^2*z^2 + 2*B^2*b^8*c^2*d^4*f^3*z^2 - 768*B^2*a^5*c^5*d^3*f^4*z^2 + 512*B^2 \\
& *a^6*c^4*d^2*f^5*z^2 + 512*B^2*a^4*c^6*d^4*f^3*z^2 - 128*B^2*a^3*c^7*d^5*f^ \\
& 2*z^2 + 80*A^2*b^4*c^6*d^5*f^2*z^2 + 31*A^2*b^6*c^4*d^4*f^3*z^2 + 14*A^2*b^ \\
& 8*c^2*d^3*f^4*z^2 - 1152*A^2*a^3*c^7*d^4*f^3*z^2 + 1008*A^2*a^4*c^6*d^3*f^4 \\
& *z^2 + 624*A^2*a^2*c^8*d^5*f^2*z^2 - 288*A^2*a^5*c^5*d^2*f^5*z^2 - 10*B^2*a
\end{aligned}$$

$$\begin{aligned}
& \cdot 2*b^8*d^2*f^5*z^2 - 48*A^2*a^6*b^2*c^2*f^7*z^2 - 16*A*B*b*c^9*d^7*z^2 + 20 \\
& *B^2*b^4*c^6*d^6*f*z^2 - 128*B^2*a^7*c^3*d*f^6*z^2 + 64*A^2*b^2*c^8*d^6*f*z \\
& ^2 - 112*A^2*a^6*c^4*d*f^6*z^2 + 3*B^2*a^4*b^6*d*f^6*z^2 + 14*A^2*a^2*b^8*d \\
& *f^6*z^2 + 12*A^2*a^5*b^4*c*f^7*z^2 - 160*A^2*a*c^9*d^6*f*z^2 + 3*B^2*b^10* \\
& d^3*f^4*z^2 - A^2*b^10*d^2*f^5*z^2 + 64*A^2*a^7*c^3*f^7*z^2 + 4*B^2*b^2*c^8 \\
& *d^7*z^2 - A^2*a^4*b^6*f^7*z^2 + 16*A^2*c^10*d^7*z^2 - 160*A*B^2*a*b*c^6*d^ \\
& 4*f^2*z + 112*A*B^2*a^4*b*c^3*d*f^5*z - 24*A*B^2*a^2*b^5*c*d*f^5*z + 480*A^ \\
& 2*B*a^2*b^2*c^4*d^2*f^4*z - 176*A*B^2*a^2*b^3*c^3*d^2*f^4*z - 10*A^2*B*a*b^ \\
& 6*c*d*f^5*z + 384*A*B^2*a^2*b*c^5*d^3*f^3*z - 352*A*B^2*a^3*b*c^4*d^2*f^4*z \\
& - 288*A^2*B*a*b^2*c^5*d^3*f^3*z - 160*A^2*B*a^3*b^2*c^3*d*f^5*z - 148*A^2* \\
& B*a*b^4*c^3*d^2*f^4*z + 112*A*B^2*a*b^3*c^4*d^3*f^3*z + 72*A^2*B*a^2*b^4*c^ \\
& 2*d*f^5*z + 72*A*B^2*a*b^5*c^2*d^2*f^4*z + 48*A*B^2*a^3*b^3*c^2*d*f^5*z + 4 \\
& 8*B^3*a*b^2*c^5*d^4*f^2*z - 36*B^3*a^4*b^2*c^2*d*f^5*z - 4*B^3*a*b^4*c^3*d^ \\
& 3*f^3*z - 480*A^3*a^2*b*c^5*d^2*f^4*z - 160*A^3*a^2*b^3*c^3*d*f^5*z + 128*A \\
& ^3*a*b^3*c^4*d^2*f^4*z + 112*A^2*B*b^4*c^4*d^3*f^3*z - 64*A*B^2*b^5*c^3*d^3 \\
& *f^3*z + 16*A^2*B*b^2*c^6*d^4*f^2*z + 16*A*B^2*b^3*c^5*d^4*f^2*z - A^2*B*b^ \\
& 6*c^2*d^2*f^4*z + 448*A^2*B*a^3*c^5*d^2*f^4*z - 352*A^2*B*a^2*c^6*d^3*f^3*z \\
& - 48*A^2*B*a^4*b^2*c^2*f^6*z + 12*B^3*a^3*b^4*c*d*f^5*z - 10*B^3*a*b^6*c*d \\
& ^2*f^4*z + 416*A^3*a^3*b*c^4*d*f^5*z + 224*A^3*a*b*c^6*d^3*f^3*z + 24*A^3*a \\
& *b^5*c^2*d*f^5*z - 2*A*B^2*b^7*c*d^2*f^4*z - 272*A^2*B*a^4*c^4*d*f^5*z + 12 \\
& 8*A^2*B*a*c^7*d^4*f^2*z + 12*A^2*B*a^3*b^4*c*f^6*z - 120*B^3*a^2*b^2*c^4*d^ \\
& 3*f^3*z + 112*B^3*a^3*b^2*c^3*d^2*f^4*z + 16*A*B^2*b*c^7*d^5*f*z + 2*A*B^2* \\
& a*b^7*d*f^5*z - 2*A^3*b^7*c*d*f^5*z - 16*A^2*B*c^8*d^5*f*z + 11*B^3*b^6*c^2 \\
& *d^3*f^3*z - 8*B^3*b^4*c^4*d^4*f^2*z - 64*A^3*b^3*c^5*d^3*f^3*z + 96*A^3*a^ \\
& 3*b^3*c^2*f^6*z - 4*B^3*b^2*c^6*d^5*f*z - 32*A^3*b*c^7*d^4*f^2*z - B^3*a^2* \\
& b^6*d*f^5*z - 128*A^3*a^4*b*c^3*f^6*z - 24*A^3*a^2*b^5*c*f^6*z + 64*A^2*B*a \\
& ^5*c^3*f^6*z - A^2*B*a^2*b^6*f^6*z + A^2*B*b^8*d*f^5*z + 2*A^3*a*b^7*f^6*z \\
& + B^3*b^8*d^2*f^4*z + 32*A^3*B*a*b*c^4*d*f^4 - 18*A^2*B^2*a*b^2*c^3*d*f^4 + \\
& 32*A*B^3*a*b*c^4*d^2*f^3 - 28*A*B^3*a^2*b*c^3*d*f^4 + 6*A*B^3*a*b^3*c^2*d* \\
& f^4 - 10*A^3*B*b^3*c^3*d*f^4 - 4*A^3*B*b*c^5*d^2*f^3 - 4*A*B^3*b*c^5*d^3*f^ \\
& 2 - 28*A^3*B*a^2*b*c^3*f^5 + 6*A^3*B*a*b^3*c^2*f^5 + 9*A^2*B^2*b^2*c^4*d^2* \\
& f^3 - 3*A^2*B^2*a^2*b^2*c^2*f^5 - 10*B^4*a*b^2*c^3*d^2*f^3 - 3*B^4*a^2*b^2* \\
& c^2*d*f^4 - 10*A*B^3*b^3*c^3*d^2*f^3 + 3*A^2*B^2*b^4*c^2*d*f^4 + 36*A^2*B^2 \\
& *a^2*c^4*d*f^4 - 24*A^2*B^2*a*c^5*d^2*f^3 + 4*A^2*B^2*c^6*d^3*f^2 + 16*A^2* \\
& B^2*a^3*c^3*f^5 + 16*B^4*a^3*c^3*d*f^4 + 8*A^4*b^2*c^4*d*f^4 - 8*A^4*a*b^2* \\
& c^3*f^5 - 24*A^4*a*c^5*d*f^4 + 3*B^4*b^4*c^2*d^2*f^3 + 4*A^4*c^6*d^2*f^3 + \\
& 36*A^4*a^2*c^4*f^5 + B^4*b^2*c^4*d^3*f^2, z, k)*(root(2560*a^3*b^2*c^9*d^8* \\
& f*z^4 - 1152*a^2*b^4*c^8*d^8*f*z^4 + 384*a^5*b^8*c*d^3*f^6*z^4 + 384*a*b^8* \\
& c^5*d^7*f^2*z^4 + 288*a^3*b^10*c*d^4*f^5*z^4 + 288*a*b^10*c^3*d^6*f^3*z^4 + \\
& 224*a^7*b^6*c*d^2*f^7*z^4 - 192*a^10*b^2*c^2*d*f^8*z^4 + 224*a*b^6*c^7*d^8 \\
& *f*z^4 + 80*a*b^12*c*d^5*f^4*z^4 + 48*a^9*b^4*c*d*f^8*z^4 - 33920*a^6*b^2*c \\
& ^6*d^5*f^4*z^4 + 27936*a^5*b^4*c^5*d^5*f^4*z^4 + 26112*a^7*b^2*c^5*d^4*f^5* \\
& z^4 + 26112*a^5*b^2*c^7*d^6*f^3*z^4 - 20352*a^6*b^4*c^4*d^4*f^5*z^4 - 20352 \\
& *a^4*b^4*c^6*d^6*f^3*z^4 - 13080*a^4*b^6*c^4*d^5*f^4*z^4 - 11520*a^8*b^2*c^ \\
& 4*d^3*f^6*z^4 - 11520*a^4*b^2*c^8*d^7*f^2*z^4 + 8736*a^5*b^6*c^3*d^4*f^5*z^
\end{aligned}$$

$$\begin{aligned}
& 4 + 8736a^3b^6c^5d^6f^3z^4 + 7488a^7b^4c^3d^3f^6z^4 + 7488a^3b^4c^7d^7f^2z^4 + 3840a^3b^8c^3d^5f^4z^4 + 2560a^9b^2c^3d^2f^7z^4 - 2416a^6b^6c^2d^3f^6z^4 - 2416a^2b^6c^6d^7f^2z^4 - 2160a^4b^8c^2d^4f^5z^4 - 2160a^2b^8c^4d^6f^3z^4 - 1152a^8b^4c^2d^2f^7z^4 - 720a^2b^10c^2d^5f^4z^4 - 16b^8c^6d^8f^6z^4 - 2048a^4c^10d^8f^6z^4 + 256a^11c^3d^8f^8z^4 - 4a^8b^6d^8f^8z^4 + 48a^8b^4c^9d^9z^4 - 24b^10c^4d^7f^2z^4 - 16b^12c^2d^6f^3z^4 + 17920a^7c^7d^5f^4z^4 - 14336a^8c^6d^4f^5z^4 - 14336a^6c^8d^6f^3z^4 + 7168a^9c^5d^3f^6z^4 + 7168a^5c^9d^7f^2z^4 - 2048a^10c^4d^2f^7z^4 - 24a^4b^10d^3f^6z^4 - 16a^6b^8d^2f^7z^4 - 16a^2b^12d^4f^5z^4 - 192a^2b^2c^10d^9z^4 - 4b^14d^5f^4z^4 - 4b^6c^8d^9z^4 + 256a^3c^11d^9z^4 + 912A^2B^2a^6b^3c^3d^2f^6z^2 + 192A^2B^2a^4b^5c^3d^2f^6z^2 + 920A^2B^2a^4b^3c^3d^2f^5z^2 - 480A^2B^2a^2b^5c^3d^3f^4z^2 - 336A^2B^2a^2b^3c^5d^4f^3z^2 - 272A^2B^2a^3b^3c^4d^3f^4z^2 + 240A^2B^2a^3b^5c^2d^2f^5z^2 + 192A^2B^2a^2b^3c^8d^6f^6z^2 - 2496A^2B^2a^5b^3c^4d^2f^5z^2 + 1872A^2B^2a^4b^3c^5d^3f^4z^2 - 744A^2B^2a^5b^3c^2d^2f^6z^2 - 720A^2B^2a^2b^3c^7d^5f^2z^2 + 504A^2B^2a^3b^3c^6d^5f^2z^2 + 256A^2B^2a^3b^3c^6d^4f^3z^2 + 168A^2B^2a^3b^7c^2d^3f^4z^2 - 144A^2B^2a^2b^7c^2d^2f^5z^2 + 144A^2B^2a^3b^5c^4d^4f^3z^2 - 56B^2a^2b^2c^7d^6f^6z^2 - 36B^2a^5b^4c^2d^2f^6z^2 - 16B^2a^2b^8c^2d^3f^4z^2 - 164A^2a^3b^6c^2d^2f^6z^2 - 16A^2a^2b^8c^2d^2f^5z^2 - 96A^2B^2a^5c^5d^5f^2z^2 - 24A^2B^2b^7c^3d^4f^3z^2 - 580B^2a^4b^2c^4d^3f^4z^2 + 536B^2a^3b^4c^3d^3f^4z^2 - 348B^2a^4b^4c^2d^2f^5z^2 + 316B^2a^2b^2c^6d^5f^2z^2 + 200B^2a^5b^2c^3d^2f^5z^2 - 120B^2a^2b^4c^4d^4f^3z^2 - 66B^2a^2b^6c^2d^3f^4z^2 - 16B^2a^3b^2c^5d^4f^3z^2 + 1952A^2a^4b^2c^4d^2f^5z^2 - 1792A^2a^3b^2c^5d^3f^4z^2 - 1272A^2a^3b^4c^3d^2f^5z^2 + 976A^2a^2b^2c^6d^4f^3z^2 + 960A^2a^2b^4c^4d^3f^4z^2 + 282A^2a^2b^6c^2d^2f^5z^2 - 72A^2B^2b^3c^7d^6f^6z^2 - 16A^2B^2b^9c^2d^3f^4z^2 - 16A^2B^2a^3b^7d^2f^6z^2 + 16A^2B^2a^3b^9d^2f^5z^2 - 180B^2a^2b^4c^5d^5f^2z^2 + 132B^2a^6b^2c^2d^2f^6z^2 + 108B^2a^3b^6c^2d^2f^5z^2 + 20B^2a^2b^6c^3d^4f^3z^2 - 736A^2a^5b^2c^3d^2f^6z^2 + 624A^2a^4b^4c^2d^2f^6z^2 - 416A^2a^2b^2c^7d^5f^2z^2 - 276A^2a^2b^4c^5d^4f^3z^2 - 196A^2a^2b^6c^3d^3f^4z^2 + 31B^2b^6c^4d^5f^2z^2 + 2B^2b^8c^2d^4f^3z^2 - 768B^2a^5c^5d^3f^4z^2 + 512B^2a^6c^4d^2f^5z^2 + 512B^2a^4c^6d^4f^3z^2 - 128B^2a^3c^7d^5f^2z^2 + 80A^2b^4c^6d^5f^2z^2 + 31A^2b^6c^4d^4f^3z^2 + 14A^2b^8c^2d^3f^4z^2 - 1152A^2a^3c^7d^4f^3z^2 + 1008A^2a^4c^6d^3f^4z^2 + 624A^2a^2c^8d^5f^2z^2 - 288A^2a^5c^5d^2f^5z^2 - 10B^2a^2b^8d^2f^5z^2 - 48A^2a^6b^2c^2f^7z^2 - 16A^2B^2b^3c^9d^7z^2 + 20B^2b^4c^6d^6f^6z^2 - 128B^2a^7c^3d^2f^6z^2 + 64A^2b^2c^8d^6f^6z^2 - 112A^2a^6c^4d^2f^6z^2 + 3B^2a^4b^6d^6f^6z^2 + 14A^2a^2b^8d^6f^6z^2 + 12A^2a^5b^4c^2f^7z^2 - 160A^2a^2c^9d^6f^6z^2 + 3B^2b^10d^3f^4z^2 - A^2b^10d^2f^5z^2 + 64A^2a^7c^3f^7z^2 + 4B^2b^2c^8d^7z^2 - A^2a^4b^6f^7z^2 + 16A^2c^10d^7z^2 - 160A^2B^2a^2b^3c^6d^4f^2z + 112A^2B^2a^4b^3c^3d^2f^5z - 24A^2B^2a^2b^5c^3d^2f^5z
\end{aligned}$$

$$\begin{aligned}
& c*d*f^5*z + 480*A^2*B*a^2*b^2*c^4*d^2*f^4*z - 176*A*B^2*a^2*b^3*c^3*d^2*f^4 \\
& *z - 10*A^2*B*a*b^6*c*d*f^5*z + 384*A*B^2*a^2*b*c^5*d^3*f^3*z - 352*A*B^2*a \\
& ^3*b*c^4*d^2*f^4*z - 288*A^2*B*a*b^2*c^5*d^3*f^3*z - 160*A^2*B*a^3*b^2*c^3* \\
& d*f^5*z - 148*A^2*B*a*b^4*c^3*d^2*f^4*z + 112*A*B^2*a*b^3*c^4*d^3*f^3*z + 7 \\
& 2*A^2*B*a^2*b^4*c^2*d*f^5*z + 72*A*B^2*a*b^5*c^2*d^2*f^4*z + 48*A*B^2*a^3*b \\
& ^3*c^2*d*f^5*z + 48*B^3*a*b^2*c^5*d^4*f^2*z - 36*B^3*a^4*b^2*c^2*d*f^5*z - \\
& 4*B^3*a*b^4*c^3*d^3*f^3*z - 480*A^3*a^2*b*c^5*d^2*f^4*z - 160*A^3*a^2*b^3*c \\
& ^3*d*f^5*z + 128*A^3*a*b^3*c^4*d^2*f^4*z + 112*A^2*B*b^4*c^4*d^3*f^3*z - 64 \\
& *A*B^2*b^5*c^3*d^3*f^3*z + 16*A^2*B*b^2*c^6*d^4*f^2*z + 16*A*B^2*b^3*c^5*d^4 \\
& f^2*z - A^2*B*b^6*c^2*d^2*f^4*z + 448*A^2*B*a^3*c^5*d^2*f^4*z - 352*A^2*B \\
& *a^2*c^6*d^3*f^3*z - 48*A^2*B*a^4*b^2*c^2*f^6*z + 12*B^3*a^3*b^4*c*d*f^5*z \\
& - 10*B^3*a*b^6*c*d^2*f^4*z + 416*A^3*a^3*b*c^4*d*f^5*z + 224*A^3*a*b*c^6*d^3 \\
& f^3*z + 24*A^3*a*b^5*c^2*d*f^5*z - 2*A*B^2*b^7*c*d^2*f^4*z - 272*A^2*B*a^4 \\
& c^4*d*f^5*z + 128*A^2*B*a*c^7*d^4*f^2*z + 12*A^2*B*a^3*b^4*c*f^6*z - 120*B^3 \\
& a^2*b^2*c^4*d^3*f^3*z + 112*B^3*a^3*b^2*c^3*d^2*f^4*z + 16*A*B^2*b*c^7*d^5 \\
& f*z + 2*A*B^2*a*b^7*d*f^5*z - 2*A^3*b^7*c*d*f^5*z - 16*A^2*B*c^8*d^5*f*z \\
& + 11*B^3*b^6*c^2*d^3*f^3*z - 8*B^3*b^4*c^4*d^4*f^2*z - 64*A^3*b^3*c^5*d^3 \\
& *f^3*z + 96*A^3*a^3*b^3*c^2*f^6*z - 4*B^3*b^2*c^6*d^5*f*z - 32*A^3*b*c^7*d^4 \\
& f^2*z - B^3*a^2*b^6*d*f^5*z - 128*A^3*a^4*b*c^3*f^6*z - 24*A^3*a^2*b^5*c \\
& f^6*z + 64*A^2*B*a^5*c^3*f^6*z - A^2*B*a^2*b^6*f^6*z + A^2*B*b^8*d*f^5*z + \\
& 2*A^3*a*b^7*f^6*z + B^3*b^8*d^2*f^4*z + 32*A^3*B*a*b*c^4*d*f^4 - 18*A^2*B^2 \\
& *a*b^2*c^3*d*f^4 + 32*A*B^3*a*b*c^4*d^2*f^3 - 28*A*B^3*a^2*b*c^3*d*f^4 + 6* \\
& A*B^3*a*b^3*c^2*d*f^4 - 10*A^3*B*b^3*c^3*d*f^4 - 4*A^3*B*b*c^5*d^2*f^3 - 4* \\
& A*B^3*b*c^5*d^3*f^2 - 28*A^3*B*a^2*b*c^3*f^5 + 6*A^3*B*a*b^3*c^2*f^5 + 9*A^2 \\
& B^2*b^2*c^4*d^2*f^3 - 3*A^2*B^2*a^2*b^2*c^2*f^5 - 10*B^4*a*b^2*c^3*d^2*f^3 \\
& - 3*B^4*a^2*b^2*c^2*d*f^4 - 10*A*B^3*b^3*c^3*d^2*f^3 + 3*A^2*B^2*b^4*c^2*d \\
& *f^4 + 36*A^2*B^2*a^2*c^4*d*f^4 - 24*A^2*B^2*a*c^5*d^2*f^3 + 4*A^2*B^2*c^6 \\
& *d^3*f^2 + 16*A^2*B^2*a^3*c^3*f^5 + 16*B^4*a^3*c^3*d*f^4 + 8*A^4*b^2*c^4*d* \\
& f^4 - 8*A^4*a*b^2*c^3*f^5 - 24*A^4*a*c^5*d*f^4 + 3*B^4*b^4*c^2*d^2*f^3 + 4* \\
& A^4*c^6*d^2*f^3 + 36*A^4*a^2*c^4*f^5 + B^4*b^2*c^4*d^3*f^2, z, k)*(root(256 \\
& 0*a^3*b^2*c^9*d^8*f*z^4 - 1152*a^2*b^4*c^8*d^8*f*z^4 + 384*a^5*b^8*c*d^3*f^ \\
& 6*z^4 + 384*a*b^8*c^5*d^7*f^2*z^4 + 288*a^3*b^10*c*d^4*f^5*z^4 + 288*a*b^10 \\
& *c^3*d^6*f^3*z^4 + 224*a^7*b^6*c*d^2*f^7*z^4 - 192*a^10*b^2*c^2*d*f^8*z^4 + \\
& 224*a*b^6*c^7*d^8*f*z^4 + 80*a*b^12*c*d^5*f^4*z^4 + 48*a^9*b^4*c*d*f^8*z^4 \\
& - 33920*a^6*b^2*c^6*d^5*f^4*z^4 + 27936*a^5*b^4*c^5*d^5*f^4*z^4 + 26112*a^7 \\
& b^2*c^5*d^4*f^5*z^4 + 26112*a^5*b^2*c^7*d^6*f^3*z^4 - 20352*a^6*b^4*c^4*d^4 \\
& f^5*z^4 - 20352*a^4*b^4*c^6*d^6*f^3*z^4 - 13080*a^4*b^6*c^4*d^5*f^4*z^4 \\
& - 11520*a^8*b^2*c^4*d^3*f^6*z^4 - 11520*a^4*b^2*c^8*d^7*f^2*z^4 + 8736*a^5* \\
& b^6*c^3*d^4*f^5*z^4 + 8736*a^3*b^6*c^5*d^6*f^3*z^4 + 7488*a^7*b^4*c^3*d^3*f^ \\
& ^6*z^4 + 7488*a^3*b^4*c^7*d^7*f^2*z^4 + 3840*a^3*b^8*c^3*d^5*f^4*z^4 + 2560 \\
& *a^9*b^2*c^3*d^2*f^7*z^4 - 2416*a^6*b^6*c^2*d^3*f^6*z^4 - 2416*a^2*b^6*c^6* \\
& d^7*f^2*z^4 - 2160*a^4*b^8*c^2*d^4*f^5*z^4 - 2160*a^2*b^8*c^4*d^6*f^3*z^4 - \\
& 1152*a^8*b^4*c^2*d^2*f^7*z^4 - 720*a^2*b^10*c^2*d^5*f^4*z^4 - 16*b^8*c^6*d^ \\
& ^8*f*z^4 - 2048*a^4*c^10*d^8*f*z^4 + 256*a^11*c^3*d*f^8*z^4 - 4*a^8*b^6*d*f^ \\
& ^8*z^4 + 48*a*b^4*c^9*d^9*z^4 - 24*b^10*c^4*d^7*f^2*z^4 - 16*b^12*c^2*d^6*f
\end{aligned}$$

$$\begin{aligned}
&^3z^4 + 17920a^7c^7d^5f^4z^4 - 14336a^8c^6d^4f^5z^4 - 14336a^6c^8d^6f^3z^4 + 7168a^9c^5d^3f^6z^4 + 7168a^5c^9d^7f^2z^4 - 2048a^{10}c^4d^2f^7z^4 - 24a^4b^{10}d^3f^6z^4 - 16a^6b^8d^2f^7z^4 - 16a^2b^{12}d^4f^5z^4 - 192a^2b^2c^{10}d^9z^4 - 4b^{14}d^5f^4z^4 - 4b^6c^8d^9z^4 + 256a^3c^{11}d^9z^4 + 912A^2B^2a^6b^3c^3d^2f^6z^2 + 192A^2B^2a^4b^5c^3d^2f^6z^2 + 920A^2B^2a^4b^3c^3d^2f^5z^2 - 480A^2B^2a^2b^5c^3d^3f^4z^2 - 336A^2B^2a^2b^3c^5d^4f^3z^2 - 272A^2B^2a^3b^3c^4d^3f^4z^2 + 240A^2B^2a^3b^5c^2d^2f^5z^2 + 192A^2B^2a^3b^3c^8d^6f^2z^2 - 2496A^2B^2a^5b^3c^4d^2f^5z^2 + 1872A^2B^2a^4b^3c^5d^3f^4z^2 - 744A^2B^2a^5b^3c^2d^2f^6z^2 - 720A^2B^2a^2b^3c^7d^5f^2z^2 + 504A^2B^2a^3b^3c^6d^5f^2z^2 + 256A^2B^2a^3b^3c^6d^4f^3z^2 + 168A^2B^2a^3b^7c^2d^3f^4z^2 - 144A^2B^2a^2b^7c^2d^2f^5z^2 + 144A^2B^2a^2b^5c^4d^4f^3z^2 - 56B^2a^2b^2c^7d^6f^2z^2 - 36B^2a^5b^4c^2d^2f^6z^2 - 16B^2a^2b^8c^2d^3f^4z^2 - 16A^2a^3b^6c^2d^2f^5z^2 - 16A^2a^2b^8c^2d^2f^5z^2 - 96A^2B^2a^5b^5c^5d^5f^2z^2 - 24A^2B^2a^7c^3d^4f^3z^2 - 580B^2a^4b^2c^4d^3f^4z^2 + 536B^2a^3b^4c^3d^3f^4z^2 - 348B^2a^4b^4c^2d^2f^5z^2 + 316B^2a^2b^2c^6d^5f^2z^2 + 200B^2a^5b^2c^3d^2f^5z^2 - 120B^2a^2b^4c^4d^4f^3z^2 - 66B^2a^2b^6c^2d^3f^4z^2 - 16B^2a^3b^2c^5d^4f^3z^2 + 1952A^2a^4b^2c^4d^2f^5z^2 - 1792A^2a^3b^2c^5d^3f^4z^2 - 1272A^2a^3b^4c^3d^2f^5z^2 + 976A^2a^2b^2c^6d^4f^3z^2 + 960A^2a^2b^4c^4d^3f^4z^2 + 282A^2a^2b^6c^2d^2f^5z^2 - 72A^2B^2a^3b^3c^7d^6f^2z^2 - 16A^2B^2a^3b^7d^2f^6z^2 + 16A^2B^2a^3b^9d^2f^5z^2 - 180B^2a^2b^4c^5d^5f^2z^2 + 132B^2a^6b^2c^2d^2f^6z^2 + 108B^2a^3b^6c^2d^2f^5z^2 + 20B^2a^2b^6c^3d^4f^3z^2 - 736A^2a^5b^2c^3d^2f^6z^2 + 624A^2a^4b^4c^2d^2f^6z^2 - 416A^2a^2b^2c^7d^5f^2z^2 - 276A^2a^2b^4c^5d^4f^3z^2 - 196A^2a^2b^6c^3d^3f^4z^2 + 31B^2a^2b^6c^4d^5f^2z^2 + 2B^2a^2b^8c^2d^4f^3z^2 - 768B^2a^5c^5d^3f^4z^2 + 512B^2a^6c^4d^2f^5z^2 + 512B^2a^4c^6d^4f^3z^2 - 128B^2a^3c^7d^5f^2z^2 + 80A^2b^4c^6d^5f^2z^2 + 31A^2b^6c^4d^4f^3z^2 + 14A^2b^8c^2d^3f^4z^2 - 1152A^2a^3c^7d^4f^3z^2 + 1008A^2a^4c^6d^3f^4z^2 + 624A^2a^2c^8d^5f^2z^2 - 288A^2a^5c^5d^2f^5z^2 - 10B^2a^2b^8d^2f^5z^2 - 48A^2a^6b^2c^2f^7z^2 - 16A^2B^2a^3b^2c^8d^6f^2z^2 + 20B^2a^2b^4c^6d^6f^2z^2 - 128B^2a^7c^3d^2f^6z^2 + 64A^2b^2c^8d^6f^2z^2 - 112A^2a^6c^4d^2f^6z^2 + 3B^2a^4b^6d^2f^6z^2 + 14A^2a^2b^8d^2f^6z^2 + 12A^2a^5b^4c^2f^7z^2 - 160A^2a^2c^9d^6f^2z^2 + 3B^2a^2b^10d^3f^4z^2 - A^2b^10d^2f^5z^2 + 64A^2a^7c^3f^7z^2 + 4B^2a^2b^2c^8d^7z^2 - A^2a^4b^6f^7z^2 + 16A^2c^10d^7z^2 - 160A^2B^2a^2b^3c^6d^4f^2z + 112A^2B^2a^4b^3c^3d^2f^5z - 24A^2B^2a^2b^5c^3d^2f^5z + 480A^2B^2a^2b^2c^4d^2f^4z - 176A^2B^2a^2b^3c^3d^2f^4z - 10A^2B^2a^2b^6c^3d^2f^4z + 384A^2B^2a^2b^3c^5d^3f^3z - 352A^2B^2a^3b^3c^4d^2f^4z - 288A^2B^2a^2b^2c^5d^3f^3z - 160A^2B^2a^3b^2c^3d^2f^5z - 148A^2B^2a^2b^4c^3d^2f^4z + 112A^2B^2a^2b^3c^4d^3f^3z + 72A^2B^2a^2b^4c^2d^2f^5z + 72A^2B^2a^2b^5c^2d^2f^4z + 48A^2B^2a^3b^3c^2d^2f^5z + 48B^3a^2b^2c^5d^4f^2z - 36B^3a^4b^2c^2d^2f^5z - 4B^3a^2b^4c^3d^3f^3z - 480A^3a^2b^3c^5d^2f^4z -
\end{aligned}$$

$$\begin{aligned}
& 160*A^3*a^2*b^3*c^3*d*f^5*z + 128*A^3*a*b^3*c^4*d^2*f^4*z + 112*A^2*B*b^4*c^4*d^3*f^3*z - 64*A*B^2*b^5*c^3*d^3*f^3*z + 16*A^2*B*b^2*c^6*d^4*f^2*z + 16*A*B^2*b^3*c^5*d^4*f^2*z - A^2*B*b^6*c^2*d^2*f^4*z + 448*A^2*B*a^3*c^5*d^2*f^4*z - 352*A^2*B*a^2*c^6*d^3*f^3*z - 48*A^2*B*a^4*b^2*c^2*f^6*z + 12*B^3*a^3*b^4*c*d*f^5*z - 10*B^3*a*b^6*c*d^2*f^4*z + 416*A^3*a^3*b*c^4*d*f^5*z + 224*A^3*a*b*c^6*d^3*f^3*z + 24*A^3*a*b^5*c^2*d*f^5*z - 2*A*B^2*b^7*c*d^2*f^4*z - 272*A^2*B*a^4*c^4*d*f^5*z + 128*A^2*B*a*c^7*d^4*f^2*z + 12*A^2*B*a^3*b^4*c*f^6*z - 120*B^3*a^2*b^2*c^4*d^3*f^3*z + 112*B^3*a^3*b^2*c^3*d^2*f^4*z + 16*A*B^2*b*c^7*d^5*f*z + 2*A*B^2*a*b^7*d*f^5*z - 2*A^3*b^7*c*d*f^5*z - 16*A^2*B*c^8*d^5*f*z + 11*B^3*b^6*c^2*d^3*f^3*z - 8*B^3*b^4*c^4*d^4*f^2*z - 64*A^3*b^3*c^5*d^3*f^3*z + 96*A^3*a^3*b^3*c^2*f^6*z - 4*B^3*b^2*c^6*d^5*f*z - 32*A^3*b*c^7*d^4*f^2*z - B^3*a^2*b^6*d*f^5*z - 128*A^3*a^4*b*c^3*f^6*z - 24*A^3*a^2*b^5*c*f^6*z + 64*A^2*B*a^5*c^3*f^6*z - A^2*B*a^2*b^6*f^6*z + A^2*B*b^8*d*f^5*z + 2*A^3*a*b^7*f^6*z + B^3*b^8*d^2*f^4*z + 32*A^3*B*a*b*c^4*d*f^4 - 18*A^2*B^2*a*b^2*c^3*d*f^4 + 32*A*B^3*a*b*c^4*d^2*f^3 - 28*A*B^3*a^2*b*c^3*d*f^4 + 6*A*B^3*a*b^3*c^2*d*f^4 - 10*A^3*B*b^3*c^3*d*f^4 - 4*A^3*B*b*c^5*d^2*f^3 - 4*A*B^3*b*c^5*d^3*f^2 - 28*A^3*B*a^2*b*c^3*f^5 + 6*A^3*B*a*b^3*c^2*f^5 + 9*A^2*B^2*b^2*c^4*d^2*f^3 - 3*A^2*B^2*a^2*b^2*c^2*f^5 - 10*B^4*a*b^2*c^3*d^2*f^3 - 3*B^4*a^2*b^2*c^2*d*f^4 - 10*A*B^3*b^3*c^3*d^2*f^3 + 3*A^2*B^2*b^4*c^2*d*f^4 + 36*A^2*B^2*a^2*c^4*d*f^4 - 24*A^2*B^2*a*c^5*d^2*f^3 + 4*A^2*B^2*c^6*d^3*f^2 + 16*A^2*B^2*a^3*c^3*f^5 + 16*B^4*a^3*c^3*d*f^4 + 8*A^4*b^2*c^4*d*f^4 - 8*A^4*a*b^2*c^3*f^5 - 24*A^4*a*c^5*d*f^4 + 3*B^4*b^4*c^2*d^2*f^3 + 4*A^4*c^6*d^2*f^3 + 36*A^4*a^2*c^4*f^5 + B^4*b^2*c^4*d^3*f^2, z, k)*((4*b^5*c^8*d^7*f^2 + 4*b^7*c^6*d^6*f^3 - 4*b^9*c^4*d^5*f^4 - 4*b^11*c^2*d^4*f^5 - 612*a^2*b^5*c^6*d^5*f^4 - 712*a^2*b^7*c^4*d^4*f^5 - 132*a^2*b^9*c^2*d^3*f^6 + 1696*a^3*b^3*c^7*d^5*f^4 + 2736*a^3*b^5*c^5*d^4*f^5 + 896*a^3*b^7*c^3*d^3*f^6 - 5120*a^4*b^3*c^6*d^4*f^5 - 3140*a^4*b^5*c^4*d^3*f^6 - 220*a^4*b^7*c^2*d^2*f^7 + 5664*a^5*b^3*c^5*d^3*f^6 + 1128*a^5*b^5*c^3*d^2*f^7 - 2560*a^6*b^3*c^4*d^2*f^7 + 8*a*b^11*c*d^3*f^6 + 8*a^5*b^7*c*d*f^8 - 448*a^8*b*c^4*d*f^8 - 32*a*b^3*c^9*d^7*f^2 - 24*a*b^5*c^7*d^6*f^3 + 88*a*b^7*c^5*d^5*f^4 + 88*a*b^9*c^3*d^4*f^5 + 64*a^2*b*c^10*d^7*f^2 + 128*a^3*b*c^9*d^6*f^3 + 16*a^3*b^9*c*d^2*f^7 - 1600*a^4*b*c^8*d^5*f^4 + 3840*a^5*b*c^7*d^4*f^5 - 4160*a^6*b*c^6*d^3*f^6 - 92*a^6*b^5*c^2*d*f^8 + 2176*a^7*b*c^5*d^2*f^7 + 352*a^7*b^3*c^3*d*f^8)/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 + 2*b^6*c^2*d^3*f + 96*a^4*c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3) + (x*(128*a^9*c^4*f^9 - 2*a^6*b^6*c*f^9 - 640*a^8*c^5*d*f^8 + 6*b^12*c*d^3*f^6 + 24*a^7*b^4*c^2*f^9 - 96*a^8*b^2*c^3*f^9 + 128*a^2*c^11*d^7*f^2 - 640*a^3*c^10*d^6*f^3 + 1152*a^4*c^9*d^5*f^4 - 640*a^5*c^8*d^4*f^5 - 640*a^6*c^7*d^3*f^6 + 1152*a^7*c^6*d^2*f^7 + 8*b^4*c^9*d^7*f^2 + 22*b^6*c^7*d^6*f^3 + 26*b^8*c^5*d^5*f^4 + 18*b^10*c^3*d^4*f^5 + 672*a^2*b^2*c^9*d^6*f^3 + 1224*a^2*b^4*c^7*d^5*f^4 + 1202*a^2*b^6*c^5*d^4*f^5 + 564*a^2*b^8*c^3*d^3*f^6 - 2048*a^3*b^2*c^8*d^5*f^4 - 2744*a^3*b^4*c^6*d^4*f^5
\end{aligned}$$

$$\begin{aligned}
& - 1736a^3b^6c^4d^3f^6 - 128a^3b^8c^2d^2f^7 + 2656a^4b^2c^7d^4f^5 + 2648a^4b^4c^5d^3f^6 + 570a^4b^6c^3d^2f^7 - 1344a^5b^2c^6d^3f^6 - 904a^5b^4c^4d^2f^7 - 160a^6b^2c^5d^2f^7 + 2a^4b^8c^*d^*f^8 - 64a^*b^2c^10d^7f^2 - 216a^*b^4c^8d^6f^3 - 300a^*b^6c^6d^5f^4 - 240a^*b^8c^4d^4f^5 - 92a^*b^10c^2d^3f^6 + 10a^2b^10c^*d^2f^7 - 12a^5b^6c^2d^*f^8 - 40a^6b^4c^3d^*f^8 + 384a^7b^2c^4d^*f^8) / (\\
& 16a^2c^6d^4 + a^4b^4f^4 + b^4c^4d^4 + 16a^6c^2f^4 + b^8d^2f^2 - 8a^*b^2c^5d^4 - 8a^5b^2c^*f^4 + 2a^2b^6d^*f^3 - 64a^3c^5d^3f - 64a^5c^3d^*f^3 + 2b^6c^2d^3f + 96a^4c^4d^2f^2 + 54a^2b^4c^2d^2f^2 - 112a^3b^2c^3d^2f^2 - 20a^*b^4c^3d^3f - 12a^*b^6c^*d^2f^2 - 20a^3b^4c^*d^*f^3 + 64a^2b^2c^4d^3f + 64a^4b^2c^2d^*f^3) + (Aa^4b^6c^*f^8 - 64Aa^7c^4f^8 - 32Aa^*c^10d^6f^2 + 352Aa^6c^5d^*f^7 + Ab^10c^*d^2f^6 - 12Aa^5b^4c^2f^8 + 48Aa^6b^2c^3f^8 + 224Aa^2c^9d^5f^3 - 640Aa^3c^8d^4f^4 + 960Aa^4c^7d^3f^5 - 800Aa^5c^6d^2f^6 + 8Ab^2c^9d^6f^2 + 16Ab^4c^7d^5f^3 + Ab^6c^5d^4f^4 - 6Ab^8c^3d^3f^5 - 4Bb^3c^8d^6f^2 - 12Bb^5c^6d^5f^3 - 4Bb^7c^4d^4f^4 + 4Bb^9c^2d^3f^5 - 120Aa^*b^2c^8d^5f^3 - 60Aa^*b^4c^6d^4f^4 + 36Aa^*b^6c^4d^3f^5 - 8Aa^*b^8c^2d^2f^6 - 20Aa^3b^6c^2d^*f^7 + 80Aa^4b^4c^3d^*f^7 - 216Aa^5b^2c^4d^*f^7 + 92Bb^3c^7d^5f^3 + 72Bb^5c^5d^4f^4 - 20Bb^7c^3d^3f^5 - 176Bb^2b^*c^8d^5f^3 + 544Bb^3b^*c^7d^4f^4 - 736Bb^4b^*c^6d^3f^5 - 4Bb^4b^5c^2d^*f^7 + 464Bb^5b^*c^5d^2f^6 + 44Bb^5b^3c^3d^*f^7 + 384Aa^2b^2c^7d^4f^4 + 32Aa^2b^4c^5d^3f^5 + 14Aa^2b^6c^3d^2f^6 - 560Aa^3b^2c^6d^3f^5 - 56Aa^3b^4c^4d^2f^6 + 456Aa^4b^2c^5d^2f^6 - 360Bb^2b^3c^6d^4f^4 - 64Bb^2b^5c^4d^3f^5 + 504Bb^3b^3c^5d^3f^5 + 40Bb^3b^5c^3d^2f^6 - 276Bb^4b^3c^4d^2f^6 + 2Aa^2b^8c^*d^*f^7 + 16Bb^*b^*c^9d^6f^2 - 112Bb^6b^*c^4d^*f^7) / (16a^2c^6d^4 + a^4b^4f^4 + b^4c^4d^4 + 16a^6c^2f^4 + b^8d^2f^2 - 8a^*b^2c^5d^4 - 8a^5b^2c^*f^4 + 2a^2b^6d^*f^3 - 64a^3c^5d^3f - 64a^5c^3d^*f^3 + 2b^6c^2d^3f + 96a^4c^4d^2f^2 + 54a^2b^4c^2d^2f^2 - 112a^3b^2c^3d^2f^2 - 20a^*b^4c^3d^3f - 12a^*b^6c^*d^2f^2 - 20a^3b^4c^*d^*f^3 + 64a^2b^2c^4d^3f + 64a^4b^2c^2d^*f^3) + (x*(64Bb^7c^4f^8 + 4Aa^3b^7c^*f^8 - 256Aa^6b^*c^4f^8 - Bb^4b^6c^*f^8 - 320Bb^6c^5d^*f^7 + 3Bb^10c^*d^2f^6 - 48Aa^4b^5c^2f^8 + 192Aa^5b^3c^3f^8 + 12Bb^5b^4c^2f^8 - 48Bb^6b^2c^3f^8 - 16Ab^3c^8d^5f^3 - 48Ab^5c^6d^4f^4 - 36Ab^7c^4d^3f^5 - 4Ab^9c^2d^2f^6 - 64Bb^2c^9d^5f^3 + 320Bb^3c^8d^4f^4 - 640Bb^4c^7d^3f^5 + 640Bb^5c^6d^2f^6 + 4Bb^4c^7d^5f^3 + 23Bb^6c^5d^4f^4 + 22Bb^8c^3d^3f^5 + 320Aa^*b^3c^7d^4f^4 + 352Aa^*b^5c^5d^3f^5 + 76Aa^*b^7c^3d^2f^6 - 512Aa^2b^*c^8d^4f^4 - 60Aa^2b^7c^2d^*f^7 + 1408Aa^3b^*c^7d^3f^5 + 352Aa^3b^5c^3d^*f^7 - 1792Aa^4b^*c^6d^2f^6 - 976Aa^4b^3c^4d^*f^7 - 132Bb^*b^4c^6d^4f^4 - 196Bb^*b^6c^4d^3f^5 - 40Bb^*b^8c^2d^2f^6 - 20Bb^3b^6c^2d^*f^7 + 52Bb^4b^4c^3d^*f^7 + 64Bb^5b^2c^4d^*f^7 + 4Aa^*b^9c^*d^*f^7 - 1184Aa^2b^3c^6d^3f^5 - 544Aa^2b^5c^4d^2f^6 + 1664Aa^3b^3c^5d^2f^6 + 80Bb^2b^2c^7d^4f^4 + 5
\end{aligned}$$

$$\begin{aligned}
& 20*B*a^2*b^4*c^5*d^3*f^5 + 210*B*a^2*b^6*c^3*d^2*f^6 - 192*B*a^3*b^2*c^6*d^3*f^5 - 456*B*a^3*b^4*c^4*d^2*f^6 + 96*B*a^4*b^2*c^5*d^2*f^6 + 64*A*a*b*c^9 \\
& *d^5*f^3 + 1088*A*a^5*b*c^5*d*f^7 + 2*B*a^2*b^8*c*d*f^7)/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 \\
& - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 + 2*b^6*c^2*d^3*f + 96*a^4*c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b \\
& ^2*c^3*d^2*f^2 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3)) + (13*A^2*a^2*b^5*c^2*f^7 - 56*A^2*a^3*b^3*c^3*f^7 + 16*A^2*b^3*c^6*d^3*f^4 + A^2*b^5*c^4*d^2*f^5 + \\
& 8*B^2*b^3*c^6*d^4*f^3 + 9*B^2*b^5*c^4*d^3*f^4 - 64*A*B*a^5*c^4*f^7 - A^2*a*b^7*c*f^7 + 80*A^2*a^4*b*c^4*f^7 + 16*A^2*b*c^8*d^4*f^3 + 2*A^2*b^7*c^2*d* \\
& f^6 - 48*A^2*a*b*c^7*d^3*f^4 - 22*A^2*a*b^5*c^3*d*f^6 - 48*A^2*a^3*b*c^5*d* \\
& f^6 - 16*B^2*a*b*c^7*d^4*f^3 - 64*B^2*a^4*b*c^4*d*f^6 - A*B*b^8*c*d*f^6 - 8 \\
& *A^2*a*b^3*c^5*d^2*f^5 + 64*A^2*a^2*b^3*c^4*d*f^6 - 56*B^2*a*b^3*c^5*d^3*f^4 + 2*B^2*a*b^5*c^3*d^2*f^5 + 96*B^2*a^2*b*c^6*d^3*f^4 - 11*B^2*a^2*b^5*c^2 \\
& *d*f^6 - 16*B^2*a^3*b*c^5*d^2*f^5 + 40*B^2*a^3*b^3*c^3*d*f^6 + A*B*a^2*b^6* \\
& c*f^7 + 32*A*B*a*c^8*d^4*f^3 + 32*A*B*a^4*c^5*d*f^6 + B^2*a*b^7*c*d*f^6 - 8 \\
& *B^2*a^2*b^3*c^4*d^2*f^5 - 12*A*B*a^3*b^4*c^2*f^7 + 48*A*B*a^4*b^2*c^3*f^7 \\
& - 160*A*B*a^2*c^7*d^3*f^4 + 160*A*B*a^3*c^6*d^2*f^5 - 24*A*B*b^2*c^7*d^4*f^3 - 24*A*B*b^4*c^5*d^3*f^4 + A*B*b^6*c^3*d^2*f^5 + 120*A*B*a*b^2*c^6*d^3*f^4 \\
& - 4*A*B*a*b^4*c^4*d^2*f^5 - 24*A*B*a^2*b^4*c^3*d*f^6 + 8*A*B*a^3*b^2*c^4* \\
& d*f^6 - 24*A*B*a^2*b^2*c^5*d^2*f^5 + 10*A*B*a*b^6*c^2*d*f^6)/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 \\
& - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 + 2*b^6*c^2*d^3*f + 96*a^4*c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c* \\
& d*f^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3) + (x*(104*A^2*a^4*c^5* \\
& f^7 - 32*B^2*a^5*c^4*f^7 + 8*A^2*c^9*d^4*f^3 + A^2*b^8*c*f^7 + 50*A^2*a^2*b^4*c^3*f^7 - 96*A^2*a^3*b^2*c^4*f^7 - 12*B^2*a^3*b^4*c^2*f^7 + 42*B^2*a^4*b^2*c^3*f^7 + 208*A^2*a^2*c^7*d^2*f^5 + 8*A^2*b^2*c^7*d^3*f^4 + 18*A^2*b^4*c^5*d^2*f^5 - 32*B^2*a^2*c^7*d^3*f^4 + 32*B^2*a^3*c^6*d^2*f^5 + 2*B^2*b^2*c^7*d^4*f^3 - 6*B^2*b^4*c^5*d^3*f^4 + 9*B^2*b^6*c^3*d^2*f^5 - 12*A^2*a*b^6*c^2*f^7 + B^2*a^2*b^6*c*f^7 - 64*A^2*a*c^8*d^3*f^4 - 256*A^2*a^3*c^6*d*f^6 + 2*A^2*b^6*c^3*d*f^6 + 32*B^2*a^4*c^5*d*f^6 - 36*A^2*a*b^4*c^4*d*f^6 - 2*B^2*a*b^6*c^2*d*f^6 - 2*A*B*a*b^7*c*f^7 - 144*A^2*a*b^2*c^6*d^2*f^5 + 168*A^2*a^2*b^2*c^5*d*f^6 + 24*B^2*a*b^2*c^6*d^3*f^4 - 64*B^2*a*b^4*c^4*d^2*f^5 + 26*B^2*a^2*b^4*c^3*d*f^6 - 88*B^2*a^3*b^2*c^4*d*f^6 + 72*A*B*a^4*b*c^4*f^7 - 8*A*B*b*c^8*d^4*f^3 + 2*A*B*b^7*c^2*d*f^6 + 84*B^2*a^2*b^2*c^5*d^2*f^5 + 24*A*B*a^2*b^5*c^2*f^7 - 84*A*B*a^3*b^3*c^3*f^7 + 4*A*B*b^3*c^6*d^3*f^4 - 20*A*B*b^5*c^4*d^2*f^5 + 148*A*B*a*b^3*c^5*d^2*f^5 - 192*A*B*a^2*b*c^6*d^2*f^5 - 4*A*B*a^2*b^3*c^4*d*f^6 + 16*A*B*a*b*c^7*d^3*f^4 - 12*A*B*a*b^5*c^3*d*f^6 + 112*A*B*a^3*b*c^5*d*f^6))/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 + 2*b^6*c^2*d^3*f + 96*a^4*c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3)
\end{aligned}$$

$$\begin{aligned}
&^3d^3f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 + 64*a^2*b^2*c^4*d^3*f + \\
&64*a^4*b^2*c^2*d*f^3) - (16*A^3*a*c^6*d*f^5 - 4*A^3*c^7*d^2*f^4 - B^3*b^3 \\
&*c^4*d^2*f^4 - 12*A^3*a^2*c^5*f^6 - 16*A*B^2*a^3*c^4*f^6 + 2*A^3*a*b^2*c^4* \\
&f^6 - 6*A^3*b^2*c^5*d*f^5 + 4*B^3*a*b*c^5*d^2*f^4 + 3*B^3*a*b^3*c^3*d*f^5 - \\
&12*B^3*a^2*b*c^4*d*f^5 + 3*A*B^2*a^2*b^2*c^3*f^6 + A*B^2*b^2*c^5*d^2*f^4 - \\
&3*A^2*B*a*b^3*c^3*f^6 + 16*A^2*B*a^2*b*c^4*f^6 - 8*A*B^2*a*c^6*d^2*f^4 + 2 \\
&4*A*B^2*a^2*c^5*d*f^5 - 3*A*B^2*b^4*c^3*d*f^5 + 4*A^2*B*b*c^6*d^2*f^4 + 9*A \\
&^2*B*b^3*c^4*d*f^5 + 4*A*B^2*a*b^2*c^4*d*f^5 - 28*A^2*B*a*b*c^5*d*f^5)/(16* \\
&a^2*c^6*d^4 + a^4*b^4*f^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8* \\
&a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 64*a^3*c^5*d^3*f - 64*a \\
&^5*c^3*d*f^3 + 2*b^6*c^2*d^3*f + 96*a^4*c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2*f^ \\
&2 - 112*a^3*b^2*c^3*d^2*f^2 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20* \\
&a^3*b^4*c*d*f^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3))*root(2560*a \\
&^3*b^2*c^9*d^8*f*z^4 - 1152*a^2*b^4*c^8*d^8*f*z^4 + 384*a^5*b^8*c*d^3*f^6*z \\
&^4 + 384*a*b^8*c^5*d^7*f^2*z^4 + 288*a^3*b^10*c*d^4*f^5*z^4 + 288*a*b^10*c^ \\
&3*d^6*f^3*z^4 + 224*a^7*b^6*c*d^2*f^7*z^4 - 192*a^10*b^2*c^2*d*f^8*z^4 + 22 \\
&4*a*b^6*c^7*d^8*f*z^4 + 80*a*b^12*c*d^5*f^4*z^4 + 48*a^9*b^4*c*d*f^8*z^4 - \\
&33920*a^6*b^2*c^6*d^5*f^4*z^4 + 27936*a^5*b^4*c^5*d^5*f^4*z^4 + 26112*a^7*b \\
&^2*c^5*d^4*f^5*z^4 + 26112*a^5*b^2*c^7*d^6*f^3*z^4 - 20352*a^6*b^4*c^4*d^4* \\
&f^5*z^4 - 20352*a^4*b^4*c^6*d^6*f^3*z^4 - 13080*a^4*b^6*c^4*d^5*f^4*z^4 - 1 \\
&1520*a^8*b^2*c^4*d^3*f^6*z^4 - 11520*a^4*b^2*c^8*d^7*f^2*z^4 + 8736*a^5*b^6 \\
&*c^3*d^4*f^5*z^4 + 8736*a^3*b^6*c^5*d^6*f^3*z^4 + 7488*a^7*b^4*c^3*d^3*f^6* \\
&z^4 + 7488*a^3*b^4*c^7*d^7*f^2*z^4 + 3840*a^3*b^8*c^3*d^5*f^4*z^4 + 2560*a^ \\
&9*b^2*c^3*d^2*f^7*z^4 - 2416*a^6*b^6*c^2*d^3*f^6*z^4 - 2416*a^2*b^6*c^6*d^7 \\
&*f^2*z^4 - 2160*a^4*b^8*c^2*d^4*f^5*z^4 - 2160*a^2*b^8*c^4*d^6*f^3*z^4 - 11 \\
&52*a^8*b^4*c^2*d^2*f^7*z^4 - 720*a^2*b^10*c^2*d^5*f^4*z^4 - 16*b^8*c^6*d^8* \\
&f*z^4 - 2048*a^4*c^10*d^8*f*z^4 + 256*a^11*c^3*d*f^8*z^4 - 4*a^8*b^6*d*f^8* \\
&z^4 + 48*a*b^4*c^9*d^9*z^4 - 24*b^10*c^4*d^7*f^2*z^4 - 16*b^12*c^2*d^6*f^3* \\
&z^4 + 17920*a^7*c^7*d^5*f^4*z^4 - 14336*a^8*c^6*d^4*f^5*z^4 - 14336*a^6*c^8 \\
&*d^6*f^3*z^4 + 7168*a^9*c^5*d^3*f^6*z^4 + 7168*a^5*c^9*d^7*f^2*z^4 - 2048*a \\
&^10*c^4*d^2*f^7*z^4 - 24*a^4*b^10*d^3*f^6*z^4 - 16*a^6*b^8*d^2*f^7*z^4 - 16 \\
&*a^2*b^12*d^4*f^5*z^4 - 192*a^2*b^2*c^10*d^9*z^4 - 4*b^14*d^5*f^4*z^4 - 4*b \\
&^6*c^8*d^9*z^4 + 256*a^3*c^11*d^9*z^4 + 912*A*B*a^6*b*c^3*d*f^6*z^2 + 192*A \\
&*B*a^4*b^5*c*d*f^6*z^2 + 920*A*B*a^4*b^3*c^3*d^2*f^5*z^2 - 480*A*B*a^2*b^5* \\
&c^3*d^3*f^4*z^2 - 336*A*B*a^2*b^3*c^5*d^4*f^3*z^2 - 272*A*B*a^3*b^3*c^4*d^3 \\
&*f^4*z^2 + 240*A*B*a^3*b^5*c^2*d^2*f^5*z^2 + 192*A*B*a*b*c^8*d^6*f*z^2 - 24 \\
&96*A*B*a^5*b*c^4*d^2*f^5*z^2 + 1872*A*B*a^4*b*c^5*d^3*f^4*z^2 - 744*A*B*a^5 \\
&*b^3*c^2*d*f^6*z^2 - 720*A*B*a^2*b*c^7*d^5*f^2*z^2 + 504*A*B*a*b^3*c^6*d^5* \\
&f^2*z^2 + 256*A*B*a^3*b*c^6*d^4*f^3*z^2 + 168*A*B*a*b^7*c^2*d^3*f^4*z^2 - 1 \\
&44*A*B*a^2*b^7*c*d^2*f^5*z^2 + 144*A*B*a*b^5*c^4*d^4*f^3*z^2 - 56*B^2*a*b^2 \\
&*c^7*d^6*f*z^2 - 36*B^2*a^5*b^4*c*d*f^6*z^2 - 16*B^2*a*b^8*c*d^3*f^4*z^2 - \\
&164*A^2*a^3*b^6*c*d*f^6*z^2 - 16*A^2*a*b^8*c*d^2*f^5*z^2 - 96*A*B*b^5*c^5*d \\
&^5*f^2*z^2 - 24*A*B*b^7*c^3*d^4*f^3*z^2 - 580*B^2*a^4*b^2*c^4*d^3*f^4*z^2 + \\
&536*B^2*a^3*b^4*c^3*d^3*f^4*z^2 - 348*B^2*a^4*b^4*c^2*d^2*f^5*z^2 + 316*B^ \\
&2*a^2*b^2*c^6*d^5*f^2*z^2 + 200*B^2*a^5*b^2*c^3*d^2*f^5*z^2 - 120*B^2*a^2*b
\end{aligned}$$

$$\begin{aligned}
&^4c^4d^4f^3z^2 - 66B^2a^2b^6c^2d^3f^4z^2 - 16B^2a^3b^2c^5d^4f^3z^2 + 1952A^2a^4b^2c^4d^2f^5z^2 - 1792A^2a^3b^2c^5d^3f^4z^2 - 1272A^2a^3b^4c^3d^2f^5z^2 + 976A^2a^2b^2c^6d^4f^3z^2 + \\
&960A^2a^2b^4c^4d^3f^4z^2 + 282A^2a^2b^6c^2d^2f^5z^2 - 72A^2B^2a^3b^3c^7d^6f^2z^2 - 16A^2B^2a^3b^9c^d^3f^4z^2 - 16A^2B^2a^3b^7d^6f^2z^2 + \\
&16A^2B^2a^3b^9d^2f^5z^2 - 180B^2a^2b^4c^5d^5f^2z^2 + 132B^2a^6b^2c^2d^6f^2z^2 + 108B^2a^3b^6c^d^2f^5z^2 + 20B^2a^2b^6c^3d^4f^3z^2 - 736A^2a^5b^2c^3d^6f^2z^2 + 624A^2a^4b^4c^2d^6f^2z^2 - 416A^2a^2b^2c^7d^5f^2z^2 - 276A^2a^2b^4c^5d^4f^3z^2 - 196A^2a^2b^6c^3d^3f^4z^2 + 31B^2b^6c^4d^5f^2z^2 + 2B^2b^8c^2d^4f^3z^2 - 768B^2a^5c^5d^3f^4z^2 + 512B^2a^6c^4d^2f^5z^2 + 512B^2a^4c^6d^4f^3z^2 - 128B^2a^3c^7d^5f^2z^2 + 80A^2b^4c^6d^5f^2z^2 + 31A^2b^6c^4d^4f^3z^2 + 14A^2b^8c^2d^3f^4z^2 - 1152A^2a^3c^7d^4f^3z^2 + 1008A^2a^4c^6d^3f^4z^2 + 624A^2a^2c^8d^5f^2z^2 - 288A^2a^5c^5d^2f^5z^2 - 10B^2a^2b^8d^2f^5z^2 - 48A^2a^6b^2c^2f^7z^2 - 16A^2B^2b^9d^7z^2 + 20B^2b^4c^6d^6f^2z^2 - 128B^2a^7c^3d^6f^2z^2 + 64A^2b^2c^8d^6f^2z^2 - 112A^2a^6c^4d^6f^2z^2 + 3B^2a^4b^6d^6f^2z^2 + 14A^2a^2b^8d^6f^2z^2 + 12A^2a^5b^4c^7z^2 - 160A^2a^2c^9d^6f^2z^2 + 3B^2b^10d^3f^4z^2 - A^2b^10d^2f^5z^2 + 64A^2a^7c^3f^7z^2 + 4B^2b^2c^8d^7z^2 - A^2a^4b^6f^7z^2 + 16A^2c^10d^7z^2 - 160A^2B^2a^2b^6c^6d^4f^2z + 112A^2B^2a^4b^6c^3d^6f^5z - 24A^2B^2a^2b^5c^d^5f^5z + 480A^2B^2a^2b^2c^4d^2f^4z - 176A^2B^2a^2b^3c^3d^2f^4z - 10A^2B^2a^2b^6c^d^5f^5z + 384A^2B^2a^2b^6c^5d^3f^3z - 352A^2B^2a^3b^6c^4d^2f^4z - 288A^2B^2a^2b^2c^5d^3f^3z - 160A^2B^2a^3b^2c^3d^6f^5z - 148A^2B^2a^2b^4c^3d^2f^4z + 112A^2B^2a^2b^3c^4d^3f^3z + 72A^2B^2a^2b^4c^2d^6f^5z + 72A^2B^2a^2b^5c^2d^2f^4z + 48A^2B^2a^3b^3c^2d^6f^5z + 48B^3a^2b^2c^5d^4f^2z - 36B^3a^4b^2c^2d^6f^5z - 4B^3a^2b^4c^3d^3f^3z - 480A^3a^2b^6c^5d^2f^4z - 160A^3a^2b^3c^3d^6f^5z + 128A^3a^2b^3c^4d^2f^4z + 112A^2B^2b^4c^4d^3f^3z - 64A^2B^2b^5c^3d^3f^3z + 16A^2B^2b^2c^6d^4f^2z + 16A^2B^2b^3c^5d^4f^2z - A^2B^2b^6c^2d^2f^4z + 448A^2B^2a^3c^5d^2f^4z - 352A^2B^2a^2c^6d^3f^3z - 48A^2B^2a^4b^2c^2f^6z + 12B^3a^3b^4c^d^5f^5z - 10B^3a^2b^6c^d^2f^4z + 416A^3a^3b^6c^4d^6f^5z + 224A^3a^2b^6c^6d^3f^3z + 24A^3a^2b^5c^2d^6f^5z - 2A^2B^2b^7c^d^2f^4z - 272A^2B^2a^4c^4d^6f^5z + 128A^2B^2a^3c^7d^4f^2z + 12A^2B^2a^3b^4c^c^f^6z - 120B^3a^2b^2c^4d^3f^3z + 112B^3a^3b^2c^3d^2f^4z + 16A^2B^2b^6c^7d^5f^2z + 2A^2B^2a^2b^7d^6f^5z - 2A^3b^7c^d^6f^5z - 16A^2B^2b^8c^8d^5f^2z + 11B^3b^6c^2d^3f^3z - 8B^3b^4c^4d^4f^2z - 64A^3b^3c^5d^3f^3z + 96A^3a^3b^3c^2f^6z - 4B^3b^2c^6d^5f^2z - 32A^3b^6c^7d^4f^2z - B^3a^2b^6d^6f^5z - 128A^3a^4b^6c^3f^6z - 24A^3a^2b^5c^f^6z + 64A^2B^2a^5c^3f^6z - A^2B^2a^2b^6f^6z + A^2B^2b^8d^6f^5z + 2A^3a^2b^7f^6z + B^3b^8d^2f^4z + 32A^3B^2a^2b^6c^4d^6f^4 - 18A^2B^2a^2b^2c^3d^6f^4 + 32A^2B^3a^2b^6c^4d^2f^3 - 28A^2B^3a^2b^6c^3d^6f^4 + 6A^2B^3a^2b^3c^2d^6f^4 - 10A^3B^2b^3c^3d^6f^4 - 4A^3B^2b^6c^5d^2f^3 - 4A^2B^3b^6c^5d^3f^2 - 28A^3B^2a^2b^6c^3f^5 + 6A^3B^2a^2b^3
\end{aligned}$$

```
*c^2*f^5 + 9*A^2*B^2*b^2*c^4*d^2*f^3 - 3*A^2*B^2*a^2*b^2*c^2*f^5 - 10*B^4*a
*b^2*c^3*d^2*f^3 - 3*B^4*a^2*b^2*c^2*d*f^4 - 10*A*B^3*b^3*c^3*d^2*f^3 + 3*A
^2*B^2*b^4*c^2*d*f^4 + 36*A^2*B^2*a^2*c^4*d*f^4 - 24*A^2*B^2*a*c^5*d^2*f^3
+ 4*A^2*B^2*c^6*d^3*f^2 + 16*A^2*B^2*a^3*c^3*f^5 + 16*B^4*a^3*c^3*d*f^4 + 8
*A^4*b^2*c^4*d*f^4 - 8*A^4*a*b^2*c^3*f^5 - 24*A^4*a*c^5*d*f^4 + 3*B^4*b^4*c
^2*d^2*f^3 + 4*A^4*c^6*d^2*f^3 + 36*A^4*a^2*c^4*f^5 + B^4*b^2*c^4*d^3*f^2,
z, k), k, 1, 4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x**2+b*x+a)**2/(f*x**2+d),x)

[Out] Timed out

$$3.6 \quad \int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d-fx^2} dx$$

Optimal. Leaf size=331

$$\frac{(B\sqrt{d} - A\sqrt{f})\sqrt{af + b(-\sqrt{d})\sqrt{f} + cd} \tanh^{-1}\left(\frac{-2a\sqrt{f} + x(2c\sqrt{d} - b\sqrt{f}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right) (A\sqrt{f} + B\sqrt{d})\sqrt{af + b\sqrt{d}\sqrt{f}}}{2\sqrt{d} f^{3/2}} +$$

[Out] $-1/2*(2*A*c+B*b)*\operatorname{arctanh}(1/2*(2*c*x+b)/c^{(1/2)}/(c*x^2+b*x+a)^{(1/2)})/f/c^{(1/2)} - B*(c*x^2+b*x+a)^{(1/2)}/f - 1/2*\operatorname{arctanh}(1/2*(b*d^{(1/2)} - 2*a*f^{(1/2)} + x*(2*c*d^{(1/2)} - b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)}/(c*d+a*f - b*d^{(1/2)}*f^{(1/2)})^{(1/2)}*(B*d^{(1/2)} - A*f^{(1/2)})*(c*d+a*f - b*d^{(1/2)}*f^{(1/2)})^{(1/2)}/f^{(3/2)}/d^{(1/2)} + 1/2*\operatorname{arctanh}(1/2*(b*d^{(1/2)} + 2*a*f^{(1/2)} + x*(2*c*d^{(1/2)} + b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)}/(c*d+a*f + b*d^{(1/2)}*f^{(1/2)})^{(1/2)}*(B*d^{(1/2)} + A*f^{(1/2)})*(c*d+a*f + b*d^{(1/2)}*f^{(1/2)})^{(1/2)}/f^{(3/2)}/d^{(1/2)}$

Rubi [A] time = 0.60, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1021, 1078, 621, 206, 1033, 724}

$$\frac{(B\sqrt{d} - A\sqrt{f})\sqrt{af + b(-\sqrt{d})\sqrt{f} + cd} \tanh^{-1}\left(\frac{-2a\sqrt{f} + x(2c\sqrt{d} - b\sqrt{f}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right) (A\sqrt{f} + B\sqrt{d})\sqrt{af + b\sqrt{d}\sqrt{f}}}{2\sqrt{d} f^{3/2}} +$$

Antiderivative was successfully verified.

[In] Int[((A + B*x)*Sqrt[a + b*x + c*x^2])/(d - f*x^2), x]

[Out] $-((B*\operatorname{Sqrt}[a + b*x + c*x^2])/f) - ((b*B + 2*A*c)*\operatorname{ArcTanh}[(b + 2*c*x)/(2*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*\operatorname{Sqrt}[c]*f) - ((B*\operatorname{Sqrt}[d] - A*\operatorname{Sqrt}[f])*\operatorname{Sqrt}[c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] - 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] - b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*\operatorname{Sqrt}[d]*f^{(3/2)}) + ((B*\operatorname{Sqrt}[d] + A*\operatorname{Sqrt}[f])*\operatorname{Sqrt}[c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] + 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] + b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*\operatorname{Sqrt}[d]*f^{(3/2)})$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

```
Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1021

```
Int[((g_) + (h_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q + 1))*x^2, x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1033

```
Int[((g_) + (h_)*(x_))/(((a_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1078

```
Int[((A_) + (B_)*(x_) + (C_)*(x_)^2)/(((a_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d-fx^2} dx &= -\frac{B\sqrt{a+bx+cx^2}}{f} + \frac{\int \frac{\frac{1}{2}(bBd+2aAf)+(Bcd+Abf+aBf)x+\frac{1}{2}(bB+2Ac)fx^2}{\sqrt{a+bx+cx^2}(d-fx^2)} dx}{f} \\
&= -\frac{B\sqrt{a+bx+cx^2}}{f} - \frac{\int \frac{-\frac{1}{2}(bB+2Ac)df-\frac{1}{2}f(bBd+2aAf)-f(Bcd+Abf+aBf)x}{\sqrt{a+bx+cx^2}(d-fx^2)} dx}{f^2} - \frac{(bB+2Ac)}{f} \\
&= -\frac{B\sqrt{a+bx+cx^2}}{f} - \frac{(bB+2Ac) \operatorname{Subst}\left(\int \frac{1}{4c-x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f} + \frac{((B\sqrt{d}-A\sqrt{f})\sqrt{cd})}{f} \\
&= -\frac{B\sqrt{a+bx+cx^2}}{f} - \frac{(bB+2Ac) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f} - \frac{((B\sqrt{d}-A\sqrt{f})(cd))}{f} \\
&= -\frac{B\sqrt{a+bx+cx^2}}{f} - \frac{(bB+2Ac) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f} - \frac{(B\sqrt{d}-A\sqrt{f})\sqrt{cd}}{f}
\end{aligned}$$

Mathematica [A] time = 0.80, size = 322, normalized size = 0.97

$$\frac{(A\sqrt{f} + B\sqrt{d})\sqrt{af + b\sqrt{d}\sqrt{f} + cd} \tanh^{-1}\left(\frac{2a\sqrt{f} + b\sqrt{d} + b\sqrt{f}x + 2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af + b\sqrt{d}\sqrt{f} + cd}}\right) - (B\sqrt{d} - A\sqrt{f})\sqrt{af + b(-\sqrt{d})\sqrt{f}}}{2\sqrt{d}f^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*x)*Sqrt[a + b*x + c*x^2])/(d - f*x^2), x]

[Out]
$$-\frac{1}{2} \left(\frac{(bB + 2Ac) \operatorname{ArcTanh}\left[\frac{b + 2cx}{2\sqrt{c}\sqrt{a + x(b + cx)}}\right]}{\sqrt{c}f} + \frac{(-2B\sqrt{d}\sqrt{f}\sqrt{a + x(b + cx)} + (B\sqrt{d} + A\sqrt{f})\sqrt{cd + b\sqrt{d}\sqrt{f} + a}) \operatorname{ArcTanh}\left[\frac{b\sqrt{d} + 2a\sqrt{f} + 2c\sqrt{d}x + b\sqrt{f}x}{2\sqrt{cd + b\sqrt{d}\sqrt{f} + a}\sqrt{a + x(b + cx)}}\right] - (B\sqrt{d} - A\sqrt{f})\sqrt{cd - b\sqrt{d}\sqrt{f} + a} \operatorname{ArcTanh}\left[\frac{-2a\sqrt{f} + 2c\sqrt{d}x + b(\sqrt{d} - \sqrt{f}x)}{2\sqrt{cd - b\sqrt{d}\sqrt{f} + a}\sqrt{a + x(b + cx)}}\right]}{2\sqrt{d}f^{3/2}} \right)$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

```
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT>Error: Bad Argument Type
```

```
maple [B] time = 0.08, size = 3358, normalized size = 10.15
```

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] 1/2/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*A-1/2/f*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*B-1/2/f*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))*c^(1/2)*A+1/2*(d*f)^(1/2)/f^2*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))*c^(1/2)*B+1/4/(d*f)^(1/2)*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/c^(1/2)*b*A-1/4/f*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/c^(1/2)*b*B+1/2/f/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*b*A-1/2*(d*f)^(1/2)/f^2/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*b*B-1/2/(d*f)^(1/2)/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(
```

$$\begin{aligned}
& d*f)^{(1/2)}+a*f+c*d)+1/f*(-2*c*(d*f)^{(1/2)}+b*f)*(x+(d*f)^{(1/2)}/f)+2*(1/f*(-b \\
& *(d*f)^{(1/2)}+a*f+c*d))^{(1/2)}*((x+(d*f)^{(1/2)}/f)^{2*c}+1/f*(-2*c*(d*f)^{(1/2)}+b \\
& *f)*(x+(d*f)^{(1/2)}/f)+1/f*(-b*(d*f)^{(1/2)}+a*f+c*d))^{(1/2)})/(x+(d*f)^{(1/2)}/f \\
&))*a*A+1/2/f/(1/f*(-b*(d*f)^{(1/2)}+a*f+c*d))^{(1/2)}*\ln((2/f*(-b*(d*f)^{(1/2)}+a \\
& *f+c*d)+1/f*(-2*c*(d*f)^{(1/2)}+b*f)*(x+(d*f)^{(1/2)}/f)+2*(1/f*(-b*(d*f)^{(1/2) \\
& +a*f+c*d))^{(1/2)}*((x+(d*f)^{(1/2)}/f)^{2*c}+1/f*(-2*c*(d*f)^{(1/2)}+b*f)*(x+(d*f) \\
& ^{(1/2)}/f)+1/f*(-b*(d*f)^{(1/2)}+a*f+c*d))^{(1/2)})/(x+(d*f)^{(1/2)}/f))*a*B-1/2/(\\
& d*f)^{(1/2)}/f/(1/f*(-b*(d*f)^{(1/2)}+a*f+c*d))^{(1/2)}*\ln((2/f*(-b*(d*f)^{(1/2)}+a \\
& *f+c*d)+1/f*(-2*c*(d*f)^{(1/2)}+b*f)*(x+(d*f)^{(1/2)}/f)+2*(1/f*(-b*(d*f)^{(1/2) \\
& +a*f+c*d))^{(1/2)}*((x+(d*f)^{(1/2)}/f)^{2*c}+1/f*(-2*c*(d*f)^{(1/2)}+b*f)*(x+(d*f) \\
& ^{(1/2)}/f)+1/f*(-b*(d*f)^{(1/2)}+a*f+c*d))^{(1/2)})/(x+(d*f)^{(1/2)}/f))*c*d*A+1/2 \\
& /f^2/(1/f*(-b*(d*f)^{(1/2)}+a*f+c*d))^{(1/2)}*\ln((2/f*(-b*(d*f)^{(1/2)}+a*f+c*d)+ \\
& 1/f*(-2*c*(d*f)^{(1/2)}+b*f)*(x+(d*f)^{(1/2)}/f)+2*(1/f*(-b*(d*f)^{(1/2)}+a*f+c*d) \\
&))^{(1/2)}*((x+(d*f)^{(1/2)}/f)^{2*c}+1/f*(-2*c*(d*f)^{(1/2)}+b*f)*(x+(d*f)^{(1/2)}/f \\
&)+1/f*(-b*(d*f)^{(1/2)}+a*f+c*d))^{(1/2)})/(x+(d*f)^{(1/2)}/f))*c*d*B-1/2/(d*f)^{(\\
& 1/2)}*((x-(d*f)^{(1/2)}/f)^{2*c}+(2*c*(d*f)^{(1/2)}+b*f)/f*(x-(d*f)^{(1/2)}/f)+(b*(d \\
& *f)^{(1/2)}+a*f+c*d)/f)^{(1/2)}*A-1/2/f*((x-(d*f)^{(1/2)}/f)^{2*c}+(2*c*(d*f)^{(1/2) \\
& +b*f)/f*(x-(d*f)^{(1/2)}/f)+(b*(d*f)^{(1/2)}+a*f+c*d)/f)^{(1/2)}*B-1/2/f*\ln((1/2* \\
& (2*c*(d*f)^{(1/2)}+b*f)/f+c*(x-(d*f)^{(1/2)}/f))/c^{(1/2)}+((x-(d*f)^{(1/2)}/f)^{2*c} \\
& +(2*c*(d*f)^{(1/2)}+b*f)/f*(x-(d*f)^{(1/2)}/f)+(b*(d*f)^{(1/2)}+a*f+c*d)/f)^{(1/2) \\
&)*c^{(1/2)}*A-1/2*(d*f)^{(1/2)}/f^2*\ln((1/2*(2*c*(d*f)^{(1/2)}+b*f)/f+c*(x-(d*f)^{(\\
& 1/2)}/f))/c^{(1/2)}+((x-(d*f)^{(1/2)}/f)^{2*c}+(2*c*(d*f)^{(1/2)}+b*f)/f*(x-(d*f)^{(\\
& 1/2)}/f)+(b*(d*f)^{(1/2)}+a*f+c*d)/f)^{(1/2))*c^{(1/2)}*B-1/4/(d*f)^{(1/2)}*\ln((1/2 \\
& *(2*c*(d*f)^{(1/2)}+b*f)/f+c*(x-(d*f)^{(1/2)}/f))/c^{(1/2)}+((x-(d*f)^{(1/2)}/f)^{2* \\
& c}+(2*c*(d*f)^{(1/2)}+b*f)/f*(x-(d*f)^{(1/2)}/f)+(b*(d*f)^{(1/2)}+a*f+c*d)/f)^{(1/2) \\
&)/c^{(1/2)}*b*A-1/4/f*\ln((1/2*(2*c*(d*f)^{(1/2)}+b*f)/f+c*(x-(d*f)^{(1/2)}/f))/c \\
& ^{(1/2)}+((x-(d*f)^{(1/2)}/f)^{2*c}+(2*c*(d*f)^{(1/2)}+b*f)/f*(x-(d*f)^{(1/2)}/f)+(b* \\
& (d*f)^{(1/2)}+a*f+c*d)/f)^{(1/2)})/c^{(1/2)}*b*B+1/2/f/((b*(d*f)^{(1/2)}+a*f+c*d)/f \\
&)^{(1/2)}*\ln((2*(b*(d*f)^{(1/2)}+a*f+c*d)/f+(2*c*(d*f)^{(1/2)}+b*f)/f*(x-(d*f)^{(1 \\
& /2)}/f)+2*((b*(d*f)^{(1/2)}+a*f+c*d)/f)^{(1/2)}*((x-(d*f)^{(1/2)}/f)^{2*c}+(2*c*(d*f) \\
&)^{(1/2)}+b*f)/f*(x-(d*f)^{(1/2)}/f)+(b*(d*f)^{(1/2)}+a*f+c*d)/f)^{(1/2)})/(x-(d*f) \\
& ^{(1/2)}/f))*b*A+1/2*(d*f)^{(1/2)}/f^2/((b*(d*f)^{(1/2)}+a*f+c*d)/f)^{(1/2)}*\ln((2* \\
& (b*(d*f)^{(1/2)}+a*f+c*d)/f+(2*c*(d*f)^{(1/2)}+b*f)/f*(x-(d*f)^{(1/2)}/f)+2*((b*(\\
& d*f)^{(1/2)}+a*f+c*d)/f)^{(1/2)}*((x-(d*f)^{(1/2)}/f)^{2*c}+(2*c*(d*f)^{(1/2)}+b*f)/f \\
& *(x-(d*f)^{(1/2)}/f)+(b*(d*f)^{(1/2)}+a*f+c*d)/f)^{(1/2)})/(x-(d*f)^{(1/2)}/f))*b*B \\
& +1/2/(d*f)^{(1/2)}/((b*(d*f)^{(1/2)}+a*f+c*d)/f)^{(1/2)}*\ln((2*(b*(d*f)^{(1/2)}+a*f \\
& +c*d)/f+(2*c*(d*f)^{(1/2)}+b*f)/f*(x-(d*f)^{(1/2)}/f)+2*((b*(d*f)^{(1/2)}+a*f+c*d) \\
&)/f)^{(1/2)}*((x-(d*f)^{(1/2)}/f)^{2*c}+(2*c*(d*f)^{(1/2)}+b*f)/f*(x-(d*f)^{(1/2)}/f) \\
& +(b*(d*f)^{(1/2)}+a*f+c*d)/f)^{(1/2)})/(x-(d*f)^{(1/2)}/f))*a*A+1/2/f/((b*(d*f)^{(\\
& 1/2)}+a*f+c*d)/f)^{(1/2)}*\ln((2*(b*(d*f)^{(1/2)}+a*f+c*d)/f+(2*c*(d*f)^{(1/2)}+b*f \\
&)/f*(x-(d*f)^{(1/2)}/f)+2*((b*(d*f)^{(1/2)}+a*f+c*d)/f)^{(1/2)}*((x-(d*f)^{(1/2)}/f \\
&)^{2*c}+(2*c*(d*f)^{(1/2)}+b*f)/f*(x-(d*f)^{(1/2)}/f)+(b*(d*f)^{(1/2)}+a*f+c*d)/f)^{(\\
& 1/2)})/(x-(d*f)^{(1/2)}/f))*a*B+1/2/(d*f)^{(1/2)}/f/((b*(d*f)^{(1/2)}+a*f+c*d)/f) \\
& ^{(1/2)}*\ln((2*(b*(d*f)^{(1/2)}+a*f+c*d)/f+(2*c*(d*f)^{(1/2)}+b*f)/f*(x-(d*f)^{(1/ \\
& 2)}/f)+2*((b*(d*f)^{(1/2)}+a*f+c*d)/f)^{(1/2)}*((x-(d*f)^{(1/2)}/f)^{2*c}+(2*c*(d*f)
\end{aligned}$$

$$\frac{(x - (d* f)^{1/2}) / f + (b * (d* f)^{1/2} + a * f + c * d) / f}{(x - (d* f)^{1/2}) / f} * c * d * A + \frac{1}{2} * \frac{1}{f^2} * \frac{(b * (d* f)^{1/2} + a * f + c * d) / f}{(x - (d* f)^{1/2}) / f} * \ln\left(\frac{2 * (b * (d* f)^{1/2} + a * f + c * d) / f + (2 * c * (d* f)^{1/2} + b * f) / f * (x - (d* f)^{1/2}) / f + 2 * ((b * (d* f)^{1/2} + a * f + c * d) / f)^{1/2} * ((x - (d* f)^{1/2}) / f)^2 * c + (2 * c * (d* f)^{1/2} + b * f) / f * (x - (d* f)^{1/2}) / f + (b * (d* f)^{1/2} + a * f + c * d) / f}{(x - (d* f)^{1/2}) / f}\right) * c * d * B$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see 'assume?' for more details)Is ((c*sqrt(4*d*f))/(2*f^2) + b/(2*f)) ^2 - (c*((b*sqrt(4*d*f)) / (2*f) + (c*d)/f+a)) / f^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + Bx) \sqrt{cx^2 + bx + a}}{d - fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/(d - f*x^2),x)

[Out] int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/(d - f*x^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{A\sqrt{a + bx + cx^2}}{-d + fx^2} dx - \int \frac{Bx\sqrt{a + bx + cx^2}}{-d + fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)

[Out] -Integral(A*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(B*x*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)

$$3.7 \quad \int \frac{A+Bx}{\sqrt{a+bx+cx^2} (d-fx^2)} dx$$

Optimal. Leaf size=249

$$\frac{\left(\frac{A\sqrt{f}}{\sqrt{d}} + B\right) \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{\left(B - \frac{A\sqrt{f}}{\sqrt{d}}\right) \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}$$

[Out] $-1/2*\operatorname{arctanh}(1/2*(b*d^{(1/2)}-2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}-b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)/(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(1/2)})*(B-A*f^{(1/2)}/d^{(1/2)})/f^{(1/2)})/(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(1/2)}+1/2*\operatorname{arctanh}(1/2*(b*d^{(1/2)}+2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}+b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)/(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(1/2)})*(B+A*f^{(1/2)}/d^{(1/2)})/f^{(1/2)})/(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(1/2)}$

Rubi [A] time = 0.20, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1033, 724, 206}

$$\frac{\left(\frac{A\sqrt{f}}{\sqrt{d}} + B\right) \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{\left(B - \frac{A\sqrt{f}}{\sqrt{d}}\right) \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(A + B*x)/(\operatorname{Sqrt}[a + b*x + c*x^2]*(d - f*x^2)), x]$

[Out] $-((B - (A*\operatorname{Sqrt}[f])/ \operatorname{Sqrt}[d])* \operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] - 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] - b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)]*\operatorname{Sqrt}[a + b*x + c*x^2]))/(2*\operatorname{Sqrt}[f]*\operatorname{Sqrt}[c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]) + ((B + (A*\operatorname{Sqrt}[f])/ \operatorname{Sqrt}[d])* \operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] + 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] + b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)]*\operatorname{Sqrt}[a + b*x + c*x^2]))/(2*\operatorname{Sqrt}[f]*\operatorname{Sqrt}[c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f])$

Rule 206

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/ \operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

Rule 724

$\operatorname{Int}[1/(((d_.) + (e_.)*(x_.)*\operatorname{Sqrt}[(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2])), x_Symbol] \rightarrow \operatorname{Dist}[-2, \operatorname{Subst}[\operatorname{Int}[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x], (2$

*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1033

Int[((g_.) + (h_.)*(x_))/(((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]

Rubi steps

$$\begin{aligned} \int \frac{A + Bx}{\sqrt{a + bx + cx^2} (d - fx^2)} dx &= \frac{1}{2} \left(B - \frac{A\sqrt{f}}{\sqrt{d}} \right) \int \frac{1}{(-\sqrt{d}\sqrt{f} - fx)\sqrt{a + bx + cx^2}} dx + \frac{1}{2} \left(B + \frac{A\sqrt{f}}{\sqrt{d}} \right) \int \frac{1}{(\sqrt{d}\sqrt{f} + fx)\sqrt{a + bx + cx^2}} dx \\ &= \left(-B - \frac{A\sqrt{f}}{\sqrt{d}} \right) \text{Subst} \left(\int \frac{1}{4cdf + 4b\sqrt{d}f^{3/2} + 4af^2 - x^2} dx, x, \frac{-b\sqrt{d}\sqrt{f} - 2ax}{\sqrt{a + bx + cx^2}} \right) \\ &\quad + \left(B + \frac{A\sqrt{f}}{\sqrt{d}} \right) \text{Subst} \left(\int \frac{1}{4cdf + 4b\sqrt{d}f^{3/2} + 4af^2 - x^2} dx, x, \frac{-b\sqrt{d}\sqrt{f} + 2ax}{\sqrt{a + bx + cx^2}} \right) \\ &= -\frac{\left(B - \frac{A\sqrt{f}}{\sqrt{d}} \right) \tanh^{-1} \left(\frac{b\sqrt{d} - 2a\sqrt{f} + (2c\sqrt{d} - b\sqrt{f})x}{2\sqrt{cd - b\sqrt{d}\sqrt{f} + af}\sqrt{a + bx + cx^2}} \right)}{2\sqrt{f}\sqrt{cd - b\sqrt{d}\sqrt{f} + af}} + \frac{\left(B + \frac{A\sqrt{f}}{\sqrt{d}} \right) \tanh^{-1} \left(\frac{b\sqrt{d} + 2a\sqrt{f} + (2c\sqrt{d} + b\sqrt{f})x}{2\sqrt{cd + b\sqrt{d}\sqrt{f} + af}\sqrt{a + bx + cx^2}} \right)}{2\sqrt{f}\sqrt{cd + b\sqrt{d}\sqrt{f} + af}} \end{aligned}$$

Mathematica [A] time = 0.24, size = 249, normalized size = 1.00

$$\frac{\frac{(B\sqrt{d} - A\sqrt{f}) \tanh^{-1} \left(\frac{-2a\sqrt{f} + b(\sqrt{d} - \sqrt{f}x) + 2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} \right)}{\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} - \frac{(A\sqrt{f} + B\sqrt{d}) \tanh^{-1} \left(\frac{-2(a\sqrt{f} + c\sqrt{d}x) - b(\sqrt{d} + \sqrt{f}x)}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} \right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x)/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)), x]

[Out] (-(((B*Sqrt[d] - A*Sqrt[f])*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + x*(b + c*x)])))/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) - ((B*Sqrt[d] + A*Sqrt[f])*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + x*(b + c*x)])))/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f))/(2*Sqrt[d]*Sqrt[f])

fricas [B] time = 147.14, size = 6113, normalized size = 24.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")

[Out] $\frac{1}{4}\sqrt{(B^2cd^2 + A^2af^2 + (B^2a - 2ABb + A^2c)d^2f + (c^2d^3f + a^2d^3f^3 - (b^2 - 2ac)d^2f^2)}\sqrt{((B^4b^2 - 4AB^3bc + 4A^2B^2c^2)d^2 - 2(2AB^3ab - A^2B^2b^2 - 2(2A^2B^2a - A^3Bb)c)d^2f + (4A^2B^2a^2 - 4A^3Bab + A^4b^2)f^2)/(c^4d^5f + a^4d^5f^5 - 2(b^2c^2 - 2ac^3)d^4f^2 + (b^4 - 4ab^2c + 6a^2c^2)d^3f^3 - 2(a^2b^2 - 2a^3c)d^2f^4)}}{(c^2d^3f + a^2d^3f^3 - (b^2 - 2ac)d^2f^2)}\log(-((B^4b^2 - 2AB^3bc)d^2 - 2(AB^3ab - A^3Bbc)d^2f + (2A^3Bab - A^4b^2)f^2 + 2((2A^3Ba - A^4b)c^2f^2 + (B^4b^2c - 2AB^3c^2)d^2 - 2(AB^3ac - A^3Bc^2)d^2f))x + 2((B^3b^2 - 3AB^2bc + 2A^2Bc^2)d^2f - (3AB^2ab - A^2Bb^2 - (4A^2Ba - A^3b)c)d^2f^2 + (2A^2Ba^2 - A^3ab)f^3 - (Bc^3d^4f - (Bb^2c - (3Ba - Ab)c^2)d^3f^2 - (Bab^2 - Ab^3 - (3Ba^2 - 2Aab)c)d^2f^3 + (Ba^3 - Aa^2b)d^2f^4)\sqrt{((B^4b^2 - 4AB^3bc + 4A^2B^2c^2)d^2 - 2(2AB^3ab - A^2B^2b^2 - 2(2A^2B^2a - A^3Bb)c)d^2f + (4A^2B^2a^2 - 4A^3Bab + A^4b^2)f^2)/(c^4d^5f + a^4d^5f^5 - 2(b^2c^2 - 2ac^3)d^4f^2 + (b^4 - 4ab^2c + 6a^2c^2)d^3f^3 - 2(a^2b^2 - 2a^3c)d^2f^4)}}{(c^2d^3f + a^2d^3f^3 - (b^2 - 2ac)d^2f^2)}\sqrt{((B^4b^2 - 4AB^3bc + 4A^2B^2c^2)d^2 - 2(2AB^3ab - A^2B^2b^2 - 2(2A^2B^2a - A^3Bb)c)d^2f + (4A^2B^2a^2 - 4A^3Bab + A^4b^2)f^2)/(c^4d^5f + a^4d^5f^5 - 2(b^2c^2 - 2ac^3)d^4f^2 + (b^4 - 4ab^2c + 6a^2c^2)d^3f^3 - 2(a^2b^2 - 2a^3c)d^2f^4)}}/x - \frac{1}{4}\sqrt{(B^2cd^2 + A^2af^2 + (B^2a - 2ABb + A^2c)d^2f + (c^2d^3f + a^2d^3f^3 - (b^2 - 2ac)d^2f^2)}\sqrt{((B^4b^2 - 4AB^3bc + 4A^2B^2c^2)d^2 - 2(2AB^3ab - A^2B^2b^2 - 2(2A^2B^2a - A^3Bb)c)d^2f + (4A^2B^2a^2 - 4A^3Bab + A^4b^2)f^2)/(c^4d^5f + a^4d^5f^5 - 2(b^2c^2 - 2ac^3)d^4f^2 + (b^4 - 4ab^2c + 6a^2c^2)d^3f^3 - 2(a^2b^2 - 2a^3c)d^2f^4)}}{(c^2d^3f + a^2d^3f^3 - (b^2 - 2ac)d^2f^2)}\log(-((B^4b^2 - 2AB^3bc)d^2 - 2(AB^3ab - A^3Bbc)d^2f + (2A^3Bab - A^4b^2)f^2 + 2((2A^3Ba - A^4b)c^2f^2 + (B^4b^2c - 2AB^3c^2)d^2 - 2(AB^3ac - A^3Bc^2)d^2f))x + 2((B^3b^2 - 3AB^2bc + 2A^2Bc^2)d^2f - (3AB^2ab - A^2Bb^2 - (4A^2Ba - A^3b)c)d^2f^2 + (2A^2Ba^2 - A^3ab)f^3 - (Bc^3d^4f - (Bb^2c - (3Ba - Ab)c^2)d^3f^2 - (Bab^2 - Ab^3 - (3Ba^2 - 2Aab)c)d^2f^3 + (Ba^3 - Aa^2b)d^2f^4)\sqrt{((B^4b^2 - 4AB^3bc + 4A^2B^2c^2)d^2 - 2(2AB^3ab - A^2B^2b^2 - 2(2A^2B^2a - A^3Bb)c)d^2f + (4A^2B^2a^2 - 4A^3Bab + A^4b^2)f^2)/(c^4d^5f + a^4d^5f^5 - 2(b^2c^2 - 2ac^3)d^4f^2 + (b^4 - 4ab^2c + 6a^2c^2)d^3f^3 - 2(a^2b^2 - 2a^3c)d^2f^4)}}{(c^2d^3f + a^2d^3f^3 - (b^2 - 2ac)d^2f^2)}$

$$\begin{aligned}
& 3*B*a*b - A^4*b^2)*f^2 + 2*((2*A^3*B*a - A^4*b)*c*f^2 + (B^4*b*c - 2*A*B^3*c^2)*d^2 - 2*(A*B^3*a*c - A^3*B*c^2)*d*f)*x - 2*((B^3*b^2 - 3*A*B^2*b*c + 2*A^2*B*c^2)*d^2*f - (3*A*B^2*a*b - A^2*B*b^2 - (4*A^2*B*a - A^3*b)*c)*d*f^2 + (2*A^2*B*a^2 - A^3*a*b)*f^3 - (B*c^3*d^4*f - (B*b^2*c - (3*B*a - A*b)*c^2)*d^3*f^2 - (B*a*b^2 - A*b^3 - (3*B*a^2 - 2*A*a*b)*c)*d^2*f^3 + (B*a^3 - A*a^2*b)*d*f^4)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))*sqrt(c*x^2 + b*x + a)*sqrt((B^2*c*d^2 + A^2*a*f^2 + (B^2*a - 2*A*B*b + A^2*c)*d*f + (c^2*d^3*f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2))*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))/(c^2*d^3*f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2)) - (2*B^2*a*c^2*d^3*f - 2*A^2*a^3*f^4 - 2*(B^2*a*b^2 - 2*B^2*a^2*c + A^2*a*c^2)*d^2*f^2 + 2*(B^2*a^3 + A^2*a*b^2 - 2*A^2*a^2*c)*d*f^3 + (B^2*b*c^2*d^3*f - A^2*a^2*b*f^4 - (B^2*b^3 - 2*B^2*a*b*c + A^2*b*c^2)*d^2*f^2 + (B^2*a^2*b + A^2*b^3 - 2*A^2*a*b*c)*d*f^3)*x)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))/x) + 1/4*sqrt((B^2*c*d^2 + A^2*a*f^2 + (B^2*a - 2*A*B*b + A^2*c)*d*f - (c^2*d^3*f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2))*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))/(c^2*d^3*f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2))*log(-((B^4*b^2 - 2*A*B^3*b*c)*d^2 - 2*(A*B^3*a*b - A^3*B*b*c)*d*f + (2*A^3*B*a*b - A^4*b^2)*f^2 + 2*((2*A^3*B*a - A^4*b)*c*f^2 + (B^4*b*c - 2*A*B^3*c^2)*d^2 - 2*(A*B^3*a*c - A^3*B*c^2)*d*f)*x + 2*((B^3*b^2 - 3*A*B^2*b*c + 2*A^2*B*c^2)*d^2*f - (3*A*B^2*a*b - A^2*B*b^2 - (4*A^2*B*a - A^3*b)*c)*d*f^2 + (2*A^2*B*a^2 - A^3*a*b)*f^3 + (B*c^3*d^4*f - (B*b^2*c - (3*B*a - A*b)*c^2)*d^3*f^2 - (B*a*b^2 - A*b^3 - (3*B*a^2 - 2*A*a*b)*c)*d^2*f^3 + (B*a^3 - A*a^2*b)*d*f^4)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))*sqrt(c*x^2 + b*x + a)*sqrt((B^2*c*d^2 + A^2*a*f^2 + (B^2*a - 2*A*B*b + A^2*c)*d*f - (c^2*d^3*f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2))*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))/(c^2*d^3*f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2))
\end{aligned}$$

$$\begin{aligned}
& 3 - (b^2 - 2ac)d^2f^2)) + (2B^2aac^2d^3f - 2A^2a^3f^4 - 2(B^2ab^2 - 2B^2a^2c + A^2ac^2)d^2f^2 + 2(B^2a^3 + A^2ab^2 - 2A^2a^2c) \\
& *d^2f^3 + (B^2bc^2d^3f - A^2a^2b^2f^4 - (B^2b^3 - 2B^2ab^2c + A^2b^2c^2)d^2f^2 + (B^2a^2b + A^2b^3 - 2A^2ab^2c)d^2f^3)*x) * \sqrt{((B^4b^2 - 4A^2B^3b^2c + 4A^2B^2c^2)d^2 - 2(2A^2B^3ab - A^2B^2b^2 - 2(2A^2B^2a - A^3B^2b)c)d^2f + (4A^2B^2a^2 - 4A^3B^2ab + A^4b^2)f^2) / (c^4d^5f + a^4d^5f - 2(b^2c^2 - 2ac^3)d^4f^2 + (b^4 - 4ab^2c + 6a^2c^2)d^3f^3 - 2(a^2b^2 - 2a^3c)d^2f^4))} / x - 1/4 * \sqrt{(B^2cd^2 + A^2af^2 + (B^2a - 2AB^2b + A^2c)d^2f - (c^2d^3f + a^2d^3f^3 - (b^2 - 2ac)d^2f^2) * \sqrt{((B^4b^2 - 4A^2B^3b^2c + 4A^2B^2c^2)d^2 - 2(2A^2B^3ab - A^2B^2b^2 - 2(2A^2B^2a - A^3B^2b)c)d^2f + (4A^2B^2a^2 - 4A^3B^2ab + A^4b^2)f^2) / (c^4d^5f + a^4d^5f - 2(b^2c^2 - 2ac^3)d^4f^2 + (b^4 - 4ab^2c + 6a^2c^2)d^3f^3 - 2(a^2b^2 - 2a^3c)d^2f^4))} / (c^2d^3f + a^2d^3f^3 - (b^2 - 2ac)d^2f^2)) * \log(-(B^4b^2 - 2A^2B^3b^2c)d^2 - 2(A^2B^3ab - A^3B^2b^2c)d^2f + (2A^3B^2ab - A^4b^2)f^2 + 2((2A^3B^2a - A^4b)c^2f^2 + (B^4b^2c - 2A^2B^3c^2)d^2 - 2(A^2B^3ac - A^3B^2c^2)d^2f)*x - 2((B^3b^2 - 3A^2B^2b^2c + 2A^2B^2c^2)d^2f - (3A^2B^2ab - A^2B^2b^2 - (4A^2B^2a - A^3b)c)d^2f^2 + (2A^2B^2a^2 - A^3ab^2)f^3 + (B^2c^3d^4f - (B^2b^2c - (3B^2a - Ab)c^2)d^3f^2 - (B^2ab^2 - Ab^3 - (3B^2a^2 - 2A^2ab)c)d^2f^3 + (B^2a^3 - A^2ab^2)d^2f^4) * \sqrt{((B^4b^2 - 4A^2B^3b^2c + 4A^2B^2c^2)d^2 - 2(2A^2B^3ab - A^2B^2b^2 - 2(2A^2B^2a - A^3B^2b)c)d^2f + (4A^2B^2a^2 - 4A^3B^2ab + A^4b^2)f^2) / (c^4d^5f + a^4d^5f - 2(b^2c^2 - 2ac^3)d^4f^2 + (b^4 - 4ab^2c + 6a^2c^2)d^3f^3 - 2(a^2b^2 - 2a^3c)d^2f^4))} * \sqrt{cx^2 + bx + a} * \sqrt{(B^2cd^2 + A^2af^2 + (B^2a - 2AB^2b + A^2c)d^2f - (c^2d^3f + a^2d^3f^3 - (b^2 - 2ac)d^2f^2) * \sqrt{((B^4b^2 - 4A^2B^3b^2c + 4A^2B^2c^2)d^2 - 2(2A^2B^3ab - A^2B^2b^2 - 2(2A^2B^2a - A^3B^2b)c)d^2f + (4A^2B^2a^2 - 4A^3B^2ab + A^4b^2)f^2) / (c^4d^5f + a^4d^5f - 2(b^2c^2 - 2ac^3)d^4f^2 + (b^4 - 4ab^2c + 6a^2c^2)d^3f^3 - 2(a^2b^2 - 2a^3c)d^2f^4))} / (c^2d^3f + a^2d^3f^3 - (b^2 - 2ac)d^2f^2)) + (2B^2aac^2d^3f - 2A^2a^3f^4 - 2(B^2ab^2 - 2B^2a^2c + A^2ac^2)d^2f^2 + 2(B^2a^3 + A^2ab^2 - 2A^2a^2c) *d^2f^3 + (B^2bc^2d^3f - A^2a^2b^2f^4 - (B^2b^3 - 2B^2ab^2c + A^2b^2c^2)d^2f^2 + (B^2a^2b + A^2b^3 - 2A^2ab^2c)d^2f^3)*x) * \sqrt{((B^4b^2 - 4A^2B^3b^2c + 4A^2B^2c^2)d^2 - 2(2A^2B^3ab - A^2B^2b^2 - 2(2A^2B^2a - A^3B^2b)c)d^2f + (4A^2B^2a^2 - 4A^3B^2ab + A^4b^2)f^2) / (c^4d^5f + a^4d^5f - 2(b^2c^2 - 2ac^3)d^4f^2 + (b^4 - 4ab^2c + 6a^2c^2)d^3f^3 - 2(a^2b^2 - 2a^3c)d^2f^4))} / x)
\end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument ValueConj Error: Bad Argument Typ
 e

maple [B] time = 0.02, size = 714, normalized size = 2.87

$$\frac{A \ln \left(\frac{\frac{2af+2cd-2\sqrt{df}b}{f} + \frac{(bf-2\sqrt{df}c)\left(x+\frac{\sqrt{df}}{f}\right)}{f} + 2\sqrt{\frac{af+cd-\sqrt{df}b}{f}} \sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2 c + \frac{(bf-2\sqrt{df}c)\left(x+\frac{\sqrt{df}}{f}\right) + \frac{af+cd-\sqrt{df}b}{f}}}{x+\frac{\sqrt{df}}{f}} \right)}{2\sqrt{df} \sqrt{\frac{af+cd-\sqrt{df}b}{f}}} + A \ln \left(\frac{2af+2cd+2\sqrt{df}b}{f} \right)}{2\sqrt{df} \sqrt{\frac{af+cd-\sqrt{df}b}{f}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x+A)/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)

[Out]
$$\begin{aligned} & -1/2/(d*f)^{(1/2)}/(1/f*(-b*(d*f)^{(1/2)}+a*f+c*d))^{(1/2)}*\ln((2/f*(-b*(d*f)^{(1/2)}+a*f+c*d)+1/f*(-2*c*(d*f)^{(1/2)}+b*f)*(x+(d*f)^{(1/2)}/f)+2*(1/f*(-b*(d*f)^{(1/2)}+a*f+c*d))^{(1/2)}*((x+(d*f)^{(1/2)}/f)^2*c+1/f*(-2*c*(d*f)^{(1/2)}+b*f)*(x+(d*f)^{(1/2)}/f)+1/f*(-b*(d*f)^{(1/2)}+a*f+c*d))^{(1/2)})/(x+(d*f)^{(1/2)}/f))*A+1/2/f/(1/f*(-b*(d*f)^{(1/2)}+a*f+c*d))^{(1/2)}*\ln((2/f*(-b*(d*f)^{(1/2)}+a*f+c*d)+1/f*(-2*c*(d*f)^{(1/2)}+b*f)*(x+(d*f)^{(1/2)}/f)+2*(1/f*(-b*(d*f)^{(1/2)}+a*f+c*d))^{(1/2)}*((x+(d*f)^{(1/2)}/f)^2*c+1/f*(-2*c*(d*f)^{(1/2)}+b*f)*(x+(d*f)^{(1/2)}/f)+1/f*(-b*(d*f)^{(1/2)}+a*f+c*d))^{(1/2)})/(x+(d*f)^{(1/2)}/f))*B+1/2/(d*f)^{(1/2)}/((b*(d*f)^{(1/2)}+a*f+c*d)/f)^{(1/2)}*\ln((2*(b*(d*f)^{(1/2)}+a*f+c*d)/f+(2*c*(d*f)^{(1/2)}+b*f)/f*(x-(d*f)^{(1/2)}/f)+2*((b*(d*f)^{(1/2)}+a*f+c*d)/f)^{(1/2)}*((x-(d*f)^{(1/2)}/f)^2*c+(2*c*(d*f)^{(1/2)}+b*f)/f*(x-(d*f)^{(1/2)}/f)+(b*(d*f)^{(1/2)}+a*f+c*d)/f)^{(1/2)})/(x-(d*f)^{(1/2)}/f))*A+1/2/f/((b*(d*f)^{(1/2)}+a*f+c*d)/f)^{(1/2)}*\ln((2*(b*(d*f)^{(1/2)}+a*f+c*d)/f+(2*c*(d*f)^{(1/2)}+b*f)/f*(x-(d*f)^{(1/2)}/f)+2*((b*(d*f)^{(1/2)}+a*f+c*d)/f)^{(1/2)}*((x-(d*f)^{(1/2)}/f)^2*c+(2*c*(d*f)^{(1/2)}+b*f)/f*(x-(d*f)^{(1/2)}/f)+(b*(d*f)^{(1/2)}+a*f+c*d)/f)^{(1/2)})/(x-(d*f)^{(1/2)}/f))*B \end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
 dditional constraints; using the 'assume' command before evaluation *may* h

elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2 - (c*((b*sqrt(4*d*f)) / (2*f) + (c*d)/f+a)) /f^2 zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + Bx}{(d - fx^2) \sqrt{cx^2 + bx + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x)/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)

[Out] int((A + B*x)/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{A}{-d\sqrt{a + bx + cx^2} + fx^2\sqrt{a + bx + cx^2}} dx - \int \frac{Bx}{-d\sqrt{a + bx + cx^2} + fx^2\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d), x)

[Out] -Integral(A/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)), x)
- Integral(B*x/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)), x)

$$3.8 \quad \int \frac{A+Bx}{(a+bx+cx^2)^{3/2}(d-fx^2)} dx$$

Optimal. Leaf size=381

$$\frac{2\left(A\left(b^3f - bc(3af + cd)\right) + cx\left(-2Ac(af + cd) + bB(cd - af) + Ab^2f\right) + aB\left(2acf + b^2(-f) + 2c^2d\right)\right) \sqrt{f}}{(b^2 - 4ac) \sqrt{a + bx + cx^2} (b^2df - (af + cd)^2)}$$

[Out] $-1/2*\operatorname{arctanh}(1/2*(b*d^{(1/2)}-2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}-b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)}/(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(1/2)})*f^{(1/2)}*(B*d^{(1/2)}-A*f^{(1/2)})/d^{(1/2)}/(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(3/2)}+1/2*\operatorname{arctanh}(1/2*(b*d^{(1/2)}+2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}+b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)}/(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(1/2)})*f^{(1/2)}*(B*d^{(1/2)}+A*f^{(1/2)})/d^{(1/2)}/(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(3/2)}-2*(a*B*(2*a*c*f-b^2*f+2*c^2*d)+A*(b^3*f-b*c*(3*a*f+cd))+c*(A*b^2*f+b*B*(-a*f+cd)-2*A*c*(a*f+cd))*x)/(-4*a*c+b^2)/(b^2*d*f-(a*f+cd)^2)/(c*x^2+b*x+a)^{(1/2)}$

Rubi [A] time = 0.80, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1018, 1033, 724, 206}

$$\frac{2\left(cx\left(-2Ac(af + cd) + bB(cd - af) + Ab^2f\right) - Abc(3af + cd) + aB\left(2acf + b^2(-f) + 2c^2d\right) + Ab^3f\right) \sqrt{f}}{(b^2 - 4ac) \sqrt{a + bx + cx^2} (b^2df - (af + cd)^2)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(A + B*x)/((a + b*x + c*x^2)^{(3/2)}*(d - f*x^2)), x]$

[Out] $(-2*(A*b^3*f - A*b*c*(c*d + 3*a*f) + a*B*(2*c^2*d - b^2*f + 2*a*c*f) + c*(A*b^2*f + b*B*(c*d - a*f) - 2*A*c*(c*d + a*f))*x)/((b^2 - 4*a*c)*(b^2*d*f - (c*d + a*f)^2)*\operatorname{Sqrt}[a + b*x + c*x^2]) - ((B*\operatorname{Sqrt}[d] - A*\operatorname{Sqrt}[f])* \operatorname{Sqrt}[f]*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] - 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] - b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*\operatorname{Sqrt}[d]*(c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)^{(3/2)}) + ((B*\operatorname{Sqrt}[d] + A*\operatorname{Sqrt}[f])* \operatorname{Sqrt}[f]*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] + 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] + b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*\operatorname{Sqrt}[d]*(c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)^{(3/2)})$

Rule 206

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*x]/\operatorname{Rt}[a, 2])]/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}[\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b] \ \&\& (Gt$

Q[a, 0] || LtQ[b, 0])

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol]
:> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1018

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol]
:> Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1)*((g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f)*x))/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p + q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] && !(IntegerQ[p] && ILtQ[q, -1])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol]
:> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx}{(a + bx + cx^2)^{3/2} (d - fx^2)} dx &= -\frac{2(Ab^3f - Abc(cd + 3af) + aB(2c^2d - b^2f + 2acf) + c(Ab^2f + bB(cd - \\
&= -\frac{2(Ab^3f - Abc(cd + 3af) + aB(2c^2d - b^2f + 2acf) + c(Ab^2f + bB(cd - \\
&= -\frac{2(Ab^3f - Abc(cd + 3af) + aB(2c^2d - b^2f + 2acf) + c(Ab^2f + bB(cd - \\
&= -\frac{2(Ab^3f - Abc(cd + 3af) + aB(2c^2d - b^2f + 2acf) + c(Ab^2f + bB(cd -
\end{aligned}$$

Mathematica [A] time = 0.65, size = 440, normalized size = 1.15

$$2 \left[\frac{B(2a^2cf + a(b^2(-f) - bcfx + 2c^2d) + bc^2dx) + A(-bc(3af + cd) - 2c^2x(af + cd) + b^3f + b^2cfx)}{\sqrt{a+x(b+cx)}} + \frac{\sqrt{f}(b^2-4ac)(A\sqrt{f}-B\sqrt{d})(af+b\sqrt{d}\sqrt{f+cd})\tanh^{-1}\left(\frac{4\sqrt{d}\sqrt{af+b(-\sqrt{d})\sqrt{f+cd}}}{(b^2-4ac)((af+cd)^2 - \dots}\right)}{4\sqrt{d}\sqrt{af+b(-\sqrt{d})\sqrt{f+cd}}} \right]$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x)/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]

[Out] (2*((A*(b^3*f - b*c*(c*d + 3*a*f) + b^2*c*f*x - 2*c^2*(c*d + a*f)*x) + B*(2*a^2*c*f + b*c^2*d*x + a*(2*c^2*d - b^2*f - b*c*f*x)))/Sqrt[a + x*(b + c*x)] + ((b^2 - 4*a*c)*(-B*Sqrt[d]) + A*Sqrt[f])*Sqrt[f]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/(4*Sqrt[d]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f)) + ((-b^2 + 4*a*c)*(B*Sqrt[d] + A*Sqrt[f])*Sqrt[f]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/(4*Sqrt[d]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f)))/((b^2 - 4*a*c)*(-b^2*d*f) + (c*d + a*f)^2))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

```
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 10.66Conj E
rror: Bad Argument Type
```

```
maple [B] time = 0.03, size = 2758, normalized size = 7.24
```

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] 1/2/(d*f)^(1/2)*f/(a*f+c*d-(d*f)^(1/2)*b)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*
f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*A-1/2/(a*f
+c*d-(d*f)^(1/2)*b)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(
1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*B+2/(a*f+c*d-(d*f)^(1/2)*b)/(4*a
*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a
*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*c^2*A-2*(d*f)^(1/2)/f/(a*f+c*d-(d*f)^(1/2)
*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)
/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*c^2*B-1/(d*f)^(1/2)*f/(a*f+c*d-(d*
f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*
f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*b*c*A+1/(a*f+c*d-(d*f)^(1/
2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/
2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*b*c*B+1/(a*f+c*d-(d*f)^(1/2)*b)/
(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/
f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b*c*A-(d*f)^(1/2)/f/(a*f+c*d-(d*f)^(1/2)
*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)
/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b*c*B-1/2/(d*f)^(1/2)*f/(a*f+c*d-(d*
f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*
f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b^2*A+1/2/(a*f+c*d-(d*f)^(1/
2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/
2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b^2*B-1/2/(d*f)^(1/2)*f/(a*f+c*d-(
```

$$\begin{aligned}
& d*f)^{(1/2)*b)/((a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2)*\ln((2*(a*f+c*d-(d*f)^{(1/2)*b)/f+(b*f-2*(d*f)^{(1/2)*c)*(x+(d*f)^{(1/2)/f)/f+2*((a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2)*((x+(d*f)^{(1/2)/f)^2*c+(b*f-2*(d*f)^{(1/2)*c)*(x+(d*f)^{(1/2)/f)/f+(a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2)))/(x+(d*f)^{(1/2)/f)})*A+1/2/(a*f+c*d-(d*f)^{(1/2)*b)/((a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2)*\ln((2*(a*f+c*d-(d*f)^{(1/2)*b)/f+(b*f-2*(d*f)^{(1/2)*c)*(x+(d*f)^{(1/2)/f)/f+2*((a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2)*((x+(d*f)^{(1/2)/f)^2*c+(b*f-2*(d*f)^{(1/2)*c)*(x+(d*f)^{(1/2)/f)/f+(a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2)))/(x+(d*f)^{(1/2)/f)})*B-1/2/(d*f)^{(1/2)*f/(a*f+c*d+(d*f)^{(1/2)*b)/((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*A-1/2/(a*f+c*d+(d*f)^{(1/2)*b)/((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*B+2/(a*f+c*d+(d*f)^{(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*x*c^2*A+2*(d*f)^{(1/2)/f/(a*f+c*d+(d*f)^{(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*x*c^2*B+1/(d*f)^{(1/2)*f/(a*f+c*d+(d*f)^{(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*x*b*c*A+1/(a*f+c*d+(d*f)^{(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*x*b*c*B+1/(a*f+c*d+(d*f)^{(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*b*c*A+(d*f)^{(1/2)/f/(a*f+c*d+(d*f)^{(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*b*c*B+1/2/(d*f)^{(1/2)*f/(a*f+c*d+(d*f)^{(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*b^2*A+1/2/(a*f+c*d+(d*f)^{(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*b^2*B+1/2/(d*f)^{(1/2)*f/(a*f+c*d+(d*f)^{(1/2)*b)/((a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*\ln((2*(a*f+c*d+(d*f)^{(1/2)*b)/f+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f)/f+2*((a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)))/(x-(d*f)^{(1/2)/f)})*A+1/2/(a*f+c*d+(d*f)^{(1/2)*b)/((a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*\ln((2*(a*f+c*d+(d*f)^{(1/2)*b)/f+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f)/f+2*((a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)))/(x-(d*f)^{(1/2)/f)})*B}
\end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a

additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see 'assume?' for more details) Is $\frac{(c\sqrt{4df})/(2f^2) + b/(2f)}{(d - fx^2)(cx^2 + bx + a)^2} - \frac{c(b\sqrt{4df})}{(2f)^2} + (cd)/(fa)$ zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + Bx}{(d - fx^2)(cx^2 + bx + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x)/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)

[Out] int((A + B*x)/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d), x)

[Out] Timed out

$$3.9 \quad \int \frac{A+Bx}{(a+bx+cx^2)^{5/2}(d-fx^2)} dx$$

Optimal. Leaf size=797

$$\frac{(B\sqrt{d} - A\sqrt{f}) \tanh^{-1}\left(\frac{-2\sqrt{f}a+(2c\sqrt{d}-b\sqrt{f})x+b\sqrt{d}}{2\sqrt{-\sqrt{d}}\sqrt{f}b+cd+af\sqrt{cx^2+bx+a}}\right) f^{3/2}}{2\sqrt{d}(-\sqrt{d}\sqrt{f}b+cd+af)^{5/2}} + \frac{(\sqrt{f}A + B\sqrt{d}) \tanh^{-1}\left(\frac{2\sqrt{f}a+(\sqrt{f}b+2c\sqrt{d})x+b\sqrt{d}}{2\sqrt{\sqrt{d}}\sqrt{f}b+cd+af\sqrt{cx^2+bx+a}}\right)}{2\sqrt{d}(\sqrt{d}\sqrt{f}b+cd+af)^{5/2}}$$

[Out] $-2/3*(a*B*(2*a*c*f-b^2*f+2*c^2*d)+A*(b^3*f-b*c*(3*a*f+c*d))+c*(A*b^2*f+b*B*(-a*f+c*d)-2*A*c*(a*f+c*d))*x)/(-4*a*c+b^2)/(b^2*d*f-(a*f+c*d)^2)/(c*x^2+b*x+a)^{(3/2)}-1/2*f^{(3/2)}*\operatorname{arctanh}(1/2*(b*d^{(1/2)}-2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}-b*f^{(1/2)}))/((c*x^2+b*x+a)^{(1/2)}*(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(1/2)})*(B*d^{(1/2)}-A*f^{(1/2)})/d^{(1/2)}+(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(5/2)}+1/2*f^{(3/2)}*\operatorname{arctanh}(1/2*(b*d^{(1/2)}+2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}+b*f^{(1/2)}))/((c*x^2+b*x+a)^{(1/2)}*(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(1/2)})*(B*d^{(1/2)}+A*f^{(1/2)})/d^{(1/2)}+(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(5/2)}-2/3*(3*b^6*B*d*f^2+24*a^2*B*c^2*f*(a*f+c*d)^2-A*b^5*f^2*(6*a*f+7*c*d)-b^4*B*f*(-3*a^2*f^2+14*a*c*d*f+7*c^2*d^2)+A*b^3*c*f*(43*a^2*f^2+46*a*c*d*f+15*c^2*d^2)+2*b^2*B*c*(-11*a^3*f^3+4*a^2*c*d*f^2+5*a*c^2*d^2*f+2*c^3*d^3)-4*A*b*c^2*(17*a^3*f^3+24*a^2*c*d*f^2+9*a*c^2*d^2*f+2*c^3*d^3)+c*(3*b^5*B*d*f^2-2*A*b^4*f^2*(3*a*f+4*c*d)-8*A*c^2*(a*f+c*d)^2*(5*a*f+2*c*d)-b^3*B*f*(-3*a^2*f^2+10*a*c*d*f+17*c^2*d^2)+2*A*b^2*c*f*(19*a^2*f^2+22*a*c*d*f+15*c^2*d^2)+4*b*B*c*(-5*a^3*f^3+4*a^2*c*d*f^2+11*a*c^2*d^2*f+2*c^3*d^3))*x)/(-4*a*c+b^2)^2/(c^2*d^2+2*a*c*d*f-f*(-a^2*f+b^2*d))^2/(c*x^2+b*x+a)^{(1/2)}$

Rubi [A] time = 1.87, antiderivative size = 796, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1018, 1064, 1033, 724, 206}

$$\frac{(B\sqrt{d} - A\sqrt{f}) \tanh^{-1}\left(\frac{-2\sqrt{f}a+(2c\sqrt{d}-b\sqrt{f})x+b\sqrt{d}}{2\sqrt{-\sqrt{d}}\sqrt{f}b+cd+af\sqrt{cx^2+bx+a}}\right) f^{3/2}}{2\sqrt{d}(-\sqrt{d}\sqrt{f}b+cd+af)^{5/2}} + \frac{(\sqrt{f}A + B\sqrt{d}) \tanh^{-1}\left(\frac{2\sqrt{f}a+(\sqrt{f}b+2c\sqrt{d})x+b\sqrt{d}}{2\sqrt{\sqrt{d}}\sqrt{f}b+cd+af\sqrt{cx^2+bx+a}}\right)}{2\sqrt{d}(\sqrt{d}\sqrt{f}b+cd+af)^{5/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(A + B*x)/((a + b*x + c*x^2)^{(5/2)}*(d - f*x^2)), x]$

[Out] $(-2*(A*b^3*f - A*b*c*(c*d + 3*a*f) + a*B*(2*c^2*d - b^2*f + 2*a*c*f) + c*(A*b^2*f + b*B*(c*d - a*f) - 2*A*c*(c*d + a*f))*x)/(3*(b^2 - 4*a*c)*(b^2*d*f - (c*d + a*f)^2)*(a + b*x + c*x^2)^{(3/2)}) - (2*(3*b^6*B*d*f^2 + 24*a^2*B*c^2*f*(c*d + a*f)^2 - A*b^5*f^2*(7*c*d + 6*a*f) - b^4*B*f*(7*c^2*d^2 + 14*a*$

$$\begin{aligned}
& c*d*f - 3*a^2*f^2) + A*b^3*c*f*(15*c^2*d^2 + 46*a*c*d*f + 43*a^2*f^2) + 2*b \\
& ^2*B*c*(2*c^3*d^3 + 5*a*c^2*d^2*f + 4*a^2*c*d*f^2 - 11*a^3*f^3) - 4*A*b*c^2 \\
& *(2*c^3*d^3 + 9*a*c^2*d^2*f + 24*a^2*c*d*f^2 + 17*a^3*f^3) + c*(3*b^5*B*d*f \\
& ^2 - 2*A*b^4*f^2*(4*c*d + 3*a*f) - 8*A*c^2*(c*d + a*f)^2*(2*c*d + 5*a*f) - \\
& b^3*B*f*(17*c^2*d^2 + 10*a*c*d*f - 3*a^2*f^2) + 2*A*b^2*c*f*(15*c^2*d^2 + 2 \\
& 2*a*c*d*f + 19*a^2*f^2) + 4*b*B*c*(2*c^3*d^3 + 11*a*c^2*d^2*f + 4*a^2*c*d*f \\
& ^2 - 5*a^3*f^3))*x)/(3*(b^2 - 4*a*c)^2*(c^2*d^2 + 2*a*c*d*f - f*(b^2*d - a \\
& ^2*f))^2*Sqrt[a + b*x + c*x^2]) - ((B*Sqrt[d] - A*Sqrt[f])*f^(3/2)*ArcTanh[\\
& (b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqr \\
& t[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2]))/(2*Sqrt[d]*(c*d - b*Sqrt[d]*Sqr \\
& rt[f] + a*f)^(5/2)) + ((B*Sqrt[d] + A*Sqrt[f])*f^(3/2)*ArcTanh[(b*Sqrt[d] + \\
& 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] \\
& + a*f]*Sqrt[a + b*x + c*x^2]))/(2*Sqrt[d]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f) \\
& ^{(5/2)})
\end{aligned}$$

Rule 206

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

```

Rule 724

```

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

```

Rule 1018

```

Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(
q + 1)*((g*c)*(-b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f
)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x)/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-b*f))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*(-b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
+ 5)*x^2, x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] && !( !IntegerQ[p
] && ILtQ[q, -1])

```


Rule 1033

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1064

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_.) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p +
1)*(d + f*x^2)^(q + 1)*((A*c - a*C)*(-(b*(c*d + a*f))) + (A*b - a*B)*(2*c^2
*d + b^2*f - c*(2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(2*a*f)) - B*(b*c*d + a
*b*f) + C*(b^2*d - 2*a*(c*d - a*f)))*x)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a
*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p +
1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)
*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + a
*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)
) - (2*f*((A*c - a*C)*(-(b*(c*d + a*f))) + (A*b - a*B)*(2*c^2*d + b^2*f - c
*(2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + a*B*f) + 2*(A*c*(c*d
- a*f) - a*(c*C*d - a*C*f)))*(b*f*(p + 1)))*x - c*f*(b^2*(C*d + A*f) - b*(
B*c*d + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(2*p + 2*q + 5)*x
^2, x], x] /; FreeQ[{a, b, c, d, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] && !(IntegerQ[p] &&
ILtQ[q, -1]) && !IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx}{(a + bx + cx^2)^{5/2} (d - fx^2)} dx &= -\frac{2(Ab^3f - Abc(cd + 3af) + aB(2c^2d - b^2f + 2acf) + c(Ab^2f + bB(cd - a))}{3(b^2 - 4ac)(b^2df - (cd + af)^2)(a + bx + cx^2)^{3/2}} \\
&= -\frac{2(Ab^3f - Abc(cd + 3af) + aB(2c^2d - b^2f + 2acf) + c(Ab^2f + bB(cd - a))}{3(b^2 - 4ac)(b^2df - (cd + af)^2)(a + bx + cx^2)^{3/2}} \\
&= -\frac{2(Ab^3f - Abc(cd + 3af) + aB(2c^2d - b^2f + 2acf) + c(Ab^2f + bB(cd - a))}{3(b^2 - 4ac)(b^2df - (cd + af)^2)(a + bx + cx^2)^{3/2}} \\
&= -\frac{2(Ab^3f - Abc(cd + 3af) + aB(2c^2d - b^2f + 2acf) + c(Ab^2f + bB(cd - a))}{3(b^2 - 4ac)(b^2df - (cd + af)^2)(a + bx + cx^2)^{3/2}} \\
&= -\frac{2(Ab^3f - Abc(cd + 3af) + aB(2c^2d - b^2f + 2acf) + c(Ab^2f + bB(cd - a))}{3(b^2 - 4ac)(b^2df - (cd + af)^2)(a + bx + cx^2)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 4.06, size = 674, normalized size = 0.85

$$2 \left(\frac{B(2a^2cf + a(b^2(-f) - bcfx + 2c^2d) + bc^2dx) + A(-bc(3af + cd) - 2c^2x(af + cd) + b^3f + b^2cfx)}{(a + x(b + cx))^{3/2}} - \frac{3f(-b^2(B(-a^2f^2 + 2acdf + c^2d^2) + 2aAcf^2x) + b^3f(Bcdx - A))}{\sqrt{a + x(b + cx)}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x)/((a + b*x + c*x^2)^(5/2)*(d - f*x^2)), x]

[Out] (2*((4*c*(-(A*b^2*f) + b*B*(-(c*d) + a*f) + 2*A*c*(c*d + a*f))*(b + 2*c*x)) / ((b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]) - (3*f*(b^4*B*d*f + 2*c*(c*d + a*f)^2*(-(a*B) + A*c*x) + b^3*f*(-(A*(c*d + 2*a*f)) + B*c*d*x) + b*c*(c*d + a*f))

$$\begin{aligned} &*(A*c*d + 5*a*A*f - 3*B*c*d*x + a*B*f*x) - b^2*(B*(c^2*d^2 + 2*a*c*d*f - a^2*f^2) + 2*a*A*c*f^2*x))/((c^2*d^2 + 2*a*c*d*f + f*(-(b^2*d) + a^2*f))*\text{Sqrt}[a + x*(b + c*x)]) + (A*(b^3*f - b*c*(c*d + 3*a*f) + b^2*c*f*x - 2*c^2*(c*d + a*f)*x) + B*(2*a^2*c*f + b*c^2*d*x + a*(2*c^2*d - b^2*f - b*c*f*x)))/(a + x*(b + c*x))^(3/2) + (3*(b^2 - 4*a*c)*f^(3/2)*(((B*\text{Sqrt}[d]) + A*\text{Sqrt}[f])*(c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f)^2*\text{ArcTanh}[(-2*a*\text{Sqrt}[f] + 2*c*\text{Sqrt}[d]*x + b*(\text{Sqrt}[d] - \text{Sqrt}[f]*x))/(2*\text{Sqrt}[c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{Sqrt}[a + x*(b + c*x)])])/\text{Sqrt}[c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f] - ((B*\text{Sqrt}[d] + A*\text{Sqrt}[f])*(c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f)^2*\text{ArcTanh}[(-2*(a*\text{Sqrt}[f] + c*\text{Sqrt}[d]*x) - b*(\text{Sqrt}[d] + \text{Sqrt}[f]*x))/(2*\text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{Sqrt}[a + x*(b + c*x)])])/\text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]))/(4*\text{Sqrt}[d]*(-(b^2*d*f) + (c*d + a*f)^2)))/(3*(b^2 - 4*a*c)*(-(b^2*d*f) + (c*d + a*f)^2)) \end{aligned}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)^(5/2)/(-f*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)^(5/2)/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x);;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 5.84Conj Error: Bad Argument Type

maple [B] time = 0.03, size = 6422, normalized size = 8.06

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x+A)/(c*x^2+b*x+a)^(5/2)/(-f*x^2+d),x)

[Out] result too large to display

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(5/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2
-(c*((b*sqrt(4*d*f)) / (2*f) + (c*d)/f+a))
/f^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + Bx}{(d - fx^2)(cx^2 + bx + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/((d - f*x^2)*(a + b*x + c*x^2)^(5/2)),x)
```

```
[Out] int((A + B*x)/((d - f*x^2)*(a + b*x + c*x^2)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)**(5/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

$$3.10 \quad \int \frac{1+2x}{(-1+x^2)\sqrt{-1+x+x^2}} dx$$

Optimal. Leaf size=47

$$\frac{3}{2} \tanh^{-1}\left(\frac{1-3x}{2\sqrt{x^2+x-1}}\right) - \frac{1}{2} \tan^{-1}\left(\frac{x+3}{2\sqrt{x^2+x-1}}\right)$$

[Out] $-1/2*\arctan(1/2*(3+x)/(x^2+x-1)^{(1/2)})+3/2*\operatorname{arctanh}(1/2*(1-3*x)/(x^2+x-1)^{(1/2)})$

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1033, 724, 206, 204}

$$\frac{3}{2} \tanh^{-1}\left(\frac{1-3x}{2\sqrt{x^2+x-1}}\right) - \frac{1}{2} \tan^{-1}\left(\frac{x+3}{2\sqrt{x^2+x-1}}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x)/((-1 + x^2)*Sqrt[-1 + x + x^2]), x]

[Out] $-\operatorname{ArcTan}[(3+x)/(2*\operatorname{Sqrt}[-1+x+x^2])]/2 + (3*\operatorname{ArcTanh}[(1-3*x)/(2*\operatorname{Sqrt}[-1+x+x^2])])/2$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\begin{aligned} \int \frac{1+2x}{(-1+x^2)\sqrt{-1+x+x^2}} dx &= \frac{1}{2} \int \frac{1}{(1+x)\sqrt{-1+x+x^2}} dx + \frac{3}{2} \int \frac{1}{(-1+x)\sqrt{-1+x+x^2}} dx \\ &= -\left(3 \operatorname{Subst}\left(\int \frac{1}{4-x^2} dx, x, \frac{-1+3x}{\sqrt{-1+x+x^2}}\right)\right) - \operatorname{Subst}\left(\int \frac{1}{-4-x^2} dx, x, \frac{-3}{\sqrt{-1+x+x^2}}\right) \\ &= \frac{1}{2} \tan^{-1}\left(\frac{-3-x}{2\sqrt{-1+x+x^2}}\right) + \frac{3}{2} \tanh^{-1}\left(\frac{1-3x}{2\sqrt{-1+x+x^2}}\right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.04

$$\frac{1}{2} \tan^{-1}\left(\frac{-x-3}{2\sqrt{x^2+x-1}}\right) - \frac{3}{2} \tanh^{-1}\left(\frac{3x-1}{2\sqrt{x^2+x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)/((-1 + x^2)*Sqrt[-1 + x + x^2]), x]
```

```
[Out] ArcTan[(-3 - x)/(2*Sqrt[-1 + x + x^2])]/2 - (3*ArcTanh[(-1 + 3*x)/(2*Sqrt[-1 + x + x^2])])/2
```

fricas [A] time = 1.12, size = 46, normalized size = 0.98

$$\arctan\left(-x + \sqrt{x^2 + x - 1} - 1\right) - \frac{3}{2} \log\left(-x + \sqrt{x^2 + x - 1} + 2\right) + \frac{3}{2} \log\left(-x + \sqrt{x^2 + x - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2-1)/(x^2+x-1)^(1/2), x, algorithm="fricas")
```

```
[Out] arctan(-x + sqrt(x^2 + x - 1) - 1) - 3/2*log(-x + sqrt(x^2 + x - 1) + 2) + 3/2*log(-x + sqrt(x^2 + x - 1))
```

giac [A] time = 0.25, size = 48, normalized size = 1.02

$$\arctan\left(-x + \sqrt{x^2 + x - 1} - 1\right) - \frac{3}{2} \log\left(\left|-x + \sqrt{x^2 + x - 1} + 2\right|\right) + \frac{3}{2} \log\left(\left|-x + \sqrt{x^2 + x - 1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x^2-1)/(x^2+x-1)^(1/2),x, algorithm="giac")

[Out] arctan(-x + sqrt(x^2 + x - 1) - 1) - 3/2*log(abs(-x + sqrt(x^2 + x - 1) + 2)) + 3/2*log(abs(-x + sqrt(x^2 + x - 1)))

maple [A] time = 0.02, size = 46, normalized size = 0.98

$$-\frac{3 \operatorname{arctanh}\left(\frac{3x-1}{2\sqrt{3x+(x-1)^2-2}}\right)}{2} + \frac{\operatorname{arctan}\left(\frac{-x-3}{2\sqrt{-x+(x+1)^2-2}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+2*x)/(x^2-1)/(x^2+x-1)^(1/2),x)

[Out] -3/2*arctanh(1/2*(-1+3*x)/((x-1)^2+3*x-2)^(1/2))+1/2*arctan(1/2*(-3-x)/((1+x)^2-x-2)^(1/2))

maxima [A] time = 0.97, size = 65, normalized size = 1.38

$$-\frac{1}{2} \arcsin\left(\frac{2\sqrt{5}x}{5|2x+2|} + \frac{6\sqrt{5}}{5|2x+2|}\right) - \frac{3}{2} \log\left(\frac{2\sqrt{x^2+x-1}}{|2x-2|} + \frac{2}{|2x-2|} + \frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x^2-1)/(x^2+x-1)^(1/2),x, algorithm="maxima")

[Out] -1/2*arcsin(2/5*sqrt(5)*x/abs(2*x + 2) + 6/5*sqrt(5)/abs(2*x + 2)) - 3/2*log(2*sqrt(x^2 + x - 1)/abs(2*x - 2) + 2/abs(2*x - 2) + 3/2)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{2x+1}{(x^2-1)\sqrt{x^2+x-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x + 1)/((x^2 - 1)*(x + x^2 - 1)^(1/2)),x)

[Out] int((2*x + 1)/((x^2 - 1)*(x + x^2 - 1)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+1}{(x-1)(x+1)\sqrt{x^2+x-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x**2-1)/(x**2+x-1)**(1/2),x)
```

```
[Out] Integral((2*x + 1)/((x - 1)*(x + 1)*sqrt(x**2 + x - 1)), x)
```


$$3.11 \quad \int \frac{1+2x}{(1+x^2)\sqrt{-1+x+x^2}} dx$$

Optimal. Leaf size=117

$$\sqrt{\frac{1}{2}}(\sqrt{5}-2) \tanh^{-1}\left(\frac{\sqrt{5}x-2\sqrt{5}+5}{\sqrt{10}(\sqrt{5}-2)\sqrt{x^2+x-1}}\right) - \sqrt{\frac{1}{2}}(2+\sqrt{5}) \tan^{-1}\left(\frac{-\sqrt{5}x+2\sqrt{5}+5}{\sqrt{10}(2+\sqrt{5})\sqrt{x^2+x-1}}\right)$$

[Out] 1/2*arctanh((5-2*5^(1/2)+x*5^(1/2))/(x^2+x-1)^(1/2)/(-20+10*5^(1/2))^(1/2)) *(-4+2*5^(1/2))^(1/2)-1/2*arctan((5+2*5^(1/2)-x*5^(1/2))/(x^2+x-1)^(1/2)/(2+10*5^(1/2))^(1/2))*(4+2*5^(1/2))^(1/2)

Rubi [A] time = 0.17, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1036, 1030, 207, 203}

$$\sqrt{\frac{1}{2}}(\sqrt{5}-2) \tanh^{-1}\left(\frac{\sqrt{5}x-2\sqrt{5}+5}{\sqrt{10}(\sqrt{5}-2)\sqrt{x^2+x-1}}\right) - \sqrt{\frac{1}{2}}(2+\sqrt{5}) \tan^{-1}\left(\frac{-\sqrt{5}x+2\sqrt{5}+5}{\sqrt{10}(2+\sqrt{5})\sqrt{x^2+x-1}}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x)/((1 + x^2)*Sqrt[-1 + x + x^2]), x]

[Out] -(Sqrt[(2 + Sqrt[5])/2]*ArcTan[(5 + 2*Sqrt[5] - Sqrt[5]*x)/(Sqrt[10*(2 + Sqrt[5])]*Sqrt[-1 + x + x^2])]) + Sqrt[(-2 + Sqrt[5])/2]*ArcTanh[(5 - 2*Sqrt[5] + Sqrt[5]*x)/(Sqrt[10*(-2 + Sqrt[5])]*Sqrt[-1 + x + x^2])])

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 207

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1030

Int[((g_) + (h_)*(x_))/(((a_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*

$e*x^2, x], x], x, \text{Simp}[a*h - g*c*x, x]/\text{Sqrt}[d + e*x + f*x^2]], x] /; \text{FreeQ}[\{a, c, d, e, f, g, h\}, x] \&\& \text{EqQ}[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]$

Rule 1036

$\text{Int}[\frac{(g_.) + (h_.)*(x_.)}{((a_.) + (c_.)*(x_.)^2)*\text{Sqrt}[(d_.) + (e_.)*(x_.) + (f_.)*(x_.)^2]}], x_Symbol] := \text{With}[\{q = \text{Rt}[(c*d - a*f)^2 + a*c*e^2, 2]\}, \text{Dist}[1/(2*q), \text{Int}[\text{Simp}[-(a*h*e) - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*e)*x, x]/((a + c*x^2)*\text{Sqrt}[d + e*x + f*x^2]), x], x] - \text{Dist}[1/(2*q), \text{Int}[\text{Simp}[-(a*h*e) - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c*x^2)*\text{Sqrt}[d + e*x + f*x^2]), x], x]] /; \text{FreeQ}[\{a, c, d, e, f, g, h\}, x] \&\& \text{NeQ}[e^2 - 4*d*f, 0] \&\& \text{NegQ}[-(a*c)]$

Rubi steps

$$\begin{aligned} \int \frac{1 + 2x}{(1 + x^2)\sqrt{-1 + x + x^2}} dx &= -\frac{\int \frac{-\sqrt{5} + (-5 - 2\sqrt{5})x}{(1 + x^2)\sqrt{-1 + x + x^2}} dx}{2\sqrt{5}} + \frac{\int \frac{\sqrt{5} + (-5 + 2\sqrt{5})x}{(1 + x^2)\sqrt{-1 + x + x^2}} dx}{2\sqrt{5}} \\ &= -\left((-5 + 2\sqrt{5}) \text{Subst}\left(\int \frac{1}{10(2 - \sqrt{5}) + x^2} dx, x, \frac{-5 + 2\sqrt{5} - \sqrt{5}x}{\sqrt{-1 + x + x^2}}\right)\right) + (5 + 2\sqrt{5}) \text{Subst}\left(\int \frac{1}{10(2 + \sqrt{5}) + x^2} dx, x, \frac{-5 + 2\sqrt{5} + \sqrt{5}x}{\sqrt{-1 + x + x^2}}\right) \\ &= -\sqrt{\frac{1}{2}}(2 + \sqrt{5}) \tan^{-1}\left(\frac{5 + 2\sqrt{5} - \sqrt{5}x}{\sqrt{10(2 + \sqrt{5})}\sqrt{-1 + x + x^2}}\right) + \sqrt{\frac{1}{2}}(-2 + \sqrt{5}) \tanh^{-1}\left(\frac{-5 + 2\sqrt{5} + \sqrt{5}x}{\sqrt{10(2 - \sqrt{5})}\sqrt{-1 + x + x^2}}\right) \end{aligned}$$

Mathematica [C] time = 0.03, size = 78, normalized size = 0.67

$$-\frac{1}{2}i \left(\sqrt{2+i} \tanh^{-1}\left(\frac{\sqrt{2+i}(x-i)}{2\sqrt{x^2+x-1}}\right) - \sqrt{2-i} \tanh^{-1}\left(\frac{\sqrt{2-i}(x+i)}{2\sqrt{x^2+x-1}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x)/((1 + x^2)*Sqrt[-1 + x + x^2]), x]

[Out] (-1/2*I)*(Sqrt[2 + I]*ArcTanh[(Sqrt[2 + I]*(-I + x))/(2*Sqrt[-1 + x + x^2])] - Sqrt[2 - I]*ArcTanh[(Sqrt[2 - I]*(I + x))/(2*Sqrt[-1 + x + x^2])])

fricas [B] time = 1.01, size = 758, normalized size = 6.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x^2+1)/(x^2+x-1)^(1/2),x, algorithm="fricas")

[Out] $\frac{1}{20}5^{1/4}\sqrt{4\sqrt{5}+10}(2\sqrt{5}-5)\log(2x^2-2\sqrt{x^2+x-1})x + \frac{1}{5}(5^{1/4}\sqrt{x^2+x-1}(2\sqrt{5}-5) - 5^{1/4}(\sqrt{5}(2x+1)-5x))\sqrt{4\sqrt{5}+10} + x + \sqrt{5}) - \frac{1}{20}5^{1/4}\sqrt{4\sqrt{5}+10}(2\sqrt{5}-5)\log(2x^2-2\sqrt{x^2+x-1})x - \frac{1}{5}(5^{1/4}\sqrt{x^2+x-1}(2\sqrt{5}-5) - 5^{1/4}(\sqrt{5}(2x+1)-5x))\sqrt{4\sqrt{5}+10} + x + \sqrt{5}) - \frac{1}{5}5^{3/4}\sqrt{4\sqrt{5}+10}\arctan(2/55\sqrt{5}(\sqrt{5}(2x-1)+3x+4) + 1/275\sqrt{10x^2-10\sqrt{x^2+x-1}}x + (5^{1/4}\sqrt{x^2+x-1}(2\sqrt{5}-5) - 5^{1/4}(\sqrt{5}(2x+1)-5x))\sqrt{4\sqrt{5}+10} + 5x + 5\sqrt{5}))((5^{3/4}(2\sqrt{5}+3) + 2*5^{1/4}(4\sqrt{5}-5))\sqrt{4\sqrt{5}+10} + 2\sqrt{5}(3\sqrt{5}+10) - 20\sqrt{5} + 80) - 2/55\sqrt{x^2+x-1}(\sqrt{5}(2\sqrt{5}+3) + 8\sqrt{5} - 10) + 1/55\sqrt{5}(16x+3) + 1/275(5^{3/4}(\sqrt{5}(3x+4) + 10x - 5) - \sqrt{x^2+x-1}(5^{3/4}(3\sqrt{5}+10) - 10*5^{1/4}(\sqrt{5}-4)) - 10*5^{1/4}(\sqrt{5}(x-6) - 4x+13))\sqrt{4\sqrt{5}+10} - 4/11x + 2/11) - 1/5*5^{3/4}\sqrt{4\sqrt{5}+10}\arctan(-2/55\sqrt{5}(\sqrt{5}(2x-1)+3x+4) + 1/275\sqrt{10x^2-10\sqrt{x^2+x-1}}x - (5^{1/4}\sqrt{x^2+x-1}(2\sqrt{5}-5) - 5^{1/4}(\sqrt{5}(2x+1)-5x))\sqrt{4\sqrt{5}+10} + 5x + 5\sqrt{5}))((5^{3/4}(2\sqrt{5}+3) + 2*5^{1/4}(4\sqrt{5}-5))\sqrt{4\sqrt{5}+10} - 2\sqrt{5}(3\sqrt{5}+10) + 20\sqrt{5} - 80) + 2/55\sqrt{x^2+x-1}(\sqrt{5}(2\sqrt{5}+3) + 8\sqrt{5} - 10) - 1/55\sqrt{5}(16x+3) + 1/275(5^{3/4}(\sqrt{5}(3x+4) + 10x - 5) - \sqrt{x^2+x-1}(5^{3/4}(3\sqrt{5}+10) - 10*5^{1/4}(\sqrt{5}-4)) - 10*5^{1/4}(\sqrt{5}(x-6) - 4x+13))\sqrt{4\sqrt{5}+10} + 4/11x - 2/11)$

giac [B] time = 0.38, size = 457, normalized size = 3.91

$$\frac{1}{4}\sqrt{2\sqrt{5}-4}\log\left(16\left(15\sqrt{5}\left(x-\sqrt{x^2+x-1}\right)+33x+5\sqrt{5}-33\sqrt{x^2+x-1}+2\sqrt{5\sqrt{5}+11}+11\right)^2+16\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x^2+1)/(x^2+x-1)^(1/2),x, algorithm="giac")

[Out] $\frac{1}{4}\sqrt{2\sqrt{5}-4}\log(16(15\sqrt{5}(x-\sqrt{x^2+x-1}))+33x+5\sqrt{5}-33\sqrt{x^2+x-1}+2\sqrt{5\sqrt{5}+11}+11)^2+16(5\sqrt{5}(x-\sqrt{x^2+x-1}))+11x-5\sqrt{5}\sqrt{5\sqrt{5}+11}-15\sqrt{5}-11\sqrt{x^2+x-1}-11\sqrt{5\sqrt{5}+11}-33)^2-\frac{1}{4}\sqrt{2\sqrt{5}-4}\log(16(15\sqrt{5}(x-\sqrt{x^2+x-1}))+33x+5\sqrt{5}-33\sqrt{x^2+x-1}-2\sqrt{5\sqrt{5}+11}+11)^2+16(5\sqrt{5}(x-\sqrt{x^2+x-1}))+11x+5\sqrt{5}\sqrt{5\sqrt{5}+11}-15\sqrt{5}-11\sqrt{x^2+x-1}+11\sqrt{5\sqrt{5}+11}-33)^2+\frac{1}{2}\sqrt{2\sqrt{5}-4}(\arctan(3)+\arctan(1/10(x-\sqrt{x^2+x-1}))(\sqrt{5}\sqrt{5\sqrt{5}+11}))$

$$\frac{(5\sqrt{5} + 11) + 4\sqrt{5} - 5\sqrt{5}(5\sqrt{5} + 11) - 7/10\sqrt{5}\sqrt{5\sqrt{5} + 11} + 1/5\sqrt{5} + 3/2\sqrt{5}\sqrt{5\sqrt{5} + 11}}{(\sqrt{5} - 2) - 1/2\sqrt{5}\sqrt{5\sqrt{5} + 11} - 4\sqrt{5} - 5\sqrt{5}(5\sqrt{5} + 11) + 7/10\sqrt{5}\sqrt{5\sqrt{5} + 11} + 1/5\sqrt{5} - 3/2\sqrt{5}\sqrt{5\sqrt{5} + 11}} \cdot (\arctan(3) + \arctan(-1/10(x - \sqrt{x^2 + x - 1})) + \arctan(3) + \arctan(-1/10(x - \sqrt{x^2 + x - 1})))$$

maple [B] time = 0.18, size = 637, normalized size = 5.44

$$\sqrt{\frac{10(x-\sqrt{5}-2)^2}{(-x-\sqrt{5}+2)^2} - \frac{5\sqrt{5}(x-\sqrt{5}-2)^2}{(-x-\sqrt{5}+2)^2} + 10 + 5\sqrt{5}} \sqrt{5} \left(\operatorname{arctanh} \left(\frac{\sqrt{\frac{10(x-\sqrt{5}-2)^2}{(-x-\sqrt{5}+2)^2} - \frac{5\sqrt{5}(x-\sqrt{5}-2)^2}{(-x-\sqrt{5}+2)^2} + 10 + 5\sqrt{5}}}{\sqrt{20+10\sqrt{5}}} \right) + \sqrt{5} \operatorname{arctan} \left(\frac{\sqrt{5}}{\dots} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1+2*x)/(x^2+1)/(x^2+x-1)^(1/2), x)`

[Out] $(10*(-5^{1/2}-2+x)^2/(-5^{1/2}+2-x)^2-5*5^{1/2}*(-5^{1/2}-2+x)^2/(-5^{1/2}+2-x)^2+10+5*5^{1/2})^{1/2}*5^{1/2}*(\arctan(1/5*5^{1/2}*((5^{1/2}-2)*(-5^{1/2}-2+x)^2/(-5^{1/2}+2-x)^2+4*5^{1/2}+9))^{1/2}*(20+10*5^{1/2})^{1/2}*(5^{1/2}*(-5^{1/2}-2+x)^2/(-5^{1/2}+2-x)^2+2*(-5^{1/2}-2+x)^2/(-5^{1/2}+2-x)^2-5^{1/2}+2)*(-5^{1/2}-2+x)/(-5^{1/2}+2-x)*(5^{1/2}-2)/((-5^{1/2}-2+x)^4/(-5^{1/2}+2-x)^4-18*(-5^{1/2}-2+x)^2/(-5^{1/2}+2-x)^2+1))*5^{1/2}+\operatorname{arctanh}((10*(-5^{1/2}-2+x)^2/(-5^{1/2}+2-x)^2-5*5^{1/2}*(-5^{1/2}-2+x)^2/(-5^{1/2}+2-x)^2+10+5*5^{1/2})^{1/2}/(20+10*5^{1/2})^{1/2})+2*\operatorname{arctan}(1/5*5^{1/2}*((5^{1/2}-2)*(-5^{1/2}-2+x)^2/(-5^{1/2}+2-x)^2+4*5^{1/2}+9))^{1/2}*(20+10*5^{1/2})^{1/2}*(5^{1/2}*(-5^{1/2}-2+x)^2/(-5^{1/2}+2-x)^2+2*(-5^{1/2}-2+x)^2/(-5^{1/2}+2-x)^2-5^{1/2}+2)*(-5^{1/2}-2+x)/(-5^{1/2}+2-x)*(5^{1/2}-2)/((-5^{1/2}-2+x)^4/(-5^{1/2}+2-x)^4-18*(-5^{1/2}-2+x)^2/(-5^{1/2}+2-x)^2+1)))/(-5*(5^{1/2}*(-5^{1/2}-2+x)^2/(-5^{1/2}+2-x)^2-2*(-5^{1/2}-2+x)^2/(-5^{1/2}+2-x)^2-5^{1/2}-2)/((-5^{1/2}-2+x)/(-5^{1/2}+2-x)+1)^2)^{1/2}/((-5^{1/2}-2+x)/(-5^{1/2}+2-x)+1)/(20+10*5^{1/2})^{1/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+1}{\sqrt{x^2+x-1}(x^2+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x^2+1)/(x^2+x-1)^(1/2),x, algorithm="maxima")

[Out] integrate((2*x + 1)/(sqrt(x^2 + x - 1)*(x^2 + 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2x+1}{(x^2+1)\sqrt{x^2+x-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x + 1)/((x^2 + 1)*(x + x^2 - 1)^(1/2)),x)

[Out] int((2*x + 1)/((x^2 + 1)*(x + x^2 - 1)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+1}{(x^2+1)\sqrt{x^2+x-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x**2+1)/(x**2+x-1)**(1/2),x)

[Out] Integral((2*x + 1)/((x**2 + 1)*sqrt(x**2 + x - 1)), x)

$$3.12 \quad \int \frac{a-c+bx}{(1+x^2)\sqrt{a+bx+cx^2}} dx$$

Optimal. Leaf size=484

$$\frac{\sqrt{-a\left(2c - \sqrt{a^2 - 2ac + b^2 + c^2}\right) + c\left(c - \sqrt{a^2 - 2ac + b^2 + c^2}\right) + a^2 + b^2} \tan^{-1}\left(\frac{b\sqrt{a^2 - 2ac + b^2 + c^2} - x}{\sqrt{2}\sqrt[4]{a^2 - 2ac + b^2 + c^2}\sqrt{-a\left(2c - \sqrt{a^2 - 2ac + b^2 + c^2}\right) + c\left(c - \sqrt{a^2 - 2ac + b^2 + c^2}\right) + a^2 + b^2}}\right)}{\sqrt{2}\sqrt[4]{a^2 - 2ac + b^2 + c^2}}$$

[Out] $-1/2*\arctan(1/2*(b*(a^2-2*a*c+b^2+c^2)^(1/2)-x*(b^2+(a-c)*(a-c+(a^2-2*a*c+b^2+c^2)^(1/2))))/(a^2-2*a*c+b^2+c^2)^(1/4)*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(a^2+b^2+c*(c-(a^2-2*a*c+b^2+c^2)^(1/2))-a*(2*c-(a^2-2*a*c+b^2+c^2)^(1/2)))^(1/2))*((a^2+b^2+c*(c-(a^2-2*a*c+b^2+c^2)^(1/2))-a*(2*c-(a^2-2*a*c+b^2+c^2)^(1/2)))^(1/2)/(a^2-2*a*c+b^2+c^2)^(1/4)*2^(1/2))-1/2*\operatorname{arctanh}(1/2*(x*(b^2+(a-c)*(a-c-(a^2-2*a*c+b^2+c^2)^(1/2))))+b*(a^2-2*a*c+b^2+c^2)^(1/2))/(a^2-2*a*c+b^2+c^2)^(1/4)*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(a^2+b^2+c*(c+(a^2-2*a*c+b^2+c^2)^(1/2))-a*(2*c+(a^2-2*a*c+b^2+c^2)^(1/2)))^(1/2))*((a^2+b^2+c*(c+(a^2-2*a*c+b^2+c^2)^(1/2))-a*(2*c+(a^2-2*a*c+b^2+c^2)^(1/2)))^(1/2)/(a^2-2*a*c+b^2+c^2)^(1/4)*2^(1/2))$

Rubi [A] time = 23.58, antiderivative size = 484, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1036, 1030, 208, 205}

$$\frac{\sqrt{-a\left(2c - \sqrt{a^2 - 2ac + b^2 + c^2}\right) + c\left(c - \sqrt{a^2 - 2ac + b^2 + c^2}\right) + a^2 + b^2} \tan^{-1}\left(\frac{b\sqrt{a^2 - 2ac + b^2 + c^2} - x}{\sqrt{2}\sqrt[4]{a^2 - 2ac + b^2 + c^2}\sqrt{-a\left(2c - \sqrt{a^2 - 2ac + b^2 + c^2}\right) + c\left(c - \sqrt{a^2 - 2ac + b^2 + c^2}\right) + a^2 + b^2}}\right)}{\sqrt{2}\sqrt[4]{a^2 - 2ac + b^2 + c^2}}$$

Antiderivative was successfully verified.

[In] Int[(a - c + b*x)/((1 + x^2)*Sqrt[a + b*x + c*x^2]), x]

[Out] $-((\operatorname{Sqrt}[a^2 + b^2 + c*(c - \operatorname{Sqrt}[a^2 + b^2 - 2*a*c + c^2])] - a*(2*c - \operatorname{Sqrt}[a^2 + b^2 - 2*a*c + c^2]))*\operatorname{ArcTan}[(b*\operatorname{Sqrt}[a^2 + b^2 - 2*a*c + c^2] - (b^2 + (a - c)*(a - c + \operatorname{Sqrt}[a^2 + b^2 - 2*a*c + c^2]))*x)/(\operatorname{Sqrt}[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*\operatorname{Sqrt}[a^2 + b^2 + c*(c - \operatorname{Sqrt}[a^2 + b^2 - 2*a*c + c^2])] - a*(2*c - \operatorname{Sqrt}[a^2 + b^2 - 2*a*c + c^2]))*\operatorname{Sqrt}[a + b*x + c*x^2]])/(\operatorname{Sqrt}[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4))) - (\operatorname{Sqrt}[a^2 + b^2 + c*(c + \operatorname{Sqrt}[a^2 + b^2 - 2*a*c + c^2])] - a*(2*c + \operatorname{Sqrt}[a^2 + b^2 - 2*a*c + c^2]))*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[a^2 + b^2 - 2*a*c + c^2] + (b^2 + (a - c)*(a - c - \operatorname{Sqrt}[a^2 + b^2 - 2*a*c + c^2]))*x)/(\operatorname{Sqrt}[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*\operatorname{Sqrt}[a^2 + b^2 + c*(c$

+ Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c + Sqrt[a^2 + b^2 - 2*a*c + c^2]))
*Sqrt[a + b*x + c*x^2]])/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4))

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1030

Int[((g_) + (h_)*(x_))/((a_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]

Rule 1036

Int[((g_) + (h_)*(x_))/((a_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist[1/(2*q), Int[Simp[-(a*h*e) - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[-(a*h*e) - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && NegQ[-(a*c)]

Rubi steps

$$\int \frac{a - c + bx}{(1 + x^2)\sqrt{a + bx + cx^2}} dx = -\frac{\int \frac{-b^2 - (a-c)(a-c + \sqrt{a^2 + b^2 - 2ac + c^2}) - b\sqrt{a^2 + b^2 - 2ac + c^2}x}{(1+x^2)\sqrt{a+bx+cx^2}} dx}{2\sqrt{a^2 + b^2 - 2ac + c^2}} + \frac{\int \frac{-b^2 - (a-c)(a-c - \sqrt{a^2 + b^2 - 2ac + c^2})}{(1+x^2)\sqrt{a+bx+cx^2}} dx}{2\sqrt{a^2 + b^2 - 2ac + c^2}}$$

$$= \left(b \left(b^2 + (a-c) \left(a-c - \sqrt{a^2 + b^2 - 2ac + c^2} \right) \right) \right) \text{Subst} \left(\int \frac{1}{-2b\sqrt{a^2 + b^2 - 2ac + c^2} + \sqrt{a^2 + b^2 + c \left(c - \sqrt{a^2 + b^2 - 2ac + c^2} \right) - a \left(2c - \sqrt{a^2 + b^2 - 2ac + c^2} \right)}} \tan^{-1} \right)$$

$$= -\frac{\sqrt{2} \sqrt{a^2 + b^2 - 2ac + c^2}}{\sqrt{2} \sqrt{a^2 + b^2 - 2ac + c^2}}$$

Mathematica [C] time = 0.08, size = 136, normalized size = 0.28

$$\frac{1}{2}i \left(\sqrt{a + ib - c} \tanh^{-1} \left(\frac{2a + b(x + i) + 2icx}{2\sqrt{a + ib - c} \sqrt{a + x(b + cx)}} \right) - \sqrt{a - ib - c} \tanh^{-1} \left(\frac{2a + b(x - i) - 2icx}{2\sqrt{a - ib - c} \sqrt{a + x(b + cx)}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a - c + b*x)/((1 + x^2)*Sqrt[a + b*x + c*x^2]),x]

[Out] (I/2)*(-(Sqrt[a - I*b - c]*ArcTanh[(2*a - (2*I)*c*x + b*(-I + x))/(2*Sqrt[a - I*b - c]*Sqrt[a + x*(b + c*x)])]) + Sqrt[a + I*b - c]*ArcTanh[(2*a + (2*I)*c*x + b*(I + x))/(2*Sqrt[a + I*b - c]*Sqrt[a + x*(b + c*x)])])

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a-c)/(x^2+1)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a-c)/(x^2+1)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the root
 of a polynomial with parameters. This might be wrong.The choice was done
 assuming [a,b,c]=[42,62,91]Warning, choosing root of [1,0,%%{-2,[2,0,0]%%
 }+%%{4,[1,0,1]%%}+%%{4,[1,0,0]%%}+%%{-2,[0,2,0]%%}+%%{-2,[0,0,2]%%}
 +%%{-4,[0,0,1]%%},%%{-8,[2,0,0]%%}+%%{16,[1,0,1]%%}+%%{-8,[0,2,0]%%
 }+%%{-8,[0,0,2]%%},%%{1,[4,0,0]%%}+%%{-4,[3,0,1]%%}+%%{4,[3,0,0]%%}
 +%%{2,[2,2,0]%%}+%%{6,[2,0,2]%%}+%%{-12,[2,0,1]%%}+%%{-4,[1,2,1]%%}
 +%%{4,[1,2,0]%%}+%%{-4,[1,0,3]%%}+%%{12,[1,0,2]%%}+%%{1,[0,4,0]%%}+
 %%{2,[0,2,2]%%}+%%{-4,[0,2,1]%%}+%%{-4,[0,2,0]%%}+%%{1,[0,0,4]%%}+
 %%{-4,[0,0,3]%%}] at parameters values [-27,26,-89]Warning, need to choose
 a branch for the root of a polynomial with parameters. This might be wrong
 .The choice was done assuming [a,b]=[-66,-52]Evaluation time: 7.83Unable to
 convert to real %%{1.0,[2]%%}+%%{132.0,[1]%%}+%%{7060.0,[0]%%} Error
 : Bad Argument Value

maple [B] time = 0.52, size = 6871419, normalized size = 14197.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a-c)/(x^2+1)/(c*x^2+b*x+a)^(1/2),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx + a - c}{\sqrt{cx^2 + bx + a}(x^2 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a-c)/(x^2+1)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")

[Out] integrate((b*x + a - c)/(sqrt(c*x^2 + b*x + a)*(x^2 + 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a - c + bx}{(x^2 + 1) \sqrt{cx^2 + bx + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a - c + b*x)/((x^2 + 1)*(a + b*x + c*x^2)^(1/2)),x)

[Out] int((a - c + b*x)/((x^2 + 1)*(a + b*x + c*x^2)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx - c}{(x^2 + 1)\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a-c)/(x**2+1)/(c*x**2+b*x+a)**(1/2),x)

[Out] Integral((a + b*x - c)/((x**2 + 1)*sqrt(a + b*x + c*x**2)), x)

$$3.13 \quad \int \frac{(A+Bx)(a+bx+cx^2)}{d+ex+fx^2} dx$$

Optimal. Leaf size=184

$$\frac{\log(d+ex+fx^2)(Af(ce-bf)-B(af^2-bef-cdf+ce^2))}{2f^3} \frac{\tanh^{-1}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)(Af(2af^2-bef-2cdf+cdf^2))}{f^3\sqrt{e^2-4df}}$$

[Out] $-(A*c*f-B*b*f+B*c*e)*x/f^2+1/2*B*c*x^2/f-1/2*(A*f*(-b*f+c*e)-B*(a*f^2-b*e*f-c*d*f+c*e^2))*\ln(f*x^2+e*x+d)/f^3-(A*f*(2*a*f^2-b*e*f-2*c*d*f+c*e^2)+B*(f*(-a*e*f-2*b*d*f+b*e^2)-c*(-3*d*e*f+e^3)))*\operatorname{arctanh}((2*f*x+e)/(-4*d*f+e^2)^{(1/2)})/f^3/(-4*d*f+e^2)^{(1/2)}$

Rubi [A] time = 0.35, antiderivative size = 182, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1628, 634, 618, 206, 628}

$$\frac{\log(d+ex+fx^2)(Bf(be-af)+Af(ce-bf)-Bc(e^2-df))}{2f^3} \frac{\tanh^{-1}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)(Af(2af^2-bef-2cdf+cdf^2))}{f^3\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

[In] Int[((A + B*x)*(a + b*x + c*x^2))/(d + e*x + f*x^2), x]

[Out] $-(((B*c*e - b*B*f - A*c*f)*x)/f^2) + (B*c*x^2)/(2*f) - ((B*f*(b*e^2 - 2*b*d*f - a*e*f) - B*c*(e^3 - 3*d*e*f) + A*f*(c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2))*\operatorname{ArcTanh}[(e + 2*f*x)/\operatorname{Sqrt}[e^2 - 4*d*f]]/(f^3*\operatorname{Sqrt}[e^2 - 4*d*f]) - ((B*f*(b*e - a*f) + A*f*(c*e - b*f) - B*c*(e^2 - d*f))*\operatorname{Log}[d + e*x + f*x^2])/(2*f^3)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p
_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p,
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned} \int \frac{(A + Bx)(a + bx + cx^2)}{d + ex + fx^2} dx &= \int \left(-\frac{Bce - bBf - Acf}{f^2} + \frac{Bcx}{f} + \frac{-Af(cd - af) + Bd(ce - bf) - (Bf(be - af) + Af(ce - bf) - Bc(e^2 - df))}{f^2(d + ex + fx^2)} \right) dx \\ &= -\frac{(Bce - bBf - Acf)x}{f^2} + \frac{Bcx^2}{2f} + \frac{\int \frac{-Af(cd - af) + Bd(ce - bf) - (Bf(be - af) + Af(ce - bf) - Bc(e^2 - df))}{d + ex + fx^2} dx}{f^2} \\ &= -\frac{(Bce - bBf - Acf)x}{f^2} + \frac{Bcx^2}{2f} + \frac{(-Bf(be - af) - Af(ce - bf) + Bc(e^2 - df)) \log(d + ex + fx^2)}{2f^3} \\ &= -\frac{(Bce - bBf - Acf)x}{f^2} + \frac{Bcx^2}{2f} - \frac{(Bf(be - af) + Af(ce - bf) - Bc(e^2 - df)) \log(d + ex + fx^2)}{2f^3} \\ &= -\frac{(Bce - bBf - Acf)x}{f^2} + \frac{Bcx^2}{2f} - \frac{(Bf(be^2 - 2bdf - aef) - Bc(e^3 - 3def) + Af(e^2 - df)) \log(d + ex + fx^2)}{f^3 \sqrt{e^2 - 4df}} \end{aligned}$$

Mathematica [A] time = 0.21, size = 175, normalized size = 0.95

$$\frac{\log(d + x(e + fx))(Bf(af - be) + Af(bf - ce) + Bc(e^2 - df)) - \frac{2 \tan^{-1}\left(\frac{e + 2fx}{\sqrt{4df - e^2}}\right)(Af(-2af^2 + bef + 2cdf - ce^2) + Bf(aef + 2bdf - ce^2))}{\sqrt{4df - e^2}}}{2f^3}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*x)*(a + b*x + c*x^2))/(d + e*x + f*x^2),x]

[Out] (2*f*(-(B*c*e) + b*B*f + A*c*f)*x + B*c*f^2*x^2 - (2*(B*f*(-(b*e^2) + 2*b*d*f + a*e*f) + B*c*(e^3 - 3*d*e*f) + A*f*(-(c*e^2) + 2*c*d*f + b*e*f - 2*a*f^2))*ArcTan[(e + 2*f*x)/Sqrt[-e^2 + 4*d*f]])/Sqrt[-e^2 + 4*d*f] + (B*f*(-(b*e) + a*f) + A*f*(-(c*e) + b*f) + B*c*(e^2 - d*f))*Log[d + x*(e + f*x)]/(2*f^3)

fricas [A] time = 1.39, size = 583, normalized size = 3.17

$$\left[\frac{(Bce^2f^2 - 4Bcdf^3)x^2 - (Bce^3 - 2Aaf^3 + (2(Bb + Ac)d + (Ba + Ab)e)f^2 - (3Bcde + (Bb + Ac)e^2)f)\sqrt{e^2 - 4d}}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] [1/2*((B*c*e^2*f^2 - 4*B*c*d*f^3)*x^2 - (B*c*e^3 - 2*A*a*f^3 + (2*(B*b + A*c)*d + (B*a + A*b)*e)*f^2 - (3*B*c*d*e + (B*b + A*c)*e^2)*f)*sqrt(e^2 - 4*d*f)*log((2*f^2*x^2 + 2*e*f*x + e^2 - 2*d*f - sqrt(e^2 - 4*d*f)*(2*f*x + e))/(f*x^2 + e*x + d)) - 2*(B*c*e^3*f + 4*(B*b + A*c)*d*f^3 - (4*B*c*d*e + (B*b + A*c)*e^2)*f^2)*x + (B*c*e^4 - 4*(B*a + A*b)*d*f^3 + (4*B*c*d^2 + 4*(B*b + A*c)*d*e + (B*a + A*b)*e^2)*f^2 - (5*B*c*d*e^2 + (B*b + A*c)*e^3)*f)*log(f*x^2 + e*x + d)/(e^2*f^3 - 4*d*f^4), 1/2*((B*c*e^2*f^2 - 4*B*c*d*f^3)*x^2 + 2*(B*c*e^3 - 2*A*a*f^3 + (2*(B*b + A*c)*d + (B*a + A*b)*e)*f^2 - (3*B*c*d*e + (B*b + A*c)*e^2)*f)*sqrt(-e^2 + 4*d*f)*arctan(-sqrt(-e^2 + 4*d*f)*(2*f*x + e)/(e^2 - 4*d*f)) - 2*(B*c*e^3*f + 4*(B*b + A*c)*d*f^3 - (4*B*c*d*e + (B*b + A*c)*e^2)*f^2)*x + (B*c*e^4 - 4*(B*a + A*b)*d*f^3 + (4*B*c*d^2 + 4*(B*b + A*c)*d*e + (B*a + A*b)*e^2)*f^2 - (5*B*c*d*e^2 + (B*b + A*c)*e^3)*f)*log(f*x^2 + e*x + d)/(e^2*f^3 - 4*d*f^4)]

giac [A] time = 0.17, size = 191, normalized size = 1.04

$$\frac{Bcfx^2 + 2Bbf x + 2Acfx - 2Bcxe}{2f^2} - \frac{(Bcdf - Baf^2 - Abf^2 + Bbfe + Acfe - Bce^2)\log(fx^2 + xe + d)}{2f^3} - \frac{(2Bbd}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] 1/2*(B*c*f*x^2 + 2*B*b*f*x + 2*A*c*f*x - 2*B*c*x*e)/f^2 - 1/2*(B*c*d*f - B*a*f^2 - A*b*f^2 + B*b*f*e + A*c*f*e - B*c*e^2)*log(f*x^2 + x*e + d)/f^3 - (

$$2B*b*d*f^2 + 2A*c*d*f^2 - 2A*a*f^3 - 3B*c*d*f*e + B*a*f^2*e + A*b*f^2*e - B*b*f*e^2 - A*c*f*e^2 + B*c*e^3) \arctan((2*f*x + e)/\sqrt{4*d*f - e^2}) / (\sqrt{4*d*f - e^2} * f^3)$$

maple [B] time = 0.01, size = 510, normalized size = 2.77

$$\frac{2Aa \arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)}{\sqrt{4df-e^2}} - \frac{Abe \arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)}{\sqrt{4df-e^2} f} - \frac{2Acd \arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)}{\sqrt{4df-e^2} f} + \frac{Ace^2 \arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)}{\sqrt{4df-e^2} f^2} - \frac{Bae \arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)}{\sqrt{4df-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x+A)*(c*x^2+b*x+a)/(f*x^2+e*x+d), x)

[Out] $\frac{1}{2}B*c/f*x^2 + A*c/f*x + B*b/f*x - 1/f^2*B*c*e*x + 1/2/f*\ln(f*x^2+e*x+d)*A*b - 1/2/f^2*\ln(f*x^2+e*x+d)*A*c*e + 1/2/f*\ln(f*x^2+e*x+d)*B*a - 1/2/f^2*\ln(f*x^2+e*x+d)*B*b*e - 1/2/f^2*\ln(f*x^2+e*x+d)*B*c*d + 1/2/f^3*\ln(f*x^2+e*x+d)*B*c*e^2 + 2/(4*d*f - e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f - e^2)^{(1/2)})*A*a - 2/f/(4*d*f - e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f - e^2)^{(1/2)})*A*c*d - 2/f/(4*d*f - e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f - e^2)^{(1/2)})*B*b*d + 3/f^2/(4*d*f - e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f - e^2)^{(1/2)})*B*c*d*e - 1/f/(4*d*f - e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f - e^2)^{(1/2)})*e*A*b + 1/f^2/(4*d*f - e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f - e^2)^{(1/2)})*e^2*A*c - 1/f/(4*d*f - e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f - e^2)^{(1/2)})*e*B*a + 1/f^2/(4*d*f - e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f - e^2)^{(1/2)})*e^2*B*b - 1/f^3/(4*d*f - e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f - e^2)^{(1/2)})*e^3*B*c$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+e*x+d), x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f - e^2 > 0)', see 'assume?' for more details) Is 4*d*f - e^2 positive or negative?

mupad [B] time = 3.85, size = 273, normalized size = 1.48

$$x \left(\frac{Ac + Bb}{f} - \frac{Bce}{f^2} \right) \frac{\ln(fx^2 + ex + d) (Bce^4 - 4Abdf^3 - 4Badf^3 - Ace^3f - Bbe^3f + Abe^2f^2 + Ba e^2)}{2(4df^4 - e^2f^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x)*(a + b*x + c*x^2))/(d + e*x + f*x^2),x)`

[Out] $x \cdot \left(\frac{A \cdot c + B \cdot b}{f} - \frac{B \cdot c \cdot e}{f^2} \right) - \frac{\log(d + e \cdot x + f \cdot x^2) \cdot (B \cdot c \cdot e^4 - 4 \cdot A \cdot b \cdot d \cdot f^3 - 4 \cdot B \cdot a \cdot d \cdot f^3 - A \cdot c \cdot e^3 \cdot f - B \cdot b \cdot e^3 \cdot f + A \cdot b \cdot e^2 \cdot f^2 + B \cdot a \cdot e^2 \cdot f^2 + 4 \cdot B \cdot c \cdot d^2 \cdot f^2 + 4 \cdot A \cdot c \cdot d \cdot e \cdot f^2 + 4 \cdot B \cdot b \cdot d \cdot e \cdot f^2 - 5 \cdot B \cdot c \cdot d \cdot e^2 \cdot f)}{2 \cdot (4 \cdot d \cdot f^4 - e^2 \cdot f^3)} - \frac{\operatorname{atan}\left(\frac{e}{4 \cdot d \cdot f - e^2}\right)^{1/2} + \frac{2 \cdot f \cdot x}{(4 \cdot d \cdot f - e^2)^{1/2}} \cdot (B \cdot c \cdot e^3 - 2 \cdot A \cdot a \cdot f^3 + A \cdot b \cdot e \cdot f^2 + 2 \cdot A \cdot c \cdot d \cdot f^2 + B \cdot a \cdot e \cdot f^2 + 2 \cdot B \cdot b \cdot d \cdot f^2 - A \cdot c \cdot e^2 \cdot f - B \cdot b \cdot e^2 \cdot f - 3 \cdot B \cdot c \cdot d \cdot e \cdot f)}{f^3 \cdot (4 \cdot d \cdot f - e^2)^{1/2}} + \frac{B \cdot c \cdot x^2}{2 \cdot f}$

sympy [B] time = 16.65, size = 1260, normalized size = 6.85

$$\frac{Bcx^2}{2f} + x \left(\frac{Ac}{f} + \frac{Bb}{f} - \frac{Bce}{f^2} \right) + \left(-\frac{\sqrt{-4df + e^2} \left(-2Aaf^3 + Abef^2 + 2Acd f^2 - Ace^2 f + Baef^2 + 2Bbd f^2 - Bbe^2 f \right)}{2f^3 (4df - e^2)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x+A)*(c*x**2+b*x+a)/(f*x**2+e*x+d),x)`

[Out] $B \cdot c \cdot x^2 / (2 \cdot f) + x \cdot \left(\frac{A \cdot c}{f} + \frac{B \cdot b}{f} - \frac{B \cdot c \cdot e}{f^2} \right) + \frac{(-\sqrt{-4 \cdot d \cdot f + e^2}) \cdot (-2 \cdot A \cdot a \cdot f^3 + A \cdot b \cdot e \cdot f^2 + 2 \cdot A \cdot c \cdot d \cdot f^2 - A \cdot c \cdot e^2 \cdot f + B \cdot a \cdot e \cdot f^2 + 2 \cdot B \cdot b \cdot d \cdot f^2 - B \cdot b \cdot e^2 \cdot f - 3 \cdot B \cdot c \cdot d \cdot e \cdot f + B \cdot c \cdot e^3)}{(2 \cdot f^3 \cdot (4 \cdot d \cdot f - e^2))} + \frac{(A \cdot b \cdot f^2 - A \cdot c \cdot e \cdot f + B \cdot a \cdot f^2 - B \cdot b \cdot e \cdot f - B \cdot c \cdot d \cdot f + B \cdot c \cdot e^2)}{(2 \cdot f^3)} \cdot \log(x + \frac{-A \cdot a \cdot e \cdot f^2 + 2 \cdot A \cdot b \cdot d \cdot f^2 - A \cdot c \cdot d \cdot e \cdot f + 2 \cdot B \cdot a \cdot d \cdot f^2 - B \cdot b \cdot d \cdot e \cdot f - 2 \cdot B \cdot c \cdot d^2 \cdot f + B \cdot c \cdot d \cdot e^2 - 4 \cdot d \cdot f^3 \cdot (-\sqrt{-4 \cdot d \cdot f + e^2}) \cdot (-2 \cdot A \cdot a \cdot f^3 + A \cdot b \cdot e \cdot f^2 + 2 \cdot A \cdot c \cdot d \cdot f^2 - A \cdot c \cdot e^2 \cdot f + B \cdot a \cdot e \cdot f^2 + 2 \cdot B \cdot b \cdot d \cdot f^2 - B \cdot b \cdot e^2 \cdot f - 3 \cdot B \cdot c \cdot d \cdot e \cdot f + B \cdot c \cdot e^3)}{(2 \cdot f^3 \cdot (4 \cdot d \cdot f - e^2))} + \frac{(A \cdot b \cdot f^2 - A \cdot c \cdot e \cdot f + B \cdot a \cdot f^2 - B \cdot b \cdot e \cdot f - B \cdot c \cdot d \cdot f + B \cdot c \cdot e^2)}{(2 \cdot f^3)} + \frac{e^2 \cdot f^2 \cdot (-\sqrt{-4 \cdot d \cdot f + e^2}) \cdot (-2 \cdot A \cdot a \cdot f^3 + A \cdot b \cdot e \cdot f^2 + 2 \cdot A \cdot c \cdot d \cdot f^2 - A \cdot c \cdot e^2 \cdot f + B \cdot a \cdot e \cdot f^2 + 2 \cdot B \cdot b \cdot d \cdot f^2 - B \cdot b \cdot e^2 \cdot f - 3 \cdot B \cdot c \cdot d \cdot e \cdot f + B \cdot c \cdot e^3)}{(2 \cdot f^3 \cdot (4 \cdot d \cdot f - e^2))} + \frac{(A \cdot b \cdot f^2 - A \cdot c \cdot e \cdot f + B \cdot a \cdot f^2 - B \cdot b \cdot e \cdot f - B \cdot c \cdot d \cdot f + B \cdot c \cdot e^2)}{(2 \cdot f^3)} \cdot \log(x + \frac{-A \cdot a \cdot e \cdot f^2 + 2 \cdot A \cdot b \cdot d \cdot f^2 - A \cdot c \cdot d \cdot e \cdot f + 2 \cdot B \cdot a \cdot d \cdot f^2 - B \cdot b \cdot d \cdot e \cdot f - 2 \cdot B \cdot c \cdot d^2 \cdot f + B \cdot c \cdot d \cdot e^2 - 4 \cdot d \cdot f^3 \cdot (\sqrt{-4 \cdot d \cdot f + e^2}) \cdot (-2 \cdot A \cdot a \cdot f^3 + A \cdot b \cdot e \cdot f^2 + 2 \cdot A \cdot c \cdot d \cdot f^2 - A \cdot c \cdot e^2 \cdot f + B \cdot a \cdot e \cdot f^2 + 2 \cdot B \cdot b \cdot d \cdot f^2 - B \cdot b \cdot e^2 \cdot f - 3 \cdot B \cdot c \cdot d \cdot e \cdot f + B \cdot c \cdot e^3)}{(2 \cdot f^3 \cdot (4 \cdot d \cdot f - e^2))} + \frac{(A \cdot b \cdot f^2 - A \cdot c \cdot e \cdot f + B \cdot a \cdot f^2 - B \cdot b \cdot e \cdot f - B \cdot c \cdot d \cdot f + B \cdot c \cdot e^2)}{(2 \cdot f^3)} + \frac{e^2 \cdot f^2 \cdot (\sqrt{-4 \cdot d \cdot f + e^2}) \cdot (-2 \cdot A \cdot a \cdot f^3 + A \cdot b \cdot e \cdot f^2 + 2 \cdot A \cdot c \cdot d \cdot f^2 - A \cdot c \cdot e^2 \cdot f + B \cdot a \cdot e \cdot f^2 + 2 \cdot B \cdot b \cdot d \cdot f^2 - B \cdot b \cdot e^2 \cdot f - 3 \cdot B \cdot c \cdot d \cdot e \cdot f + B \cdot c \cdot e^3)}{(2 \cdot f^3 \cdot (4 \cdot d \cdot f - e^2))} + \frac{(A \cdot b \cdot f^2 - A \cdot c \cdot e \cdot f + B \cdot a \cdot f^2 - B \cdot b \cdot e \cdot f - B \cdot c \cdot d \cdot f + B \cdot c \cdot e^2)}{(2 \cdot f^3)}$

$$\frac{\quad}{(-2A^2af^3 + A^2bef^2 + 2Acdf^2 - A^2ce^2f + B^2aef^2 + 2B^2bdf^2 - B^2be^2f - 3B^2cdef + B^2ce^3)}$$

$$3.14 \quad \int \frac{(A+Bx)(a+bx+cx^2)^2}{d+ex+fx^2} dx$$

Optimal. Leaf size=542

$$\frac{\log(d+ex+fx^2) \left(B(-f^2(-a^2f^2+2abef - (b^2(e^2-df)))) + 2cf(af(e^2-df) - b(e^3-2def)) + c^2(d^2f^2 - 2f^5) \right)}{2f^5}$$

[Out] (B*(-b*f+c*e)*(f*(-2*a*f+b*e)-c*(-2*d*f+e^2))+A*f*(b^2*f^2-2*c*f*(-a*f+b*e)+c^2*(-d*f+e^2)))*x/f^4-1/2*(A*c*f*(-2*b*f+c*e)-B*(b^2*f^2-2*c*f*(-a*f+b*e)+c^2*(-d*f+e^2)))*x^2/f^3-1/3*c*(-A*c*f-2*B*b*f+B*c*e)*x^3/f^2+1/4*B*c^2*x^4/f+1/2*(A*f*(-b*f+c*e)*(f*(-2*a*f+b*e)-c*(-2*d*f+e^2))+B*(c^2*(d^2*f^2-3*d*e^2*f+e^4)-f^2*(2*a*b*e*f-a^2*f^2-b^2*(-d*f+e^2))+2*c*f*(a*f*(-d*f+e^2)-b*(-2*d*e*f+e^3))))*ln(f*x^2+e*x+d)/f^5-(A*f*(c^2*(2*d^2*f^2-4*d*e^2*f+e^4)-f^2*(2*a*b*e*f-2*a^2*f^2-b^2*(-2*d*f+e^2))+2*c*f*(a*f*(-2*d*f+e^2)-b*(-3*d*e*f+e^3)))-B*(c^2*(5*d^2*e*f^2-5*d*e^3*f+e^5)+f^2*(a^2*e*f^2-2*a*b*f*(-2*d*f+e^2)+b^2*(-3*d*e*f+e^3))+2*c*f*(a*e*f*(-3*d*f+e^2)-b*(2*d^2*f^2-4*d*e^2*f+e^4))))*arctanh((2*f*x+e)/(-4*d*f+e^2)^(1/2))/f^5/(-4*d*f+e^2)^(1/2)

Rubi [A] time = 1.10, antiderivative size = 542, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1011, 634, 618, 206, 628}

$$\frac{\log(d+ex+fx^2) \left(B(-f^2(-a^2f^2+2abef + b^2(-(e^2-df)))) + 2cf(af(e^2-df) - b(e^3-2def)) + c^2(d^2f^2 - 2f^5) \right)}{2f^5}$$

Antiderivative was successfully verified.

[In] Int[((A + B*x)*(a + b*x + c*x^2)^2)/(d + e*x + f*x^2), x]

[Out] ((B*(c*e - b*f)*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) + A*f*(b^2*f^2 - 2*c*f*(b*e - a*f) + c^2*(e^2 - d*f)))*x)/f^4 - (((A*c*f*(c*e - 2*b*f) - B*(b^2*f^2 - 2*c*f*(b*e - a*f) + c^2*(e^2 - d*f)))*x^2)/(2*f^3) - (c*(B*c*e - 2*b*B*f - A*c*f)*x^3)/(3*f^2) + (B*c^2*x^4)/(4*f) - ((A*f*(c^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2) - f^2*(2*a*b*e*f - 2*a^2*f^2 - b^2*(e^2 - 2*d*f)) + 2*c*f*(a*f*(e^2 - 2*d*f) - b*(e^3 - 3*d*e*f))) - B*(c^2*(e^5 - 5*d*e^3*f + 5*d^2*e*f^2) + f^2*(a^2*e*f^2 - 2*a*b*f*(e^2 - 2*d*f) + b^2*(e^3 - 3*d*e*f)) + 2*c*f*(a*e*f*(e^2 - 3*d*f) - b*(e^4 - 4*d*e^2*f + 2*d^2*f^2))))*ArcTanh[(e + 2*f*x)/Sqrt[e^2 - 4*d*f]]/(f^5*Sqrt[e^2 - 4*d*f]) + ((A*f*(c*e - b*f)*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) + B*(c^2*(e^4 - 3*d*e^2*f + d^2*f^2) - f^2*(2*a*b*e*f - a^2*f^2 - b^2*(e^2 - d*f)) + 2*c*f*(a*f*(e^2 - d*f) - b*(e^3 - 2*d*e*f))))*Log[d + e*x + f*x^2])/(2*f^5)

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1011

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Int[ExpandIntegrand[(a + b*x +
c*x^2)^p*(d + e*x + f*x^2)^q*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && IGtQ[p, 0] && In
tegerQ[q]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx)(a+bx+cx^2)^2}{d+ex+fx^2} dx &= \int \left(\frac{B(ce-bf)(f(be-2af)-c(e^2-2df)) + Af(b^2f^2-2cf(be-af)+c^2(e^2-2df))}{f^4} \right) dx \\
&= \frac{(B(ce-bf)(f(be-2af)-c(e^2-2df)) + Af(b^2f^2-2cf(be-af)+c^2(e^2-2df)))}{f^4} \\
&= \frac{(B(ce-bf)(f(be-2af)-c(e^2-2df)) + Af(b^2f^2-2cf(be-af)+c^2(e^2-2df)))}{f^4} \\
&= \frac{(B(ce-bf)(f(be-2af)-c(e^2-2df)) + Af(b^2f^2-2cf(be-af)+c^2(e^2-2df)))}{f^4} \\
&= \frac{(B(ce-bf)(f(be-2af)-c(e^2-2df)) + Af(b^2f^2-2cf(be-af)+c^2(e^2-2df)))}{f^4}
\end{aligned}$$

Mathematica [A] time = 0.60, size = 535, normalized size = 0.99

$$6 \log(d + x(e + fx)) \left(B(f^2(a^2f^2 - 2abef + b^2(e^2 - df))) - 2cf(af(df - e^2) + b(e^3 - 2def)) + c^2(d^2f^2 - 3def) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*x)*(a + b*x + c*x^2)^2)/(d + e*x + f*x^2),x]

[Out] (12*f*(-(B*(c*e - b*f)*(f*(-(b*e) + 2*a*f) + c*(e^2 - 2*d*f))) + A*f*(b^2*f^2 + 2*c*f*(-(b*e) + a*f) + c^2*(e^2 - d*f)))*x + 6*f^2*(A*c*f*(-(c*e) + 2*b*f) + B*(b^2*f^2 + 2*c*f*(-(b*e) + a*f) + c^2*(e^2 - d*f)))*x^2 + 4*c*f^3*(-(B*c*e) + 2*b*B*f + A*c*f)*x^3 + 3*B*c^2*f^4*x^4 - (12*(-(A*f*(c^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2) + f^2*(-2*a*b*e*f + 2*a^2*f^2 + b^2*(e^2 - 2*d*f)) + 2*c*f*(a*f*(e^2 - 2*d*f) - b*(e^3 - 3*d*e*f)))) + B*(c^2*(e^5 - 5*d*e^3*f + 5*d^2*e*f^2) + f^2*(a^2*e*f^2 + 2*a*b*f*(-e^2 + 2*d*f) + b^2*(e^3 - 3*d*e*f)) - 2*c*f*(-(a*e*f*(e^2 - 3*d*f)) + b*(e^4 - 4*d*e^2*f + 2*d^2*f^2))))*ArcTan[(e + 2*f*x)/Sqrt[-e^2 + 4*d*f]]/Sqrt[-e^2 + 4*d*f] + 6*(A*f*(-(c*e) + b*f)*(f*(-(b*e) + 2*a*f) + c*(e^2 - 2*d*f)) + B*(c^2*(e^4 - 3*d*e^2*f + d^2*f^2) + f^2*(-2*a*b*e*f + a^2*f^2 + b^2*(e^2 - d*f)) - 2*c*f*(a*f*(-e^2 + d*f) + b*(e^3 - 2*d*e*f))))*Log[d + x*(e + f*x)]/(12*f^5)

fricas [A] time = 1.96, size = 1837, normalized size = 3.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] [1/12*(3*(B*c^2*e^2*f^4 - 4*B*c^2*d*f^5)*x^4 - 4*(B*c^2*e^3*f^3 + 4*(2*B*b*c + A*c^2)*d*f^5 - (4*B*c^2*d*e + (2*B*b*c + A*c^2)*e^2)*f^4)*x^3 + 6*(B*c^2*e^4*f^2 - 4*(B*b^2 + 2*(B*a + A*b)*c)*d*f^5 + (4*B*c^2*d^2 + 4*(2*B*b*c + A*c^2)*d*e + (B*b^2 + 2*(B*a + A*b)*c)*e^2)*f^4 - (5*B*c^2*d*e^2 + (2*B*b*c + A*c^2)*e^3)*f^3)*x^2 - 6*(B*c^2*e^5 - 2*A*a^2*f^5 + (2*(2*B*a*b + A*b^2 + 2*A*a*c)*d + (B*a^2 + 2*A*a*b)*e)*f^4 - (2*(2*B*b*c + A*c^2)*d^2 + 3*(B*b^2 + 2*(B*a + A*b)*c)*d*e + (2*B*a*b + A*b^2 + 2*A*a*c)*e^2)*f^3 + (5*B*c^2*d^2*e + 4*(2*B*b*c + A*c^2)*d*e^2 + (B*b^2 + 2*(B*a + A*b)*c)*e^3)*f^2 - (5*B*c^2*d*e^3 + (2*B*b*c + A*c^2)*e^4)*f)*sqrt(e^2 - 4*d*f)*log((2*f^2*x^2 + 2*e*f*x + e^2 - 2*d*f - sqrt(e^2 - 4*d*f)*(2*f*x + e))/(f*x^2 + e*x + d)) - 12*(B*c^2*e^5*f + 4*(2*B*a*b + A*b^2 + 2*A*a*c)*d*f^5 - (4*(2*B*b*c + A*c^2)*d^2 + 4*(B*b^2 + 2*(B*a + A*b)*c)*d*e + (2*B*a*b + A*b^2 + 2*A*a*c)*e^2)*f^4 + (8*B*c^2*d^2*e + 5*(2*B*b*c + A*c^2)*d*e^2 + (B*b^2 + 2*(B*a + A*b)*c)*e^3)*f^3 - (6*B*c^2*d*e^3 + (2*B*b*c + A*c^2)*e^4)*f^2)*x + 6*(B*c^2*e^6 - 4*(B*a^2 + 2*A*a*b)*d*f^5 + (4*(B*b^2 + 2*(B*a + A*b)*c)*d^2 + 4*(2*B*a*b + A*b^2 + 2*A*a*c)*d*e + (B*a^2 + 2*A*a*b)*e^2)*f^4 - (4*B*c^2*d^3 + 8*(2*B*b*c + A*c^2)*d^2*e + 5*(B*b^2 + 2*(B*a + A*b)*c)*d*e^2 + (2*B*a*b + A*b^2 + 2*A*a*c)*e^3)*f^3 + (13*B*c^2*d^2*e^2 + 6*(2*B*b*c + A*c^2)*d*e^3 + (B*b^2 + 2*(B*a + A*b)*c)*e^4)*f^2 - (7*B*c^2*d*e^4 + (2*B*b*c + A*c^2)*e^5)*f)*log(f*x^2 + e*x + d))/(e^2*f^5 - 4*d*f^6), 1/12*(3*(B*c^2*e^2*f^4 - 4*B*c^2*d*f^5)*x^4 - 4*(B*c^2*e^3*f^3 + 4*(2*B*b*c + A*c^2)*d*f^5 - (4*B*c^2*d*e + (2*B*b*c + A*c^2)*e^2)*f^4)*x^3 + 6*(B*c^2*e^4*f^2 - 4*(B*b^2 + 2*(B*a + A*b)*c)*d*f^5 + (4*B*c^2*d^2 + 4*(2*B*b*c + A*c^2)*d*e + (B*b^2 + 2*(B*a + A*b)*c)*e^2)*f^4 - (5*B*c^2*d*e^2 + (2*B*b*c + A*c^2)*e^3)*f^3)*x^2 + 12*(B*c^2*e^5 - 2*A*a^2*f^5 + (2*(2*B*a*b + A*b^2 + 2*A*a*c)*d + (B*a^2 + 2*A*a*b)*e)*f^4 - (2*(2*B*b*c + A*c^2)*d^2 + 3*(B*b^2 + 2*(B*a + A*b)*c)*d*e + (2*B*a*b + A*b^2 + 2*A*a*c)*e^2)*f^3 + (5*B*c^2*d^2*e + 4*(2*B*b*c + A*c^2)*d*e^2 + (B*b^2 + 2*(B*a + A*b)*c)*e^3)*f^2 - (5*B*c^2*d*e^3 + (2*B*b*c + A*c^2)*e^4)*f)*sqrt(-e^2 + 4*d*f)*arctan(-sqrt(-e^2 + 4*d*f)*(2*f*x + e)/(e^2 - 4*d*f)) - 12*(B*c^2*e^5*f + 4*(2*B*a*b + A*b^2 + 2*A*a*c)*d*f^5 - (4*(2*B*b*c + A*c^2)*d^2 + 4*(B*b^2 + 2*(B*a + A*b)*c)*d*e + (2*B*a*b + A*b^2 + 2*A*a*c)*e^2)*f^4 + (8*B*c^2*d^2*e + 5*(2*B*b*c + A*c^2)*d*e^2 + (B*b^2 + 2*(B*a + A*b)*c)*e^3)*f^3 - (6*B*c^2*d*e^3 + (2*B*b*c + A*c^2)*e^4)*f^2)*x + 6*(B*c^2*e^6 - 4*(B*a^2 + 2*A*a*b)*d*f^5 + (4*(B*b^2 + 2*(B*a + A*b)*c)*d^2 + 4*(2*B*a*b + A*b^2 + 2*A*a*c)*d*e + (B*a^2 + 2*A*a*b)*e^2)*f^4 - (4*B*c^2*d^3 + 8*(2*B*b*c + A*c^2)*d^2*e + 5*(B*b^2 + 2*(B*a + A*b)*c)*d*e^2 + (2*B*a*b + A*b^2 + 2*A*a*c)*e^3)*f^3 + (13*B*c^2*d^2*e^2 + 6*(2*B*b*c + A*c^2)*d*e^3 + (B*b^2 + 2*(B*a + A*b)*c)*e^4)*f^2 - (7*B*c^2*d*e^4 + (2*B*b*c + A*c^2)*e^5)*f)*log(f*x^2 + e*x + d))/(e^2*f^5 - 4*d*f^6)]

giac [A] time = 0.25, size = 738, normalized size = 1.36

$$\frac{3Bc^2f^3x^4 + 8Bbcf^3x^3 + 4Ac^2f^3x^3 - 4Bc^2f^2x^3e - 6Bc^2df^2x^2 + 6Bb^2f^3x^2 + 12Bacf^3x^2 + 12Abcf^3x^2 - 12B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="giac")

[Out] $\frac{1}{12}*(3B*c^2*f^3*x^4 + 8*B*b*c*f^3*x^3 + 4*A*c^2*f^3*x^3 - 4*B*c^2*f^2*x^3 * e - 6*B*c^2*d*f^2*x^2 + 6*B*b^2*f^3*x^2 + 12*B*a*c*f^3*x^2 + 12*A*b*c*f^3*x^2 - 12*B*b*c*f^2*x^2*e - 6*A*c^2*f^2*x^2*e - 24*B*b*c*d*f^2*x - 12*A*c^2*d*f^2*x + 24*B*a*b*f^3*x + 12*A*b^2*f^3*x + 24*A*a*c*f^3*x + 6*B*c^2*f*x^2*e^2 + 24*B*c^2*d*f*x*e - 12*B*b^2*f^2*x*e - 24*B*a*c*f^2*x*e - 24*A*b*c*f^2*x*x*e + 24*B*b*c*f*x*e^2 + 12*A*c^2*f*x*e^2 - 12*B*c^2*x*e^3)/f^4 + \frac{1}{2}*(B*c^2*d^2*f^2 - B*b^2*d*f^3 - 2*B*a*c*d*f^3 - 2*A*b*c*d*f^3 + B*a^2*f^4 + 2*A*a*b*f^4 + 4*B*b*c*d*f^2*e + 2*A*c^2*d*f^2*e - 2*B*a*b*f^3*e - A*b^2*f^3*e - 2*A*a*c*f^3*e - 3*B*c^2*d*f*e^2 + B*b^2*f^2*e^2 + 2*B*a*c*f^2*e^2 + 2*A*b*c*f^2*e^2 - 2*B*b*c*f*e^3 - A*c^2*f*e^3 + B*c^2*e^4)*\log(f*x^2 + x*e + d)/f^5 + (4*B*b*c*d^2*f^3 + 2*A*c^2*d^2*f^3 - 4*B*a*b*d*f^4 - 2*A*b^2*d*f^4 - 4*A*a*c*d*f^4 + 2*A*a^2*f^5 - 5*B*c^2*d^2*f^2*e + 3*B*b^2*d*f^3*e + 6*B*a*c*d*f^3*e + 6*A*b*c*d*f^3*e - B*a^2*f^4*e - 2*A*a*b*f^4*e - 8*B*b*c*d*f^2*e^2 - 4*A*c^2*d*f^2*e^2 + 2*B*a*b*f^3*e^2 + A*b^2*f^3*e^2 + 2*A*a*c*f^3*e^2 + 5*B*c^2*d*f*e^3 - B*b^2*f^2*e^3 - 2*B*a*c*f^2*e^3 - 2*A*b*c*f^2*e^3 + 2*B*b*c*f*e^4 + A*c^2*f*e^4 - B*c^2*e^5)*\arctan((2*f*x + e)/\sqrt{4*d*f - e^2})/(\sqrt{4*d*f - e^2})*f^5)$

maple [B] time = 0.01, size = 1672, normalized size = 3.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x)

[Out] $\frac{1}{4}*B*c^2/f*x^4 - 2*B*b*c*d/f^2*x + 1/2/f*\ln(f*x^2+e*x+d)*B*a^2 + 2/(4*d*f - e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f - e^2)^{(1/2)})*A*a^2 + 1/3*A*c^2/f*x^3 + 1/2*B*b^2/f*x^2 + A*b^2/f*x - A*c^2*d/f^2*x + 2*B*a*b/f*x + B*a*c/f*x^2 - 1/2*B*c^2*d/f^2*x^2 + 2*A*a*c/f*x + 2/3*B*b*c/f*x^3 + A*b*c/f*x^2 - 8/f^3/(4*d*f - e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f - e^2)^{(1/2)})*B*b*c*d*e^2 + 6/f^2/(4*d*f - e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f - e^2)^{(1/2)})*A*b*c*d*e + 6/f^2/(4*d*f - e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f - e^2)^{(1/2)})*B*a*c*d*e + 1/f^3*A*c^2*e^2*x - 1/f^2*B*b^2*e*x - 1/f^4*B*c^2*e^3*x - 1/2/f^2*A*x^2*c^2*e + 1/2/f^3*B*x^2*c^2*e^2 - 1/3/f^2*B*x^3*c^2*e - 1/2/f^2*\ln(f*x^2+e*x+d)*A*b^2*e - 1/2/f^4*\ln(f*x^2+e*x+d)*A*c^2*e^3 - 1/2/f^2*\ln(f*x^2+e*x+d)*B*b^2*d + 1/2/f^3*\ln(f*x^2+e*x+d)*B*b^2*e^2 + 1/2/f^3*\ln(f*x^2+e*x+d)*B*c^2*$

```

d^2+1/2/f^5*ln(f*x^2+e*x+d)*B*c^2*e^4+1/f*ln(f*x^2+e*x+d)*A*a*b-1/f^2*B*x^2
*b*c*e-2/f^2*A*b*c*e*x-2/f^2*B*a*c*e*x+2/f^3*B*b*c*e^2*x+2/f^3*B*c^2*d*e*x+
1/f^2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^2*A*b^2-1/f/(
4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e*B*a^2-1/f^3/(4*d*f-e
^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^3*B*b^2-1/f^5/(4*d*f-e^2)^(
1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^5*B*c^2-2/f/(4*d*f-e^2)^(1/2)*ar
ctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*A*b^2*d+2/f^2/(4*d*f-e^2)^(1/2)*arctan((2
*f*x+e)/(4*d*f-e^2)^(1/2))*A*c^2*d^2-1/f^2*ln(f*x^2+e*x+d)*A*a*c*e-1/f^2*ln
(f*x^2+e*x+d)*A*b*c*d+1/f^3*ln(f*x^2+e*x+d)*A*b*c*e^2+1/f^3*ln(f*x^2+e*x+d)
*A*c^2*d*e-1/f^2*ln(f*x^2+e*x+d)*B*a*b*e-1/f^2*ln(f*x^2+e*x+d)*B*a*c*d+1/f^
3*ln(f*x^2+e*x+d)*B*a*c*e^2-1/f^4*ln(f*x^2+e*x+d)*B*b*c*e^3-3/2/f^4*ln(f*x^
2+e*x+d)*B*c^2*d*e^2+1/f^4/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(
1/2))*e^4*A*c^2-2/f^3/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))
*e^3*A*b*c+2/f^2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^2*
B*a*b-2/f^3/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^3*B*a*c
+2/f^3*ln(f*x^2+e*x+d)*B*b*c*d*e-4/f/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*
d*f-e^2)^(1/2))*A*a*c*d-4/f^3/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2
)^(1/2))*A*c^2*d*e^2-4/f/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/
2))*B*a*b*d+3/f^2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*B*b
^2*d*e+4/f^2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*B*b*c*d^
2-5/f^3/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*B*c^2*d^2*e-2
/f/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e*A*a*b+5/f^4/(4*d
*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*B*c^2*d*e^3+2/f^4/(4*d*f-
e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^4*B*b*c+2/f^2/(4*d*f-e^2)^(
1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^2*A*a*c

```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details)Is 4*d*f-e^2 positive or negative?

mupad [B] time = 4.85, size = 893, normalized size = 1.65

$$x^3 \left(\frac{Ac^2 + 2Bbc}{3f} - \frac{Bc^2e}{3f^2} \right) + x \left(\frac{Ab^2 + 2Bab + 2Aac}{f} - \frac{d \left(\frac{Ac^2 + 2Bbc}{f} - \frac{Bc^2e}{f^2} \right)}{f} + \frac{e \left(\frac{Ac^2 + 2Bbc}{f} - \frac{Bc^2e}{f^2} \right) - \frac{Bb^2 + 2Ac}{f}}{f} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(((A + Bx)*(a + bx + cx^2)^2)/(d + ex + fx^2), x)$

[Out] $x^3*((A*c^2 + 2*B*b*c)/(3*f) - (B*c^2*e)/(3*f^2)) + x*((A*b^2 + 2*A*a*c + 2*B*a*b)/f - (d*((A*c^2 + 2*B*b*c)/f - (B*c^2*e)/f^2))/f + (e*((e*((A*c^2 + 2*B*b*c)/f - (B*c^2*e)/f^2))/f - (B*b^2 + 2*A*b*c + 2*B*a*c)/f + (B*c^2*d)/f^2))/f - x^2*((e*((A*c^2 + 2*B*b*c)/f - (B*c^2*e)/f^2))/(2*f) - (B*b^2 + 2*A*b*c + 2*B*a*c)/(2*f) + (B*c^2*d)/(2*f^2)) - (\log(d + ex + fx^2)*(B*c^2*e^6 - 4*B*a^2*d*f^5 - A*c^2*e^5*f - A*b^2*e^3*f^3 + B*a^2*e^2*f^4 + 4*B*b^2*d^2*f^4 + B*b^2*e^4*f^2 - 4*B*c^2*d^3*f^3 + 6*A*c^2*d*e^3*f^2 - 8*A*c^2*d^2*e*f^3 - 5*B*b^2*d*e^2*f^3 - 8*A*a*b*d*f^5 - 2*B*b*c*e^5*f + 13*B*c^2*d^2*e^2*f^2 + 2*A*a*b*e^2*f^4 - 2*A*a*c*e^3*f^3 + 8*A*b*c*d^2*f^4 - 2*B*a*b*e^3*f^3 + 8*B*a*c*d^2*f^4 + 2*A*b*c*e^4*f^2 + 2*B*a*c*e^4*f^2 + 4*A*b^2*d*e*f^4 - 7*B*c^2*d*e^4*f - 10*A*b*c*d*e^2*f^3 - 10*B*a*c*d*e^2*f^3 + 12*B*b*c*d*e^3*f^2 - 16*B*b*c*d^2*e*f^3 + 8*A*a*c*d*e*f^4 + 8*B*a*b*d*e*f^4))/(2*(4*d*f^6 - e^2*f^5)) + (B*c^2*x^4)/(4*f) + (\text{atan}(e/(4*d*f - e^2))^{1/2} + (2*f*x)/(4*d*f - e^2)^{1/2})*(2*A*a^2*f^5 - B*c^2*e^5 - 2*A*b^2*d*f^4 - B*a^2*e*f^4 + A*c^2*e^4*f + A*b^2*e^2*f^3 + 2*A*c^2*d^2*f^3 - B*b^2*e^3*f^2 - 4*A*c^2*d*e^2*f^2 - 5*B*c^2*d^2*e*f^2 - 2*A*a*b*e*f^4 - 4*A*a*c*d*f^4 - 4*B*a*b*d*f^4 + 2*B*b*c*e^4*f + 2*A*a*c*e^2*f^3 + 2*B*a*b*e^2*f^3 - 2*A*b*c*e^3*f^2 - 2*B*a*c*e^3*f^2 + 4*B*b*c*d^2*f^3 + 3*B*b^2*d*e*f^3 + 5*B*c^2*d*e^3*f - 8*B*b*c*d*e^2*f^2 + 6*A*b*c*d*e*f^3 + 6*B*a*c*d*e*f^3))/(f^5*(4*d*f - e^2)^{1/2}))$

sympy [B] time = 145.64, size = 4663, normalized size = 8.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((B*x+A)*(c*x**2+b*x+a)**2/(f*x**2+e*x+d), x)$

[Out] $B*c**2*x**4/(4*f) + x**3*(A*c**2/(3*f) + 2*B*b*c/(3*f) - B*c**2*e/(3*f**2)) + x**2*(A*b*c/f - A*c**2*e/(2*f**2) + B*a*c/f + B*b**2/(2*f) - B*b*c*e/f**2 - B*c**2*d/(2*f**2) + B*c**2*e**2/(2*f**3)) + x*(2*A*a*c/f + A*b**2/f - 2*A*b*c*e/f**2 - A*c**2*d/f**2 + A*c**2*e**2/f**3 + 2*B*a*b/f - 2*B*a*c*e/f**2 - B*b**2*e/f**2 - 2*B*b*c*d/f**2 + 2*B*b*c*e**2/f**3 + 2*B*c**2*d*e/f**3 - B*c**2*e**3/f**4) + (-sqrt(-4*d*f + e**2))*(-2*A*a**2*f**5 + 2*A*a*b*e*f**4 + 4*A*a*c*d*f**4 - 2*A*a*c*e**2*f**3 + 2*A*b**2*d*f**4 - A*b**2*e**2*f**3 - 6*A*b*c*d*e*f**3 + 2*A*b*c*e**3*f**2 - 2*A*c**2*d**2*f**3 + 4*A*c**2*d*e**2*f**2 - A*c**2*e**4*f + B*a**2*e*f**4 + 4*B*a*b*d*f**4 - 2*B*a*b*e**2*f**3 - 6*B*a*c*d*e*f**3 + 2*B*a*c*e**3*f**2 - 3*B*b**2*d*e*f**3 + B*b**2*e**3*f**2 - 4*B*b*c*d**2*f**3 + 8*B*b*c*d*e**2*f**2 - 2*B*b*c*e**4*f + 5*B*c**2*d**2*e*f**2 - 5*B*c**2*d*e**3*f + B*c**2*e**5)/(2*f**5*(4*d*f - e**2)) + (2*A*a*b*f**4 - 2*A*a*c*e*f**3 - A*b**2*e*f**3 - 2*A*b*c*d*f**3 + 2*A*b*c*e$

$$\begin{aligned}
& **2*f**2 + 2*A*c**2*d*e*f**2 - A*c**2*e**3*f + B*a**2*f**4 - 2*B*a*b*e*f**3 \\
& - 2*B*a*c*d*f**3 + 2*B*a*c*e**2*f**2 - B*b**2*d*f**3 + B*b**2*e**2*f**2 + \\
& 4*B*b*c*d*e*f**2 - 2*B*b*c*e**3*f + B*c**2*d**2*f**2 - 3*B*c**2*d*e**2*f + \\
& B*c**2*e**4)/(2*f**5))*\log(x + (-A*a**2*e*f**4 + 4*A*a*b*d*f**4 - 2*A*a*c*d \\
& *e*f**3 - A*b**2*d*e*f**3 - 4*A*b*c*d**2*f**3 + 2*A*b*c*d*e**2*f**2 + 3*A*c \\
& **2*d**2*e*f**2 - A*c**2*d*e**3*f + 2*B*a**2*d*f**4 - 2*B*a*b*d*e*f**3 - 4* \\
& B*a*c*d**2*f**3 + 2*B*a*c*d*e**2*f**2 - 2*B*b**2*d**2*f**3 + B*b**2*d*e**2* \\
& f**2 + 6*B*b*c*d**2*e*f**2 - 2*B*b*c*d*e**3*f + 2*B*c**2*d**3*f**2 - 4*B*c* \\
& *2*d**2*e**2*f + B*c**2*d*e**4 - 4*d*f**5*(-\sqrt{-4*d*f + e**2})*(-2*A*a**2* \\
& f**5 + 2*A*a*b*e*f**4 + 4*A*a*c*d*f**4 - 2*A*a*c*e**2*f**3 + 2*A*b**2*d*f** \\
& 4 - A*b**2*e**2*f**3 - 6*A*b*c*d*e*f**3 + 2*A*b*c*e**3*f**2 - 2*A*c**2*d**2 \\
& *f**3 + 4*A*c**2*d*e**2*f**2 - A*c**2*e**4*f + B*a**2*e*f**4 + 4*B*a*b*d*f* \\
& *4 - 2*B*a*b*e**2*f**3 - 6*B*a*c*d*e*f**3 + 2*B*a*c*e**3*f**2 - 3*B*b**2*d* \\
& e*f**3 + B*b**2*e**3*f**2 - 4*B*b*c*d**2*f**3 + 8*B*b*c*d*e**2*f**2 - 2*B*b \\
& *c*e**4*f + 5*B*c**2*d**2*e*f**2 - 5*B*c**2*d*e**3*f + B*c**2*e**5)/(2*f**5 \\
& *(4*d*f - e**2)) + (2*A*a*b*f**4 - 2*A*a*c*e*f**3 - A*b**2*e*f**3 - 2*A*b*c \\
& *d*f**3 + 2*A*b*c*e**2*f**2 + 2*A*c**2*d*e*f**2 - A*c**2*e**3*f + B*a**2*f* \\
& *4 - 2*B*a*b*e*f**3 - 2*B*a*c*d*f**3 + 2*B*a*c*e**2*f**2 - B*b**2*d*f**3 + \\
& B*b**2*e**2*f**2 + 4*B*b*c*d*e*f**2 - 2*B*b*c*e**3*f + B*c**2*d**2*f**2 - 3 \\
& *B*c**2*d*e**2*f + B*c**2*e**4)/(2*f**5)) + e**2*f**4*(-\sqrt{-4*d*f + e**2}) \\
& *(-2*A*a**2*f**5 + 2*A*a*b*e*f**4 + 4*A*a*c*d*f**4 - 2*A*a*c*e**2*f**3 + 2* \\
& A*b**2*d*f**4 - A*b**2*e**2*f**3 - 6*A*b*c*d*e*f**3 + 2*A*b*c*e**3*f**2 - 2 \\
& *A*c**2*d**2*f**3 + 4*A*c**2*d*e**2*f**2 - A*c**2*e**4*f + B*a**2*e*f**4 + \\
& 4*B*a*b*d*f**4 - 2*B*a*b*e**2*f**3 - 6*B*a*c*d*e*f**3 + 2*B*a*c*e**3*f**2 - \\
& 3*B*b**2*d*e*f**3 + B*b**2*e**3*f**2 - 4*B*b*c*d**2*f**3 + 8*B*b*c*d*e**2* \\
& f**2 - 2*B*b*c*e**4*f + 5*B*c**2*d**2*e*f**2 - 5*B*c**2*d*e**3*f + B*c**2*e \\
& **5)/(2*f**5*(4*d*f - e**2)) + (2*A*a*b*f**4 - 2*A*a*c*e*f**3 - A*b**2*e*f* \\
& *3 - 2*A*b*c*d*f**3 + 2*A*b*c*e**2*f**2 + 2*A*c**2*d*e*f**2 - A*c**2*e**3*f \\
& + B*a**2*f**4 - 2*B*a*b*e*f**3 - 2*B*a*c*d*f**3 + 2*B*a*c*e**2*f**2 - B*b* \\
& **2*d*f**3 + B*b**2*e**2*f**2 + 4*B*b*c*d*e*f**2 - 2*B*b*c*e**3*f + B*c**2*d \\
& **2*f**2 - 3*B*c**2*d*e**2*f + B*c**2*e**4)/(2*f**5)))/(-2*A*a**2*f**5 + 2* \\
& A*a*b*e*f**4 + 4*A*a*c*d*f**4 - 2*A*a*c*e**2*f**3 + 2*A*b**2*d*f**4 - A*b** \\
& 2*e**2*f**3 - 6*A*b*c*d*e*f**3 + 2*A*b*c*e**3*f**2 - 2*A*c**2*d**2*f**3 + 4 \\
& *A*c**2*d*e**2*f**2 - A*c**2*e**4*f + B*a**2*e*f**4 + 4*B*a*b*d*f**4 - 2*B* \\
& a*b*e**2*f**3 - 6*B*a*c*d*e*f**3 + 2*B*a*c*e**3*f**2 - 3*B*b**2*d*e*f**3 + \\
& B*b**2*e**3*f**2 - 4*B*b*c*d**2*f**3 + 8*B*b*c*d*e**2*f**2 - 2*B*b*c*e**4*f \\
& + 5*B*c**2*d**2*e*f**2 - 5*B*c**2*d*e**3*f + B*c**2*e**5)) + (\sqrt{-4*d*f \\
& + e**2})*(-2*A*a**2*f**5 + 2*A*a*b*e*f**4 + 4*A*a*c*d*f**4 - 2*A*a*c*e**2*f* \\
& *3 + 2*A*b**2*d*f**4 - A*b**2*e**2*f**3 - 6*A*b*c*d*e*f**3 + 2*A*b*c*e**3*f \\
& **2 - 2*A*c**2*d**2*f**3 + 4*A*c**2*d*e**2*f**2 - A*c**2*e**4*f + B*a**2*e* \\
& f**4 + 4*B*a*b*d*f**4 - 2*B*a*b*e**2*f**3 - 6*B*a*c*d*e*f**3 + 2*B*a*c*e**3 \\
& *f**2 - 3*B*b**2*d*e*f**3 + B*b**2*e**3*f**2 - 4*B*b*c*d**2*f**3 + 8*B*b*c* \\
& d*e**2*f**2 - 2*B*b*c*e**4*f + 5*B*c**2*d**2*e*f**2 - 5*B*c**2*d*e**3*f + B \\
& *c**2*e**5)/(2*f**5*(4*d*f - e**2)) + (2*A*a*b*f**4 - 2*A*a*c*e*f**3 - A*b* \\
& **2*e*f**3 - 2*A*b*c*d*f**3 + 2*A*b*c*e**2*f**2 + 2*A*c**2*d*e*f**2 - A*c**2
\end{aligned}$$

$$\begin{aligned}
& e^{3f} + B a^{2f} - 2 B a b e^f - 2 B a c d f^3 + 2 B a c e^{2f} \\
& - B b^2 d f^3 + B b^2 e^{2f} + 4 B b c d e^f - 2 B b c e^{3f} + B \\
& c^2 d^2 f^2 - 3 B c^2 d e^{2f} + B c^2 e^4 / (2 f^5) \log(x + (-A a \\
& e^{2f} + 4 A a b d f^4 - 2 A a c d e^f - A b^2 d e^f - 4 A b c d \\
& e^{2f} + 2 A b c d e^{2f} + 3 A c^2 d^2 e^f - A c^2 d e^{3f} + 2 \\
& B a^2 d f^4 - 2 B a b d e^f - 4 B a c d^2 f^3 + 2 B a c d e^{2f} \\
& - 2 B b^2 d^2 f^3 + B b^2 d e^{2f} + 6 B b c d^2 e^f - 2 B b c \\
& d e^{3f} + 2 B c^2 d^3 f^2 - 4 B c^2 d^2 e^{2f} + B c^2 d e^4 - 4 d \\
& f^5 (\sqrt{-4 d f + e^2}) (-2 A a^2 f^5 + 2 A a b e^f + 4 A a c d f^4 \\
& - 2 A a c e^{2f} + 2 A b^2 d f^4 - A b^2 e^{2f} - 6 A b c d e^f \\
& + 2 A b c e^{3f} - 2 A c^2 d^2 f^3 + 4 A c^2 d e^{2f} - A c^2 e \\
& e^{4f} + B a^2 e^f + 4 B a b d f^4 - 2 B a b e^{2f} - 6 B a c d e^f \\
& + 2 B a c e^{3f} - 3 B b^2 d e^f + B b^2 e^{3f} - 4 B b c d \\
& e^{2f} + 8 B b c d e^{2f} - 2 B b c e^{4f} + 5 B c^2 d^2 e^f - 5 B \\
& c^2 d e^{3f} + B c^2 e^5) / (2 f^5 (4 d f - e^2)) + (2 A a b f^4 - 2 A \\
& a c e^f - A b^2 e^f - 2 A b c d f^3 + 2 A b c e^{2f} + 2 A c^2 \\
& d e^f - A c^2 e^{3f} + B a^2 f^4 - 2 B a b e^f - 2 B a c d f^3 + \\
& 2 B a c e^{2f} - B b^2 d f^3 + B b^2 e^{2f} + 4 B b c d e^f - \\
& 2 B b c e^{3f} + B c^2 d^2 f^2 - 3 B c^2 d e^{2f} + B c^2 e^4) / (2 f^5) \\
& + e^{2f} (4 (\sqrt{-4 d f + e^2}) (-2 A a^2 f^5 + 2 A a b e^f + 4 A \\
& a c d f^4 - 2 A a c e^{2f} + 2 A b^2 d f^4 - A b^2 e^{2f} - 6 A \\
& b c d e^f + 2 A b c e^{3f} - 2 A c^2 d^2 f^3 + 4 A c^2 d e^{2f} \\
& - A c^2 e^{4f} + B a^2 e^f + 4 B a b d f^4 - 2 B a b e^{2f} - 6 \\
& B a c d e^f + 2 B a c e^{3f} - 3 B b^2 d e^f + B b^2 e^{3f} - \\
& 4 B b c d^2 f^3 + 8 B b c d e^{2f} - 2 B b c e^{4f} + 5 B c^2 d^2 e \\
& f^2 - 5 B c^2 d e^{3f} + B c^2 e^5) / (2 f^5 (4 d f - e^2)) + (2 A a b \\
& f^4 - 2 A a c e^f - A b^2 e^f - 2 A b c d f^3 + 2 A b c e^{2f} \\
& + 2 A c^2 d e^f - A c^2 e^{3f} + B a^2 f^4 - 2 B a b e^f - 2 B a \\
& c d f^3 + 2 B a c e^{2f} - B b^2 d f^3 + B b^2 e^{2f} + 4 B b c \\
& d e^f - 2 B b c e^{3f} + B c^2 d^2 f^2 - 3 B c^2 d e^{2f} + B c^2 e \\
& e^4) / (2 f^5) / (-2 A a^2 f^5 + 2 A a b e^f + 4 A a c d f^4 - 2 A a c \\
& e^{2f} + 2 A b^2 d f^4 - A b^2 e^{2f} - 6 A b c d e^f + 2 A b c \\
& e^{3f} - 2 A c^2 d^2 f^3 + 4 A c^2 d e^{2f} - A c^2 e^{4f} + B \\
& a^2 e^f + 4 B a b d f^4 - 2 B a b e^{2f} - 6 B a c d e^f + 2 B \\
& a c e^{3f} - 3 B b^2 d e^f + B b^2 e^{3f} - 4 B b c d^2 f^3 + \\
& 8 B b c d e^{2f} - 2 B b c e^{4f} + 5 B c^2 d^2 e^f - 5 B c^2 d e \\
& e^{3f} + B c^2 e^5)
\end{aligned}$$

$$3.15 \quad \int \frac{A+Bx}{(a+bx+cx^2)(d+ex+fx^2)} dx$$

Optimal. Leaf size=406

$$\frac{\tanh^{-1}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)(B(aef-2bdf+cde)-A(2af^2-bef-2cdf+ce^2))}{\sqrt{e^2-4df}(f(a^2f-abe+b^2d)-c(bde-a(e^2-2df))+c^2d^2)} + \frac{\log(a+bx+cx^2)(-aBf+Abf-}{2(f(a^2f-abe+b^2d)-c(bde-a(e^2-2df))$$

[Out] $1/2*(A*b*f-A*c*e-B*a*f+B*c*d)*\ln(c*x^2+b*x+a)/(c^2*d^2+f*(a^2*f-a*b*e+b^2*d)-c*(b*d*e-a*(-2*d*f+e^2)))-1/2*(A*b*f-A*c*e-B*a*f+B*c*d)*\ln(f*x^2+e*x+d)/(c^2*d^2+f*(a^2*f-a*b*e+b^2*d)-c*(b*d*e-a*(-2*d*f+e^2)))-(A*b^2*f+2*c*(-A*a*f+A*c*d+B*a*e)-b*(A*c*e+B*a*f+B*c*d))*\operatorname{arctanh}((2*c*x+b)/(-4*a*c+b^2)^{(1/2)})/(c^2*d^2+f*(a^2*f-a*b*e+b^2*d)-c*(b*d*e-a*(-2*d*f+e^2)))/(-4*a*c+b^2)^{(1/2)}+(B*(a*e*f-2*b*d*f+c*d*e)-A*(2*a*f^2-b*e*f-2*c*d*f+c*e^2))*\operatorname{arctanh}((2*f*x+e)/(-4*d*f+e^2)^{(1/2)})/(c^2*d^2+f*(a^2*f-a*b*e+b^2*d)-c*(b*d*e-a*(-2*d*f+e^2)))/(-4*d*f+e^2)^{(1/2)}$

Rubi [A] time = 0.48, antiderivative size = 398, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1022, 634, 618, 206, 628}

$$\frac{\tanh^{-1}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)(B(aef-2bdf+cde)-A(2af^2-bef-2cdf+ce^2))}{\sqrt{e^2-4df}(f(a^2f-abe+b^2d)+ac(e^2-2df)-bcde+c^2d^2)} + \frac{\log(a+bx+cx^2)(-aBf+Abf-}{2(f(a^2f-abe+b^2d)+ac(e^2-2df))$$

Antiderivative was successfully verified.

[In] Int[(A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)), x]

[Out] $-(((A*b^2*f+2*c*(A*c*d+a*B*e-a*A*f))-b*(B*c*d+A*c*e+a*B*f))*\operatorname{ArcTanh}[(b+2*c*x)/\operatorname{Sqrt}[b^2-4*a*c]])/(\operatorname{Sqrt}[b^2-4*a*c]*(c^2*d^2-b*c*d*e+f*(b^2*d-a*b*e+a^2*f)+a*c*(e^2-2*d*f)))+((B*(c*d*e-2*b*d*f+a*e*f)-A*(c*e^2-2*c*d*f-b*e*f+2*a*f^2))*\operatorname{ArcTanh}[(e+2*f*x)/\operatorname{Sqrt}[e^2-4*d*f]])/(\operatorname{Sqrt}[e^2-4*d*f]*(c^2*d^2-b*c*d*e+f*(b^2*d-a*b*e+a^2*f)+a*c*(e^2-2*d*f)))+((B*c*d-A*c*e+A*b*f-a*B*f)*\operatorname{Log}[a+b*x+c*x^2])/(2*(c^2*d^2-b*c*d*e+f*(b^2*d-a*b*e+a^2*f)+a*c*(e^2-2*d*f)))-((B*c*d-A*c*e+A*b*f-a*B*f)*\operatorname{Log}[d+e*x+f*x^2])/(2*(c^2*d^2-b*c*d*e+f*(b^2*d-a*b*e+a^2*f)+a*c*(e^2-2*d*f))$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1022

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] := With[{q = Simplify[c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2]}, Dist[1/q, Int[Simp[g*c^2*d - g*b*c*e + a*h*c*e + g*b^2*f - a*b*h*f - a*g*c*f + c*(h*c*d - g*c*e + g*b*f - a*h*f)*x, x]/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[Simp[-(h*c*d*e) + g*c*e^2 + b*h*d*f - g*c*d*f - g*b*e*f + a*g*f^2 - f*(h*c*d - g*c*e + g*b*f - a*h*f)*x, x]/(d + e*x + f*x^2), x], x] /; NeQ[q, 0] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx}{(a + bx + cx^2)(d + ex + fx^2)} dx &= \frac{\int \frac{aB(ce-bf)+A(c^2d+b^2f-c(be+af))+c(Bcd-Ace+Abf-aBf)x}{a+bx+cx^2} dx}{c^2d^2 - bcde + f(b^2d - abe + a^2f) + ac(e^2 - 2df)} + \frac{\int \frac{-Af(be-af)+Ac(e^2-d)}{c^2d^2 - bcde + f(b^2d - abe + a^2f) + ac(e^2 - 2df)} dx}{c^2d^2 - bcde + f(b^2d - abe + a^2f) + ac(e^2 - 2df)} \\
&= \frac{(Bcd - Ace + Abf - aBf) \int \frac{b+2cx}{a+bx+cx^2} dx}{2(c^2d^2 - bcde + f(b^2d - abe + a^2f) + ac(e^2 - 2df))} - \frac{(Bcd - Ace + Abf - aBf)}{2(c^2d^2 - bcde + f(b^2d - abe + a^2f) + ac(e^2 - 2df))} \\
&= \frac{(Bcd - Ace + Abf - aBf) \log(a + bx + cx^2)}{2(c^2d^2 - bcde + f(b^2d - abe + a^2f) + ac(e^2 - 2df))} - \frac{(Bcd - Ace + Abf - aBf)}{2(c^2d^2 - bcde + f(b^2d - abe + a^2f) + ac(e^2 - 2df))} \\
&= -\frac{(Ab^2f + 2c(Acd + aBe - aAf) - b(Bcd + Ace + aBf)) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2 - 4ac} (c^2d^2 - bcde + f(b^2d - abe + a^2f) + ac(e^2 - 2df))}
\end{aligned}$$

Mathematica [A] time = 0.46, size = 267, normalized size = 0.66

$$\frac{2 \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)(-b(aBf+Ace+Bcd)+2c(-aAf+aBe+Ac d)+Ab^2f)}{\sqrt{4ac-b^2}} - \frac{2 \tan^{-1}\left(\frac{e+2fx}{\sqrt{4df-e^2}}\right)(A(-2af^2+bef+2cdf-ce^2)+B(aef-2bdf+cde))}{\sqrt{4df-e^2}} + \log(a)$$

$$\frac{2(f(a^2f - abe + b^2d) + ac(e^2 - 2df))}{2(f(a^2f - abe + b^2d) + ac(e^2 - 2df))}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)),x]

[Out] ((2*(A*b^2*f + 2*c*(A*c*d + a*B*e - a*A*f) - b*(B*c*d + A*c*e + a*B*f))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - (2*(B*(c*d*e - 2*b*d*f + a*e*f) + A*(-(c*e^2) + 2*c*d*f + b*e*f - 2*a*f^2))*ArcTan[(e + 2*f*x)/Sqrt[-e^2 + 4*d*f]])/Sqrt[-e^2 + 4*d*f] + (B*c*d - A*c*e + A*b*f - a*B*f)*Log[a + x*(b + c*x)] + ((-B*c*d) + A*c*e - A*b*f + a*B*f)*Log[d + x*(e + f*x)]/(2*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f)))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.17, size = 416, normalized size = 1.02

$$\frac{(Bcd - Baf + Abf - Ace) \log(cx^2 + bx + a)}{2(c^2d^2 + b^2df - 2acdf + a^2f^2 - bcde - abfe + ace^2)} - \frac{(Bcd - Baf + Abf - Ace) \log(fx^2 + xe + d)}{2(c^2d^2 + b^2df - 2acdf + a^2f^2 - bcde - abfe + ace^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] 1/2*(B*c*d - B*a*f + A*b*f - A*c*e)*log(c*x^2 + b*x + a)/(c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2 - b*c*d*e - a*b*f*e + a*c*e^2) - 1/2*(B*c*d - B*a*f + A*b*f - A*c*e)*log(f*x^2 + x*e + d)/(c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2 - b*c*d*e - a*b*f*e + a*c*e^2) - (B*b*c*d - 2*A*c^2*d + B*a*b*f - A*b^2*f + 2*A*a*c*f - 2*B*a*c*e + A*b*c*e)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2 - b*c*d*e - a*b*f*e + a*c*e^2)*sqrt(-b^2 + 4*a*c) + (2*B*b*d*f - 2*A*c*d*f + 2*A*a*f^2 - B*c*d*e - B*a*f*e - A*b*f*e + A*c*e^2)*arctan((2*f*x + e)/sqrt(4*d*f - e^2))/(c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2 - b*c*d*e - a*b*f*e + a*c*e^2)*sqrt(4*d*f - e^2)

maple [B] time = 0.01, size = 1698, normalized size = 4.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d),x)

[Out] 1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)*ln(c*x^2+b*x+a)*A*b*f-1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)*c*ln(c*x^2+b*x+a)*A*e-1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)*ln(c*x^2+b*x+a)*B*a*f+1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)*c*ln(c*x^2+b*x+a)*B*d-2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*A*a*c*f+1/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*A*b^2*f-1/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*A*b*c*e+2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*A*c^2*d-1/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*B*a*b*f+2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*B*a*c*e-1/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*B*b*c*d-1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)

```

*f*ln(f*x^2+e*x+d)*A*b+1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d
*e+c^2*d^2)*ln(f*x^2+e*x+d)*A*c*e+1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^
2*d*f-b*c*d*e+c^2*d^2)*f*ln(f*x^2+e*x+d)*B*a-1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f
+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)*ln(f*x^2+e*x+d)*B*c*d+2/(a^2*f^2-a*b*e*f-
2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*d*f-e^2)^(1/2)*arctan((2*f*x+
e)/(4*d*f-e^2)^(1/2))*A*a*f^2-1/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-
b*c*d*e+c^2*d^2)/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*A*b*
e*f-2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*d*f-e^
2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*A*c*d*f+1/(a^2*f^2-a*b*e*f-2*a
*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/
(4*d*f-e^2)^(1/2))*A*c*e^2-1/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c
*d*e+c^2*d^2)/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*B*a*e*f
+2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*d*f-e^2)^
(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*B*b*d*f-1/(a^2*f^2-a*b*e*f-2*a*c*
d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*
d*f-e^2)^(1/2))*B*c*d*e

```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive or negative?
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

$$3.16 \quad \int \frac{A+Bx}{(a+bx+cx^2)^2(d+ex+fx^2)} dx$$

Optimal. Leaf size=1075

$$\frac{Ac(2ace - b(cd + af)) + (Ab - aB)(fb^2 + 2c^2d - c(be + 2af)) + c(Afb^2 - (Bcd + Ace + aBf)b + 2c(Acd + a))}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))(cx^2 + bx + a)}$$

[Out] $(-A*c*(2*a*c*e-b*(a*f+c*d))-(A*b-B*a)*(2*c^2*d+b^2*f-c*(2*a*f+b*e))-c*(A*b^2*f+2*c*(-A*a*f+A*c*d+B*a*e)-b*(A*c*e+B*a*f+B*c*d))*x)/(-4*a*c+b^2)/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))/(c*x^2+b*x+a)-(b^5*(-A*e+B*d)*f^2-2*b^4*f*(B*c*d*e-A*(a*f^2-c*d*f+c*e^2))-4*c^2*(A*(c^3*d^3-3*a^3*f^3-a^2*c*f*(-7*d*f+e^2))+a*c^2*d*(-5*d*f+3*e^2))-a*B*e*(c^2*d^2-3*a^2*f^2-a*c*(-2*d*f+e^2)))-4*b^2*c*(B*c^2*d^2*e+A*f*(2*c^2*d^2+3*a^2*f^2+3*a*c*(-d*f+e^2)))+2*b*c*(B*(c^3*d^3+3*a^3*f^3+a*c^2*d*(-7*d*f+e^2))+3*a^2*c*f*(d*f+e^2))+A*c*e*(3*c^2*d^2+3*a^2*f^2+a*c*(2*d*f+3*e^2)))-b^3*(A*c*e*(-4*a*f^2-2*c*d*f+c*e^2)+B*(4*a*c*d*f^2+a^2*f^3-c^2*d*(5*d*f+e^2)))*\operatorname{arctanh}((2*c*x+b)/(-4*a*c+b^2)^{(1/2)})/(-4*a*c+b^2)^{(3/2)}/(c^2*d^2+f*(a^2*f-a*b*e+b^2*d)-c*(b*d*e-a*(-2*d*f+e^2)))^2+1/2*(A*(-b*f+c*e)*(f*(-2*a*f+b*e)-c*(-2*d*f+e^2))-B*(2*c*d*f*(-a*f+b*e)-f^2*(-a^2*f+b^2*d)-c^2*d*(-d*f+e^2)))*\ln(c*x^2+b*x+a)/(c^2*d^2+f*(a^2*f-a*b*e+b^2*d)-c*(b*d*e-a*(-2*d*f+e^2)))^2-1/2*(A*(-b*f+c*e)*(f*(-2*a*f+b*e)-c*(-2*d*f+e^2))-B*(2*c*d*f*(-a*f+b*e)-f^2*(-a^2*f+b^2*d)-c^2*d*(-d*f+e^2)))*\ln(f*x^2+e*x+d)/(c^2*d^2+f*(a^2*f-a*b*e+b^2*d)-c*(b*d*e-a*(-2*d*f+e^2)))^2+(B*(c^2*d*e*(-3*d*f+e^2)-2*c*d*f*(-a*e*f-2*b*d*f+b*e^2)+f^2*(a^2*e*f-4*a*b*d*f+b^2*d*e))-A*(c^2*(2*d^2*f^2-4*d*e^2*f+e^4)-f^2*(2*a*b*e*f-2*a^2*f^2-b^2*(-2*d*f+e^2))+2*c*f*(a*f*(-2*d*f+e^2)-b*(-3*d*e*f+e^3)))*\operatorname{arctanh}((2*f*x+e)/(-4*d*f+e^2)^{(1/2)})/(c^2*d^2+f*(a^2*f-a*b*e+b^2*d)-c*(b*d*e-a*(-2*d*f+e^2)))^2/(-4*d*f+e^2)^{(1/2)}$

Rubi [A] time = 4.18, antiderivative size = 1067, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1016, 1072, 634, 618, 206, 628}

$$\frac{Ac(2ace - b(cd + af)) + (Ab - aB)(fb^2 + 2c^2d - c(be + 2af)) + c(Afb^2 - (Bcd + Ace + aBf)b + 2c(Acd + a))}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))(cx^2 + bx + a)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*x)/((a + b*x + c*x^2)^2*(d + e*x + f*x^2)), x]$

[Out] $-((A*c*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*b^2*f + 2*c*(A*c*d + a*B*e - a*A*f) - b*(B*c*d + A*c*e + a*B$

$$\begin{aligned} & *f)) * x) / ((b^2 - 4*a*c) * ((c*d - a*f)^2 - (b*d - a*e) * (c*e - b*f)) * (a + b*x + \\ & c*x^2)) - ((b^5 * (B*d - A*e) * f^2 - 2*b^4 * f * (B*c*d*e - a*A*f^2 - A*c*(e^2 - \\ & d*f)) - 4*c^2 * (A*(c^3*d^3 - 3*a^3*f^3 - a^2*c*f*(e^2 - 7*d*f)) + a*c^2*d*(3 \\ & *e^2 - 5*d*f)) - a*B*e*(c^2*d^2 - 3*a^2*f^2 - a*c*(e^2 - 2*d*f))) - 4*b^2 * (\\ & B*c^3*d^2*e + A*c*f*(2*c^2*d^2 + 3*a^2*f^2 + 3*a*c*(e^2 - d*f))) + 2*b*c*(B \\ & *(c^3*d^3 + 3*a^3*f^3 + a*c^2*d*(e^2 - 7*d*f)) + 3*a^2*c*f*(e^2 + d*f)) + A \\ & c*e*(3*c^2*d^2 + 3*a^2*f^2 + a*c*(3*e^2 + 2*d*f))) - b^3 * (A*c*e*(c*e^2 - 2* \\ & c*d*f - 4*a*f^2) + B*(4*a*c*d*f^2 + a^2*f^3 - c^2*d*(e^2 + 5*d*f)))) * ArcTan \\ & h[(b + 2*c*x) / Sqrt[b^2 - 4*a*c]] / ((b^2 - 4*a*c)^(3/2) * (c^2*d^2 - b*c*d*e + \\ & f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))^2) + ((B*(c^2*d*e*(e^2 - 3* \\ & d*f) - 2*c*d*f*(b*e^2 - 2*b*d*f - a*e*f) + f^2*(b^2*d*e - 4*a*b*d*f + a^2*e \\ & *f)) - A*(c^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2) - f^2*(2*a*b*e*f - 2*a^2*f^2 - \\ & b^2*(e^2 - 2*d*f)) + 2*c*f*(a*f*(e^2 - 2*d*f) - b*(e^3 - 3*d*e*f)))) * ArcTan \\ & h[(e + 2*f*x) / Sqrt[e^2 - 4*d*f]] / (Sqrt[e^2 - 4*d*f] * (c^2*d^2 - b*c*d*e + f \\ & *(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))^2) + ((A*(c*e - b*f) * (f*(b*e \\ & - 2*a*f) - c*(e^2 - 2*d*f)) - B*(2*c*d*f*(b*e - a*f) - f^2*(b^2*d - a^2*f) \\ & - c^2*d*(e^2 - d*f))) * Log[a + b*x + c*x^2]) / (2*(c^2*d^2 - b*c*d*e + f*(b^2* \\ & d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))^2) - ((A*(c*e - b*f) * (f*(b*e - 2*a* \\ & f) - c*(e^2 - 2*d*f)) - B*(2*c*d*f*(b*e - a*f) - f^2*(b^2*d - a^2*f) - c^2* \\ & d*(e^2 - d*f))) * Log[d + e*x + f*x^2]) / (2*(c^2*d^2 - b*c*d*e + f*(b^2*d - a* \\ & b*e + a^2*f) + a*c*(e^2 - 2*d*f))^2) \end{aligned}$$

Rule 206

$$\text{Int}[(a + (b \cdot x)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1 * \text{ArcTanh}[\text{Rt}[-b, 2] * x] / \text{Rt}[a, 2]) / (\text{Rt}[a, 2] * \text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$$

Rule 618

$$\text{Int}[(a + (b \cdot x) + (c \cdot x)^2)^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}\{a, b, c, x\} \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$$

Rule 628

$$\text{Int}[(d + (e \cdot x)) / ((a + (b \cdot x) + (c \cdot x)^2), x_Symbol] \rightarrow \text{Simp}[(d * \text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]) / b, x] /; \text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$$

Rule 634

$$\text{Int}[(d + (e \cdot x)) / ((a + (b \cdot x) + (c \cdot x)^2), x_Symbol] \rightarrow \text{Dist}[(2*c*d - b*e) / (2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e / (2*c), \text{Int}[(b + 2*c*x) / (a + b*x + c*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{NeQ}$$

$[2*c*d - b*e, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4*a*c]$

Rule 1016

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1) * (d + e*x + f*x^2)^(q + 1) * (g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*c*d - 2*a*c*e + a*b*f))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))^(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))^(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))^(p + 1) + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !(IntegerQ[p] && ILtQ[q, -1])
```

Rule 1072

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2) * ((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] := With[{q = c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx}{(a + bx + cx^2)^2 (d + ex + fx^2)} dx &= -\frac{Ac(2ace - b(cd + af)) + (Ab - aB)(2c^2d + b^2f - c(be + 2af)) + c(Ab^2 - a^2)}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - ad))} \\
&= -\frac{Ac(2ace - b(cd + af)) + (Ab - aB)(2c^2d + b^2f - c(be + 2af)) + c(Ab^2 - a^2)}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - ad))} \\
&= -\frac{Ac(2ace - b(cd + af)) + (Ab - aB)(2c^2d + b^2f - c(be + 2af)) + c(Ab^2 - a^2)}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - ad))} \\
&= -\frac{Ac(2ace - b(cd + af)) + (Ab - aB)(2c^2d + b^2f - c(be + 2af)) + c(Ab^2 - a^2)}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - ad))} \\
&= -\frac{Ac(2ace - b(cd + af)) + (Ab - aB)(2c^2d + b^2f - c(be + 2af)) + c(Ab^2 - a^2)}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - ad))}
\end{aligned}$$

Mathematica [A] time = 6.70, size = 952, normalized size = 0.89

$$-\frac{2(c^2d^2 - bced + f(fa^2 - bea + b^2d) + ac(e^2 - 2df))(A(fb^3 + c(fx - e)b^2 + c(c(d - ex) - 3af)b + 2c^2(cdx + a(e - fx))) + B(2cfa^2 - (fb^2 + c(fx - e)b + 2c^2(d - ex))a - bc^2d)}{(b^2 - 4ac)(a + x(b + cx))}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x)/((a + b*x + c*x^2)^2*(d + e*x + f*x^2)),x]

[Out] ((-2*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))*(A*(b^3*f + b^2*c*(-e + f*x) + b*c*(-3*a*f + c*(d - e*x)) + 2*c^2*(c*d*x + a*(e - f*x))) + B*(2*a^2*c*f - b*c^2*d*x - a*(b^2*f + 2*c^2*(d - e*x) + b*c*(-e + f*x)))))/((b^2 - 4*a*c)*(a + x*(b + c*x))) - (2*(b^5*(B*d - A*e)*f^2 + 2*b^4*f*(-(B*c*d*e) + a*A*f^2 + A*c*(e^2 - d*f)) - 4*b^2*(B*c^3*d^2*e + A*c*f*(2*c^2*d^2 + 3*a^2*f^2 + 3*a*c*(e^2 - d*f))) + 2*b*c*(B*(c^3*d^3 + 3*a^3*f^3 + a*c^2*d*(e^2 - 7*d*f) + 3*a^2*c*f*(e^2 + d*f)) + A*c*e*(3*c^2*d^2 + 3*a^2*f^2 + a*c*(3*e^2 + 2*d*f))) + 4*c^2*(a*B*e*(c^2*d^2 - 3*a^2*f^2 - a*c*(e^2 - 2*d*f)) + A*(-(c^3*d^3) + 3*a^3*f^3 + a^2*c*f*(e^2 - 7*d*f) + a*c^2*d*(-3*e^2 + 5*d*f))) + b^3*(A*c*e*(-(c*e^2) + 2*c*d*f + 4*a*f^2) + B*(-4*a*c*d*f^2 - a^2*f^3 + c^2*d*(e^2 + 5*d*f))))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +

$$\frac{4ac}{(-b^2 + 4ac)^{3/2}} + (2(B(c^2de(-e^2 + 3df) - 2cdf(-b^2e^2 + 2bdf + aef) + f^2(-b^2de + 4abd - a^2ef)) + A(c^2(e^4 - 4de^2f + 2d^2f^2) + f^2(-2abef + 2a^2f^2 + b^2(e^2 - 2df)) + 2cf(aef(e^2 - 2df) - b(e^3 - 3def)))) * \text{ArcTan}[(e + 2fx) / \sqrt{-e^2 + 4df}] / \sqrt{-e^2 + 4df} - (A(c(e - b)f)(f(-be) + 2af) + c(e^2 - 2df)) + B(2cdf(b - a) + f^2(-b^2d + a^2f) + c^2d(-e^2 + df))) * \text{Log}[a + x(b + cx)] + (A(c(e - b)f)(f(-be) + 2af) + c(e^2 - 2df)) + B(2cdf(b - a) + f^2(-b^2d + a^2f) + c^2d(-e^2 + df))) * \text{Log}[d + x(e + fx)] / (2(c^2d^2 - bcd + f(b^2d - abe + a^2f) + ac(e^2 - 2df))^2)$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [B] time = 0.45, size = 3226, normalized size = 3.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/2*(B*c^2*d^2*f - B*b^2*d*f^2 - 2*B*a*c*d*f^2 + 2*A*b*c*d*f^2 + B*a^2*f^3 \\ & - 2*A*a*b*f^3 + 2*B*b*c*d*f*e - 2*A*c^2*d*f*e + A*b^2*f^2*e + 2*A*a*c*f^2*e \\ & e - B*c^2*d*e^2 - 2*A*b*c*f*e^2 + A*c^2*e^3)*\log(c*x^2 + b*x + a)/(c^4*d^4 \\ & + 2*b^2*c^2*d^3*f - 4*a*c^3*d^3*f + b^4*d^2*f^2 - 4*a*b^2*c*d^2*f^2 + 6*a^2 \\ & *c^2*d^2*f^2 + 2*a^2*b^2*d*f^3 - 4*a^3*c*d*f^3 + a^4*f^4 - 2*b*c^3*d^3*e - \\ & 2*b^3*c*d^2*f*e + 2*a*b*c^2*d^2*f*e - 2*a*b^3*d*f^2*e + 2*a^2*b*c*d*f^2*e - \\ & 2*a^3*b*f^3*e + b^2*c^2*d^2*e^2 + 2*a*c^3*d^2*e^2 + 4*a*b^2*c*d*f*e^2 - 4* \\ & a^2*c^2*d*f*e^2 + a^2*b^2*f^2*e^2 + 2*a^3*c*f^2*e^2 - 2*a*b*c^2*d*e^3 - 2*a \\ & ^2*b*c*f*e^3 + a^2*c^2*e^4) + 1/2*(B*c^2*d^2*f - B*b^2*d*f^2 - 2*B*a*c*d*f^2 \\ & + 2*A*b*c*d*f^2 + B*a^2*f^3 - 2*A*a*b*f^3 + 2*B*b*c*d*f*e - 2*A*c^2*d*f*e \\ & + A*b^2*f^2*e + 2*A*a*c*f^2*e - B*c^2*d*e^2 - 2*A*b*c*f*e^2 + A*c^2*e^3)*\log(f*x^2 + x*e + d)/(c^4*d^4 + 2*b^2*c^2*d^3*f - 4*a*c^3*d^3*f + b^4*d^2*f^2 \\ & - 4*a*b^2*c*d^2*f^2 + 6*a^2*c^2*d^2*f^2 + 2*a^2*b^2*d*f^3 - 4*a^3*c*d*f^3 \\ & + a^4*f^4 - 2*b*c^3*d^3*e - 2*b^3*c*d^2*f*e + 2*a*b*c^2*d^2*f*e - 2*a*b^3* \\ & d*f^2*e + 2*a^2*b*c*d*f^2*e - 2*a^3*b*f^3*e + b^2*c^2*d^2*e^2 + 2*a*c^3*d^2 \\ & *e^2 + 4*a*b^2*c*d*f*e^2 - 4*a^2*c^2*d*f*e^2 + a^2*b^2*f^2*e^2 + 2*a^3*c*f^2 \\ & *e^2 - 2*a*b*c^2*d*e^3 - 2*a^2*b*c*f*e^3 + a^2*c^2*e^4) + (2*B*b*c^4*d^3 - \\ & 4*A*c^5*d^3 + 5*B*b^3*c^2*d^2*f - 14*B*a*b*c^3*d^2*f - 8*A*b^2*c^3*d^2*f + \end{aligned}$$

$$\begin{aligned}
&20A^2ac^4d^2f + B^2b^5d^2f^2 - 4B^2ab^3c^2d^2f^2 - 2A^2b^4c^2d^2f^2 + 6B^2 \\
&a^2b^2c^2d^2f^2 + 12A^2ab^2c^2d^2f^2 - 28A^2a^2c^3d^2f^2 - B^2a^2b^3f^3 + 2A^2ab^4f^3 + 6B^2a^3b^2c^2f^3 - 12A^2a^2b^2c^2f^3 + 12A^2a^3c^2f^3 \\
&- 4B^2b^2c^3d^2e + 4B^2a^2c^4d^2e + 6A^2b^2c^4d^2e - 2B^2b^4c^2d^2f^2e + 2A^2b^3c^2d^2f^2e + 8B^2a^2c^3d^2f^2e + 4A^2ab^2c^3d^2f^2e - A^2b^5f^2e + \\
&4A^2ab^3c^2f^2e - 12B^2a^3c^2f^2e + 6A^2a^2b^2c^2f^2e + B^2b^3c^2d^2 \\
&e^2 + 2B^2a^2b^2c^3d^2e^2 - 12A^2ac^4d^2e^2 + 2A^2b^4c^2f^2e^2 + 6B^2a^2b^2c^2 \\
&f^2e^2 - 12A^2ab^2c^2f^2e^2 + 4A^2a^2c^3f^2e^2 - A^2b^3c^2e^3 - 4B^2a^2 \\
&c^3e^3 + 6A^2ab^2c^3e^3) * \arctan((2cx + b)/\sqrt{-b^2 + 4ac}) / ((b^2c^4d^4 - \\
&4ac^5d^4 + 2b^4c^2d^3f - 12ab^2c^3d^3f + 16a^2c^4d^3f + b^6d^2f^2 - 8a^2b^4c^2d^2f^2 + \\
&22a^2b^2c^2d^2f^2 - 24a^3c^3d^2f^2 + 2a^2b^4d^2f^3 - 12a^3b^2c^2d^2f^3 + 16a^4c^2d^2f^3 + \\
&a^4b^2f^4 - 4a^5c^2f^4 - 2b^3c^3d^3e + 8a^2b^2c^4d^3e - 2b^5c^2d^2f^2e + \\
&10a^2b^3c^2d^2f^2e - 8a^2b^2c^3d^2f^2e - 2a^2b^5d^2f^2e + 10a^2b^3c^2d^2f^2e - \\
&8a^3b^2c^2d^2f^2e - 2a^3b^3f^3e + 8a^4b^2c^2f^3e + b^4c^2d^2e^2 - 2a^2b^2c^3d^2e^2 - \\
&8a^2c^4d^2e^2 + 4a^2b^4c^2d^2f^2e^2 - 20a^2b^2c^2d^2f^2e^2 + 16a^3c^3d^2f^2e^2 + a^2b^4f^2e^2 - \\
&2a^3b^2c^2f^2e^2 - 8a^4c^2f^2e^2 - 2a^2b^3c^2d^2e^3 + 8a^2b^2c^3d^2e^3 - 2a^2 \\
&b^3c^2f^2e^3 + 8a^3b^2c^2f^2e^3 + a^2b^2c^2e^4 - 4a^3c^3e^4) * \sqrt{-b^2 + 4ac} - \\
&(4B^2b^2c^2d^2f^2 - 2A^2c^2d^2f^2 - 4B^2a^2b^2d^2f^3 + 2A^2b^2d^2f^3 + 4A^2ac^2d^2f^3 - \\
&2A^2a^2f^4 - 3B^2c^2d^2f^2e + B^2b^2d^2f^2e + 2B^2a^2c^2d^2f^2e - 6A^2b^2c^2d^2f^2e + \\
&B^2a^2f^3e + 2A^2ab^2f^3e - 2B^2b^2c^2d^2f^2e + 4A^2c^2d^2f^2e^2 - A^2b^2f^2e^2 - \\
&2A^2ac^2f^2e^2 + B^2c^2d^2e^3 + 2A^2b^2c^2f^2e^3 - A^2c^2e^4) * \arctan((2fx + e)/\sqrt{4d^2f - e^2}) / ((c^4d^4 + \\
&2b^2c^2d^3f - 4ac^3d^3f + b^4d^2f^2 - 4ab^2c^2d^2f^2 + 6a^2c^2d^2f^2 + 2a^2b^2d^2f^3 - \\
&4a^3c^2d^2f^3 + a^4f^4 - 2b^2c^3d^3e - 2b^3c^2d^2f^2e + 2a^2b^2c^2d^2f^2e - \\
&2a^2b^3d^2f^2e + 2a^2b^2c^2d^2f^2e - 2a^2b^3d^2f^2e + 2a^2b^2c^2d^2f^2e - 2a^3b^2f^3e + \\
&b^2c^2d^2e^2 + 2a^2c^3d^2e^2 + 4a^2b^2c^2d^2f^2e - 4a^2c^2d^2f^2e^2 + a^2b^2f^2e^2 + \\
&2a^3c^2f^2e^2 - 2a^2b^2c^2d^2e^3 - 2a^2b^2c^2f^2e^3 + a^2c^2e^4) * \sqrt{4d^2f - e^2} + (2B^2a^2c^4d^3 - \\
&A^2b^2c^4d^3 + 3B^2a^2b^2c^2d^2f - 2A^2b^3c^2d^2f - 6B^2a^2c^3d^2f + 5A^2a^2b^2c^3 \\
&d^2f + B^2ab^4d^2f^2 - A^2b^5d^2f^2 - 4B^2a^2b^2c^2d^2f^2 + 5A^2a^2b^3c^2d^2f^2 + \\
&6B^2a^3c^2d^2f^2 - 7A^2a^2b^2c^2d^2f^2 + B^2a^3b^2f^3 - A^2a^2b^3f^3 - 2B^2a^4c^2f^3 + \\
&3A^2a^3b^2c^2f^3 - 3B^2a^2b^2c^3d^2e + 2A^2b^2c^3d^2e - 2A^2a^2c^4d^2e - 2B^2a^2b^3c^2d^2f^2e + \\
&2A^2b^4c^2d^2f^2e + 2B^2a^2b^2c^2d^2f^2e - 6A^2ab^2c^2d^2f^2e + 4A^2a^2c^3d^2f^2e - \\
&B^2a^2b^3f^2e + A^2ab^4f^2e + B^2a^3b^2c^2f^2e - 2A^2a^2b^2c^2f^2e - 2A^2a^3c^2f^2e + \\
&B^2a^2b^2c^2d^2e^2 - A^2b^3c^2d^2e^2 + 2B^2a^2c^3d^2e^2 + A^2ab^2c^3d^2e^2 + 2B^2a^2 \\
&b^2c^2f^2e^2 - 2A^2a^2b^3c^2f^2e^2 - 2B^2a^3c^2f^2e^2 + 5A^2a^2b^2c^2f^2e^2 - B^2a^2b^2c^2e^3 + \\
&A^2ab^2c^2e^3 - 2A^2a^2c^3e^3 + (B^2b^2c^4d^3 - 2A^2c^5d^3 + B^2b^3c^2d^2f - B^2a^2b^2c^3d^2f - \\
&3A^2b^2c^3d^2f + 6A^2ac^4d^2f + B^2ab^3c^2d^2f^2 - A^2b^4c^2d^2f^2 - B^2a^2b^2c^2d^2f^2 + \\
&4A^2a^2b^2c^2d^2f^2 - 6A^2a^2c^3d^2f^2 + B^2a^3b^2c^2f^3 - A^2a^2b^2c^2f^3 + 2A^2a^3c^2f^3 - \\
&B^2b^2c^3d^2e - 2B^2a^2c^4d^2e + 3A^2b^2c^4d^2e - 4B^2a^2b^2c^2d^2f^2e + 2A^2b^3c^2d^2f^2e + \\
&4B^2a^2c^3d^2f^2e - 2A^2a^2b^2c^3d^2f^2e - B^2a^2b^2c^3d^2f^2e - B^2a^2b^2c^3d^2f^2e - B^2a^2b^2c^3d^2f^2e - B^2a^2b^2c^3d^2f^2e - B^2a^2b^2c^3d^2f^2e
\end{aligned}$$

$$\begin{aligned} &^2*b^2*c*f^2*e + A*a*b^3*c*f^2*e - 2*B*a^3*c^2*f^2*e - A*a^2*b*c^2*f^2*e + \\ &3*B*a*b*c^3*d*e^2 - A*b^2*c^3*d*e^2 - 2*A*a*c^4*d*e^2 + 3*B*a^2*b*c^2*f*e^2 \\ &- 2*A*a*b^2*c^2*f*e^2 + 2*A*a^2*c^3*f*e^2 - 2*B*a^2*c^3*e^3 + A*a*b*c^3*e^3 \\ &3)*x)/((c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2 - b*c*d*e - a*b*f*e + a*c*e^2)^2*(c*x^2 + b*x + a)*(b^2 - 4*a*c)) \end{aligned}$$

maple [B] time = 0.05, size = 51470, normalized size = 47.88

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x)

[Out] result too large to display

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* h elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo re details)Is 4*d*f-e^2 positive or negative?

mupad [B] time = 30.31, size = 118429, normalized size = 110.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x)/((a + b*x + c*x^2)^2*(d + e*x + f*x^2)),x)

[Out] symsum(log((x*(4*A^3*b^3*c^4*f^6 + 16*B^3*a^3*c^4*f^6 - 3*B^3*a^2*b^2*c^3*f^6 + 4*B^3*a^2*c^5*e^2*f^4 + B^3*b^2*c^5*d^2*f^4 - 16*A^3*a*b*c^5*f^6 + 16*A^3*a*c^6*e*f^5 + 20*A^2*B*a^2*c^5*f^6 - 3*A^2*B*b^4*c^3*f^6 + 4*A^2*B*c^7*d^2*f^4 - 16*B^3*a^2*c^5*d*f^5 - 4*A^3*b^2*c^5*e*f^5 + 6*B^3*a*b^2*c^4*d*f^5 - 4*B^3*a^2*b*c^4*e*f^5 + A^2*B*b^2*c^5*e^2*f^4 - 24*A^2*B*a*c^6*d*f^5 + 6*A*B^2*a*b^3*c^3*f^6 - 28*A*B^2*a^2*b*c^4*f^6 + 8*A^2*B*a*b^2*c^4*f^6 - 4*A*B^2*b*c^6*d^2*f^4 + 8*A*B^2*a^2*c^5*e*f^5 - 6*A*B^2*b^3*c^4*d*f^5 + 8*A^2*B*b^2*c^5*d*f^5 + 2*A^2*B*b^3*c^4*e*f^5 - 4*B^3*a*b*c^5*d*e*f^4 - 4*A*B^2*a*b*c^5*e^2*f^4 + 2*A*B^2*a*b^2*c^4*e*f^5 + 2*A*B^2*b^2*c^5*d*e*f^4 + 16*A*B^2*a*b*c^5*d*f^5 - 12*A^2*B*a*b*c^5*e*f^5 + 8*A*B^2*a*c^6*d*e*f^4 - 4*A^2*

$$\begin{aligned}
& B*b*c^6*d*e*f^4)) / (16*a^2*c^6*d^4 + a^4*b^4*f^4 + 16*a^4*c^4*e^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 2*a^3*b^5*e*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 - 2*b^5*c^3*d^3*e + 2*b^6*c^2*d^3*f + a^2*b^4*c^2*e^4 - 8*a^3*b^2*c^3*e^4 + 32*a^3*c^5*d^2*e^2 + a^2*b^6*e^2*f^2 + 96*a^4*c^4*d^2*f^2 + b^6*c^2*d^2*e^2 + 3*2*a^5*c^3*e^2*f^2 - 2*a*b^7*d*e*f^2 - 2*b^7*c*d^2*e*f + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 + 16*a*b^3*c^4*d^3*e - 2*a*b^5*c^2*d*e^3 - 32*a^2*b*c^5*d^3*e - 32*a^3*b*c^4*d*e^3 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 - 2*a^2*b^5*c*e^3*f - 32*a^4*b*c^3*e^3*f + 16*a^4*b^3*c*e*f^3 - 32*a^5*b*c^2*e*f^3 - 64*a^4*c^4*d*e^2*f - 6*a*b^4*c^3*d^2*e^2 + 16*a^2*b^3*c^3*d*e^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3 + 16*a^3*b^3*c^2*e^3*f - 6*a^3*b^4*c*e^2*f^2 - 48*a^2*b^3*c^3*d^2*e*f - 36*a^2*b^4*c^2*d*e^2*f + 96*a^3*b^2*c^3*d*e^2*f - 48*a^3*b^3*c^2*d*e*f^2 + 4*a*b^6*c*d*e^2*f + 18*a*b^5*c^2*d^2*e*f + 18*a^2*b^5*c*d*e*f^2 + 32*a^3*b*c^4*d^2*e*f + 32*a^4*b*c^3*d*e*f^2) - \text{root}(48416*a^6*b^2*c^6*d^4*e^2*f^4*z^4 - 41544*a^5*b^4*c^5*d^4*e^2*f^4*z^4 - 31872*a^7*b^2*c^5*d^3*e^2*f^5*z^4 - 31872*a^5*b^2*c^7*d^5*e^2*f^3*z^4 - 29184*a^6*b^2*c^6*d^3*e^4*f^3*z^4 + 28800*a^5*b^4*c^5*d^3*e^4*f^3*z^4 + 21510*a^4*b^6*c^4*d^4*e^2*f^4*z^4 + 21408*a^6*b^4*c^4*d^3*e^2*f^5*z^4 + 21408*a^4*b^4*c^6*d^5*e^2*f^3*z^4 - 18112*a^7*b^3*c^4*d^2*e^3*f^5*z^4 - 18112*a^4*b^3*c^7*d^5*e^3*f^2*z^4 - 15600*a^5*b^5*c^4*d^3*e^3*f^4*z^4 - 15600*a^4*b^5*c^5*d^4*e^3*f^3*z^4 + 15296*a^6*b^3*c^5*d^3*e^3*f^4*z^4 + 15296*a^5*b^3*c^6*d^4*e^3*f^3*z^4 + 14016*a^7*b^2*c^5*d^2*e^4*f^4*z^4 + 14016*a^5*b^2*c^7*d^4*e^4*f^2*z^4 - 13920*a^4*b^6*c^4*d^3*e^4*f^3*z^4 - 11648*a^6*b^3*c^5*d^2*e^5*f^3*z^4 - 11648*a^5*b^3*c^6*d^3*e^5*f^2*z^4 + 10432*a^6*b^2*c^6*d^2*e^6*f^2*z^4 + 9008*a^6*b^5*c^3*d^2*e^3*f^5*z^4 + 9008*a^3*b^5*c^6*d^5*e^3*f^2*z^4 + 8544*a^5*b^5*c^4*d^2*e^5*f^3*z^4 + 8544*a^4*b^5*c^5*d^3*e^5*f^2*z^4 - 8496*a^5*b^4*c^5*d^2*e^6*f^2*z^4 + 7488*a^8*b^2*c^4*d^2*e^2*f^6*z^4 + 7488*a^4*b^2*c^8*d^6*e^2*f^2*z^4 + 7380*a^4*b^7*c^3*d^3*e^3*f^4*z^4 + 7380*a^3*b^7*c^4*d^4*e^3*f^3*z^4 - 6720*a^3*b^8*c^3*d^4*e^2*f^4*z^4 - 5784*a^5*b^6*c^3*d^3*e^2*f^5*z^4 - 5784*a^3*b^6*c^5*d^5*e^2*f^3*z^4 - 3440*a^6*b^4*c^4*d^2*e^4*f^4*z^4 - 3440*a^4*b^4*c^6*d^4*e^4*f^2*z^4 + 3360*a^3*b^8*c^3*d^3*e^4*f^3*z^4 + 3140*a^4*b^6*c^4*d^2*e^6*f^2*z^4 - 2760*a^4*b^7*c^3*d^2*e^5*f^3*z^4 - 2760*a^3*b^7*c^4*d^3*e^5*f^2*z^4 - 1764*a^5*b^7*c^2*d^2*e^3*f^5*z^4 - 1764*a^2*b^7*c^5*d^5*e^3*f^2*z^4 - 1640*a^3*b^9*c^2*d^3*e^3*f^4*z^4 - 1640*a^2*b^9*c^3*d^4*e^3*f^3*z^4 - 1604*a^6*b^6*c^2*d^2*e^2*f^6*z^4 - 1604*a^2*b^6*c^6*d^6*e^2*f^2*z^4 - 1500*a^5*b^6*c^3*d^2*e^4*f^4*z^4 - 1500*a^3*b^6*c^5*d^4*e^4*f^2*z^4 + 1140*a^2*b^10*c^2*d^4*e^2*f^4*z^4 + 810*a^4*b^8*c^2*d^2*e^4*f^4*z^4 + 810*a^2*b^8*c^4*d^4*e^4*f^2*z^4 - 544*a^3*b^8*c^3*d^2*e^6*f^2*z^4 + 416*a^3*b^9*c^2*d^2*e^5*f^3*z^4 + 416*a^2*b^9*c^3*d^3*e^5*f^2*z^4 - 384*a^2*b^10*c^2*d^3*e^4*f^3*z^4 + 180*a^4*b^8*c^2*d^3*e^2*f^5*z^4 + 180*a^2*b^8*c^4*d^5*e^2*f^3*z^4 + 48*a^7*b^4*c^3*d^2*e^2*f^6*z^4 + 48*a^3*b^4*c^7*d^6*e^2*f^2*z^4 + 36*a^2*b^10*c^2*d^2*e^6*f^2*z^4 - 1024*a^10*b*c^3*d*e*f^8*z^4 - 1024*a^3*b*c^10*d^8*e*f*z^4 - 192*a^8*b^5*c*d*e*f^8*z^4 - 192*a*b^5*c^8*d^8*e*f*z^4 + 16128*a^7*b^3*c^4*d^3*e*f^6*z^4 + 16128*a^4*b^3*c^7*d^6*e*f^3*z^4 - 11712*a^6*b^5*c^3*d^3*e*f^6*z^4
\end{aligned}$$

$$\begin{aligned}
& 4 - 11712a^3b^5c^6d^6e^3f^3z^4 + 11520a^8b^5c^5d^2e^3f^5z^4 + 11520a^5b^5c^8d^5e^3f^2z^4 - 9984a^6b^3c^5d^4e^3f^5z^4 - 9984a^5b^3c^6d^5e^3f^4z^4 + 8640a^5b^5c^4d^4e^3f^5z^4 + 8640a^4b^5c^5d^5e^3f^4z^4 - 7424a^7b^3c^6d^3e^3f^4z^4 - 7424a^6b^3c^7d^4e^3f^3z^4 - 6912a^8b^3c^3d^2e^3f^7z^4 - 6912a^3b^3c^8d^7e^3f^2z^4 + 4800a^7b^3c^4d^2e^5f^4z^4 + 4800a^4b^3c^7d^4e^5f^3z^4 + 4608a^7b^3c^6d^2e^5f^3z^4 + 4608a^6b^3c^7d^3e^5f^2z^4 - 4560a^4b^7c^3d^4e^3f^5z^4 - 4560a^3b^7c^4d^5e^3f^4z^4 + 4176a^5b^7c^2d^3e^3f^6z^4 + 4176a^2b^7c^5d^6e^3f^3z^4 + 3264a^7b^5c^2d^2e^3f^7z^4 + 3264a^2b^5c^7d^7e^3f^2z^4 + 3008a^8b^3c^3d^2e^3f^6z^4 + 3008a^3b^3c^8d^6e^3f^5z^4 + 2880a^6b^3c^5d^2e^7f^2z^4 + 2880a^5b^3c^6d^2e^7f^2z^4 - 2240a^7b^4c^3d^2e^4f^5z^4 - 2240a^3b^4c^7d^5e^4f^5z^4 - 1488a^5b^5c^4d^2e^7f^2z^4 - 1488a^4b^5c^5d^2e^7f^2z^4 + 1440a^3b^9c^2d^4e^3f^5z^4 + 1440a^2b^9c^3d^5e^3f^4z^4 - 1328a^6b^5c^3d^2e^5f^4z^4 - 1328a^3b^5c^6d^4e^5f^3z^4 - 1152a^7b^2c^5d^2e^6f^3z^4 - 1152a^5b^2c^7d^3e^6f^3z^4 - 1120a^6b^4c^4d^2e^6f^3z^4 - 1120a^4b^4c^6d^3e^6f^3z^4 + 912a^6b^6c^2d^2e^4f^5z^4 + 912a^2b^6c^6d^5e^4f^5z^4 + 872a^5b^6c^3d^2e^6f^3z^4 + 872a^3b^6c^5d^3e^6f^3z^4 + 768a^8b^2c^4d^2e^4f^5z^4 + 768a^4b^2c^8d^5e^4f^5z^4 - 672a^8b^4c^2d^2e^2f^7z^4 - 672a^2b^4c^8d^7e^2f^7z^4 - 624a^7b^5c^2d^2e^3f^6z^4 - 624a^2b^5c^7d^6e^3f^6z^4 + 480a^5b^8c^2d^2e^2f^6z^4 + 480a^2b^8c^5d^6e^2f^2z^4 + 316a^4b^7c^3d^2e^7f^2z^4 + 316a^3b^7c^4d^2e^7f^2z^4 - 204a^4b^8c^2d^2e^6f^3z^4 - 204a^2b^8c^4d^3e^6f^3z^4 + 168a^3b^10c^3d^3e^2f^5z^4 + 168a^2b^10c^3d^5e^2f^3z^4 + 156a^2b^11c^3d^3e^3f^4z^4 + 156a^2b^11c^2d^4e^3f^3z^4 + 128a^9b^2c^3d^2e^2f^7z^4 + 128a^3b^2c^9d^7e^2f^7z^4 - 124a^3b^10c^3d^2e^4f^4z^4 - 124a^2b^10c^3d^4e^4f^2z^4 + 100a^4b^9c^3d^2e^3f^5z^4 + 100a^2b^9c^4d^5e^3f^2z^4 + 36a^5b^7c^2d^2e^5f^4z^4 + 36a^2b^7c^5d^4e^5f^4z^4 - 24a^3b^9c^2d^2e^7f^2z^4 - 24a^2b^11c^2d^2e^5f^3z^4 - 24a^2b^9c^3d^2e^7f^2z^4 - 24a^2b^11c^2d^3e^5f^2z^4 - 9216a^8b^3c^5d^3e^3f^6z^4 - 9216a^5b^3c^8d^6e^3f^3z^4 - 5376a^8b^3c^5d^2e^5f^4z^4 - 5376a^5b^3c^8d^4e^5f^3z^4 + 5120a^9b^3c^4d^2e^3f^7z^4 + 5120a^7b^3c^6d^4e^3f^5z^4 + 5120a^6b^3c^7d^5e^3f^4z^4 + 5120a^4b^3c^9d^7e^3f^2z^4 - 4352a^9b^3c^4d^2e^3f^6z^4 - 4352a^4b^3c^9d^6e^3f^3z^4 - 1792a^7b^3c^6d^2e^7f^2z^4 - 1792a^6b^3c^7d^2e^7f^2z^4 - 1600a^6b^2c^6d^2e^8f^3z^4 + 912a^5b^4c^5d^2e^8f^3z^4 + 768a^9b^3c^2d^2e^3f^8z^4 + 768a^2b^3c^9d^8e^3f^3z^4 - 720a^4b^9c^3d^3e^3f^6z^4 - 720a^2b^9c^4d^6e^3f^3z^4 - 656a^6b^7c^3d^2e^3f^7z^4 - 656a^2b^7c^6d^7e^3f^2z^4 - 240a^2b^11c^3d^4e^3f^5z^4 - 240a^2b^11c^2d^5e^3f^4z^4 + 216a^7b^6c^3d^2e^2f^7z^4 + 216a^2b^6c^7d^7e^2f^7z^4 - 204a^4b^6c^4d^2e^8f^3z^4 - 144a^5b^8c^3d^2e^4f^5z^4 - 144a^2b^8c^5d^5e^4f^5z^4 - 84a^2b^12c^3d^4e^2f^4z^4 + 36a^4b^9c^3d^2e^5f^4z^4 + 36a^2b^9c^4d^4e^5f^3z^4 + 20a^6b^7c^6d^6e^3f^3z^4 + 20a^2b^7c^6d^6e^3f^3z^4 + 16a^3b^10c^3d^2e^6f^3z^4 + 16a^3b^8c^3d^2e^8f^3z^4 + 16a^2b^12c^3d^3e^4f^3z^4 + 16a^2b^10c^3d^3e^6f^3z^4 + 48b^11c^3d^6e^3f^3z^4 + 48b^
\end{aligned}$$

$$\begin{aligned}
& b^9c^5d^7ef^2z^4 - 20b^8c^6d^7e^2fz^4 + 8b^{10}c^4d^5e^4fz^4 \\
& - 4b^{13}c^d^4e^3f^3z^4 - 4b^{11}c^3d^4e^5fz^4 + 4b^9c^5d^6e^3fz^4 + 3072a^9c^5d^4e^4f^5z^4 + 3072a^5c^9d^5e^4fz^4 + 2560a^8c^6d^4e^6f^3z^4 + 2560a^6c^8d^3e^6fz^4 + 1536a^{10}c^4d^4e^2f^7z^4 + 1536a^4c^{10}d^7e^2fz^4 + 48a^5b^9d^2ef^7z^4 + 48a^3b^{11}d^3ef^6z^4 - 20a^6b^8d^4e^2f^7z^4 + 8a^4b^{10}d^4ef^5z^4 + 4a^5b^9d^4e^3f^6z^4 - 4a^3b^{11}d^4e^5f^4z^4 - 4a^2b^{13}d^3e^3f^4z^4 + 768a^9b^8c^4e^5f^5z^4 + 768a^8b^8c^5e^7f^3z^4 + 256a^{10}b^8c^3e^3f^7z^4 - 192a^6b^3c^5e^9fz^4 - 68a^7b^6c^4ef^6z^4 + 48a^8b^5c^3e^3f^7z^4 + 48a^5b^5c^4e^9fz^4 + 36a^6b^7c^5ef^5z^4 - 12a^9b^4c^4e^2f^8z^4 - 4a^4b^9c^4ef^7z^4 - 4a^4b^7c^3e^9fz^4 + 384a^5b^8c^3d^3f^7z^4 + 384a^2b^8c^5d^7f^3z^4 + 288a^3b^{10}c^4d^4f^6z^4 + 288a^2b^{10}c^3d^6f^4z^4 + 224a^7b^6c^2d^2f^8z^4 + 224a^2b^6c^7d^8f^2z^4 - 192a^{10}b^2c^2d^2f^9z^4 - 192a^2b^2c^{10}d^9fz^4 + 768a^5b^8c^8d^3e^7z^4 + 768a^4b^8c^9d^5e^5z^4 + 256a^3b^8c^{10}d^7e^3z^4 - 192a^5b^3c^6d^4e^9z^4 - 68a^2b^6c^7d^6e^4z^4 + 48a^4b^5c^5d^4e^9z^4 + 48a^2b^5c^8d^7e^3z^4 + 36a^2b^7c^6d^5e^5z^4 - 12a^2b^4c^9d^8e^2z^4 - 4a^3b^7c^4d^4e^9z^4 - 4a^2b^9c^4d^3e^7z^4 + 16b^{13}c^d^5ef^4z^4 + 16b^7c^7d^8efz^4 + 768a^7c^7d^8efz^4 + 16a^7b^7d^8ef^8z^4 + 16a^2b^{13}d^4ef^5z^4 + 256a^7b^8c^6e^9fz^4 + 80a^2b^{12}c^d^5f^5z^4 + 48a^9b^4c^2d^2f^9z^4 + 48a^2b^4c^9d^9fz^4 + 256a^6b^8c^7d^4e^9z^4 - 42b^{10}c^4d^6e^2f^2z^4 - 20b^{12}c^2d^5e^2f^3z^4 + 6b^{12}c^2d^4e^4f^2z^4 + 4b^{11}c^3d^5e^3f^2z^4 - 24960a^7c^7d^4e^2f^4z^4 + 18944a^8c^6d^3e^2f^5z^4 + 18944a^6c^8d^5e^2f^3z^4 + 14336a^7c^7d^3e^4f^3z^4 - 9984a^8c^6d^2e^4f^4z^4 - 9984a^6c^8d^4e^4f^2z^4 - 7936a^9c^5d^2e^2f^6z^4 - 7936a^5c^9d^6e^2f^2z^4 - 4352a^7c^7d^2e^6f^2z^4 - 42a^4b^{10}d^2e^2f^6z^4 - 20a^2b^{12}d^3e^2f^5z^4 + 6a^2b^{12}d^2e^4f^4z^4 + 4a^3b^{11}d^2e^3f^5z^4 - 480a^8b^2c^4e^6f^4z^4 + 440a^7b^4c^3e^6f^4z^4 - 320a^8b^3c^3e^5f^5z^4 - 320a^7b^3c^4e^7f^3z^4 + 240a^8b^4c^2e^4f^6z^4 + 240a^6b^4c^4e^8f^2z^4 - 192a^9b^3c^2e^3f^7z^4 - 192a^9b^2c^3e^4f^6z^4 - 192a^7b^2c^5e^8f^2z^4 - 90a^6b^6c^2e^6f^4z^4 - 68a^5b^6c^3e^8f^2z^4 + 48a^{10}b^2c^2e^2f^8z^4 - 48a^7b^5c^2e^5f^5z^4 - 48a^6b^5c^3e^7f^3z^4 + 36a^5b^7c^2e^7f^3z^4 + 6a^4b^8c^2e^8f^2z^4 - 33920a^6b^2c^6d^5f^5z^4 + 27936a^5b^4c^5d^5f^5z^4 + 26112a^7b^2c^5d^4f^6z^4 + 26112a^5b^2c^7d^6f^4z^4 - 20352a^6b^4c^4d^4f^6z^4 - 20352a^4b^4c^6d^6f^4z^4 - 13080a^4b^6c^4d^5f^5z^4 - 11520a^8b^2c^4d^3f^7z^4 - 11520a^4b^2c^8d^7f^3z^4 + 8736a^5b^6c^3d^4f^6z^4 + 8736a^3b^6c^5d^6f^4z^4 + 7488a^7b^4c^3d^3f^7z^4 + 7488a^3b^4c^7d^7f^3z^4 + 3840a^3b^8c^3d^5f^5z^4 + 2560a^9b^2c^3d^2f^8z^4 + 2560a^3b^2c^9d^8f^2z^4 - 2416a^6b^6c^2d^3f^7z^4 - 2416a^2b^6c^6d^7f^3z^4 - 2160a^4b^8c^2d^4f^6z^4 - 2160a^2b^8c^4d^6f^4z^4 - 1152a^8b^4c^2d^2f^8z^4 - 1152a^2b^4c^8d^8f^2z^4 - 720a^2b^{10}c^2d^5f^5z^4 - 480a^4b^2c^8d^4e^6z^4 + 440a^3b^4c^7d^4e^6z^4
\end{aligned}$$

$$\begin{aligned}
& z^4 - 320a^4b^3c^7d^3e^7z^4 - 320a^3b^3c^8d^5e^5z^4 + 240a^4b^4c^6d^2e^8z^4 + 240a^2b^4c^8d^6e^4z^4 - 192a^5b^2c^7d^2e^8z^4 - 192a^3b^2c^9d^6e^4z^4 - 192a^2b^3c^9d^7e^3z^4 - 90a^2b^6c^6d^4e^6z^4 - 68a^3b^6c^5d^2e^8z^4 - 48a^3b^5c^6d^3e^7z^4 - 48a^2b^5c^7d^5e^5z^4 + 48a^2b^2c^{10}d^8e^2z^4 + 36a^2b^7c^5d^3e^7z^4 + 6a^2b^8c^4d^2e^8z^4 - 4b^6c^8d^9fz^4 + 256a^{11}c^3df^9z^4 + 256a^3c^{11}d^9fz^4 - 4a^8b^6df^9z^4 - 384a^9c^5e^6f^4z^4 - 256a^{10}c^4e^4f^6z^4 - 256a^8c^6e^8f^2z^4 - 64a^{11}c^3e^2f^8z^4 - 24b^{10}c^4d^7f^3z^4 - 16b^{12}c^2d^6f^4z^4 - 16b^8c^6d^8f^2z^4 + 17920a^7c^7d^5f^5z^4 - 14336a^8c^6d^4f^6z^4 - 14336a^6c^8d^6f^4z^4 + 7168a^9c^5d^3f^7z^4 + 7168a^5c^9d^7f^3z^4 - 2048a^{10}c^4d^2f^8z^4 - 2048a^4c^{10}d^8f^2z^4 + 6b^8c^6d^6e^4z^4 + 6a^6b^8e^4f^6z^4 - 4b^9c^5d^5e^5z^4 - 4b^7c^7d^7e^3z^4 - 4a^7b^7e^3f^7z^4 - 4a^5b^9e^5f^5z^4 - 384a^5c^9d^4e^6z^4 - 256a^6c^8d^2e^8z^4 - 256a^4c^{10}d^6e^4z^4 - 64a^3c^{11}d^8e^2z^4 - 24a^4b^{10}d^3f^7z^4 - 16a^6b^8d^2f^8z^4 - 16a^2b^{12}d^4f^6z^4 + 48a^6b^2c^6e^{10}z^4 - 12a^5b^4c^5e^{10}z^4 - 4b^{14}d^5f^5z^4 - 64a^7c^7e^{10}z^4 + b^{14}d^4e^2f^4z^4 + b^{10}c^4d^4e^6z^4 + b^6c^8d^8e^2z^4 + a^8b^6e^2f^8z^4 + a^4b^{10}e^6f^4z^4 + a^4b^6c^4e^{10}z^4 - 4820A^2B^2a^4b^3c^5d^2e^2f^4z^2 + 2976A^2B^2a^3b^3c^6d^3e^2f^3z^2 - 2328A^2B^2a^3b^3c^6d^2e^4f^2z^2 + 1848A^2B^2a^2b^4c^4d^3e^2f^4z^2 - 1768A^2B^2a^3b^4c^3d^2e^2f^5z^2 + 1528A^2B^2a^4b^2c^4d^2e^2f^5z^2 - 1136A^2B^2a^3b^2c^5d^3e^2f^4z^2 - 974A^2B^2a^4b^3c^3d^2e^2f^5z^2 + 692A^2B^2a^2b^3c^7d^4e^2f^2z^2 + 588A^2B^2a^2b^6c^3d^2e^3f^3z^2 - 580A^2B^2a^3b^3c^4d^2e^4f^3z^2 + 488A^2B^2a^3b^4c^3d^2e^3f^4z^2 - 444A^2B^2a^2b^2c^6d^2e^5f^2z^2 - 412A^2B^2a^2b^5c^4d^2e^4f^2z^2 + 366A^2B^2a^2b^6c^2d^2e^5f^2z^2 - 352A^2B^2a^2b^2c^6d^4e^3f^3z^2 + 326A^2B^2a^2b^4c^4d^2e^5f^2z^2 + 324A^2B^2a^2b^5c^4d^3e^2f^3z^2 - 302A^2B^2a^2b^3c^6d^4e^2f^2z^2 - 296A^2B^2a^2b^7c^2d^2e^2f^4z^2 + 122A^2B^2a^4b^2c^4d^2e^3f^4z^2 - 122A^2B^2a^2b^6c^2d^2e^3f^4z^2 - 84A^2B^2a^3b^2c^5d^2e^5f^2z^2 + 72A^2B^2a^2b^4c^5d^3e^3f^2z^2 - 64A^2B^2a^2b^5c^3d^2e^4f^3z^2 + 60A^2B^2a^3b^5c^2d^2e^2f^5z^2 + 1312A^2B^2a^5b^3c^4d^2e^2f^5z^2 + 1040A^2B^2a^4b^3c^5d^2e^4f^3z^2 - 500A^2B^2a^2b^6c^3d^3e^2f^4z^2 - 376A^2B^2a^2b^2c^7d^5e^2f^2z^2 + 276A^2B^2a^4b^4c^2d^2e^6f^2z^2 - 262A^2B^2a^2b^3c^5d^2e^6f^2z^2 + 238A^2B^2a^2b^2c^7d^4e^3f^2z^2 + 232A^2B^2a^5b^2c^3d^2e^6f^2z^2 - 176A^2B^2a^2b^3c^7d^3e^4f^2z^2 - 120A^2B^2a^2b^6c^3d^2e^5f^2z^2 - 108A^2B^2a^2b^4c^5d^4e^2f^3z^2 + 68A^2B^2a^2b^7c^2d^2e^4f^3z^2 + 68A^2B^2a^2b^4c^5d^2e^5f^2z^2 + 46A^2B^2a^2b^7c^2d^2e^2f^5z^2 - 36A^2B^2a^2b^3c^6d^3e^4f^2z^2 - 1932A^2B^2a^2b^3c^5d^3e^2f^3z^2 - 1818A^2B^2a^2b^4c^4d^2e^3f^3z^2 + 1620A^2B^2a^3b^3c^4d^2e^2f^4z^2 + 1560A^2B^2a^2b^3c^5d^2e^4f^2z^2 + 1244A^2B^2a^3b^2c^5d^2e^3f^3z^2 + 820A^2B^2a^2b^2c^6d^3e^3f^2z^2 + 480A^2B^2a^2b^5c^3d^2e^2f^4z^2 + 352A^2B^2a^3b^3c^6d^2e^6f^2z^2 - 108A^2B^2a^3b^6c^2d^2e^6f^2z^2 + 82A^2B^2a^2b^5c^4d^2e^6f^2z^2 - 64A^2B^2a^2b^8c^2d^5e^2f^2z^2 + 16A^2B^2a^2b^8c^2d^2e^2f^5z^2 - 4A^2B^2a^2b^8c^2d^6e^2f^5z^2 + 16B^2a^2b^8c^8d^6e^2f^5z^2
\end{aligned}$$

$$\begin{aligned}
&^2 + 56*A*B*b^2*c^8*d^6*e*f*z^2 - 8*A*B*b^9*c*d*e^4*f^3*z^2 - 8*A*B*b^7*c^3 \\
&*d*e^6*f*z^2 - 800*A*B*a^6*c^4*d*e*f^6*z^2 + 10*A*B*a^2*b^8*d*e*f^6*z^2 - 6 \\
&*A*B*a*b^9*d*e^2*f^5*z^2 - 12*A*B*a^5*b^4*c*e*f^7*z^2 + 912*A*B*a^6*b*c^3*d \\
&*f^7*z^2 + 192*A*B*a^4*b^5*c*d*f^7*z^2 + 192*A*B*a*b*c^8*d^6*f^2*z^2 - 20*A \\
&*B*a*b^4*c^5*d*e^7*z^2 + 4*A*B*a*b*c^8*d^4*e^4*z^2 + 2144*B^2*a^4*b*c^5*d^3 \\
&*e*f^4*z^2 - 1120*B^2*a^3*b*c^6*d^4*e*f^3*z^2 - 688*B^2*a^5*b*c^4*d^2*e*f^5 \\
&*z^2 - 256*B^2*a^3*b*c^6*d^2*e^5*f*z^2 + 152*B^2*a*b^3*c^6*d^5*e*f^2*z^2 + \\
&120*B^2*a^5*b^3*c^2*d*e*f^6*z^2 - 116*B^2*a^5*b*c^4*d*e^3*f^4*z^2 + 110*B^2 \\
&*a*b^7*c^2*d^3*e*f^4*z^2 - 80*B^2*a^2*b*c^7*d^5*e*f^2*z^2 - 72*B^2*a*b^5*c^ \\
&4*d^4*e*f^3*z^2 - 48*B^2*a^4*b*c^5*d*e^5*f^2*z^2 - 46*B^2*a*b^3*c^6*d^4*e^3 \\
&*f*z^2 - 44*B^2*a*b^4*c^5*d^3*e^4*f*z^2 - 34*B^2*a*b^5*c^4*d^2*e^5*f*z^2 + \\
&20*B^2*a^2*b*c^7*d^4*e^3*f*z^2 - 10*B^2*a^3*b^6*c*d*e^2*f^5*z^2 - 10*B^2*a^ \\
&2*b^7*c*d^2*e*f^5*z^2 - 10*B^2*a*b^2*c^7*d^5*e^2*f*z^2 - 7*B^2*a^2*b^4*c^4* \\
&d*e^6*f*z^2 - 6*B^2*a^3*b^2*c^5*d*e^6*f*z^2 + 4*B^2*a*b^8*c*d^2*e^2*f^4*z^2 \\
&- 2*B^2*a^2*b^7*c*d*e^3*f^4*z^2 + 3196*A^2*a^4*b*c^5*d*e^3*f^4*z^2 - 3184* \\
&A^2*a^4*b*c^5*d^2*e*f^5*z^2 + 1568*A^2*a^3*b*c^6*d^3*e*f^4*z^2 + 1504*A^2*a \\
&^3*b*c^6*d*e^5*f^2*z^2 - 656*A^2*a^4*b^3*c^3*d*e*f^6*z^2 - 400*A^2*a*b^6*c^ \\
&3*d*e^4*f^3*z^2 + 314*A^2*a*b^5*c^4*d*e^5*f^2*z^2 - 264*A^2*a^3*b^5*c^2*d*e \\
&*f^6*z^2 + 240*A^2*a^2*b^2*c^6*d*e^6*f*z^2 - 224*A^2*a^2*b*c^7*d^4*e*f^3*z^ \\
&2 + 216*A^2*a*b^5*c^4*d^3*e*f^4*z^2 - 192*A^2*a^2*b*c^7*d^2*e^5*f*z^2 + 178 \\
&*A^2*a*b^7*c^2*d*e^3*f^4*z^2 - 154*A^2*a*b^7*c^2*d^2*e*f^5*z^2 + 128*A^2*a* \\
&b^3*c^6*d^4*e*f^3*z^2 + 106*A^2*a*b^3*c^6*d^2*e^5*f*z^2 - 12*A^2*a*b^2*c^7* \\
&d^3*e^4*f*z^2 - 58*A*B*b^8*c^2*d^2*e^3*f^3*z^2 + 40*A*B*b^7*c^3*d^2*e^4*f^2 \\
&*z^2 - 28*A*B*b^7*c^3*d^3*e^2*f^3*z^2 - 24*A*B*b^5*c^5*d^4*e^2*f^2*z^2 - 20 \\
&*A*B*b^6*c^4*d^3*e^3*f^2*z^2 + 2768*A*B*a^4*c^6*d^2*e^3*f^3*z^2 - 1712*A*B* \\
&a^3*c^7*d^3*e^3*f^2*z^2 - 156*A*B*a^4*b^2*c^4*e^5*f^3*z^2 + 146*A*B*a^4*b^3 \\
&*c^3*e^4*f^4*z^2 - 106*A*B*a^5*b^2*c^3*e^3*f^5*z^2 + 90*A*B*a^5*b^3*c^2*e^2 \\
&*f^6*z^2 + 38*A*B*a^3*b^3*c^4*e^6*f^2*z^2 - 36*A*B*a^3*b^5*c^2*e^4*f^4*z^2 \\
&+ 16*A*B*a^3*b^4*c^3*e^5*f^3*z^2 - 9*A*B*a^4*b^4*c^2*e^3*f^5*z^2 - 8*A*B*a^ \\
&2*b^5*c^3*e^6*f^2*z^2 + 2*A*B*a^2*b^6*c^2*e^5*f^3*z^2 + 920*A*B*a^4*b^3*c^3 \\
&*d^2*f^6*z^2 - 480*A*B*a^2*b^5*c^3*d^3*f^5*z^2 - 336*A*B*a^2*b^3*c^5*d^4*f^ \\
&4*z^2 - 272*A*B*a^3*b^3*c^4*d^3*f^5*z^2 + 240*A*B*a^3*b^5*c^2*d^2*f^6*z^2 - \\
&32*A*B*a*c^9*d^6*e*f*z^2 - 792*B^2*a^2*b^3*c^5*d^3*e^3*f^2*z^2 + 714*B^2*a \\
&^2*b^4*c^4*d^3*e^2*f^3*z^2 - 572*B^2*a^3*b^2*c^5*d^3*e^2*f^3*z^2 - 475*B^2* \\
&a^2*b^2*c^6*d^4*e^2*f^2*z^2 + 265*B^2*a^4*b^2*c^4*d^2*e^2*f^4*z^2 + 260*B^2 \\
&*a^3*b^3*c^4*d^2*e^3*f^3*z^2 - 212*B^2*a^3*b^4*c^3*d^2*e^2*f^4*z^2 + 180*B^ \\
&2*a^3*b^2*c^5*d^2*e^4*f^2*z^2 - 158*B^2*a^2*b^4*c^4*d^2*e^4*f^2*z^2 + 47*B^ \\
&2*a^2*b^6*c^2*d^2*e^2*f^4*z^2 + 16*B^2*a^2*b^5*c^3*d^2*e^3*f^3*z^2 + 2752*A \\
&^2*a^3*b^2*c^5*d^2*e^2*f^4*z^2 - 2148*A^2*a^2*b^4*c^4*d^2*e^2*f^4*z^2 + 206 \\
&4*A^2*a^2*b^3*c^5*d^2*e^3*f^3*z^2 - 424*A^2*a^2*b^2*c^6*d^3*e^2*f^3*z^2 - 1 \\
&98*A^2*a^2*b^2*c^6*d^2*e^4*f^2*z^2 - 272*B^2*a^6*b*c^3*d*e*f^6*z^2 - 24*B^2 \\
&*a^4*b^5*c*d*e*f^6*z^2 + 1808*A^2*a^5*b*c^4*d*e*f^6*z^2 - 244*A^2*a*b*c^8*d \\
&^4*e^3*f*z^2 + 208*A^2*a*b*c^8*d^5*e*f^2*z^2 + 134*A^2*a^2*b^7*c*d*e*f^6*z^ \\
&2 - 76*A^2*a*b^4*c^5*d*e^6*f*z^2 + 4*A^2*a*b^8*c*d*e^2*f^5*z^2 + 148*A*B*b^ \\
&4*c^6*d^5*e*f^2*z^2 + 65*A*B*b^6*c^4*d^4*e*f^3*z^2 + 46*A*B*b^8*c^2*d^3*e*f
\end{aligned}$$

$$\begin{aligned}
&^4z^2 - 38*A*B*b^3*c^7*d^5*e^2*f*z^2 + 34*A*B*b^9*c*d^2*e^2*f^4*z^2 - 29*A \\
&*B*b^4*c^6*d^4*e^3*f*z^2 + 20*A*B*b^5*c^5*d^3*e^4*f*z^2 + 12*A*B*b^8*c^2*d* \\
&e^5*f^2*z^2 - 7*A*B*b^6*c^4*d^2*e^5*f*z^2 - 2880*A*B*a^4*c^6*d^3*e*f^4*z^2 \\
&+ 2784*A*B*a^5*c^5*d^2*e*f^5*z^2 - 1112*A*B*a^5*c^5*d*e^3*f^4*z^2 + 896*A*B \\
&*a^3*c^7*d^4*e*f^3*z^2 + 848*A*B*a^3*c^7*d^2*e^5*f*z^2 - 560*A*B*a^4*c^6*d* \\
&e^5*f^2*z^2 + 96*A*B*a^2*c^8*d^5*e*f^2*z^2 - 88*A*B*a^2*c^8*d^4*e^3*f*z^2 - \\
&100*A*B*a^6*b*c^3*e^2*f^6*z^2 - 76*A*B*a^5*b*c^4*e^4*f^4*z^2 + 48*A*B*a^6* \\
&b^2*c^2*e*f^7*z^2 - 42*A*B*a^3*b^2*c^5*e^7*f*z^2 + 36*A*B*a^4*b*c^5*e^6*f^2 \\
&*z^2 - 24*A*B*a^4*b^5*c*e^2*f^6*z^2 + 10*A*B*a^3*b^6*c*e^3*f^5*z^2 + 7*A*B* \\
&a^2*b^4*c^4*e^7*f*z^2 + 2*A*B*a^2*b^7*c*e^4*f^4*z^2 - 2496*A*B*a^5*b*c^4*d^ \\
&2*f^6*z^2 + 1872*A*B*a^4*b*c^5*d^3*f^5*z^2 - 744*A*B*a^5*b^3*c^2*d*f^7*z^2 \\
&- 720*A*B*a^2*b*c^7*d^5*f^3*z^2 + 504*A*B*a*b^3*c^6*d^5*f^3*z^2 + 256*A*B*a \\
&^3*b*c^6*d^4*f^4*z^2 + 168*A*B*a*b^7*c^2*d^3*f^5*z^2 - 144*A*B*a^2*b^7*c*d^ \\
&2*f^6*z^2 + 144*A*B*a*b^5*c^4*d^4*f^4*z^2 + 66*A*B*a^2*b^2*c^6*d*e^7*z^2 - \\
&36*A*B*a*b^2*c^7*d^3*e^5*z^2 + 20*A*B*a*b^3*c^6*d^2*e^6*z^2 + 12*A*B*a^2*b* \\
&c^7*d^2*e^6*z^2 + 1208*B^2*a^3*b*c^6*d^3*e^3*f^2*z^2 - 848*B^2*a^3*b^3*c^4* \\
&d^3*e*f^4*z^2 + 672*B^2*a^2*b^3*c^5*d^4*e*f^3*z^2 - 632*B^2*a^4*b*c^5*d^2*e \\
&^3*f^3*z^2 + 432*B^2*a^4*b^3*c^3*d^2*e*f^5*z^2 + 276*B^2*a^2*b^2*c^6*d^3*e^ \\
&4*f*z^2 - 196*B^2*a*b^6*c^3*d^3*e^2*f^3*z^2 - 168*B^2*a^2*b^5*c^3*d^3*e*f^4 \\
&*z^2 + 154*B^2*a^2*b^3*c^5*d^2*e^5*f*z^2 + 148*B^2*a*b^5*c^4*d^3*e^3*f^2*z^ \\
&2 + 96*B^2*a*b^4*c^5*d^4*e^2*f^2*z^2 - 72*B^2*a^3*b^5*c^2*d^2*e*f^5*z^2 + 7 \\
&0*B^2*a^5*b^2*c^3*d*e^2*f^5*z^2 - 60*B^2*a^4*b^3*c^3*d*e^3*f^4*z^2 + 52*B^2 \\
&*a*b^6*c^3*d^2*e^4*f^2*z^2 + 36*B^2*a^4*b^2*c^4*d*e^4*f^3*z^2 - 32*B^2*a*b^ \\
&7*c^2*d^2*e^3*f^3*z^2 + 24*B^2*a^3*b^5*c^2*d*e^3*f^4*z^2 + 15*B^2*a^4*b^4*c \\
&^2*d*e^2*f^5*z^2 - 8*B^2*a^3*b^4*c^3*d*e^4*f^3*z^2 + 8*B^2*a^2*b^5*c^3*d*e^ \\
&5*f^2*z^2 - 2*B^2*a^3*b^3*c^4*d*e^5*f^2*z^2 - 2*B^2*a^2*b^6*c^2*d*e^4*f^3*z \\
&^2 - 3176*A^2*a^3*b*c^6*d^2*e^3*f^3*z^2 - 2252*A^2*a^4*b^2*c^4*d*e^2*f^5*z^ \\
&2 + 1952*A^2*a^3*b^4*c^3*d*e^2*f^5*z^2 - 1496*A^2*a^3*b^3*c^4*d*e^3*f^4*z^2 \\
&+ 1378*A^2*a^2*b^4*c^4*d*e^4*f^3*z^2 + 1184*A^2*a^3*b^3*c^4*d^2*e*f^5*z^2 \\
&- 1166*A^2*a^2*b^3*c^5*d*e^5*f^2*z^2 - 1164*A^2*a^3*b^2*c^5*d*e^4*f^3*z^2 - \\
&1152*A^2*a^2*b^3*c^5*d^3*e*f^4*z^2 + 578*A^2*a*b^6*c^3*d^2*e^2*f^4*z^2 - 5 \\
&48*A^2*a*b^5*c^4*d^2*e^3*f^3*z^2 + 440*A^2*a*b^2*c^7*d^4*e^2*f^2*z^2 - 412* \\
&A^2*a^2*b^6*c^2*d*e^2*f^5*z^2 - 360*A^2*a*b^3*c^6*d^3*e^3*f^2*z^2 + 312*A^2 \\
&*a*b^4*c^5*d^3*e^2*f^3*z^2 + 248*A^2*a^2*b*c^7*d^3*e^3*f^2*z^2 - 224*A^2*a^ \\
&2*b^5*c^3*d*e^3*f^4*z^2 + 216*A^2*a^2*b^5*c^3*d^2*e*f^5*z^2 + 52*A^2*a*b^4* \\
&c^5*d^2*e^4*f^2*z^2 - 16*B^2*b^3*c^7*d^6*e*f*z^2 - 14*B^2*b^9*c*d^3*e*f^4*z \\
&^2 + 32*B^2*a^4*c^6*d*e^6*f*z^2 - 20*A^2*b^9*c*d*e^3*f^4*z^2 + 18*A^2*b^9*c \\
&*d^2*e*f^5*z^2 + 8*A^2*b^6*c^4*d*e^6*f*z^2 - 360*A^2*a^3*c^7*d*e^6*f*z^2 + \\
&136*A^2*a*c^9*d^5*e^2*f*z^2 + 2*B^2*a^3*b^7*d*e*f^6*z^2 + 2*B^2*a*b^9*d^2*e \\
&*f^5*z^2 + 12*B^2*a^4*b*c^5*e^7*f*z^2 - 204*A^2*a^3*b*c^6*e^7*f*z^2 - 128*A \\
&^2*a^6*b*c^3*e*f^7*z^2 - 48*A^2*a*b^5*c^4*e^7*f*z^2 - 36*B^2*a^5*b^4*c*d*f^ \\
&7*z^2 - 24*A^2*a^4*b^5*c*e*f^7*z^2 - 16*B^2*a*b^8*c*d^3*f^5*z^2 - 164*A^2*a \\
&^3*b^6*c*d*f^7*z^2 - 16*A^2*a*b^8*c*d^2*f^6*z^2 + 4*B^2*a^3*b*c^6*d*e^7*z^2 \\
&- 4*B^2*a*b*c^8*d^5*e^3*z^2 + 48*A^2*a*b*c^8*d^3*e^5*z^2 + 36*A^2*a^2*b*c^ \\
&7*d*e^7*z^2 - 6*A^2*a*b^3*c^6*d*e^7*z^2 + 136*A*B*a^6*c^4*e^3*f^5*z^2 - 96*
\end{aligned}$$

$$\begin{aligned}
& A*B*b^5*c^5*d^5*f^3*z^2 + 80*A*B*a^5*c^5*e^5*f^3*z^2 - 72*A*B*b^3*c^7*d^6*f^2*z^2 - 24*A*B*b^7*c^3*d^4*f^4*z^2 + 14*A*B*b^3*c^7*d^4*e^4*z^2 - 14*A*B*b^2*c^8*d^5*e^3*z^2 - 2*A*B*b^5*c^5*d^2*e^6*z^2 - 2*A*B*b^4*c^6*d^3*e^5*z^2 \\
& + 2*A*B*a^3*b^7*e^2*f^6*z^2 - A*B*a^2*b^8*e^3*f^5*z^2 + 16*A*B*a^2*c^8*d^3*e^5*z^2 - 2*A*B*a^2*b^3*c^5*e^8*z^2 + 22*B^2*b^8*c^2*d^3*e^2*f^3*z^2 - 12*B^2*b^7*c^3*d^3*e^3*f^2*z^2 + 12*B^2*b^6*c^4*d^4*e^2*f^2*z^2 - 6*B^2*b^8*c^2*d^2*e^4*f^2*z^2 - 864*B^2*a^4*c^6*d^3*e^2*f^3*z^2 + 496*B^2*a^3*c^7*d^4*e^2*f^2*z^2 + 224*B^2*a^5*c^5*d^2*e^2*f^4*z^2 + 136*B^2*a^4*c^6*d^2*e^4*f^2*z^2 - 53*A^2*b^8*c^2*d^2*e^2*f^4*z^2 + 52*A^2*b^7*c^3*d^2*e^3*f^3*z^2 + 52*A^2*b^5*c^5*d^3*e^3*f^2*z^2 - 36*A^2*b^6*c^4*d^3*e^2*f^3*z^2 - 12*A^2*b^4*c^6*d^4*e^2*f^2*z^2 - 9*A^2*b^6*c^4*d^2*e^4*f^2*z^2 + 836*A^2*a^4*c^6*d^2*e^2*f^4*z^2 - 668*A^2*a^2*c^8*d^4*e^2*f^2*z^2 + 656*A^2*a^3*c^7*d^2*e^4*f^2*z^2 + 368*A^2*a^3*c^7*d^3*e^2*f^3*z^2 - 45*B^2*a^6*b^2*c^2*e^2*f^6*z^2 - 18*B^2*a^5*b^2*c^3*e^4*f^4*z^2 - 9*B^2*a^4*b^2*c^4*e^6*f^2*z^2 - 6*B^2*a^5*b^3*c^2*e^3*f^5*z^2 + 3*B^2*a^4*b^4*c^2*e^4*f^4*z^2 - 2*B^2*a^4*b^3*c^3*e^5*f^3*z^2 - 580*B^2*a^4*b^2*c^4*d^3*f^5*z^2 + 536*B^2*a^3*b^4*c^3*d^3*f^5*z^2 + 471*A^2*a^4*b^2*c^4*e^4*f^4*z^2 - 436*A^2*a^3*b^4*c^3*e^4*f^4*z^2 - 348*B^2*a^4*b^4*c^2*d^2*f^6*z^2 + 316*B^2*a^2*b^2*c^6*d^5*f^3*z^2 + 310*A^2*a^3*b^3*c^4*e^5*f^3*z^2 + 232*A^2*a^5*b^2*c^3*e^2*f^6*z^2 - 229*A^2*a^2*b^4*c^4*e^6*f^2*z^2 - 216*A^2*a^4*b^4*c^2*e^2*f^6*z^2 + 204*A^2*a^4*b^3*c^3*e^3*f^5*z^2 + 200*B^2*a^5*b^2*c^3*d^2*f^6*z^2 + 150*A^2*a^3*b^2*c^5*e^6*f^2*z^2 - 120*B^2*a^2*b^4*c^4*d^4*f^4*z^2 + 91*A^2*a^2*b^6*c^2*e^4*f^4*z^2 + 72*A^2*a^3*b^5*c^2*e^3*f^5*z^2 - 66*B^2*a^2*b^6*c^2*d^3*f^5*z^2 + 44*A^2*a^2*b^5*c^3*e^5*f^3*z^2 - 16*B^2*a^3*b^2*c^5*d^4*f^4*z^2 + 1952*A^2*a^4*b^2*c^4*d^2*f^6*z^2 - 1792*A^2*a^3*b^2*c^5*d^3*f^5*z^2 - 1272*A^2*a^3*b^4*c^3*d^2*f^6*z^2 + 976*A^2*a^2*b^2*c^6*d^4*f^4*z^2 + 960*A^2*a^2*b^4*c^4*d^3*f^5*z^2 + 282*A^2*a^2*b^6*c^2*d^2*f^6*z^2 - 45*B^2*a^2*b^2*c^6*d^2*e^6*z^2 - 48*A^2*b*c^9*d^6*e*f*z^2 - 14*A^2*a*b^9*d*e*f^6*z^2 - 7*A*B*b^10*d^2*e*f^5*z^2 + 2*A*B*b^10*d*e^3*f^4*z^2 - 64*A*B*a^7*c^3*e*f^7*z^2 - 16*A*B*b^9*c*d^3*f^5*z^2 + 8*A*B*a^4*c^6*e^7*f*z^2 + 4*A*B*b*c^9*d^6*e^2*z^2 + 2*A*B*b^6*c^4*d*e^7*z^2 - 120*A*B*a^3*c^7*d*e^7*z^2 - 16*A*B*a^3*b^7*d*f^7*z^2 + 16*A*B*a*b^9*d^2*f^6*z^2 + 8*A*B*a*c^9*d^5*e^3*z^2 + 12*A*B*a^3*b*c^6*e^8*z^2 - 48*B^2*b^5*c^5*d^5*e*f^2*z^2 + 15*B^2*b^4*c^6*d^5*e^2*f*z^2 - 14*B^2*b^7*c^3*d^4*e*f^3*z^2 + 4*B^2*b^9*c*d^2*e^3*f^3*z^2 + 4*B^2*b^7*c^3*d^2*e^5*f*z^2 + 4*B^2*b^5*c^5*d^4*e^3*f*z^2 - B^2*b^6*c^4*d^3*e^4*f*z^2 - 336*B^2*a^3*c^7*d^3*e^4*f*z^2 + 112*B^2*a^5*c^5*d*e^4*f^3*z^2 - 112*A^2*b^3*c^7*d^5*e*f^2*z^2 + 80*B^2*a^6*c^4*d*e^2*f^5*z^2 - 48*A^2*b^5*c^5*d^4*e*f^3*z^2 + 36*A^2*b^8*c^2*d*e^4*f^3*z^2 + 36*A^2*b^3*c^7*d^4*e^3*f*z^2 - 28*A^2*b^7*c^3*d*e^5*f^2*z^2 + 20*A^2*b^2*c^8*d^5*e^2*f*z^2 + 16*B^2*a^2*c^8*d^5*e^2*f*z^2 - 14*A^2*b^7*c^3*d^3*e*f^4*z^2 - 14*A^2*b^4*c^6*d^3*e^4*f*z^2 - 10*A^2*b^5*c^5*d^2*e^5*f*z^2 - 1008*A^2*a^4*c^6*d*e^4*f^3*z^2 - 760*A^2*a^5*c^5*d*e^2*f^5*z^2 + 272*A^2*a^2*c^8*d^3*e^4*f*z^2 + 48*B^2*a^5*b*c^4*e^5*f^3*z^2 + 36*B^2*a^6*b*c^3*e^3*f^5*z^2 + 12*B^2*a^5*b^4*c*e^2*f^6*z^2 - 624*A^2*a^4*b*c^5*e^5*f^3*z^2 - 548*A^2*a^5*b*c^4*e^3*f^5*z^2 + 182*A^2*a^2*b^3*c^5*e^7*f*z^2 - 180*B^2*a*b^4*c^5*d^5*f^3*z^2 + 132*B^2*a^6*b^2*c^2*d*f^7*z^2 + 108*B^2*a^3*b^6*c*d^
\end{aligned}$$

$$\begin{aligned}
& 2*f^6*z^2 + 96*A^2*a^5*b^3*c^2*e*f^7*z^2 + 68*A^2*a*b^6*c^3*e^6*f^2*z^2 + 5 \\
& 8*A^2*a^3*b^6*c*e^2*f^6*z^2 - 56*B^2*a*b^2*c^7*d^6*f^2*z^2 - 38*A^2*a^2*b^7 \\
& *c*e^3*f^5*z^2 - 36*A^2*a*b^7*c^2*e^5*f^3*z^2 + 20*B^2*a*b^6*c^3*d^4*f^4*z^2 \\
& - 736*A^2*a^5*b^2*c^3*d*f^7*z^2 + 624*A^2*a^4*b^4*c^2*d*f^7*z^2 - 416*A^2 \\
& *a*b^2*c^7*d^5*f^3*z^2 - 276*A^2*a*b^4*c^5*d^4*f^4*z^2 - 196*A^2*a*b^6*c^3* \\
& d^3*f^5*z^2 + 8*B^2*a*b^4*c^5*d^2*e^6*z^2 + 6*B^2*a*b^2*c^7*d^4*e^4*z^2 + 2 \\
& *B^2*a^2*b^3*c^5*d*e^7*z^2 + 2*B^2*a*b^3*c^6*d^3*e^5*z^2 - 18*A^2*a*b^2*c^7 \\
& *d^2*e^6*z^2 - 16*A*B*b*c^9*d^7*f*z^2 - B^2*b^10*d^2*e^2*f^4*z^2 + 48*B^2*a \\
& ^7*c^3*e^2*f^6*z^2 - 36*B^2*a^6*c^4*e^4*f^4*z^2 + 31*B^2*b^6*c^4*d^5*f^3*z^2 \\
& - 24*B^2*a^5*c^5*e^6*f^2*z^2 + 20*B^2*b^4*c^6*d^6*f^2*z^2 - 6*A^2*b^8*c^2 \\
& *e^6*f^2*z^2 + 2*B^2*b^8*c^2*d^4*f^4*z^2 - 768*B^2*a^5*c^5*d^3*f^5*z^2 + 51 \\
& 2*B^2*a^6*c^4*d^2*f^6*z^2 + 512*B^2*a^4*c^6*d^4*f^4*z^2 + 232*A^2*a^5*c^5*e \\
& ^4*f^4*z^2 + 188*A^2*a^4*c^6*e^6*f^2*z^2 - 128*B^2*a^3*c^7*d^5*f^3*z^2 + 92 \\
& *A^2*a^6*c^4*e^2*f^6*z^2 + 80*A^2*b^4*c^6*d^5*f^3*z^2 + 64*A^2*b^2*c^8*d^6* \\
& f^2*z^2 + 31*A^2*b^6*c^4*d^4*f^4*z^2 + 14*A^2*b^8*c^2*d^3*f^5*z^2 - 5*B^2*b \\
& ^4*c^6*d^4*e^4*z^2 + 4*B^2*b^3*c^7*d^5*e^3*z^2 + 2*B^2*b^5*c^5*d^3*e^5*z^2 \\
& - B^2*b^6*c^4*d^2*e^6*z^2 - B^2*b^2*c^8*d^6*e^2*z^2 - B^2*a^4*b^6*e^2*f^6*z \\
& ^2 - 1152*A^2*a^3*c^7*d^4*f^4*z^2 + 1008*A^2*a^4*c^6*d^3*f^5*z^2 + 624*A^2* \\
& a^2*c^8*d^5*f^3*z^2 - 288*A^2*a^5*c^5*d^2*f^6*z^2 + 56*B^2*a^3*c^7*d^2*e^6* \\
& z^2 - 10*B^2*a^2*b^8*d^2*f^6*z^2 - 9*A^2*b^2*c^8*d^4*e^4*z^2 - 5*A^2*a^2*b^ \\
& 8*e^2*f^6*z^2 - 4*B^2*a^2*c^8*d^4*e^4*z^2 + 3*A^2*b^4*c^6*d^2*e^6*z^2 - 2*A \\
& ^2*b^3*c^7*d^3*e^5*z^2 - 36*A^2*a^2*c^8*d^2*e^6*z^2 - 48*A^2*a^6*b^2*c^2*f^ \\
& 8*z^2 - 45*A^2*a^2*b^2*c^6*e^8*z^2 + 4*A^2*b^10*d*e^2*f^5*z^2 + 4*B^2*b^2*c \\
& ^8*d^7*f*z^2 + 4*A^2*b^9*c*e^5*f^3*z^2 + 4*A^2*b^7*c^3*e^7*f*z^2 - 128*B^2* \\
& a^7*c^3*d*f^7*z^2 - 160*A^2*a*c^9*d^6*f^2*z^2 - 112*A^2*a^6*c^4*d*f^7*z^2 + \\
& 12*A^2*b*c^9*d^5*e^3*z^2 + 4*A^2*a*b^9*e^3*f^5*z^2 + 3*B^2*a^4*b^6*d*f^7*z \\
& ^2 + 2*A^2*a^3*b^7*e*f^7*z^2 - 24*A^2*a*c^9*d^4*e^4*z^2 + 14*A^2*a^2*b^8*d* \\
& f^7*z^2 + 12*A^2*a^5*b^4*c*f^8*z^2 + 12*A^2*a*b^4*c^5*e^8*z^2 + A*B*a^4*b^6 \\
& *e*f^7*z^2 + B^2*a^2*b^8*d*e^2*f^5*z^2 + 16*A^2*c^10*d^7*f*z^2 + 3*B^2*b^10 \\
& *d^3*f^5*z^2 - A^2*b^10*e^4*f^4*z^2 - 4*A^2*c^10*d^6*e^2*z^2 - A^2*b^10*d^2 \\
& *f^6*z^2 + 64*A^2*a^7*c^3*f^8*z^2 - 4*B^2*a^4*c^6*e^8*z^2 - A^2*b^6*c^4*e^8 \\
& *z^2 + 48*A^2*a^3*c^7*e^8*z^2 - A^2*a^4*b^6*f^8*z^2 + 720*A^2*B*a*b^2*c^5*d \\
& ^2*e^2*f^3*z - 600*A^2*B*a^2*b^2*c^4*d*e^2*f^4*z + 576*A*B^2*a^2*b^2*c^4*d^ \\
& 2*e*f^4*z + 348*A*B^2*a*b^2*c^5*d^2*e^3*f^2*z - 336*A*B^2*a^2*b*c^5*d^2*e^2 \\
& *f^3*z - 260*A*B^2*a*b^3*c^4*d^2*e^2*f^3*z - 240*A*B^2*a^2*b^2*c^4*d*e^3*f^ \\
& 3*z + 196*A*B^2*a^2*b^3*c^3*d*e^2*f^4*z + 172*A^2*B*a*b*c^6*d*e^5*f*z + 20* \\
& A*B^2*a*b^6*c*d*e*f^5*z - 912*A^2*B*a^2*b*c^5*d^2*e*f^4*z - 644*A^2*B*a*b*c \\
& ^6*d^2*e^3*f^2*z - 432*A*B^2*a*b^2*c^5*d^3*e*f^3*z + 372*A^2*B*a^2*b*c^5*d* \\
& e^3*f^3*z - 330*A^2*B*a*b^2*c^5*d*e^4*f^2*z + 312*A*B^2*a*b*c^6*d^3*e^2*f^2 \\
& *z - 208*A*B^2*a^3*b^2*c^3*d*e*f^5*z + 192*A^2*B*a^2*b^3*c^3*d*e*f^5*z + 17 \\
& 2*A^2*B*a*b^3*c^4*d*e^3*f^3*z + 108*A*B^2*a^2*b*c^5*d*e^4*f^2*z + 104*A*B^2 \\
& *a^3*b*c^4*d*e^2*f^4*z - 80*A^2*B*a*b^3*c^4*d^2*e*f^4*z + 68*A^2*B*a*b^4*c^ \\
& 3*d*e^2*f^4*z - 60*A*B^2*a*b^5*c^2*d*e^2*f^4*z + 58*A*B^2*a*b^3*c^4*d*e^4*f \\
& ^2*z - 36*A*B^2*a*b^4*c^3*d^2*e*f^4*z - 24*A*B^2*a^2*b^4*c^2*d*e*f^5*z + 24 \\
& *A*B^2*a*b^4*c^3*d*e^3*f^3*z + 592*A^2*B*a*b*c^6*d^3*e*f^3*z + 240*A^2*B*a^
\end{aligned}$$

$$\begin{aligned}
& 3*b*c^4*d*e*f^5*z - 132*A*B^2*a*b*c^6*d^2*e^4*f*z - 60*A*B^2*a*b^2*c^5*d*e^5*f*z - 48*A^2*B*a*b^5*c^2*d*e*f^5*z + 20*B^3*a*b*c^6*d^3*e^3*f*z + 16*B^3*a^4*b*c^3*d*e*f^5*z - 16*B^3*a*b*c^6*d^4*e*f^2*z + 12*B^3*a^2*b*c^5*d*e^5*f*z + 320*A^3*a*b*c^6*d*e^4*f^2*z + 40*A^3*a*b^4*c^3*d*e*f^5*z - 48*A^2*B*b*c^7*d^4*e*f^2*z - 44*A^2*B*b^3*c^5*d*e^5*f*z - 20*A*B^2*b*c^7*d^4*e^2*f*z + 14*A*B^2*b^4*c^4*d*e^5*f*z + 12*A^2*B*b*c^7*d^3*e^3*f*z + 4*A*B^2*b^7*c*d*e^2*f^4*z + 160*A*B^2*a^4*c^4*d*e*f^5*z + 152*A^2*B*a*c^7*d^2*e^4*f*z - 40*A*B^2*a*c^7*d^3*e^3*f*z + 32*A*B^2*a*c^7*d^4*e*f^2*z - 16*A*B^2*a^2*c^6*d*e^5*f*z + 128*A^2*B*a^4*b*c^3*e*f^6*z + 42*A^2*B*a*b^2*c^5*e^6*f*z + 24*A^2*B*a^2*b^5*c*e*f^6*z - 12*A*B^2*a^3*b^4*c*e*f^6*z - 12*A*B^2*a^2*b*c^5*e^6*f*z - 10*A^2*B*a*b^6*c*e^2*f^5*z - 160*A*B^2*a*b*c^6*d^4*f^3*z + 112*A*B^2*a^4*b*c^3*d*f^6*z - 24*A*B^2*a^2*b^5*c*d*f^6*z - 84*B^3*a*b^2*c^5*d^3*e^2*f^2*z - 80*B^3*a^2*b^3*c^3*d^2*e*f^4*z - 60*B^3*a^2*b*c^5*d^2*e^3*f^2*z - 20*B^3*a^3*b^2*c^3*d*e^2*f^4*z - 20*B^3*a*b^3*c^4*d^2*e^3*f^2*z - 9*B^3*a^2*b^2*c^4*d*e^4*f^2*z - 8*B^3*a*b^4*c^3*d^2*e^2*f^3*z + 6*B^3*a^2*b^4*c^2*d*e^2*f^4*z - 4*B^3*a^2*b^3*c^3*d*e^3*f^3*z - 216*A^2*B*b^4*c^4*d^2*e^2*f^3*z + 196*A^2*B*b^3*c^5*d^2*e^3*f^2*z - 108*A*B^2*b^3*c^5*d^3*e^2*f^2*z - 94*A*B^2*b^4*c^4*d^2*e^3*f^2*z + 88*A^2*B*b^2*c^6*d^3*e^2*f^2*z + 80*A*B^2*b^5*c^3*d^2*e^2*f^3*z + 360*A^2*B*a^2*c^6*d^2*e^2*f^3*z + 8*A*B^2*a^2*c^6*d^2*e^3*f^2*z + 153*A^2*B*a^2*b^2*c^4*e^4*f^3*z - 144*A^2*B*a^2*b^3*c^3*e^3*f^4*z + 80*A^2*B*a^3*b^2*c^3*e^2*f^5*z + 36*A*B^2*a^3*b^2*c^3*e^3*f^4*z + 12*A^2*B*a^2*b^4*c^2*e^2*f^5*z + 12*A*B^2*a^3*b^3*c^2*e^2*f^5*z + 9*A*B^2*a^2*b^2*c^4*e^5*f^2*z - 6*A*B^2*a^2*b^4*c^2*e^3*f^4*z + 4*A*B^2*a^2*b^3*c^3*e^4*f^3*z + 480*A^2*B*a^2*b^2*c^4*d^2*f^5*z - 176*A*B^2*a^2*b^3*c^3*d^2*f^5*z - 10*A^2*B*a*b^6*c*d*f^6*z + 16*A*B^2*a*b*c^6*d*e^6*z + 80*B^3*a*b^3*c^4*d^3*e*f^3*z - 48*B^3*a^3*b*c^4*d^2*e*f^4*z + 48*B^3*a^2*b*c^5*d^3*e*f^3*z + 44*B^3*a^3*b*c^4*d*e^3*f^3*z + 24*B^3*a*b^5*c^2*d^2*e*f^4*z + 18*B^3*a*b^2*c^5*d^2*e^4*f*z + 696*A^3*a^2*b*c^5*d*e^2*f^4*z - 504*A^3*a*b*c^6*d^2*e^2*f^3*z - 192*A^3*a*b^2*c^5*d*e^3*f^3*z - 144*A^3*a^2*b^2*c^4*d*e*f^5*z + 96*A^3*a*b^2*c^5*d^2*e*f^4*z - 72*A^3*a*b^3*c^4*d*e^2*f^4*z - 208*A^2*B*b^3*c^5*d^3*e*f^3*z + 152*A*B^2*b^4*c^4*d^3*e*f^3*z + 80*A^2*B*b^5*c^3*d^2*e*f^4*z + 75*A^2*B*b^4*c^4*d*e^4*f^2*z - 59*A^2*B*b^2*c^6*d^2*e^4*f*z - 52*A^2*B*b^5*c^3*d*e^3*f^3*z + 42*A*B^2*b^3*c^5*d^2*e^4*f*z - 21*A*B^2*b^6*c^2*d^2*e*f^4*z - 16*A*B^2*b^5*c^3*d*e^4*f^2*z + 16*A*B^2*b^2*c^6*d^4*e*f^2*z + 16*A*B^2*b^2*c^6*d^3*e^3*f*z + 11*A^2*B*b^6*c^2*d*e^2*f^4*z + 4*A*B^2*b^6*c^2*d*e^3*f^3*z - 256*A^2*B*a*c^7*d^3*e^2*f^2*z - 96*A*B^2*a^3*c^5*d^2*e*f^4*z - 36*A^2*B*a^2*c^6*d*e^4*f^2*z - 32*A^2*B*a^3*c^5*d*e^2*f^4*z - 32*A*B^2*a^2*c^6*d^3*e*f^3*z + 8*A*B^2*a^3*c^5*d*e^3*f^3*z - 96*A^2*B*a^3*b^3*c^2*e*f^6*z + 68*A^2*B*a^3*b*c^4*e^3*f^4*z - 60*A*B^2*a^4*b*c^3*e^2*f^5*z - 60*A*B^2*a^3*b*c^4*e^4*f^3*z + 48*A*B^2*a^4*b^2*c^2*e*f^6*z - 38*A^2*B*a*b^3*c^4*e^5*f^2*z - 36*A^2*B*a^2*b*c^5*e^5*f^2*z + 36*A^2*B*a*b^5*c^2*e^3*f^4*z - 16*A^2*B*a*b^4*c^3*e^4*f^3*z + 384*A*B^2*a^2*b*c^5*d^3*f^4*z - 352*A*B^2*a^3*b*c^4*d^2*f^5*z - 288*A^2*B*a*b^2*c^5*d^3*f^4*z - 160*A^2*B*a^3*b^2*c^3*d*f^6*z - 148*A^2*B*a*b^4*c^3*d^2*f^5*z + 112*A*B^2*a*b^3*c^4*d^3*f^4*z + 72*A^2*B*a^2*b^4*c^2*d*f^6*z + 72*A*B^2*a*b^5*c^2*d^2*f^5*z + 48*A*B^2*a^3*b^3*c^2*d*f^
\end{aligned}$$

$$\begin{aligned}
& 6*z + 102*B^3*a^2*b^2*c^4*d^2*e^2*f^3*z - 32*B^3*b^5*c^3*d^3*e*f^3*z - 8*B^3 \\
& 3*b^3*c^5*d^3*e^3*f*z - 7*B^3*b^4*c^4*d^2*e^4*f*z + 5*B^3*b^2*c^6*d^4*e^2*f \\
& *z + 80*A^3*b^2*c^6*d^3*e*f^3*z - 74*A^3*b^3*c^5*d*e^4*f^2*z - 64*A^3*b^4*c \\
& ^4*d^2*e*f^4*z + 60*A^3*b^4*c^4*d*e^3*f^3*z - 48*B^3*a^4*c^4*d*e^2*f^4*z - \\
& 24*B^3*a^3*c^5*d*e^4*f^2*z + 20*B^3*a^2*c^6*d^2*e^4*f*z - 16*A^3*b^5*c^3*d* \\
& e^2*f^4*z + 8*A^3*b*c^7*d^3*e^2*f^2*z + 480*A^3*a^2*c^6*d^2*e*f^4*z - 392*A \\
& ^3*a^2*c^6*d*e^3*f^3*z + 280*A^3*a*c^7*d^2*e^3*f^2*z - 4*B^3*a^4*b*c^3*e^3*f \\
& f^4*z - 200*A^3*a^3*b*c^4*e^2*f^5*z - 144*A^3*a^2*b*c^5*e^4*f^3*z + 48*B^3* \\
& a*b^2*c^5*d^4*f^3*z + 42*A^3*a*b^2*c^5*e^5*f^2*z - 36*B^3*a^4*b^2*c^2*d*f^6 \\
& *z - 32*A^3*a^3*b^2*c^3*e*f^6*z - 24*A^3*a^2*b^4*c^2*e*f^6*z - 24*A^3*a*b^5 \\
& *c^2*e^2*f^5*z + 10*A^3*a*b^3*c^4*e^4*f^3*z - 4*B^3*a*b^4*c^3*d^3*f^4*z - 4 \\
& *A^3*a*b^4*c^3*e^3*f^4*z - 480*A^3*a^2*b*c^5*d^2*f^5*z - 160*A^3*a^2*b^3*c^ \\
& 3*d*f^6*z + 128*A^3*a*b^3*c^4*d^2*f^5*z + 8*A^2*B*b^5*c^3*e^5*f^2*z - 2*A^2 \\
& *B*b^6*c^2*e^4*f^3*z + 112*A^2*B*b^4*c^4*d^3*f^4*z - 92*A^2*B*a^4*c^4*e^2*f \\
& ^5*z - 64*A^2*B*a^3*c^5*e^4*f^3*z - 64*A*B^2*b^5*c^3*d^3*f^4*z + 24*A*B^2*a \\
& ^4*c^4*e^3*f^4*z + 24*A*B^2*a^3*c^5*e^5*f^2*z + 16*A^2*B*b^2*c^6*d^4*f^3*z \\
& + 16*A*B^2*b^3*c^5*d^4*f^3*z - A^2*B*b^6*c^2*d^2*f^5*z + 448*A^2*B*a^3*c^5* \\
& d^2*f^5*z - 352*A^2*B*a^2*c^6*d^3*f^4*z - 5*A*B^2*b^2*c^6*d^2*e^5*z - 48*A^ \\
& 2*B*a^4*b^2*c^2*f^7*z - 2*B^3*b^7*c*d^2*e*f^4*z + 34*A^3*b^2*c^6*d*e^5*f*z \\
& + 16*A^3*b*c^7*d^2*e^4*f*z + 2*A^3*b^6*c^2*d*e*f^5*z - 416*A^3*a^3*c^5*d*e* \\
& f^5*z - 224*A^3*a*c^7*d^3*e*f^3*z + 12*B^3*a^3*b^4*c*d*f^6*z - 10*B^3*a*b^6 \\
& *c*d^2*f^5*z + 416*A^3*a^3*b*c^4*d*f^6*z + 224*A^3*a*b*c^6*d^3*f^4*z + 24*A \\
& ^3*a*b^5*c^2*d*f^6*z - 4*B^3*a*b*c^6*d^2*e^5*z + 20*A^2*B*c^8*d^4*e^2*f*z - \\
& 7*A^2*B*b^4*c^4*e^6*f*z - 2*A^2*B*b^7*c*e^3*f^4*z - 64*A*B^2*a^5*c^3*e*f^6 \\
& *z + 16*A*B^2*b*c^7*d^5*f^2*z - 8*A^2*B*a^2*c^6*e^6*f*z - 2*A*B^2*b^7*c*d^2 \\
& *f^5*z - 272*A^2*B*a^4*c^4*d*f^6*z + 128*A^2*B*a*c^7*d^4*f^3*z + 9*A^2*B*b^ \\
& 2*c^6*d*e^6*z - 4*A*B^2*b^3*c^5*d*e^6*z + 4*A*B^2*b*c^7*d^3*e^4*z + 8*A*B^2 \\
& *a*c^7*d^2*e^5*z + 12*A^2*B*a^3*b^4*c*f^7*z + 30*B^3*b^4*c^4*d^3*e^2*f^2*z \\
& + 8*B^3*b^5*c^3*d^2*e^3*f^2*z - 2*B^3*b^6*c^2*d^2*e^2*f^3*z + 152*A^3*b^3*c \\
& ^5*d^2*e^2*f^3*z - 108*A^3*b^2*c^6*d^2*e^3*f^2*z + 48*B^3*a^3*c^5*d^2*e^2*f \\
& ^3*z - 16*B^3*a^2*c^6*d^3*e^2*f^2*z - 3*B^3*a^4*b^2*c^2*e^2*f^5*z - 120*B^3 \\
& *a^2*b^2*c^4*d^3*f^4*z + 112*B^3*a^3*b^2*c^3*d^2*f^5*z + 112*A^3*a^2*b^3*c^ \\
& 3*e^2*f^5*z + 12*A^3*a^2*b^2*c^4*e^3*f^4*z - 120*A^3*a*c^7*d*e^5*f*z - 52*A \\
& ^3*a*b*c^6*e^6*f*z + 10*A^3*a*b^6*c*e*f^6*z - 2*A*B^2*b^8*d*e*f^5*z - 2*A^2 \\
& *B*a*b^7*e*f^6*z - 24*A^2*B*a*c^7*d*e^6*z + 2*A*B^2*a*b^7*d*f^6*z - 12*A^2* \\
& B*a*b*c^6*e^7*z - 2*A^3*b^7*c*d*f^6*z - 4*A^3*b*c^7*d*e^6*z + 16*B^3*a^5*c^ \\
& 3*e^2*f^5*z + 11*B^3*b^6*c^2*d^3*f^4*z - 11*A^3*b^4*c^4*e^5*f^2*z - 8*B^3*b \\
& ^4*c^4*d^4*f^3*z - 4*B^3*b^2*c^6*d^5*f^2*z + 4*B^3*a^4*c^4*e^4*f^3*z + 4*A^ \\
& 3*b^5*c^3*e^4*f^3*z - A^3*b^6*c^2*e^3*f^4*z + 136*A^3*a^3*c^5*e^3*f^4*z + 6 \\
& 8*A^3*a^2*c^6*e^5*f^2*z - 64*A^3*b^3*c^5*d^3*f^4*z + 2*B^3*b^3*c^5*d^2*e^5* \\
& z - B^3*b^2*c^6*d^3*e^4*z + 96*A^3*a^3*b^3*c^2*f^7*z + A*B^2*a^2*b^6*e*f^6* \\
& z + 32*A^3*c^8*d^4*e*f^2*z - 24*A^3*c^8*d^3*e^3*f*z + 10*A^3*b^3*c^5*e^6*f* \\
& z + 2*A^3*b^7*c*e^2*f^5*z + 128*A^3*a^4*c^4*e*f^6*z - 32*A^3*b*c^7*d^4*f^3* \\
& z - 4*B^3*a^2*c^6*d*e^6*z - B^3*a^2*b^6*d*f^6*z - 128*A^3*a^4*b*c^3*f^7*z - \\
& 24*A^3*a^2*b^5*c*f^7*z - 16*A^2*B*c^8*d^5*f^2*z - 4*A^2*B*c^8*d^3*e^4*z +
\end{aligned}$$

$$\begin{aligned}
& 64*A^2*B*a^5*c^3*f^7*z + 2*A^2*B*b^3*c^5*e^7*z + 4*A*B^2*a^2*c^6*e^7*z - A^2*B*a^2*b^6*f^7*z + 4*A^3*c^8*d^2*e^5*z - 3*A^3*b^2*c^6*e^7*z + A^2*B*b^8*d \\
& *f^6*z - A^3*b^8*e*f^6*z + 16*A^3*a*c^7*e^7*z + 2*A^3*a*b^7*f^7*z + A^2*B*b^8*e^2*f^5*z + B^3*b^8*d^2*f^5*z - 48*A^2*B^2*a*b*c^4*d*e*f^4 + 28*A*B^3*a* \\
& b^2*c^3*d*e*f^4 - 16*A*B^3*a*b*c^4*d*e^2*f^3 + 16*A^3*B*a*c^5*d*e*f^4 + 32* \\
& A^3*B*a*b*c^4*d*f^5 + 12*A^2*B^2*b^3*c^3*d*e*f^4 + 5*A*B^3*b^2*c^4*d^2*e*f^ \\
& 3 + 4*A*B^3*b^3*c^3*d*e^2*f^3 + 24*A^2*B^2*a*c^5*d*e^2*f^3 + 24*A^2*B^2*a^2 \\
& *b*c^3*e*f^5 + 12*A^2*B^2*a*b*c^4*e^3*f^3 - 6*A^2*B^2*a*b^3*c^2*e*f^5 + 4*A \\
& *B^3*a^2*b*c^3*e^2*f^4 + 3*A*B^3*a^2*b^2*c^2*e*f^5 - 18*A^2*B^2*a*b^2*c^3*d \\
& *f^5 - 4*B^4*a^2*b*c^3*d*e*f^4 + 4*B^4*a*b*c^4*d^2*e*f^3 - 6*A*B^3*b^4*c^2* \\
& d*e*f^4 + 4*A^3*B*b*c^5*d*e^2*f^3 - 2*A^3*B*b^2*c^4*d*e*f^4 - 8*A*B^3*a^2*c \\
& ^4*d*e*f^4 - 8*A*B^3*a*c^5*d^2*e*f^3 + 26*A^3*B*a*b^2*c^3*e*f^5 + 8*A^3*B*a \\
& *b*c^4*e^2*f^4 + 32*A*B^3*a*b*c^4*d^2*f^4 - 28*A*B^3*a^2*b*c^3*d*f^5 + 6*A* \\
& B^3*a*b^3*c^2*d*f^5 - 9*A^2*B^2*b^2*c^4*d*e^2*f^3 - 18*A^2*B^2*a*b^2*c^3*e^ \\
& 2*f^4 - 4*A^3*B*c^6*d^2*e*f^3 - 3*A^3*B*b^4*c^2*e*f^5 - 44*A^3*B*a^2*c^4*e* \\
& f^5 - 16*A^3*B*a*c^5*e^3*f^3 - 16*A*B^3*a^3*c^3*e*f^5 - 10*A^3*B*b^3*c^3*d* \\
& f^5 - 4*A^3*B*b*c^5*d^2*f^4 - 4*A*B^3*b*c^5*d^3*f^3 - 28*A^3*B*a^2*b*c^3*f^ \\
& 6 + 6*A^3*B*a*b^3*c^2*f^6 - 4*A^4*b*c^5*d*e*f^4 - 20*A^4*a*b*c^4*e*f^5 + 3* \\
& A^2*B^2*b^4*c^2*e^2*f^4 - 2*A^2*B^2*b^3*c^3*e^3*f^3 + 12*A^2*B^2*a^2*c^4*e^ \\
& 2*f^4 + 9*A^2*B^2*b^2*c^4*d^2*f^4 - 3*A^2*B^2*a^2*b^2*c^2*f^6 - 2*B^4*b^3*c \\
& ^3*d^2*e*f^3 + 4*B^4*a^2*c^4*d*e^2*f^3 - 10*B^4*a*b^2*c^3*d^2*f^4 - 3*B^4*a \\
& ^2*b^2*c^2*d*f^5 + 3*A^3*B*b^2*c^4*e^3*f^3 - 2*A^3*B*b^3*c^3*e^2*f^4 - 10*A \\
& *B^3*b^3*c^3*d^2*f^4 - 4*A*B^3*a^2*c^4*e^3*f^3 + 3*A^2*B^2*b^4*c^2*d*f^5 + \\
& 36*A^2*B^2*a^2*c^4*d*f^5 - 24*A^2*B^2*a*c^5*d^2*f^4 + 4*A^2*B^2*c^6*d^3*f^3 \\
& + 16*A^2*B^2*a^3*c^3*f^6 + 4*A^4*b^3*c^3*e*f^5 + 16*B^4*a^3*c^3*d*f^5 + 16 \\
& *A^4*a*c^5*e^2*f^4 + 8*A^4*b^2*c^4*d*f^5 - 8*A^4*a*b^2*c^3*f^6 - 24*A^4*a*c \\
& ^5*d*f^5 + 3*B^4*b^4*c^2*d^2*f^4 - 3*A^4*b^2*c^4*e^2*f^4 + 4*A^4*c^6*d^2*f^ \\
& 4 + 36*A^4*a^2*c^4*f^6 + B^4*b^2*c^4*d^3*f^3, z, k)*(root(48416*a^6*b^2*c^6 \\
& *d^4*e^2*f^4*z^4 - 41544*a^5*b^4*c^5*d^4*e^2*f^4*z^4 - 31872*a^7*b^2*c^5*d^ \\
& 3*e^2*f^5*z^4 - 31872*a^5*b^2*c^7*d^5*e^2*f^3*z^4 - 29184*a^6*b^2*c^6*d^3*e \\
& ^4*f^3*z^4 + 28800*a^5*b^4*c^5*d^3*e^4*f^3*z^4 + 21510*a^4*b^6*c^4*d^4*e^2* \\
& f^4*z^4 + 21408*a^6*b^4*c^4*d^3*e^2*f^5*z^4 + 21408*a^4*b^4*c^6*d^5*e^2*f^3 \\
& *z^4 - 18112*a^7*b^3*c^4*d^2*e^3*f^5*z^4 - 18112*a^4*b^3*c^7*d^5*e^3*f^2*z^ \\
& 4 - 15600*a^5*b^5*c^4*d^3*e^3*f^4*z^4 - 15600*a^4*b^5*c^5*d^4*e^3*f^3*z^4 + \\
& 15296*a^6*b^3*c^5*d^3*e^3*f^4*z^4 + 15296*a^5*b^3*c^6*d^4*e^3*f^3*z^4 + 14 \\
& 016*a^7*b^2*c^5*d^2*e^4*f^4*z^4 + 14016*a^5*b^2*c^7*d^4*e^4*f^2*z^4 - 13920 \\
& *a^4*b^6*c^4*d^3*e^4*f^3*z^4 - 11648*a^6*b^3*c^5*d^2*e^5*f^3*z^4 - 11648*a^ \\
& 5*b^3*c^6*d^3*e^5*f^2*z^4 + 10432*a^6*b^2*c^6*d^2*e^6*f^2*z^4 + 9008*a^6*b^ \\
& 5*c^3*d^2*e^3*f^5*z^4 + 9008*a^3*b^5*c^6*d^5*e^3*f^2*z^4 + 8544*a^5*b^5*c^4 \\
& *d^2*e^5*f^3*z^4 + 8544*a^4*b^5*c^5*d^3*e^5*f^2*z^4 - 8496*a^5*b^4*c^5*d^2* \\
& e^6*f^2*z^4 + 7488*a^8*b^2*c^4*d^2*e^2*f^6*z^4 + 7488*a^4*b^2*c^8*d^6*e^2*f \\
& ^2*z^4 + 7380*a^4*b^7*c^3*d^3*e^3*f^4*z^4 + 7380*a^3*b^7*c^4*d^4*e^3*f^3*z^ \\
& 4 - 6720*a^3*b^8*c^3*d^4*e^2*f^4*z^4 - 5784*a^5*b^6*c^3*d^3*e^2*f^5*z^4 - 5 \\
& 784*a^3*b^6*c^5*d^5*e^2*f^3*z^4 - 3440*a^6*b^4*c^4*d^2*e^4*f^4*z^4 - 3440*a \\
& ^4*b^4*c^6*d^4*e^4*f^2*z^4 + 3360*a^3*b^8*c^3*d^3*e^4*f^3*z^4 + 3140*a^4*b^
\end{aligned}$$

$$\begin{aligned}
&6*c^4*d^2*e^6*f^2*z^4 - 2760*a^4*b^7*c^3*d^2*e^5*f^3*z^4 - 2760*a^3*b^7*c^4 \\
&*d^3*e^5*f^2*z^4 - 1764*a^5*b^7*c^2*d^2*e^3*f^5*z^4 - 1764*a^2*b^7*c^5*d^5* \\
&e^3*f^2*z^4 - 1640*a^3*b^9*c^2*d^3*e^3*f^4*z^4 - 1640*a^2*b^9*c^3*d^4*e^3*f \\
&^3*z^4 - 1604*a^6*b^6*c^2*d^2*e^2*f^6*z^4 - 1604*a^2*b^6*c^6*d^6*e^2*f^2*z^ \\
&4 - 1500*a^5*b^6*c^3*d^2*e^4*f^4*z^4 - 1500*a^3*b^6*c^5*d^4*e^4*f^2*z^4 + 1 \\
&140*a^2*b^10*c^2*d^4*e^2*f^4*z^4 + 810*a^4*b^8*c^2*d^2*e^4*f^4*z^4 + 810*a^ \\
&2*b^8*c^4*d^4*e^4*f^2*z^4 - 544*a^3*b^8*c^3*d^2*e^6*f^2*z^4 + 416*a^3*b^9*c \\
&^2*d^2*e^5*f^3*z^4 + 416*a^2*b^9*c^3*d^3*e^5*f^2*z^4 - 384*a^2*b^10*c^2*d^3 \\
&*e^4*f^3*z^4 + 180*a^4*b^8*c^2*d^3*e^2*f^5*z^4 + 180*a^2*b^8*c^4*d^5*e^2*f^ \\
&3*z^4 + 48*a^7*b^4*c^3*d^2*e^2*f^6*z^4 + 48*a^3*b^4*c^7*d^6*e^2*f^2*z^4 + 3 \\
&6*a^2*b^10*c^2*d^2*e^6*f^2*z^4 - 1024*a^10*b*c^3*d*e*f^8*z^4 - 1024*a^3*b*c \\
&^10*d^8*e*f*z^4 - 192*a^8*b^5*c*d*e*f^8*z^4 - 192*a*b^5*c^8*d^8*e*f*z^4 + 1 \\
&6128*a^7*b^3*c^4*d^3*e*f^6*z^4 + 16128*a^4*b^3*c^7*d^6*e*f^3*z^4 - 11712*a^ \\
&6*b^5*c^3*d^3*e*f^6*z^4 - 11712*a^3*b^5*c^6*d^6*e*f^3*z^4 + 11520*a^8*b*c^5 \\
&*d^2*e^3*f^5*z^4 + 11520*a^5*b*c^8*d^5*e^3*f^2*z^4 - 9984*a^6*b^3*c^5*d^4*e \\
&*f^5*z^4 - 9984*a^5*b^3*c^6*d^5*e*f^4*z^4 + 8640*a^5*b^5*c^4*d^4*e*f^5*z^4 \\
&+ 8640*a^4*b^5*c^5*d^5*e*f^4*z^4 - 7424*a^7*b*c^6*d^3*e^3*f^4*z^4 - 7424*a^ \\
&6*b*c^7*d^4*e^3*f^3*z^4 - 6912*a^8*b^3*c^3*d^2*e*f^7*z^4 - 6912*a^3*b^3*c^8 \\
&*d^7*e*f^2*z^4 + 4800*a^7*b^3*c^4*d*e^5*f^4*z^4 + 4800*a^4*b^3*c^7*d^4*e^5* \\
&f*z^4 + 4608*a^7*b*c^6*d^2*e^5*f^3*z^4 + 4608*a^6*b*c^7*d^3*e^5*f^2*z^4 - 4 \\
&560*a^4*b^7*c^3*d^4*e*f^5*z^4 - 4560*a^3*b^7*c^4*d^5*e*f^4*z^4 + 4176*a^5*b \\
&^7*c^2*d^3*e*f^6*z^4 + 4176*a^2*b^7*c^5*d^6*e*f^3*z^4 + 3264*a^7*b^5*c^2*d^ \\
&2*e*f^7*z^4 + 3264*a^2*b^5*c^7*d^7*e*f^2*z^4 + 3008*a^8*b^3*c^3*d*e^3*f^6*z \\
&^4 + 3008*a^3*b^3*c^8*d^6*e^3*f*z^4 + 2880*a^6*b^3*c^5*d*e^7*f^2*z^4 + 2880 \\
&*a^5*b^3*c^6*d^2*e^7*f*z^4 - 2240*a^7*b^4*c^3*d*e^4*f^5*z^4 - 2240*a^3*b^4* \\
&c^7*d^5*e^4*f*z^4 - 1488*a^5*b^5*c^4*d*e^7*f^2*z^4 - 1488*a^4*b^5*c^5*d^2*e \\
&^7*f*z^4 + 1440*a^3*b^9*c^2*d^4*e*f^5*z^4 + 1440*a^2*b^9*c^3*d^5*e*f^4*z^4 \\
&- 1328*a^6*b^5*c^3*d*e^5*f^4*z^4 - 1328*a^3*b^5*c^6*d^4*e^5*f*z^4 - 1152*a^ \\
&7*b^2*c^5*d*e^6*f^3*z^4 - 1152*a^5*b^2*c^7*d^3*e^6*f*z^4 - 1120*a^6*b^4*c^4 \\
&*d*e^6*f^3*z^4 - 1120*a^4*b^4*c^6*d^3*e^6*f*z^4 + 912*a^6*b^6*c^2*d*e^4*f^5 \\
&*z^4 + 912*a^2*b^6*c^6*d^5*e^4*f*z^4 + 872*a^5*b^6*c^3*d*e^6*f^3*z^4 + 872* \\
&a^3*b^6*c^5*d^3*e^6*f*z^4 + 768*a^8*b^2*c^4*d*e^4*f^5*z^4 + 768*a^4*b^2*c^8 \\
&*d^5*e^4*f*z^4 - 672*a^8*b^4*c^2*d*e^2*f^7*z^4 - 672*a^2*b^4*c^8*d^7*e^2*f* \\
&z^4 - 624*a^7*b^5*c^2*d*e^3*f^6*z^4 - 624*a^2*b^5*c^7*d^6*e^3*f*z^4 + 480*a \\
&^5*b^8*c*d^2*e^2*f^6*z^4 + 480*a*b^8*c^5*d^6*e^2*f^2*z^4 + 316*a^4*b^7*c^3* \\
&d*e^7*f^2*z^4 + 316*a^3*b^7*c^4*d^2*e^7*f*z^4 - 204*a^4*b^8*c^2*d*e^6*f^3*z \\
&^4 - 204*a^2*b^8*c^4*d^3*e^6*f*z^4 + 168*a^3*b^10*c*d^3*e^2*f^5*z^4 + 168*a \\
&*b^10*c^3*d^5*e^2*f^3*z^4 + 156*a^2*b^11*c*d^3*e^3*f^4*z^4 + 156*a*b^11*c^2 \\
&*d^4*e^3*f^3*z^4 + 128*a^9*b^2*c^3*d*e^2*f^7*z^4 + 128*a^3*b^2*c^9*d^7*e^2* \\
&f*z^4 - 124*a^3*b^10*c*d^2*e^4*f^4*z^4 - 124*a*b^10*c^3*d^4*e^4*f^2*z^4 + 1 \\
&00*a^4*b^9*c*d^2*e^3*f^5*z^4 + 100*a*b^9*c^4*d^5*e^3*f^2*z^4 + 36*a^5*b^7*c \\
&^2*d*e^5*f^4*z^4 + 36*a^2*b^7*c^5*d^4*e^5*f*z^4 - 24*a^3*b^9*c^2*d*e^7*f^2* \\
&z^4 - 24*a^2*b^11*c*d^2*e^5*f^3*z^4 - 24*a^2*b^9*c^3*d^2*e^7*f*z^4 - 24*a*b \\
&^11*c^2*d^3*e^5*f^2*z^4 - 9216*a^8*b*c^5*d^3*e*f^6*z^4 - 9216*a^5*b*c^8*d^6 \\
&*e*f^3*z^4 - 5376*a^8*b*c^5*d*e^5*f^4*z^4 - 5376*a^5*b*c^8*d^4*e^5*f*z^4 +
\end{aligned}$$

$$\begin{aligned}
& 5120*a^9*b*c^4*d^2*e*f^7*z^4 + 5120*a^7*b*c^6*d^4*e*f^5*z^4 + 5120*a^6*b*c^7*d^5*e*f^4*z^4 + 5120*a^4*b*c^9*d^7*e*f^2*z^4 - 4352*a^9*b*c^4*d*e^3*f^6*z^4 \\
& - 4352*a^4*b*c^9*d^6*e^3*f*z^4 - 1792*a^7*b*c^6*d*e^7*f^2*z^4 - 1792*a^6*b*c^7*d^2*e^7*f*z^4 - 1600*a^6*b^2*c^6*d*e^8*f*z^4 + 912*a^5*b^4*c^5*d*e^8 \\
& *f*z^4 + 768*a^9*b^3*c^2*d*e*f^8*z^4 + 768*a^2*b^3*c^9*d^8*e*f*z^4 - 720*a^4*b^9*c*d^3*e*f^6*z^4 - 720*a*b^9*c^4*d^6*e*f^3*z^4 - 656*a^6*b^7*c*d^2*e*f \\
& ^7*z^4 - 656*a*b^7*c^6*d^7*e*f^2*z^4 - 240*a^2*b^11*c*d^4*e*f^5*z^4 - 240*a*b^11*c^2*d^5*e*f^4*z^4 + 216*a^7*b^6*c*d*e^2*f^7*z^4 + 216*a*b^6*c^7*d^7*e \\
& ^2*f*z^4 - 204*a^4*b^6*c^4*d*e^8*f*z^4 - 144*a^5*b^8*c*d*e^4*f^5*z^4 - 144*a*b^8*c^5*d^5*e^4*f*z^4 - 84*a*b^12*c*d^4*e^2*f^4*z^4 + 36*a^4*b^9*c*d*e^5* \\
& f^4*z^4 + 36*a*b^9*c^4*d^4*e^5*f*z^4 + 20*a^6*b^7*c*d*e^3*f^6*z^4 + 20*a*b^7*c^6*d^6*e^3*f*z^4 + 16*a^3*b^10*c*d*e^6*f^3*z^4 + 16*a^3*b^8*c^3*d*e^8*f* \\
& z^4 + 16*a*b^12*c*d^3*e^4*f^3*z^4 + 16*a*b^10*c^3*d^3*e^6*f*z^4 + 48*b^11*c^3*d^6*e*f^3*z^4 + 48*b^9*c^5*d^7*e*f^2*z^4 - 20*b^8*c^6*d^7*e^2*f*z^4 + 8* \\
& b^10*c^4*d^5*e^4*f*z^4 - 4*b^13*c*d^4*e^3*f^3*z^4 - 4*b^11*c^3*d^4*e^5*f*z^4 + 4*b^9*c^5*d^6*e^3*f*z^4 + 3072*a^9*c^5*d*e^4*f^5*z^4 + 3072*a^5*c^9*d^5 \\
& *e^4*f*z^4 + 2560*a^8*c^6*d*e^6*f^3*z^4 + 2560*a^6*c^8*d^3*e^6*f*z^4 + 1536*a^10*c^4*d*e^2*f^7*z^4 + 1536*a^4*c^10*d^7*e^2*f*z^4 + 48*a^5*b^9*d^2*e*f^ \\
& 7*z^4 + 48*a^3*b^11*d^3*e*f^6*z^4 - 20*a^6*b^8*d*e^2*f^7*z^4 + 8*a^4*b^10*d \\
& *e^4*f^5*z^4 + 4*a^5*b^9*d*e^3*f^6*z^4 - 4*a^3*b^11*d*e^5*f^4*z^4 - 4*a*b^13*d^3*e^3*f^4*z^4 + 768*a^9*b*c^4*e^5*f^5*z^4 + 768*a^8*b*c^5*e^7*f^3*z^4 + \\
& 256*a^10*b*c^3*e^3*f^7*z^4 - 192*a^6*b^3*c^5*e^9*f*z^4 - 68*a^7*b^6*c*e^4* \\
& f^6*z^4 + 48*a^8*b^5*c*e^3*f^7*z^4 + 48*a^5*b^5*c^4*e^9*f*z^4 + 36*a^6*b^7* \\
& c*e^5*f^5*z^4 - 12*a^9*b^4*c*e^2*f^8*z^4 - 4*a^4*b^9*c*e^7*f^3*z^4 - 4*a^4* \\
& b^7*c^3*e^9*f*z^4 + 384*a^5*b^8*c*d^3*f^7*z^4 + 384*a*b^8*c^5*d^7*f^3*z^4 + \\
& 288*a^3*b^10*c*d^4*f^6*z^4 + 288*a*b^10*c^3*d^6*f^4*z^4 + 224*a^7*b^6*c*d^ \\
& 2*f^8*z^4 + 224*a*b^6*c^7*d^8*f^2*z^4 - 192*a^10*b^2*c^2*d*f^9*z^4 - 192*a^ \\
& 2*b^2*c^10*d^9*f*z^4 + 768*a^5*b*c^8*d^3*e^7*z^4 + 768*a^4*b*c^9*d^5*e^5*z^ \\
& 4 + 256*a^3*b*c^10*d^7*e^3*z^4 - 192*a^5*b^3*c^6*d*e^9*z^4 - 68*a*b^6*c^7*d \\
& ^6*e^4*z^4 + 48*a^4*b^5*c^5*d*e^9*z^4 + 48*a*b^5*c^8*d^7*e^3*z^4 + 36*a*b^7 \\
& *c^6*d^5*e^5*z^4 - 12*a*b^4*c^9*d^8*e^2*z^4 - 4*a^3*b^7*c^4*d*e^9*z^4 - 4*a \\
& *b^9*c^4*d^3*e^7*z^4 + 16*b^13*c*d^5*e*f^4*z^4 + 16*b^7*c^7*d^8*e*f*z^4 + 7 \\
& 68*a^7*c^7*d*e^8*f*z^4 + 16*a^7*b^7*d*e*f^8*z^4 + 16*a*b^13*d^4*e*f^5*z^4 + \\
& 256*a^7*b*c^6*e^9*f*z^4 + 80*a*b^12*c*d^5*f^5*z^4 + 48*a^9*b^4*c*d*f^9*z^4 \\
& + 48*a*b^4*c^9*d^9*f*z^4 + 256*a^6*b*c^7*d*e^9*z^4 - 42*b^10*c^4*d^6*e^2*f \\
& ^2*z^4 - 20*b^12*c^2*d^5*e^2*f^3*z^4 + 6*b^12*c^2*d^4*e^4*f^2*z^4 + 4*b^11* \\
& c^3*d^5*e^3*f^2*z^4 - 24960*a^7*c^7*d^4*e^2*f^4*z^4 + 18944*a^8*c^6*d^3*e^2 \\
& *f^5*z^4 + 18944*a^6*c^8*d^5*e^2*f^3*z^4 + 14336*a^7*c^7*d^3*e^4*f^3*z^4 - \\
& 9984*a^8*c^6*d^2*e^4*f^4*z^4 - 9984*a^6*c^8*d^4*e^4*f^2*z^4 - 7936*a^9*c^5* \\
& d^2*e^2*f^6*z^4 - 7936*a^5*c^9*d^6*e^2*f^2*z^4 - 4352*a^7*c^7*d^2*e^6*f^2*z \\
& ^4 - 42*a^4*b^10*d^2*e^2*f^6*z^4 - 20*a^2*b^12*d^3*e^2*f^5*z^4 + 6*a^2*b^12 \\
& *d^2*e^4*f^4*z^4 + 4*a^3*b^11*d^2*e^3*f^5*z^4 - 480*a^8*b^2*c^4*e^6*f^4*z^4 \\
& + 440*a^7*b^4*c^3*e^6*f^4*z^4 - 320*a^8*b^3*c^3*e^5*f^5*z^4 - 320*a^7*b^3* \\
& c^4*e^7*f^3*z^4 + 240*a^8*b^4*c^2*e^4*f^6*z^4 + 240*a^6*b^4*c^4*e^8*f^2*z^4 \\
& - 192*a^9*b^3*c^2*e^3*f^7*z^4 - 192*a^9*b^2*c^3*e^4*f^6*z^4 - 192*a^7*b^2*
\end{aligned}$$

$$\begin{aligned}
& c^5 e^8 f^2 z^4 - 90 a^6 b^6 c^2 e^6 f^4 z^4 - 68 a^5 b^6 c^3 e^8 f^2 z^4 + \\
& 48 a^{10} b^2 c^2 e^2 f^8 z^4 - 48 a^7 b^5 c^2 e^5 f^5 z^4 - 48 a^6 b^5 c^3 e^7 f^3 z^4 + 36 a^5 b^7 c^2 e^7 f^3 z^4 + 6 a^4 b^8 c^2 e^8 f^2 z^4 - 3392 \\
& 0 a^6 b^2 c^6 d^5 f^5 z^4 + 27936 a^5 b^4 c^5 d^5 f^5 z^4 + 26112 a^7 b^2 c^5 d^4 f^6 z^4 + 26112 a^5 b^2 c^7 d^6 f^4 z^4 - 20352 a^6 b^4 c^4 d^4 f^6 z^4 - \\
& 20352 a^4 b^4 c^6 d^6 f^4 z^4 - 13080 a^4 b^6 c^4 d^5 f^5 z^4 - 11520 a^8 b^2 c^4 d^3 f^7 z^4 - 11520 a^4 b^2 c^8 d^7 f^3 z^4 + 8736 a^5 b^6 c^3 \\
& d^4 f^6 z^4 + 8736 a^3 b^6 c^5 d^6 f^4 z^4 + 7488 a^7 b^4 c^3 d^3 f^7 z^4 + 7488 a^3 b^4 c^7 d^7 f^3 z^4 + 3840 a^3 b^8 c^3 d^5 f^5 z^4 + 2560 a^9 b^2 \\
& c^3 d^2 f^8 z^4 + 2560 a^3 b^2 c^9 d^8 f^2 z^4 - 2416 a^6 b^6 c^2 d^3 f^7 z^4 - 2416 a^2 b^6 c^6 d^7 f^3 z^4 - 2160 a^4 b^8 c^2 d^4 f^6 z^4 - 2160 a^2 \\
& b^8 c^4 d^6 f^4 z^4 - 1152 a^8 b^4 c^2 d^2 f^8 z^4 - 1152 a^2 b^4 c^8 d^8 f^2 z^4 - 720 a^2 b^{10} c^2 d^5 f^5 z^4 - 480 a^4 b^2 c^8 d^4 e^6 z^4 + 44 \\
& 0 a^3 b^4 c^7 d^4 e^6 z^4 - 320 a^4 b^3 c^7 d^3 e^7 z^4 - 320 a^3 b^3 c^8 d^5 e^5 z^4 + 240 a^4 b^4 c^6 d^2 e^8 z^4 + 240 a^2 b^4 c^8 d^6 e^4 z^4 - 19 \\
& 2 a^5 b^2 c^7 d^2 e^8 z^4 - 192 a^3 b^2 c^9 d^6 e^4 z^4 - 192 a^2 b^3 c^9 d^7 e^3 z^4 - 90 a^2 b^6 c^6 d^4 e^6 z^4 - 68 a^3 b^6 c^5 d^2 e^8 z^4 - 48 a^3 \\
& b^5 c^6 d^3 e^7 z^4 - 48 a^2 b^5 c^7 d^5 e^5 z^4 + 48 a^2 b^2 c^{10} d^8 e^2 z^4 + 36 a^2 b^7 c^5 d^3 e^7 z^4 + 6 a^2 b^8 c^4 d^2 e^8 z^4 - 4 b^6 c^8 \\
& d^9 f z^4 + 256 a^{11} c^3 d f^9 z^4 + 256 a^3 c^{11} d^9 f z^4 - 4 a^8 b^6 d f^9 z^4 - 384 a^9 c^5 e^6 f^4 z^4 - 256 a^{10} c^4 e^4 f^6 z^4 - 256 a^8 c^6 e^8 f^2 z^4 - \\
& 64 a^{11} c^3 e^2 f^8 z^4 - 24 b^{10} c^4 d^7 f^3 z^4 - 16 b^{12} c^2 d^6 f^4 z^4 - 16 b^8 c^6 d^8 f^2 z^4 + 17920 a^7 c^7 d^5 f^5 z^4 - 14336 \\
& a^8 c^6 d^4 f^6 z^4 - 14336 a^6 c^8 d^6 f^4 z^4 + 7168 a^9 c^5 d^3 f^7 z^4 + 7168 a^5 c^9 d^7 f^3 z^4 - 2048 a^{10} c^4 d^2 f^8 z^4 - 2048 a^4 c^{10} d^8 \\
& f^2 z^4 + 6 b^8 c^6 d^6 e^4 z^4 + 6 a^6 b^8 e^4 f^6 z^4 - 4 b^9 c^5 d^5 e^5 z^4 - 4 b^7 c^7 d^7 e^3 z^4 - 4 a^7 b^7 e^3 f^7 z^4 - 4 a^5 b^9 e^5 f^5 z^4 - \\
& 384 a^5 c^9 d^4 e^6 z^4 - 256 a^6 c^8 d^2 e^8 z^4 - 256 a^4 c^{10} d^6 e^4 z^4 - 64 a^3 c^{11} d^8 e^2 z^4 - 24 a^4 b^{10} d^3 f^7 z^4 - 16 a^6 b^8 d^2 \\
& f^8 z^4 - 16 a^2 b^{12} d^4 f^6 z^4 + 48 a^6 b^2 c^6 e^{10} z^4 - 12 a^5 b^4 c^5 e^{10} z^4 - 4 b^{14} d^5 f^5 z^4 - 64 a^7 c^7 e^{10} z^4 + b^{14} d^4 e^2 f^4 z^4 \\
& + b^{10} c^4 d^4 e^6 z^4 + b^6 c^8 d^8 e^2 z^4 + a^8 b^6 e^2 f^8 z^4 + a^4 b^{10} e^6 f^4 z^4 + a^4 b^6 c^4 e^{10} z^4 - 4820 A B a^4 b^6 c^5 d^2 e^2 f^4 z^2 \\
& + 2976 A B a^3 b^6 c^6 d^3 e^2 f^3 z^2 - 2328 A B a^3 b^6 c^6 d^2 e^4 f^2 z^2 + 1848 A B a^2 b^4 c^4 d^3 e f^4 z^2 - 1768 A B a^3 b^4 c^3 d^2 e f^5 z^2 \\
& + 1528 A B a^4 b^2 c^4 d^2 e f^5 z^2 - 1136 A B a^3 b^2 c^5 d^3 e f^4 z^2 - 974 A B a^4 b^3 c^3 d e^2 f^5 z^2 + 692 A B a^2 b^6 c^7 d^4 e^2 f^2 z^2 + 5 \\
& 88 A B a^6 b^6 c^3 d^2 e^3 f^3 z^2 - 580 A B a^3 b^3 c^4 d e^4 f^3 z^2 + 488 A B a^3 b^4 c^3 d e^3 f^4 z^2 - 444 A B a^2 b^2 c^6 d^2 e^5 f z^2 - 412 A B \\
& a^5 b^5 c^4 d^2 e^4 f^2 z^2 + 366 A B a^2 b^6 c^2 d^2 e f^5 z^2 - 352 A B a^2 b^2 c^6 d^4 e f^3 z^2 + 326 A B a^2 b^4 c^4 d e^5 f^2 z^2 + 324 A B a^5 b^5 \\
& c^4 d^3 e^2 f^3 z^2 - 302 A B a^3 b^3 c^6 d^4 e^2 f^2 z^2 - 296 A B a^6 b^7 c^2 d^2 e^2 f^4 z^2 + 122 A B a^4 b^2 c^4 d e^3 f^4 z^2 - 122 A B a^2 b^6 c^2 \\
& d e^3 f^4 z^2 - 84 A B a^3 b^2 c^5 d e^5 f^2 z^2 + 72 A B a^6 b^4 c^5 d^3 e^3 f^2 z^2 - 64 A B a^2 b^5 c^3 d e^4 f^3 z^2 + 60 A B a^3 b^5 c^2 d e^2 f^5
\end{aligned}$$

$$\begin{aligned}
& *z^2 + 1312*A*B*a^5*b*c^4*d*e^2*f^5*z^2 + 1040*A*B*a^4*b*c^5*d*e^4*f^3*z^2 \\
& - 500*A*B*a*b^6*c^3*d^3*e*f^4*z^2 - 376*A*B*a*b^2*c^7*d^5*e*f^2*z^2 + 276*A \\
& *B*a^4*b^4*c^2*d*e*f^6*z^2 - 262*A*B*a^2*b^3*c^5*d*e^6*f*z^2 + 238*A*B*a*b^ \\
& 2*c^7*d^4*e^3*f*z^2 + 232*A*B*a^5*b^2*c^3*d*e*f^6*z^2 - 176*A*B*a^2*b*c^7*d \\
& ^3*e^4*f*z^2 - 120*A*B*a*b^6*c^3*d*e^5*f^2*z^2 - 108*A*B*a*b^4*c^5*d^4*e*f^ \\
& 3*z^2 + 68*A*B*a*b^7*c^2*d*e^4*f^3*z^2 + 68*A*B*a*b^4*c^5*d^2*e^5*f*z^2 + 4 \\
& 6*A*B*a^2*b^7*c*d*e^2*f^5*z^2 - 36*A*B*a*b^3*c^6*d^3*e^4*f*z^2 - 1932*A*B*a \\
& ^2*b^3*c^5*d^3*e^2*f^3*z^2 - 1818*A*B*a^2*b^4*c^4*d^2*e^3*f^3*z^2 + 1620*A* \\
& B*a^3*b^3*c^4*d^2*e^2*f^4*z^2 + 1560*A*B*a^2*b^3*c^5*d^2*e^4*f^2*z^2 + 1244 \\
& *A*B*a^3*b^2*c^5*d^2*e^3*f^3*z^2 + 820*A*B*a^2*b^2*c^6*d^3*e^3*f^2*z^2 + 48 \\
& 0*A*B*a^2*b^5*c^3*d^2*e^2*f^4*z^2 + 352*A*B*a^3*b*c^6*d*e^6*f*z^2 - 108*A*B \\
& *a^3*b^6*c*d*e*f^6*z^2 + 82*A*B*a*b^5*c^4*d*e^6*f*z^2 - 64*A*B*a*b*c^8*d^5* \\
& e^2*f*z^2 + 16*A*B*a*b^8*c*d^2*e*f^5*z^2 - 4*A*B*a*b^8*c*d*e^3*f^4*z^2 + 16 \\
& *B^2*a*b*c^8*d^6*e*f*z^2 + 56*A*B*b^2*c^8*d^6*e*f*z^2 - 8*A*B*b^9*c*d*e^4*f \\
& ^3*z^2 - 8*A*B*b^7*c^3*d*e^6*f*z^2 - 800*A*B*a^6*c^4*d*e*f^6*z^2 + 10*A*B*a \\
& ^2*b^8*d*e*f^6*z^2 - 6*A*B*a*b^9*d*e^2*f^5*z^2 - 12*A*B*a^5*b^4*c*e*f^7*z^2 \\
& + 912*A*B*a^6*b*c^3*d*f^7*z^2 + 192*A*B*a^4*b^5*c*d*f^7*z^2 + 192*A*B*a*b* \\
& c^8*d^6*f^2*z^2 - 20*A*B*a*b^4*c^5*d*e^7*z^2 + 4*A*B*a*b*c^8*d^4*e^4*z^2 + \\
& 2144*B^2*a^4*b*c^5*d^3*e*f^4*z^2 - 1120*B^2*a^3*b*c^6*d^4*e*f^3*z^2 - 688*B \\
& ^2*a^5*b*c^4*d^2*e*f^5*z^2 - 256*B^2*a^3*b*c^6*d^2*e^5*f*z^2 + 152*B^2*a*b^ \\
& 3*c^6*d^5*e*f^2*z^2 + 120*B^2*a^5*b^3*c^2*d*e*f^6*z^2 - 116*B^2*a^5*b*c^4*d \\
& *e^3*f^4*z^2 + 110*B^2*a*b^7*c^2*d^3*e*f^4*z^2 - 80*B^2*a^2*b*c^7*d^5*e*f^2 \\
& *z^2 - 72*B^2*a*b^5*c^4*d^4*e*f^3*z^2 - 48*B^2*a^4*b*c^5*d*e^5*f^2*z^2 - 46 \\
& *B^2*a*b^3*c^6*d^4*e^3*f*z^2 - 44*B^2*a*b^4*c^5*d^3*e^4*f*z^2 - 34*B^2*a*b^ \\
& 5*c^4*d^2*e^5*f*z^2 + 20*B^2*a^2*b*c^7*d^4*e^3*f*z^2 - 10*B^2*a^3*b^6*c*d*e \\
& ^2*f^5*z^2 - 10*B^2*a^2*b^7*c*d^2*e*f^5*z^2 - 10*B^2*a*b^2*c^7*d^5*e^2*f*z^ \\
& 2 - 7*B^2*a^2*b^4*c^4*d*e^6*f*z^2 - 6*B^2*a^3*b^2*c^5*d*e^6*f*z^2 + 4*B^2*a \\
& *b^8*c*d^2*e^2*f^4*z^2 - 2*B^2*a^2*b^7*c*d*e^3*f^4*z^2 + 3196*A^2*a^4*b*c^5 \\
& *d*e^3*f^4*z^2 - 3184*A^2*a^4*b*c^5*d^2*e*f^5*z^2 + 1568*A^2*a^3*b*c^6*d^3* \\
& e*f^4*z^2 + 1504*A^2*a^3*b*c^6*d*e^5*f^2*z^2 - 656*A^2*a^4*b^3*c^3*d*e*f^6* \\
& z^2 - 400*A^2*a*b^6*c^3*d*e^4*f^3*z^2 + 314*A^2*a*b^5*c^4*d*e^5*f^2*z^2 - 2 \\
& 64*A^2*a^3*b^5*c^2*d*e*f^6*z^2 + 240*A^2*a^2*b^2*c^6*d*e^6*f*z^2 - 224*A^2* \\
& a^2*b*c^7*d^4*e*f^3*z^2 + 216*A^2*a*b^5*c^4*d^3*e*f^4*z^2 - 192*A^2*a^2*b*c \\
& ^7*d^2*e^5*f*z^2 + 178*A^2*a*b^7*c^2*d*e^3*f^4*z^2 - 154*A^2*a*b^7*c^2*d^2* \\
& e*f^5*z^2 + 128*A^2*a*b^3*c^6*d^4*e*f^3*z^2 + 106*A^2*a*b^3*c^6*d^2*e^5*f*z \\
& ^2 - 12*A^2*a*b^2*c^7*d^3*e^4*f*z^2 - 58*A*B*b^8*c^2*d^2*e^3*f^3*z^2 + 40*A \\
& *B*b^7*c^3*d^2*e^4*f^2*z^2 - 28*A*B*b^7*c^3*d^3*e^2*f^3*z^2 - 24*A*B*b^5*c^ \\
& 5*d^4*e^2*f^2*z^2 - 20*A*B*b^6*c^4*d^3*e^3*f^2*z^2 + 2768*A*B*a^4*c^6*d^2*e \\
& ^3*f^3*z^2 - 1712*A*B*a^3*c^7*d^3*e^3*f^2*z^2 - 156*A*B*a^4*b^2*c^4*e^5*f^3 \\
& *z^2 + 146*A*B*a^4*b^3*c^3*e^4*f^4*z^2 - 106*A*B*a^5*b^2*c^3*e^3*f^5*z^2 + \\
& 90*A*B*a^5*b^3*c^2*e^2*f^6*z^2 + 38*A*B*a^3*b^3*c^4*e^6*f^2*z^2 - 36*A*B*a^ \\
& 3*b^5*c^2*e^4*f^4*z^2 + 16*A*B*a^3*b^4*c^3*e^5*f^3*z^2 - 9*A*B*a^4*b^4*c^2* \\
& e^3*f^5*z^2 - 8*A*B*a^2*b^5*c^3*e^6*f^2*z^2 + 2*A*B*a^2*b^6*c^2*e^5*f^3*z^2 \\
& + 920*A*B*a^4*b^3*c^3*d^2*f^6*z^2 - 480*A*B*a^2*b^5*c^3*d^3*f^5*z^2 - 336* \\
& A*B*a^2*b^3*c^5*d^4*f^4*z^2 - 272*A*B*a^3*b^3*c^4*d^3*f^5*z^2 + 240*A*B*a^3
\end{aligned}$$

$$\begin{aligned}
& *b^5*c^2*d^2*f^6*z^2 - 32*A*B*a*c^9*d^6*e*f*z^2 - 792*B^2*a^2*b^3*c^5*d^3*e \\
& ^3*f^2*z^2 + 714*B^2*a^2*b^4*c^4*d^3*e^2*f^3*z^2 - 572*B^2*a^3*b^2*c^5*d^3* \\
& e^2*f^3*z^2 - 475*B^2*a^2*b^2*c^6*d^4*e^2*f^2*z^2 + 265*B^2*a^4*b^2*c^4*d^2 \\
& *e^2*f^4*z^2 + 260*B^2*a^3*b^3*c^4*d^2*e^3*f^3*z^2 - 212*B^2*a^3*b^4*c^3*d^ \\
& 2*e^2*f^4*z^2 + 180*B^2*a^3*b^2*c^5*d^2*e^4*f^2*z^2 - 158*B^2*a^2*b^4*c^4*d \\
& ^2*e^4*f^2*z^2 + 47*B^2*a^2*b^6*c^2*d^2*e^2*f^4*z^2 + 16*B^2*a^2*b^5*c^3*d^ \\
& 2*e^3*f^3*z^2 + 2752*A^2*a^3*b^2*c^5*d^2*e^2*f^4*z^2 - 2148*A^2*a^2*b^4*c^4 \\
& *d^2*e^2*f^4*z^2 + 2064*A^2*a^2*b^3*c^5*d^2*e^3*f^3*z^2 - 424*A^2*a^2*b^2*c \\
& ^6*d^3*e^2*f^3*z^2 - 198*A^2*a^2*b^2*c^6*d^2*e^4*f^2*z^2 - 272*B^2*a^6*b*c^ \\
& 3*d*e*f^6*z^2 - 24*B^2*a^4*b^5*c*d*e*f^6*z^2 + 1808*A^2*a^5*b*c^4*d*e*f^6*z \\
& ^2 - 244*A^2*a*b*c^8*d^4*e^3*f*z^2 + 208*A^2*a*b*c^8*d^5*e*f^2*z^2 + 134*A^ \\
& 2*a^2*b^7*c*d*e*f^6*z^2 - 76*A^2*a*b^4*c^5*d*e^6*f*z^2 + 4*A^2*a*b^8*c*d*e^ \\
& 2*f^5*z^2 + 148*A*B*b^4*c^6*d^5*e*f^2*z^2 + 65*A*B*b^6*c^4*d^4*e*f^3*z^2 + \\
& 46*A*B*b^8*c^2*d^3*e*f^4*z^2 - 38*A*B*b^3*c^7*d^5*e^2*f*z^2 + 34*A*B*b^9*c \\
& d^2*e^2*f^4*z^2 - 29*A*B*b^4*c^6*d^4*e^3*f*z^2 + 20*A*B*b^5*c^5*d^3*e^4*f*z \\
& ^2 + 12*A*B*b^8*c^2*d*e^5*f^2*z^2 - 7*A*B*b^6*c^4*d^2*e^5*f*z^2 - 2880*A*B* \\
& a^4*c^6*d^3*e*f^4*z^2 + 2784*A*B*a^5*c^5*d^2*e*f^5*z^2 - 1112*A*B*a^5*c^5*d \\
& *e^3*f^4*z^2 + 896*A*B*a^3*c^7*d^4*e*f^3*z^2 + 848*A*B*a^3*c^7*d^2*e^5*f*z^ \\
& 2 - 560*A*B*a^4*c^6*d*e^5*f^2*z^2 + 96*A*B*a^2*c^8*d^5*e*f^2*z^2 - 88*A*B*a \\
& ^2*c^8*d^4*e^3*f*z^2 - 100*A*B*a^6*b*c^3*e^2*f^6*z^2 - 76*A*B*a^5*b*c^4*e^4 \\
& *f^4*z^2 + 48*A*B*a^6*b^2*c^2*e*f^7*z^2 - 42*A*B*a^3*b^2*c^5*e^7*f*z^2 + 36 \\
& *A*B*a^4*b*c^5*e^6*f^2*z^2 - 24*A*B*a^4*b^5*c*e^2*f^6*z^2 + 10*A*B*a^3*b^6* \\
& c*e^3*f^5*z^2 + 7*A*B*a^2*b^4*c^4*e^7*f*z^2 + 2*A*B*a^2*b^7*c*e^4*f^4*z^2 - \\
& 2496*A*B*a^5*b*c^4*d^2*f^6*z^2 + 1872*A*B*a^4*b*c^5*d^3*f^5*z^2 - 744*A*B* \\
& a^5*b^3*c^2*d*f^7*z^2 - 720*A*B*a^2*b*c^7*d^5*f^3*z^2 + 504*A*B*a*b^3*c^6*d \\
& ^5*f^3*z^2 + 256*A*B*a^3*b*c^6*d^4*f^4*z^2 + 168*A*B*a*b^7*c^2*d^3*f^5*z^2 \\
& - 144*A*B*a^2*b^7*c*d^2*f^6*z^2 + 144*A*B*a*b^5*c^4*d^4*f^4*z^2 + 66*A*B*a^ \\
& 2*b^2*c^6*d*e^7*z^2 - 36*A*B*a*b^2*c^7*d^3*e^5*z^2 + 20*A*B*a*b^3*c^6*d^2*e \\
& ^6*z^2 + 12*A*B*a^2*b*c^7*d^2*e^6*z^2 + 1208*B^2*a^3*b*c^6*d^3*e^3*f^2*z^2 \\
& - 848*B^2*a^3*b^3*c^4*d^3*e*f^4*z^2 + 672*B^2*a^2*b^3*c^5*d^4*e*f^3*z^2 - 6 \\
& 32*B^2*a^4*b*c^5*d^2*e^3*f^3*z^2 + 432*B^2*a^4*b^3*c^3*d^2*e*f^5*z^2 + 276* \\
& B^2*a^2*b^2*c^6*d^3*e^4*f*z^2 - 196*B^2*a*b^6*c^3*d^3*e^2*f^3*z^2 - 168*B^2 \\
& *a^2*b^5*c^3*d^3*e*f^4*z^2 + 154*B^2*a^2*b^3*c^5*d^2*e^5*f*z^2 + 148*B^2*a* \\
& b^5*c^4*d^3*e^3*f^2*z^2 + 96*B^2*a*b^4*c^5*d^4*e^2*f^2*z^2 - 72*B^2*a^3*b^5 \\
& *c^2*d^2*e*f^5*z^2 + 70*B^2*a^5*b^2*c^3*d*e^2*f^5*z^2 - 60*B^2*a^4*b^3*c^3* \\
& d*e^3*f^4*z^2 + 52*B^2*a*b^6*c^3*d^2*e^4*f^2*z^2 + 36*B^2*a^4*b^2*c^4*d*e^4 \\
& *f^3*z^2 - 32*B^2*a*b^7*c^2*d^2*e^3*f^3*z^2 + 24*B^2*a^3*b^5*c^2*d*e^3*f^4* \\
& z^2 + 15*B^2*a^4*b^4*c^2*d*e^2*f^5*z^2 - 8*B^2*a^3*b^4*c^3*d*e^4*f^3*z^2 + \\
& 8*B^2*a^2*b^5*c^3*d*e^5*f^2*z^2 - 2*B^2*a^3*b^3*c^4*d*e^5*f^2*z^2 - 2*B^2*a \\
& ^2*b^6*c^2*d*e^4*f^3*z^2 - 3176*A^2*a^3*b*c^6*d^2*e^3*f^3*z^2 - 2252*A^2*a^ \\
& 4*b^2*c^4*d*e^2*f^5*z^2 + 1952*A^2*a^3*b^4*c^3*d*e^2*f^5*z^2 - 1496*A^2*a^3 \\
& *b^3*c^4*d*e^3*f^4*z^2 + 1378*A^2*a^2*b^4*c^4*d*e^4*f^3*z^2 + 1184*A^2*a^3* \\
& b^3*c^4*d^2*e*f^5*z^2 - 1166*A^2*a^2*b^3*c^5*d*e^5*f^2*z^2 - 1164*A^2*a^3*b \\
& ^2*c^5*d*e^4*f^3*z^2 - 1152*A^2*a^2*b^3*c^5*d^3*e*f^4*z^2 + 578*A^2*a*b^6*c \\
& ^3*d^2*e^2*f^4*z^2 - 548*A^2*a*b^5*c^4*d^2*e^3*f^3*z^2 + 440*A^2*a*b^2*c^7*
\end{aligned}$$

$$\begin{aligned}
& d^4 e^2 f^2 z^2 - 412 A^2 a^2 b^6 c^2 d e^2 f^5 z^2 - 360 A^2 a b^3 c^6 d^3 e^3 f^2 z^2 + 312 A^2 a b^4 c^5 d^3 e^2 f^3 z^2 + 248 A^2 a^2 b c^7 d^3 e^3 f^2 z^2 - 224 A^2 a^2 b^5 c^3 d e^3 f^4 z^2 + 216 A^2 a^2 b^5 c^3 d^2 e f^5 z^2 + 52 A^2 a b^4 c^5 d^2 e^4 f^2 z^2 - 16 B^2 b^3 c^7 d^6 e f z^2 - 14 B^2 b^9 c d^3 e f^4 z^2 + 32 B^2 a^4 c^6 d e^6 f z^2 - 20 A^2 b^9 c d e^3 f^4 z^2 + 18 A^2 b^9 c d^2 e f^5 z^2 + 8 A^2 b^6 c^4 d e^6 f z^2 - 360 A^2 a^3 c^7 d e^6 f z^2 + 136 A^2 a c^9 d^5 e^2 f z^2 + 2 B^2 a^3 b^7 d e f^6 z^2 + 2 B^2 a b^9 d^2 e f^5 z^2 + 12 B^2 a^4 b c^5 e^7 f z^2 - 204 A^2 a^3 b c^6 e^7 f z^2 - 128 A^2 a^6 b c^3 e f^7 z^2 - 48 A^2 a b^5 c^4 e^7 f z^2 - 36 B^2 a^5 b^4 c d f^7 z^2 - 24 A^2 a^4 b^5 c e f^7 z^2 - 16 B^2 a b^8 c d^3 f^5 z^2 - 164 A^2 a^3 b^6 c d f^7 z^2 - 16 A^2 a b^8 c d^2 f^6 z^2 + 4 B^2 a^3 b c^6 d e^7 z^2 - 4 B^2 a b c^8 d^5 e^3 z^2 + 48 A^2 a b c^8 d^3 e^5 z^2 + 36 A^2 a^2 b c^7 d e^7 z^2 - 6 A^2 a b^3 c^6 d e^7 z^2 + 136 A B a^6 c^4 e^3 f^5 z^2 - 96 A B b^5 c^5 d^5 f^3 z^2 + 80 A B a^5 c^5 e^5 f^3 z^2 - 72 A B b^3 c^7 d^6 f^2 z^2 - 24 A B b^7 c^3 d^4 f^4 z^2 + 14 A B b^3 c^7 d^4 e^4 z^2 - 14 A B b^2 c^8 d^5 e^3 z^2 - 2 A B b^5 c^5 d^2 e^6 z^2 - 2 A B b^4 c^6 d^3 e^5 z^2 + 2 A B a^3 b^7 e^2 f^6 z^2 - A B a^2 b^8 e^3 f^5 z^2 + 16 A B a^2 c^8 d^3 e^5 z^2 - 2 A B a^2 b^3 c^5 e^8 z^2 + 22 B^2 b^8 c^2 d^3 e^2 f^3 z^2 - 12 B^2 b^7 c^3 d^3 e^3 f^2 z^2 + 12 B^2 b^6 c^4 d^4 e^2 f^2 z^2 - 6 B^2 b^8 c^2 d^2 e^4 f^2 z^2 - 864 B^2 a^4 c^6 d^3 e^2 f^3 z^2 + 496 B^2 a^3 c^7 d^4 e^2 f^2 z^2 + 224 B^2 a^5 c^5 d^2 e^2 f^4 z^2 + 136 B^2 a^4 c^6 d^2 e^4 f^2 z^2 - 53 A^2 b^8 c^2 d^2 e^2 f^4 z^2 + 52 A^2 b^7 c^3 d^2 e^3 f^3 z^2 + 52 A^2 b^5 c^5 d^3 e^3 f^2 z^2 - 36 A^2 b^6 c^4 d^3 e^2 f^3 z^2 - 12 A^2 b^4 c^6 d^4 e^2 f^2 z^2 - 9 A^2 b^6 c^4 d^2 e^4 f^2 z^2 + 836 A^2 a^4 c^6 d^2 e^2 f^4 z^2 - 668 A^2 a^2 c^8 d^4 e^2 f^2 z^2 + 656 A^2 a^3 c^7 d^2 e^4 f^2 z^2 + 368 A^2 a^3 c^7 d^3 e^2 f^3 z^2 - 45 B^2 a^6 b^2 c^2 e^2 f^6 z^2 - 18 B^2 a^5 b^2 c^3 e^4 f^4 z^2 - 9 B^2 a^4 b^2 c^4 e^6 f^2 z^2 - 6 B^2 a^5 b^3 c^2 e^3 f^5 z^2 + 3 B^2 a^4 b^4 c^2 e^4 f^4 z^2 - 2 B^2 a^4 b^3 c^3 e^5 f^3 z^2 - 580 B^2 a^4 b^2 c^4 d^3 f^5 z^2 + 536 B^2 a^3 b^4 c^3 d^3 f^5 z^2 + 471 A^2 a^4 b^2 c^4 e^4 f^4 z^2 - 436 A^2 a^3 b^4 c^3 e^4 f^4 z^2 - 348 B^2 a^4 b^4 c^2 d^2 f^6 z^2 + 316 B^2 a^2 b^2 c^6 d^5 f^3 z^2 + 310 A^2 a^3 b^3 c^4 e^5 f^3 z^2 + 232 A^2 a^5 b^2 c^3 e^2 f^6 z^2 - 229 A^2 a^2 b^4 c^4 e^6 f^2 z^2 - 216 A^2 a^4 b^4 c^2 e^2 f^6 z^2 + 204 A^2 a^4 b^3 c^3 e^3 f^5 z^2 + 200 B^2 a^5 b^2 c^3 d^2 f^6 z^2 + 150 A^2 a^3 b^2 c^5 e^6 f^2 z^2 - 120 B^2 a^2 b^4 c^4 d^4 f^4 z^2 + 91 A^2 a^2 b^6 c^2 e^4 f^4 z^2 + 72 A^2 a^3 b^5 c^2 e^3 f^5 z^2 - 66 B^2 a^2 b^6 c^2 d^3 f^5 z^2 + 44 A^2 a^2 b^5 c^3 e^5 f^3 z^2 - 16 B^2 a^3 b^2 c^5 d^4 f^4 z^2 + 1952 A^2 a^4 b^2 c^4 d^2 f^6 z^2 - 1792 A^2 a^3 b^2 c^5 d^3 f^5 z^2 - 1272 A^2 a^3 b^4 c^3 d^2 f^6 z^2 + 976 A^2 a^2 b^2 c^6 d^4 f^4 z^2 + 960 A^2 a^2 b^4 c^4 d^3 f^5 z^2 + 282 A^2 a^2 b^6 c^2 d^2 f^6 z^2 - 45 B^2 a^2 b^2 c^6 d^2 e^6 z^2 - 48 A^2 b c^9 d^6 e f z^2 - 14 A^2 a b^9 d e f^6 z^2 - 7 A B b^10 d^2 e f^5 z^2 + 2 A B b^10 d e^3 f^4 z^2 - 64 A B a^7 c^3 e f^7 z^2 - 16 A B b^9 c d^3 f^5 z^2 + 8 A B a^4 c^6 e^7 f z^2 + 4 A B b c^9 d^6 e^2 z^2 + 2 A B b^6 c^4 d e^7 z^2 - 120 A B a^3 c^7 d e^7 z^2 - 16 A B a^3 b^7 d f^7 z^2 + 16 A B a b^9 d^2 f^6 z^2 + 8 A B a c^9 d^5 e^3 z^2 + 12 A B a^3 b c^6
\end{aligned}$$

$$\begin{aligned}
& e^8 z^2 - 48 B^2 b^5 c^5 d^5 e^2 f^2 z^2 + 15 B^2 b^4 c^6 d^5 e^2 f z^2 - 14 B^2 b^7 c^3 d^4 e^2 f^3 z^2 + 4 B^2 b^9 c^2 d^2 e^3 f^3 z^2 + 4 B^2 b^7 c^3 d^2 e^5 f z^2 + 4 B^2 b^5 c^5 d^4 e^3 f z^2 - B^2 b^6 c^4 d^3 e^4 f z^2 - 336 B^2 a^3 c^7 d^3 e^4 f z^2 + 112 B^2 a^5 c^5 d e^4 f^3 z^2 - 112 A^2 b^3 c^7 d^5 e^2 f z^2 + 80 B^2 a^6 c^4 d e^2 f^5 z^2 - 48 A^2 b^5 c^5 d^4 e^2 f^3 z^2 + 36 A^2 b^8 c^2 d e^4 f^3 z^2 + 36 A^2 b^3 c^7 d^4 e^3 f z^2 - 28 A^2 b^7 c^3 d e^5 f^2 z^2 + 20 A^2 b^2 c^8 d^5 e^2 f z^2 + 16 B^2 a^2 c^8 d^5 e^2 f z^2 - 14 A^2 b^7 c^3 d^3 e^4 f z^2 - 14 A^2 b^4 c^6 d^3 e^4 f z^2 - 10 A^2 b^5 c^5 d^2 e^5 f z^2 - 1008 A^2 a^4 c^6 d e^4 f^3 z^2 - 760 A^2 a^5 c^5 d e^2 f^5 z^2 + 272 A^2 a^2 c^8 d^3 e^4 f z^2 + 48 B^2 a^5 b^2 c^4 e^5 f^3 z^2 + 36 B^2 a^6 b^2 c^3 e^3 f^5 z^2 + 12 B^2 a^5 b^4 c^2 e^2 f^6 z^2 - 624 A^2 a^4 b^2 c^5 e^5 f^3 z^2 - 548 A^2 a^5 b^2 c^4 e^3 f^5 z^2 + 182 A^2 a^2 b^3 c^5 e^7 f z^2 - 180 B^2 a^2 b^4 c^5 d^5 e^2 f^3 z^2 + 132 B^2 a^6 b^2 c^2 d e^4 f^7 z^2 + 108 B^2 a^3 b^6 c^2 d^2 e^2 f^6 z^2 + 96 A^2 a^5 b^3 c^2 e^2 f^7 z^2 + 68 A^2 a^2 b^6 c^3 e^6 f^2 z^2 + 58 A^2 a^3 b^6 c^2 e^2 f^6 z^2 - 56 B^2 a^2 b^2 c^7 d^6 e^2 f^2 z^2 - 38 A^2 a^2 b^7 c^2 e^3 f^5 z^2 - 36 A^2 a^2 b^7 c^2 e^5 f^3 z^2 + 20 B^2 a^2 b^6 c^3 d^4 e^2 f^4 z^2 - 736 A^2 a^5 b^2 c^3 d e^4 f^7 z^2 + 624 A^2 a^4 b^4 c^2 d e^4 f^7 z^2 - 416 A^2 a^2 b^2 c^7 d^5 e^2 f^3 z^2 - 276 A^2 a^2 b^4 c^5 d^4 e^2 f^4 z^2 - 196 A^2 a^2 b^6 c^3 d^3 e^2 f^5 z^2 + 8 B^2 a^2 b^4 c^5 d^2 e^6 z^2 + 6 B^2 a^2 b^2 c^7 d^4 e^4 z^2 + 2 B^2 a^2 b^3 c^5 d e^7 z^2 + 2 B^2 a^2 b^3 c^6 d^3 e^5 z^2 - 18 A^2 a^2 b^2 c^7 d^2 e^6 z^2 - 16 A^2 b^2 c^9 d^7 e^2 f z^2 - B^2 b^10 d^2 e^2 f^4 z^2 + 48 B^2 a^7 c^3 e^2 f^6 z^2 - 36 B^2 a^6 c^4 e^4 f^4 z^2 + 31 B^2 b^6 c^4 d^5 e^2 f^3 z^2 - 24 B^2 a^5 c^5 e^6 f^2 z^2 + 20 B^2 b^4 c^6 d^6 e^2 f^2 z^2 - 6 A^2 b^8 c^2 e^6 f^2 z^2 + 2 B^2 b^8 c^2 d^4 e^4 z^2 - 768 B^2 a^5 c^5 d^3 e^4 f^5 z^2 + 512 B^2 a^6 c^4 d^2 e^2 f^6 z^2 + 512 B^2 a^4 c^6 d^4 e^2 f^4 z^2 + 232 A^2 a^5 c^5 e^4 f^4 z^2 + 188 A^2 a^4 c^6 e^6 f^2 z^2 - 128 B^2 a^3 c^7 d^5 e^2 f^3 z^2 + 92 A^2 a^6 c^4 e^2 f^6 z^2 + 80 A^2 b^4 c^6 d^5 e^2 f^3 z^2 + 64 A^2 b^2 c^8 d^6 e^2 f^2 z^2 + 31 A^2 b^6 c^4 d^4 e^2 f^4 z^2 + 14 A^2 b^8 c^2 d^3 e^2 f^5 z^2 - 5 B^2 b^4 c^6 d^4 e^4 z^2 + 4 B^2 b^3 c^7 d^5 e^3 z^2 + 2 B^2 b^5 c^5 d^3 e^5 z^2 - B^2 b^6 c^4 d^2 e^6 z^2 - B^2 b^2 c^8 d^6 e^2 z^2 - B^2 a^4 b^6 e^2 f^6 z^2 - 1152 A^2 a^3 c^7 d^4 e^2 f^4 z^2 + 1008 A^2 a^4 c^6 d^3 e^2 f^5 z^2 + 624 A^2 a^2 c^8 d^5 e^2 f^3 z^2 - 288 A^2 a^5 c^5 d^2 e^2 f^6 z^2 + 56 B^2 a^3 c^7 d^2 e^6 z^2 - 10 B^2 a^2 b^8 d^2 e^2 f^6 z^2 - 9 A^2 b^2 c^8 d^4 e^4 z^2 - 5 A^2 a^2 b^8 e^2 f^6 z^2 - 4 B^2 a^2 c^8 d^4 e^4 z^2 + 3 A^2 b^4 c^6 d^2 e^6 z^2 - 2 A^2 b^3 c^7 d^3 e^5 z^2 - 36 A^2 a^2 c^8 d^2 e^6 z^2 - 48 A^2 a^6 b^2 c^2 e^2 f^8 z^2 - 45 A^2 a^2 b^2 c^6 e^8 z^2 + 4 A^2 b^10 d e^2 f^5 z^2 + 4 B^2 b^2 c^8 d^7 e^2 f z^2 + 4 A^2 b^9 c^2 e^5 f^3 z^2 + 4 A^2 b^7 c^3 e^7 f z^2 - 128 B^2 a^7 c^3 d e^4 f^7 z^2 - 160 A^2 a^2 c^9 d^6 e^2 f^2 z^2 - 112 A^2 a^6 c^4 d e^4 f^7 z^2 + 12 A^2 b^2 c^9 d^5 e^3 z^2 + 4 A^2 a^2 b^9 e^3 f^5 z^2 + 3 B^2 a^4 b^6 d e^4 f^7 z^2 + 2 A^2 a^3 b^7 e^2 f^7 z^2 - 24 A^2 a^2 c^9 d^4 e^4 z^2 + 14 A^2 a^2 b^8 d e^4 f^7 z^2 + 12 A^2 a^5 b^4 c^2 e^8 z^2 + 12 A^2 a^2 b^4 c^5 e^8 z^2 + A^2 b^4 c^6 e^2 f^7 z^2 + B^2 a^2 b^8 d e^2 f^5 z^2 + 16 A^2 c^10 d^7 e^2 f z^2 + 3 B^2 b^10 d^3 e^2 f^5 z^2 - A^2 b^10 e^4 f^4 z^2 - 4 A^2 c^10 d^6 e^2 z^2 - A^2 b^10 d^2 e^2 f^6 z^2 + 64 A^2 a^7 c^3 e^8 z^2 - 4 B^2 a^4 c^6 e^8 z^2 - A^2 b^6 c^4 e^8 z^2 + 48 A^2 a^3 c^7 e^8 z^2 - A^2 a^4 b^6 e^8 z^2 +
\end{aligned}$$

$$\begin{aligned}
& 720A^2B^2a^2b^2c^5d^2e^2f^3z - 600A^2B^2a^2b^2c^4d^2e^2f^4z + 57 \\
& 6A^2B^2a^2b^2c^4d^2e^2f^4z + 348A^2B^2a^2b^2c^5d^2e^3f^2z - 336A \\
& B^2a^2b^2c^5d^2e^2f^3z - 260A^2B^2a^2b^3c^4d^2e^2f^3z - 240A^2B^ \\
& 2a^2b^2c^4d^2e^3f^3z + 196A^2B^2a^2b^3c^3d^2e^2f^4z + 172A^2B^2a \\
& b^2c^6d^2e^5f^2z + 20A^2B^2a^2b^6c^2d^2e^2f^5z - 912A^2B^2a^2b^2c^5d^2e^2f \\
& ^4z - 644A^2B^2a^2b^2c^6d^2e^3f^2z - 432A^2B^2a^2b^2c^5d^3e^2f^3z + \\
& 372A^2B^2a^2b^2c^5d^2e^3f^3z - 330A^2B^2a^2b^2c^5d^2e^4f^2z + 312A^2B \\
& ^2a^2b^2c^6d^3e^2f^2z - 208A^2B^2a^3b^2c^3d^2e^2f^5z + 192A^2B^2a^2 \\
& b^3c^3d^2e^2f^5z + 172A^2B^2a^2b^3c^4d^2e^3f^3z + 108A^2B^2a^2b^2c^5d \\
& e^4f^2z + 104A^2B^2a^3b^2c^4d^2e^2f^4z - 80A^2B^2a^2b^3c^4d^2e^2f^4 \\
& z + 68A^2B^2a^2b^4c^3d^2e^2f^4z - 60A^2B^2a^2b^5c^2d^2e^2f^4z + 58A \\
& B^2a^2b^3c^4d^2e^4f^2z - 36A^2B^2a^2b^4c^3d^2e^2f^4z - 24A^2B^2a^2 \\
& b^4c^2d^2e^2f^5z + 24A^2B^2a^2b^4c^3d^2e^3f^3z + 592A^2B^2a^2b^2c^6d^3 \\
& e^2f^3z + 240A^2B^2a^3b^2c^4d^2e^2f^5z - 132A^2B^2a^2b^2c^6d^2e^4f^2z - 6 \\
& 0A^2B^2a^2b^2c^5d^2e^5f^2z - 48A^2B^2a^2b^5c^2d^2e^2f^5z + 20B^3a^2b^2c^6 \\
& d^3e^3f^2z + 16B^3a^4b^2c^3d^2e^2f^5z - 16B^3a^2b^2c^6d^4e^2f^2z + 12 \\
& B^3a^2b^2c^5d^2e^5f^2z + 320A^3a^2b^2c^6d^2e^4f^2z + 40A^3a^2b^4c^3d \\
& e^2f^5z - 48A^2B^2b^2c^7d^4e^2f^2z - 44A^2B^2b^3c^5d^2e^5f^2z - 20A^2B \\
& ^2b^2c^7d^4e^2f^2z + 14A^2B^2b^4c^4d^2e^5f^2z + 12A^2B^2b^2c^7d^3e^3 \\
& f^2z + 4A^2B^2b^7c^2d^2e^2f^4z + 160A^2B^2a^4c^4d^2e^2f^5z + 152A^2B^2a \\
& c^7d^2e^4f^2z - 40A^2B^2a^2c^7d^3e^3f^2z + 32A^2B^2a^2c^7d^4e^2f^2z \\
& - 16A^2B^2a^2c^6d^2e^5f^2z + 128A^2B^2a^4b^2c^3e^2f^6z + 42A^2B^2a^2b^2 \\
& c^5e^6f^2z + 24A^2B^2a^2b^5c^2e^2f^6z - 12A^2B^2a^3b^4c^2e^2f^6z - 12 \\
& A^2B^2a^2b^2c^5e^6f^2z - 10A^2B^2a^2b^6c^2e^2f^5z - 160A^2B^2a^2b^2c^6d \\
& ^4f^3z + 112A^2B^2a^4b^2c^3d^2f^6z - 24A^2B^2a^2b^5c^2d^2f^6z - 84B^ \\
& 3a^2b^2c^5d^3e^2f^2z - 80B^3a^2b^3c^3d^2e^2f^4z - 60B^3a^2b^2c^ \\
& ^5d^2e^3f^2z - 20B^3a^3b^2c^3d^2e^2f^4z - 20B^3a^2b^3c^4d^2e^ \\
& 3f^2z - 9B^3a^2b^2c^4d^2e^4f^2z - 8B^3a^2b^4c^3d^2e^2f^3z + 6 \\
& B^3a^2b^4c^2d^2e^2f^4z - 4B^3a^2b^3c^3d^2e^3f^3z - 216A^2B^2b^ \\
& 4c^4d^2e^2f^3z + 196A^2B^2b^3c^5d^2e^3f^2z - 108A^2B^2b^3c^5d \\
& ^3e^2f^2z - 94A^2B^2b^4c^4d^2e^3f^2z + 88A^2B^2b^2c^6d^3e^2f^ \\
& 2z + 80A^2B^2b^5c^3d^2e^2f^3z + 360A^2B^2a^2c^6d^2e^2f^3z + 8 \\
& A^2B^2a^2c^6d^2e^3f^2z + 153A^2B^2a^2b^2c^4e^4f^3z - 144A^2B^2a \\
& ^2b^3c^3e^3f^4z + 80A^2B^2a^3b^2c^3e^2f^5z + 36A^2B^2a^3b^2c^ \\
& 3e^3f^4z + 12A^2B^2a^2b^4c^2e^2f^5z + 12A^2B^2a^3b^3c^2e^2f^5 \\
& z + 9A^2B^2a^2b^2c^4e^5f^2z - 6A^2B^2a^2b^4c^2e^3f^4z + 4A^2B^ \\
& 2a^2b^3c^3e^4f^3z + 480A^2B^2a^2b^2c^4d^2f^5z - 176A^2B^2a^2b \\
& ^3c^3d^2f^5z - 10A^2B^2a^2b^6c^2d^2f^6z + 16A^2B^2a^2b^2c^6d^2e^6z + 80 \\
& B^3a^2b^3c^4d^3e^2f^3z - 48B^3a^3b^2c^4d^2e^2f^4z + 48B^3a^2b^2c^ \\
& 5d^3e^2f^3z + 44B^3a^3b^2c^4d^2e^3f^3z + 24B^3a^2b^5c^2d^2e^2f^4z \\
& + 18B^3a^2b^2c^5d^2e^4f^2z + 696A^3a^2b^2c^5d^2e^2f^4z - 504A^3a \\
& b^2c^6d^2e^2f^3z - 192A^3a^2b^2c^5d^2e^3f^3z - 144A^3a^2b^2c^4 \\
& d^2e^2f^5z + 96A^3a^2b^2c^5d^2e^2f^4z - 72A^3a^2b^3c^4d^2e^2f^4z - 2 \\
& 08A^2B^2b^3c^5d^3e^2f^3z + 152A^2B^2b^4c^4d^3e^2f^3z + 80A^2B^2b^5 \\
& c^3d^2e^2f^4z + 75A^2B^2b^4c^4d^2e^4f^2z - 59A^2B^2b^2c^6d^2e^4z
\end{aligned}$$

$$\begin{aligned}
& f*z - 52*A^2*B*b^5*c^3*d*e^3*f^3*z + 42*A*B^2*b^3*c^5*d^2*e^4*f*f*z - 21*A*B^2*b^6*c^2*d^2*e*f^4*z - 16*A*B^2*b^5*c^3*d*e^4*f^2*z + 16*A*B^2*b^2*c^6*d^4 \\
& *e*f^2*z + 16*A*B^2*b^2*c^6*d^3*e^3*f*f*z + 11*A^2*B*b^6*c^2*d*e^2*f^4*z + 4* \\
& A*B^2*b^6*c^2*d*e^3*f^3*z - 256*A^2*B*a*c^7*d^3*e^2*f^2*z - 96*A*B^2*a^3*c^5 \\
& *d^2*e*f^4*z - 36*A^2*B*a^2*c^6*d*e^4*f^2*z - 32*A^2*B*a^3*c^5*d*e^2*f^4*z \\
& - 32*A*B^2*a^2*c^6*d^3*e*f^3*z + 8*A*B^2*a^3*c^5*d*e^3*f^3*z - 96*A^2*B*a^3 \\
& *b^3*c^2*e*f^6*z + 68*A^2*B*a^3*b*c^4*e^3*f^4*z - 60*A*B^2*a^4*b*c^3*e^2*f^5 \\
& *z - 60*A*B^2*a^3*b*c^4*e^4*f^3*z + 48*A*B^2*a^4*b^2*c^2*e*f^6*z - 38*A^2 \\
& *B*a*b^3*c^4*e^5*f^2*z - 36*A^2*B*a^2*b*c^5*e^5*f^2*z + 36*A^2*B*a*b^5*c^2* \\
& e^3*f^4*z - 16*A^2*B*a*b^4*c^3*e^4*f^3*z + 384*A*B^2*a^2*b*c^5*d^3*f^4*z - \\
& 352*A*B^2*a^3*b*c^4*d^2*f^5*z - 288*A^2*B*a*b^2*c^5*d^3*f^4*z - 160*A^2*B*a \\
& ^3*b^2*c^3*d*f^6*z - 148*A^2*B*a*b^4*c^3*d^2*f^5*z + 112*A*B^2*a*b^3*c^4*d^3 \\
& *f^4*z + 72*A^2*B*a^2*b^4*c^2*d*f^6*z + 72*A*B^2*a*b^5*c^2*d^2*f^5*z + 48* \\
& A*B^2*a^3*b^3*c^2*d*f^6*z + 102*B^3*a^2*b^2*c^4*d^2*e^2*f^3*z - 32*B^3*b^5* \\
& c^3*d^3*e*f^3*z - 8*B^3*b^3*c^5*d^3*e^3*f*f*z - 7*B^3*b^4*c^4*d^2*e^4*f*f*z + 5 \\
& *B^3*b^2*c^6*d^4*e^2*f*f*z + 80*A^3*b^2*c^6*d^3*e*f^3*z - 74*A^3*b^3*c^5*d*e^4 \\
& *f^2*z - 64*A^3*b^4*c^4*d^2*e*f^4*z + 60*A^3*b^4*c^4*d*e^3*f^3*z - 48*B^3*a \\
& ^4*c^4*d*e^2*f^4*z - 24*B^3*a^3*c^5*d*e^4*f^2*z + 20*B^3*a^2*c^6*d^2*e^4*f \\
& *z - 16*A^3*b^5*c^3*d*e^2*f^4*z + 8*A^3*b*c^7*d^3*e^2*f^2*z + 480*A^3*a^2*c^6 \\
& *d^2*e*f^4*z - 392*A^3*a^2*c^6*d*e^3*f^3*z + 280*A^3*a*c^7*d^2*e^3*f^2*z - \\
& 4*B^3*a^4*b*c^3*e^3*f^4*z - 200*A^3*a^3*b*c^4*e^2*f^5*z - 144*A^3*a^2*b*c^5 \\
& *e^4*f^3*z + 48*B^3*a*b^2*c^5*d^4*f^3*z + 42*A^3*a*b^2*c^5*e^5*f^2*z - 36 \\
& *B^3*a^4*b^2*c^2*d*f^6*z - 32*A^3*a^3*b^2*c^3*e*f^6*z - 24*A^3*a^2*b^4*c^2* \\
& e*f^6*z - 24*A^3*a*b^5*c^2*e^2*f^5*z + 10*A^3*a*b^3*c^4*e^4*f^3*z - 4*B^3*a \\
& *b^4*c^3*d^3*f^4*z - 4*A^3*a*b^4*c^3*e^3*f^4*z - 480*A^3*a^2*b*c^5*d^2*f^5* \\
& z - 160*A^3*a^2*b^3*c^3*d*f^6*z + 128*A^3*a*b^3*c^4*d^2*f^5*z + 8*A^2*B*b^5 \\
& *c^3*e^5*f^2*z - 2*A^2*B*b^6*c^2*e^4*f^3*z + 112*A^2*B*b^4*c^4*d^3*f^4*z - \\
& 92*A^2*B*a^4*c^4*e^2*f^5*z - 64*A^2*B*a^3*c^5*e^4*f^3*z - 64*A*B^2*b^5*c^3* \\
& d^3*f^4*z + 24*A*B^2*a^4*c^4*e^3*f^4*z + 24*A*B^2*a^3*c^5*e^5*f^2*z + 16*A^2 \\
& *B*b^2*c^6*d^4*f^3*z + 16*A*B^2*b^3*c^5*d^4*f^3*z - A^2*B*b^6*c^2*d^2*f^5* \\
& z + 448*A^2*B*a^3*c^5*d^2*f^5*z - 352*A^2*B*a^2*c^6*d^3*f^4*z - 5*A*B^2*b^2 \\
& *c^6*d^2*e^5*z - 48*A^2*B*a^4*b^2*c^2*f^7*z - 2*B^3*b^7*c*d^2*e*f^4*z + 34* \\
& A^3*b^2*c^6*d*e^5*f*f*z + 16*A^3*b*c^7*d^2*e^4*f*f*z + 2*A^3*b^6*c^2*d*e*f^5*z \\
& - 416*A^3*a^3*c^5*d*e*f^5*z - 224*A^3*a*c^7*d^3*e*f^3*z + 12*B^3*a^3*b^4*c* \\
& d*f^6*z - 10*B^3*a*b^6*c*d^2*f^5*z + 416*A^3*a^3*b*c^4*d*f^6*z + 224*A^3*a* \\
& b*c^6*d^3*f^4*z + 24*A^3*a*b^5*c^2*d*f^6*z - 4*B^3*a*b*c^6*d^2*e^5*z + 20*A \\
& ^2*B*c^8*d^4*e^2*f*f*z - 7*A^2*B*b^4*c^4*e^6*f*f*z - 2*A^2*B*b^7*c*e^3*f^4*z - \\
& 64*A*B^2*a^5*c^3*e*f^6*z + 16*A*B^2*b*c^7*d^5*f^2*z - 8*A^2*B*a^2*c^6*e^6*f \\
& *z - 2*A*B^2*b^7*c*d^2*f^5*z - 272*A^2*B*a^4*c^4*d*f^6*z + 128*A^2*B*a*c^7* \\
& d^4*f^3*z + 9*A^2*B*b^2*c^6*d*e^6*z - 4*A*B^2*b^3*c^5*d*e^6*z + 4*A*B^2*b*c^7 \\
& *d^3*e^4*z + 8*A*B^2*a*c^7*d^2*e^5*z + 12*A^2*B*a^3*b^4*c*f^7*z + 30*B^3* \\
& b^4*c^4*d^3*e^2*f^2*z + 8*B^3*b^5*c^3*d^2*e^3*f^2*z - 2*B^3*b^6*c^2*d^2*e^2 \\
& *f^3*z + 152*A^3*b^3*c^5*d^2*e^2*f^3*z - 108*A^3*b^2*c^6*d^2*e^3*f^2*z + 48 \\
& *B^3*a^3*c^5*d^2*e^2*f^3*z - 16*B^3*a^2*c^6*d^3*e^2*f^2*z - 3*B^3*a^4*b^2*c^2 \\
& *e^2*f^5*z - 120*B^3*a^2*b^2*c^4*d^3*f^4*z + 112*B^3*a^3*b^2*c^3*d^2*f^5*
\end{aligned}$$

$$\begin{aligned}
& z + 112A^3a^2b^3c^3e^2f^5z + 12A^3a^2b^2c^4e^3f^4z - 120A^3a^2c^7de^5fz - 52A^3a^2b^3c^6e^6fz + 10A^3a^2b^6c^6e^6fz - 2A^2B^2 \\
& *b^8d^6e^5fz - 2A^2B^2a^2b^7e^6fz - 24A^2B^2a^2c^7d^6e^6z + 2A^2B^2a^2b^7d^6fz - 12A^2B^2a^2b^3c^6e^7z - 2A^3b^7c^6d^6fz - 4A^3b^3c^7d \\
& *e^6z + 16B^3a^5c^3e^2f^5z + 11B^3b^6c^2d^3f^4z - 11A^3b^4c^4e^5f^2z - 8B^3b^4c^4d^4f^3z - 4B^3b^2c^6d^5f^2z + 4B^3a^4 \\
& *c^4e^4f^3z + 4A^3b^5c^3e^4f^3z - A^3b^6c^2e^3f^4z + 136A^3a^3c^5e^3f^4z + 68A^3a^2c^6e^5f^2z - 64A^3b^3c^5d^3f^4z + \\
& 2B^3b^3c^5d^2e^5z - B^3b^2c^6d^3e^4z + 96A^3a^3b^3c^2f^7z + AB^2a^2b^6e^6fz + 32A^3c^8d^4e^6fz - 24A^3c^8d^3e^3fz + \\
& 10A^3b^3c^5e^6fz + 2A^3b^7c^6e^2f^5z + 128A^3a^4c^4e^6fz - 32A^3b^3c^7d^4f^3z - 4B^3a^2c^6d^6e^6z - B^3a^2b^6d^6fz - 128 \\
& *A^3a^4b^3c^3f^7z - 24A^3a^2b^5c^6f^7z - 16A^2B^3c^8d^5f^2z - 4A^2B^3c^8d^3e^4z + 64A^2B^3a^5c^3f^7z + 2A^2B^3b^3c^5e^7z + 4A^2 \\
& *B^2a^2c^6e^7z - A^2B^2a^2b^6f^7z + 4A^3c^8d^2e^5z - 3A^3b^2c^6e^7z + A^2B^2b^8d^6fz - A^3b^8e^6fz + 16A^3a^2c^7e^7z + 2A^3 \\
& *a^2b^7f^7z + A^2B^2b^8e^2f^5z + B^3b^8d^2f^5z - 48A^2B^2a^2b^3c^4d^6e^6fz + 28A^2B^3a^2b^2c^3d^6e^6fz - 16A^2B^3a^2b^3c^4d^6e^2f^3 + 16A^3 \\
& *B^2a^2c^5d^6e^6fz + 32A^3B^2a^2b^3c^4d^6fz + 12A^2B^2b^3c^3d^6e^6fz + 5A^2B^3b^2c^4d^2e^6fz + 4A^2B^3b^3c^3d^6e^2f^3 + 24A^2B^2a^2c^5d^6e^ \\
& 2f^3 + 24A^2B^2a^2b^2c^3e^6fz + 12A^2B^2a^2b^3c^4e^3f^3 - 6A^2B^2a^2b^3c^2e^6fz + 4A^2B^3a^2b^2c^3e^2f^4 + 3A^2B^3a^2b^2c^2e^6fz - \\
& 18A^2B^2a^2b^2c^3d^6fz - 4B^4a^2b^2c^3d^6e^6fz + 4B^4a^2b^2c^4d^2e^6fz - 6A^2B^3b^4c^2d^6e^6fz + 4A^3B^2b^2c^4d^6e^6fz - 2A^3B^2b^2c^4d^6 \\
& *e^6fz - 8A^2B^3a^2c^4d^6e^6fz - 8A^2B^3a^2c^5d^2e^6fz + 26A^3B^2a^2b^2c^3e^6fz + 8A^3B^2a^2b^3c^4e^2f^4 + 32A^2B^3a^2b^3c^4d^2f^4 - 28A^2B^3a^2 \\
& *b^3c^3d^6fz + 6A^2B^3a^2b^3c^2d^6fz - 9A^2B^2b^2c^4d^6e^2f^3 - 18A^2B^2a^2b^2c^3e^2f^4 - 4A^3B^2c^6d^2e^6fz - 3A^3B^2b^4c^2e^6fz - \\
& 44A^3B^2a^2c^4e^6fz - 16A^3B^2a^2c^5e^3f^3 - 16A^2B^3a^3c^3e^6fz - 10A^3B^2b^3c^3d^6fz - 4A^3B^2b^3c^5d^2f^4 - 4A^2B^3b^3c^5d^3f^3 - \\
& 28A^3B^2a^2b^3c^3f^6 + 6A^3B^2a^2b^3c^2f^6 - 4A^4b^3c^5d^6e^6fz - 20A^4a^2b^3c^4e^6fz + 3A^2B^2b^4c^2e^2f^4 - 2A^2B^2b^3c^3e^3f^3 + \\
& 12A^2B^2a^2c^4e^2f^4 + 9A^2B^2b^2c^4d^2f^4 - 3A^2B^2a^2b^2c^2f^6 - 2B^4b^3c^3d^2e^6fz + 4B^4a^2c^4d^6e^2f^3 - 10B^4a^2b^2c^3d^2f^4 - 3B^4a^2b^2c^2d^6fz \\
& + 3A^3B^2b^2c^4e^3f^3 - 2A^3B^2b^3c^3e^2f^4 - 10A^2B^3b^3c^3d^2f^4 - 4A^2B^3a^2c^4e^3f^3 + 3A^2B^2b^4c^2d^6fz + 36A^2B^2a^2c^4d^6fz - 24A^2B^2a^2c^5d^2f^4 + \\
& 4A^2B^2c^6d^3f^3 + 16A^2B^2a^3c^3f^6 + 4A^4b^3c^3e^6fz + 16B^4a^3c^3d^6fz + 16A^4a^2c^5e^2f^4 + 8A^4b^2c^4d^6fz - 8A^4a^2b^2c^3f^6 - 24A^4a^2c^5d^6fz \\
& + 3B^4b^4c^2d^2f^4 - 3A^4b^2c^4e^2f^4 + 4A^4c^6d^2f^4 + 36A^4a^2c^4f^6 + B^4b^2c^4d^3f^3, z, k) * \\
& \text{root}(48416a^6b^2c^6d^4e^2f^4z^4 - 41544a^5b^4c^5d^4e^2f^4z^4 - 31872a^7b^2c^5d^3e^2f^5z^4 - 31872a^5b^2c^7d^5e^2f^3z^4 - 2 \\
& 9184a^6b^2c^6d^3e^4f^3z^4 + 28800a^5b^4c^5d^3e^4f^3z^4 + 21510a^4b^6c^4d^4e^2f^4z^4 + 21408a^6b^4c^4d^3e^2f^5z^4 + 21408a
\end{aligned}$$

$$\begin{aligned}
&^4*b^4*c^6*d^5*e^2*f^3*z^4 - 18112*a^7*b^3*c^4*d^2*e^3*f^5*z^4 - 18112*a^4* \\
&b^3*c^7*d^5*e^3*f^2*z^4 - 15600*a^5*b^5*c^4*d^3*e^3*f^4*z^4 - 15600*a^4*b^5 \\
&*c^5*d^4*e^3*f^3*z^4 + 15296*a^6*b^3*c^5*d^3*e^3*f^4*z^4 + 15296*a^5*b^3*c^ \\
&6*d^4*e^3*f^3*z^4 + 14016*a^7*b^2*c^5*d^2*e^4*f^4*z^4 + 14016*a^5*b^2*c^7*d \\
&^4*e^4*f^2*z^4 - 13920*a^4*b^6*c^4*d^3*e^4*f^3*z^4 - 11648*a^6*b^3*c^5*d^2* \\
&e^5*f^3*z^4 - 11648*a^5*b^3*c^6*d^3*e^5*f^2*z^4 + 10432*a^6*b^2*c^6*d^2*e^6 \\
&*f^2*z^4 + 9008*a^6*b^5*c^3*d^2*e^3*f^5*z^4 + 9008*a^3*b^5*c^6*d^5*e^3*f^2* \\
&z^4 + 8544*a^5*b^5*c^4*d^2*e^5*f^3*z^4 + 8544*a^4*b^5*c^5*d^3*e^5*f^2*z^4 - \\
&8496*a^5*b^4*c^5*d^2*e^6*f^2*z^4 + 7488*a^8*b^2*c^4*d^2*e^2*f^6*z^4 + 7488 \\
&*a^4*b^2*c^8*d^6*e^2*f^2*z^4 + 7380*a^4*b^7*c^3*d^3*e^3*f^4*z^4 + 7380*a^3* \\
&b^7*c^4*d^4*e^3*f^3*z^4 - 6720*a^3*b^8*c^3*d^4*e^2*f^4*z^4 - 5784*a^5*b^6*c \\
&^3*d^3*e^2*f^5*z^4 - 5784*a^3*b^6*c^5*d^5*e^2*f^3*z^4 - 3440*a^6*b^4*c^4*d^ \\
&2*e^4*f^4*z^4 - 3440*a^4*b^4*c^6*d^4*e^4*f^2*z^4 + 3360*a^3*b^8*c^3*d^3*e^4 \\
&*f^3*z^4 + 3140*a^4*b^6*c^4*d^2*e^6*f^2*z^4 - 2760*a^4*b^7*c^3*d^2*e^5*f^3* \\
&z^4 - 2760*a^3*b^7*c^4*d^3*e^5*f^2*z^4 - 1764*a^5*b^7*c^2*d^2*e^3*f^5*z^4 - \\
&1764*a^2*b^7*c^5*d^5*e^3*f^2*z^4 - 1640*a^3*b^9*c^2*d^3*e^3*f^4*z^4 - 1640 \\
&*a^2*b^9*c^3*d^4*e^3*f^3*z^4 - 1604*a^6*b^6*c^2*d^2*e^2*f^6*z^4 - 1604*a^2* \\
&b^6*c^6*d^6*e^2*f^2*z^4 - 1500*a^5*b^6*c^3*d^2*e^4*f^4*z^4 - 1500*a^3*b^6*c \\
&^5*d^4*e^4*f^2*z^4 + 1140*a^2*b^10*c^2*d^4*e^2*f^4*z^4 + 810*a^4*b^8*c^2*d^ \\
&2*e^4*f^4*z^4 + 810*a^2*b^8*c^4*d^4*e^4*f^2*z^4 - 544*a^3*b^8*c^3*d^2*e^6*f \\
&^2*z^4 + 416*a^3*b^9*c^2*d^2*e^5*f^3*z^4 + 416*a^2*b^9*c^3*d^3*e^5*f^2*z^4 \\
&- 384*a^2*b^10*c^2*d^3*e^4*f^3*z^4 + 180*a^4*b^8*c^2*d^3*e^2*f^5*z^4 + 180* \\
&a^2*b^8*c^4*d^5*e^2*f^3*z^4 + 48*a^7*b^4*c^3*d^2*e^2*f^6*z^4 + 48*a^3*b^4*c \\
&^7*d^6*e^2*f^2*z^4 + 36*a^2*b^10*c^2*d^2*e^6*f^2*z^4 - 1024*a^10*b*c^3*d*e* \\
&f^8*z^4 - 1024*a^3*b*c^10*d^8*e*f*z^4 - 192*a^8*b^5*c*d*e*f^8*z^4 - 192*a*b \\
&^5*c^8*d^8*e*f*z^4 + 16128*a^7*b^3*c^4*d^3*e*f^6*z^4 + 16128*a^4*b^3*c^7*d^ \\
&6*e*f^3*z^4 - 11712*a^6*b^5*c^3*d^3*e*f^6*z^4 - 11712*a^3*b^5*c^6*d^6*e*f^3 \\
&*z^4 + 11520*a^8*b*c^5*d^2*e^3*f^5*z^4 + 11520*a^5*b*c^8*d^5*e^3*f^2*z^4 - \\
&9984*a^6*b^3*c^5*d^4*e*f^5*z^4 - 9984*a^5*b^3*c^6*d^5*e*f^4*z^4 + 8640*a^5* \\
&b^5*c^4*d^4*e*f^5*z^4 + 8640*a^4*b^5*c^5*d^5*e*f^4*z^4 - 7424*a^7*b*c^6*d^3 \\
&*e^3*f^4*z^4 - 7424*a^6*b*c^7*d^4*e^3*f^3*z^4 - 6912*a^8*b^3*c^3*d^2*e*f^7* \\
&z^4 - 6912*a^3*b^3*c^8*d^7*e*f^2*z^4 + 4800*a^7*b^3*c^4*d*e^5*f^4*z^4 + 480 \\
&0*a^4*b^3*c^7*d^4*e^5*f*z^4 + 4608*a^7*b*c^6*d^2*e^5*f^3*z^4 + 4608*a^6*b*c \\
&^7*d^3*e^5*f^2*z^4 - 4560*a^4*b^7*c^3*d^4*e*f^5*z^4 - 4560*a^3*b^7*c^4*d^5* \\
&e*f^4*z^4 + 4176*a^5*b^7*c^2*d^3*e*f^6*z^4 + 4176*a^2*b^7*c^5*d^6*e*f^3*z^4 \\
&+ 3264*a^7*b^5*c^2*d^2*e*f^7*z^4 + 3264*a^2*b^5*c^7*d^7*e*f^2*z^4 + 3008*a \\
&^8*b^3*c^3*d*e^3*f^6*z^4 + 3008*a^3*b^3*c^8*d^6*e^3*f*z^4 + 2880*a^6*b^3*c^ \\
&5*d*e^7*f^2*z^4 + 2880*a^5*b^3*c^6*d^2*e^7*f*z^4 - 2240*a^7*b^4*c^3*d*e^4*f \\
&^5*z^4 - 2240*a^3*b^4*c^7*d^5*e^4*f*z^4 - 1488*a^5*b^5*c^4*d*e^7*f^2*z^4 - \\
&1488*a^4*b^5*c^5*d^2*e^7*f*z^4 + 1440*a^3*b^9*c^2*d^4*e*f^5*z^4 + 1440*a^2* \\
&b^9*c^3*d^5*e*f^4*z^4 - 1328*a^6*b^5*c^3*d*e^5*f^4*z^4 - 1328*a^3*b^5*c^6*d \\
&^4*e^5*f*z^4 - 1152*a^7*b^2*c^5*d*e^6*f^3*z^4 - 1152*a^5*b^2*c^7*d^3*e^6*f* \\
&z^4 - 1120*a^6*b^4*c^4*d*e^6*f^3*z^4 - 1120*a^4*b^4*c^6*d^3*e^6*f*z^4 + 912 \\
&*a^6*b^6*c^2*d*e^4*f^5*z^4 + 912*a^2*b^6*c^6*d^5*e^4*f*z^4 + 872*a^5*b^6*c^ \\
&3*d*e^6*f^3*z^4 + 872*a^3*b^6*c^5*d^3*e^6*f*z^4 + 768*a^8*b^2*c^4*d*e^4*f^5
\end{aligned}$$

$$\begin{aligned}
& *z^4 + 768*a^4*b^2*c^8*d^5*e^4*f*z^4 - 672*a^8*b^4*c^2*d*e^2*f^7*z^4 - 672* \\
& a^2*b^4*c^8*d^7*e^2*f*z^4 - 624*a^7*b^5*c^2*d*e^3*f^6*z^4 - 624*a^2*b^5*c^7 \\
& *d^6*e^3*f*z^4 + 480*a^5*b^8*c*d^2*e^2*f^6*z^4 + 480*a*b^8*c^5*d^6*e^2*f^2* \\
& z^4 + 316*a^4*b^7*c^3*d*e^7*f^2*z^4 + 316*a^3*b^7*c^4*d^2*e^7*f*z^4 - 204*a \\
& ^4*b^8*c^2*d*e^6*f^3*z^4 - 204*a^2*b^8*c^4*d^3*e^6*f*z^4 + 168*a^3*b^10*c*d \\
& ^3*e^2*f^5*z^4 + 168*a*b^10*c^3*d^5*e^2*f^3*z^4 + 156*a^2*b^11*c*d^3*e^3*f^ \\
& 4*z^4 + 156*a*b^11*c^2*d^4*e^3*f^3*z^4 + 128*a^9*b^2*c^3*d*e^2*f^7*z^4 + 12 \\
& 8*a^3*b^2*c^9*d^7*e^2*f*z^4 - 124*a^3*b^10*c*d^2*e^4*f^4*z^4 - 124*a*b^10*c \\
& ^3*d^4*e^4*f^2*z^4 + 100*a^4*b^9*c*d^2*e^3*f^5*z^4 + 100*a*b^9*c^4*d^5*e^3* \\
& f^2*z^4 + 36*a^5*b^7*c^2*d*e^5*f^4*z^4 + 36*a^2*b^7*c^5*d^4*e^5*f*z^4 - 24* \\
& a^3*b^9*c^2*d*e^7*f^2*z^4 - 24*a^2*b^11*c*d^2*e^5*f^3*z^4 - 24*a^2*b^9*c^3* \\
& d^2*e^7*f*z^4 - 24*a*b^11*c^2*d^3*e^5*f^2*z^4 - 9216*a^8*b*c^5*d^3*e*f^6*z^ \\
& 4 - 9216*a^5*b*c^8*d^6*e*f^3*z^4 - 5376*a^8*b*c^5*d*e^5*f^4*z^4 - 5376*a^5* \\
& b*c^8*d^4*e^5*f*z^4 + 5120*a^9*b*c^4*d^2*e*f^7*z^4 + 5120*a^7*b*c^6*d^4*e*f \\
& ^5*z^4 + 5120*a^6*b*c^7*d^5*e*f^4*z^4 + 5120*a^4*b*c^9*d^7*e*f^2*z^4 - 4352 \\
& *a^9*b*c^4*d*e^3*f^6*z^4 - 4352*a^4*b*c^9*d^6*e^3*f*z^4 - 1792*a^7*b*c^6*d* \\
& e^7*f^2*z^4 - 1792*a^6*b*c^7*d^2*e^7*f*z^4 - 1600*a^6*b^2*c^6*d*e^8*f*z^4 + \\
& 912*a^5*b^4*c^5*d*e^8*f*z^4 + 768*a^9*b^3*c^2*d*e*f^8*z^4 + 768*a^2*b^3*c^ \\
& 9*d^8*e*f*z^4 - 720*a^4*b^9*c*d^3*e*f^6*z^4 - 720*a*b^9*c^4*d^6*e*f^3*z^4 - \\
& 656*a^6*b^7*c*d^2*e*f^7*z^4 - 656*a*b^7*c^6*d^7*e*f^2*z^4 - 240*a^2*b^11*c \\
& *d^4*e*f^5*z^4 - 240*a*b^11*c^2*d^5*e*f^4*z^4 + 216*a^7*b^6*c*d*e^2*f^7*z^4 \\
& + 216*a*b^6*c^7*d^7*e^2*f*z^4 - 204*a^4*b^6*c^4*d*e^8*f*z^4 - 144*a^5*b^8* \\
& c*d*e^4*f^5*z^4 - 144*a*b^8*c^5*d^5*e^4*f*z^4 - 84*a*b^12*c*d^4*e^2*f^4*z^4 \\
& + 36*a^4*b^9*c*d*e^5*f^4*z^4 + 36*a*b^9*c^4*d^4*e^5*f*z^4 + 20*a^6*b^7*c*d \\
& *e^3*f^6*z^4 + 20*a*b^7*c^6*d^6*e^3*f*z^4 + 16*a^3*b^10*c*d*e^6*f^3*z^4 + 1 \\
& 6*a^3*b^8*c^3*d*e^8*f*z^4 + 16*a*b^12*c*d^3*e^4*f^3*z^4 + 16*a*b^10*c^3*d^3 \\
& *e^6*f*z^4 + 48*b^11*c^3*d^6*e*f^3*z^4 + 48*b^9*c^5*d^7*e*f^2*z^4 - 20*b^8* \\
& c^6*d^7*e^2*f*z^4 + 8*b^10*c^4*d^5*e^4*f*z^4 - 4*b^13*c*d^4*e^3*f^3*z^4 - 4 \\
& *b^11*c^3*d^4*e^5*f*z^4 + 4*b^9*c^5*d^6*e^3*f*z^4 + 3072*a^9*c^5*d*e^4*f^5* \\
& z^4 + 3072*a^5*c^9*d^5*e^4*f*z^4 + 2560*a^8*c^6*d*e^6*f^3*z^4 + 2560*a^6*c^ \\
& 8*d^3*e^6*f*z^4 + 1536*a^10*c^4*d*e^2*f^7*z^4 + 1536*a^4*c^10*d^7*e^2*f*z^4 \\
& + 48*a^5*b^9*d^2*e*f^7*z^4 + 48*a^3*b^11*d^3*e*f^6*z^4 - 20*a^6*b^8*d*e^2* \\
& f^7*z^4 + 8*a^4*b^10*d*e^4*f^5*z^4 + 4*a^5*b^9*d*e^3*f^6*z^4 - 4*a^3*b^11*d \\
& *e^5*f^4*z^4 - 4*a*b^13*d^3*e^3*f^4*z^4 + 768*a^9*b*c^4*e^5*f^5*z^4 + 768*a \\
& ^8*b*c^5*e^7*f^3*z^4 + 256*a^10*b*c^3*e^3*f^7*z^4 - 192*a^6*b^3*c^5*e^9*f*z \\
& ^4 - 68*a^7*b^6*c*e^4*f^6*z^4 + 48*a^8*b^5*c*e^3*f^7*z^4 + 48*a^5*b^5*c^4*e \\
& ^9*f*z^4 + 36*a^6*b^7*c*e^5*f^5*z^4 - 12*a^9*b^4*c*e^2*f^8*z^4 - 4*a^4*b^9* \\
& c*e^7*f^3*z^4 - 4*a^4*b^7*c^3*e^9*f*z^4 + 384*a^5*b^8*c*d^3*f^7*z^4 + 384*a \\
& *b^8*c^5*d^7*f^3*z^4 + 288*a^3*b^10*c*d^4*f^6*z^4 + 288*a*b^10*c^3*d^6*f^4* \\
& z^4 + 224*a^7*b^6*c*d^2*f^8*z^4 + 224*a*b^6*c^7*d^8*f^2*z^4 - 192*a^10*b^2* \\
& c^2*d*f^9*z^4 - 192*a^2*b^2*c^10*d^9*f*z^4 + 768*a^5*b*c^8*d^3*e^7*z^4 + 76 \\
& 8*a^4*b*c^9*d^5*e^5*z^4 + 256*a^3*b*c^10*d^7*e^3*z^4 - 192*a^5*b^3*c^6*d*e^ \\
& 9*z^4 - 68*a*b^6*c^7*d^6*e^4*z^4 + 48*a^4*b^5*c^5*d*e^9*z^4 + 48*a*b^5*c^8* \\
& d^7*e^3*z^4 + 36*a*b^7*c^6*d^5*e^5*z^4 - 12*a*b^4*c^9*d^8*e^2*z^4 - 4*a^3*b \\
& ^7*c^4*d*e^9*z^4 - 4*a*b^9*c^4*d^3*e^7*z^4 + 16*b^13*c*d^5*e*f^4*z^4 + 16*b
\end{aligned}$$

$$\begin{aligned}
& 7c^7d^8efz^4 + 768a^7c^7d^8e^8fz^4 + 16a^7b^7d^8e^8fz^4 + 16a^7b^7c^7d^8e^8fz^4 + 16a^7b^7c^7d^8e^8fz^4 + 16a^7b^7c^7d^8e^8fz^4 \\
& + 256a^7b^7c^6e^9fz^4 + 80a^7b^7c^6d^5f^5z^4 + 48a^9b^4c^4d^5f^9z^4 + 48a^7b^4c^9d^9fz^4 + 256a^6b^7c^7d^8e^9fz^4 - \\
& 42b^10c^4d^6e^2f^2z^4 - 20b^12c^2d^5e^2f^3z^4 + 6b^12c^2d^4e^4f^2z^4 + 4b^11c^3d^5e^3f^2z^4 - 24960a^7c^7d^4e^2f^4z^4 + \\
& 18944a^8c^6d^3e^2f^5z^4 + 18944a^6c^8d^5e^2f^3z^4 + 14336a^7c^7d^3e^4f^3z^4 - 9984a^8c^6d^2e^4f^4z^4 - 9984a^6c^8d^4e^4f^2z^4 - \\
& 7936a^9c^5d^2e^2f^6z^4 - 7936a^5c^9d^6e^2f^2z^4 - 4352a^7c^7d^2e^6f^2z^4 - 42a^4b^10d^2e^2f^6z^4 - 20a^2b^12d^3e^2f^5z^4 + 6a^2b^12d^2e^4f^4z^4 + \\
& 4a^3b^11d^2e^3f^5z^4 - 480a^8b^2c^4e^6f^4z^4 + 440a^7b^4c^3e^6f^4z^4 - 320a^8b^3c^3e^5f^5z^4 - 320a^7b^3c^4e^7f^3z^4 + \\
& 240a^8b^4c^2e^4f^6z^4 + 240a^6b^4c^4e^8f^2z^4 - 192a^9b^3c^2e^3f^7z^4 - 192a^9b^2c^3e^4f^6z^4 - 192a^7b^2c^5e^8f^2z^4 - \\
& 90a^6b^6c^2e^6f^4z^4 - 68a^5b^6c^3e^8f^2z^4 + 48a^10b^2c^2e^2f^8z^4 - 48a^7b^5c^2e^5f^5z^4 - 48a^6b^5c^3e^7f^3z^4 + \\
& 36a^5b^7c^2e^7f^3z^4 + 6a^4b^8c^2e^8f^2z^4 - 33920a^6b^2c^6d^5f^5z^4 + 27936a^5b^4c^5d^5f^5z^4 + 26112a^7b^2c^5d^4f^6z^4 + \\
& 26112a^5b^2c^7d^6f^4z^4 - 20352a^6b^4c^4d^4f^6z^4 - 20352a^4b^4c^6d^6f^4z^4 - 13080a^4b^6c^4d^5f^5z^4 - 11520a^8b^2c^4d^3f^7z^4 - \\
& 11520a^4b^2c^8d^7f^3z^4 + 8736a^5b^6c^3d^4f^6z^4 + 8736a^3b^6c^5d^6f^4z^4 + 7488a^7b^4c^3d^3f^7z^4 + 7488a^3b^4c^7d^7f^3z^4 + \\
& 3840a^3b^8c^3d^5f^5z^4 + 2560a^9b^2c^3d^2f^8z^4 + 2560a^3b^2c^9d^8f^2z^4 - 2416a^6b^6c^2d^3f^7z^4 - 2416a^2b^6c^6d^7f^3z^4 - \\
& 2160a^4b^8c^2d^4f^6z^4 - 2160a^2b^8c^4d^6f^4z^4 - 1152a^8b^4c^2d^2f^8z^4 - 1152a^2b^4c^8d^8f^2z^4 - 720a^2b^10c^2d^5f^5z^4 - \\
& 480a^4b^2c^8d^4e^6z^4 + 440a^3b^4c^7d^4e^6z^4 - 320a^4b^3c^7d^3e^7z^4 - 320a^3b^3c^8d^5e^5z^4 + 240a^4b^4c^6d^2e^8z^4 + \\
& 240a^2b^4c^8d^6e^4z^4 - 192a^5b^2c^7d^2e^8z^4 - 192a^3b^2c^9d^6e^4z^4 - 192a^2b^3c^9d^7e^3z^4 - 90a^2b^6c^6d^4e^6z^4 - \\
& 68a^3b^6c^5d^2e^8z^4 - 48a^3b^5c^6d^3e^7z^4 - 48a^2b^5c^7d^5e^5z^4 + 48a^2b^2c^10d^8e^2z^4 + 36a^2b^7c^5d^3e^7z^4 + \\
& 6a^2b^8c^4d^2e^8z^4 - 4b^6c^8d^9fz^4 + 256a^11c^3d^5f^9z^4 + 256a^3c^11d^9fz^4 - 4a^8b^6d^5f^9z^4 - 384a^9c^5e^6f^4z^4 - \\
& 256a^10c^4e^4f^6z^4 - 256a^8c^6e^8f^2z^4 - 64a^11c^3e^2f^8z^4 - 24b^10c^4d^7f^3z^4 - 16b^12c^2d^6f^4z^4 - \\
& 16b^8c^6d^8f^2z^4 + 17920a^7c^7d^5f^5z^4 - 14336a^8c^6d^4f^6z^4 - 14336a^6c^8d^6f^4z^4 + 7168a^9c^5d^3f^7z^4 + \\
& 7168a^5c^9d^7f^3z^4 - 2048a^10c^4d^2f^8z^4 - 2048a^4c^10d^8f^2z^4 + 6b^8c^6d^6e^4z^4 + 6a^6b^8e^4f^6z^4 - \\
& 4b^9c^5d^5e^5z^4 - 4b^7c^7d^7e^3z^4 - 4a^7b^7e^3f^7z^4 - 4a^5b^9e^5f^5z^4 - 384a^5c^9d^4e^6z^4 - 256a^6c^8d^2e^8z^4 - \\
& 256a^4c^10d^6e^4z^4 - 64a^3c^11d^8e^2z^4 - 24a^4b^10d^3f^7z^4 - 16a^6b^8d^2f^8z^4 - 16a^2b^12d^4f^6z^4 + \\
& 48a^6b^2c^6e^10z^4 - 12a^5b^4c^5e^10z^4 - 4b^14d^5f^5z^4 - 64a^7c^7e^10z^4 + b^14d^4e^2f^4z^4 + b^10c^4d^4e^6z^4 + \\
& b^6c^8d^8e^2z^4 + a^8
\end{aligned}$$

$$\begin{aligned}
& *b^6e^2f^8z^4 + a^4b^{10}e^6f^4z^4 + a^4b^6c^4e^{10}z^4 - 4820*ABa^4b^6c^5d^2e^2f^4z^2 + 2976*ABa^3b^6c^6d^3e^2f^3z^2 - 2328*ABa^3b^6c^6d^2e^4f^2z^2 + 1848*ABa^2b^4c^4d^3e^4f^4z^2 - 1768*ABa^3b^4c^3d^2e^5f^5z^2 + 1528*ABa^4b^2c^4d^2e^5f^5z^2 - 1136*ABa^3b^2c^5d^3e^4f^4z^2 - 974*ABa^4b^3c^3d^2e^2f^5z^2 + 692*ABa^2b^6c^7d^4e^2f^2z^2 + 588*ABa^6b^6c^3d^2e^3f^3z^2 - 580*ABa^3b^3c^4d^4e^4f^3z^2 + 488*ABa^3b^4c^3d^2e^3f^4z^2 - 444*ABa^2b^2c^6d^2e^5f^4z^2 - 412*ABa^5b^5c^4d^2e^4f^2z^2 + 366*ABa^2b^6c^2d^2e^5f^5z^2 - 352*ABa^2b^2c^6d^4e^3f^3z^2 + 326*ABa^2b^4c^4d^2e^5f^2z^2 + 324*ABa^5b^5c^4d^3e^2f^3z^2 - 302*ABa^3b^3c^6d^4e^2f^2z^2 - 296*ABa^7b^7c^2d^2e^2f^4z^2 + 122*ABa^4b^2c^4d^2e^3f^4z^2 - 122*ABa^2b^6c^2d^2e^3f^4z^2 - 84*ABa^3b^2c^5d^2e^5f^2z^2 + 72*ABa^6b^4c^5d^3e^3f^2z^2 - 64*ABa^2b^5c^3d^2e^4f^3z^2 + 60*ABa^3b^5c^2d^2e^2f^5z^2 + 1312*ABa^5b^6c^4d^2e^2f^5z^2 + 1040*ABa^4b^6c^5d^4e^4f^3z^2 - 500*ABa^6b^6c^3d^3e^4f^4z^2 - 376*ABa^6b^2c^7d^5e^2f^2z^2 + 276*ABa^4b^4c^2d^2e^6f^6z^2 - 262*ABa^2b^3c^5d^2e^6f^6z^2 + 238*ABa^6b^2c^7d^4e^3f^6z^2 + 232*ABa^5b^2c^3d^2e^6f^6z^2 - 176*ABa^2b^6c^7d^3e^4f^6z^2 - 120*ABa^6b^6c^3d^2e^5f^2z^2 - 108*ABa^6b^4c^5d^4e^3f^3z^2 + 68*ABa^6b^7c^2d^2e^4f^3z^2 + 68*ABa^6b^4c^5d^2e^5f^4z^2 + 46*ABa^2b^7c^2d^2e^2f^5z^2 - 36*ABa^6b^3c^6d^3e^4f^4z^2 - 1932*ABa^2b^3c^5d^3e^2f^3z^2 - 1818*ABa^2b^4c^4d^2e^3f^3z^2 + 1620*ABa^3b^3c^4d^2e^2f^4z^2 + 1560*ABa^2b^3c^5d^2e^4f^2z^2 + 1244*ABa^3b^2c^5d^2e^3f^3z^2 + 820*ABa^2b^2c^6d^3e^3f^2z^2 + 480*ABa^2b^5c^3d^2e^2f^4z^2 + 352*ABa^3b^6c^6d^2e^6f^6z^2 - 108*ABa^3b^6c^6d^2e^6f^6z^2 + 82*ABa^5b^5c^4d^2e^6f^6z^2 - 64*ABa^6b^6c^8d^5e^2f^6z^2 + 16*ABa^6b^8c^2d^2e^5f^5z^2 - 4*ABa^6b^8c^2d^2e^3f^4z^2 + 16*B^2a^6b^6c^8d^6e^6f^6z^2 + 56*ABb^2c^8d^6e^6f^6z^2 - 8*ABb^9c^2d^2e^4f^3z^2 - 8*ABb^7c^3d^2e^6f^6z^2 - 800*ABa^6c^4d^2e^6f^6z^2 + 10*ABa^2b^8d^2e^6f^6z^2 - 6*ABa^6b^9d^2e^2f^5z^2 - 12*ABa^5b^4c^2e^7f^7z^2 + 912*ABa^6b^6c^3d^2f^7z^2 + 192*ABa^4b^5c^2d^2f^7z^2 + 192*ABa^6b^6c^8d^6f^2z^2 - 20*ABa^6b^4c^5d^2e^7z^2 + 4*ABa^6b^6c^8d^4e^4z^2 + 2144*B^2a^4b^6c^5d^3e^4f^4z^2 - 1120*B^2a^3b^6c^6d^4e^4f^3z^2 - 688*B^2a^5b^6c^4d^2e^5f^5z^2 - 256*B^2a^3b^6c^6d^2e^5f^5z^2 + 152*B^2a^6b^3c^6d^5e^5f^2z^2 + 120*B^2a^5b^3c^2d^2e^6f^6z^2 - 116*B^2a^5b^6c^4d^2e^3f^4z^2 + 110*B^2a^6b^7c^2d^3e^4f^4z^2 - 80*B^2a^2b^6c^7d^5e^5f^2z^2 - 72*B^2a^6b^5c^4d^4e^5f^3z^2 - 48*B^2a^4b^6c^5d^2e^5f^2z^2 - 46*B^2a^6b^3c^6d^4e^3f^3z^2 - 44*B^2a^6b^4c^5d^3e^4f^4z^2 - 34*B^2a^6b^5c^4d^2e^5f^5z^2 + 20*B^2a^2b^6c^7d^4e^3f^6z^2 - 10*B^2a^3b^6c^6d^2e^2f^5z^2 - 10*B^2a^2b^7c^2d^2e^5f^5z^2 - 10*B^2a^6b^2c^7d^5e^2f^6z^2 - 7*B^2a^2b^4c^4d^2e^6f^6z^2 - 6*B^2a^3b^2c^5d^2e^6f^6z^2 + 4*B^2a^6b^8c^2d^2e^2f^4z^2 - 2*B^2a^2b^7c^2d^2e^3f^4z^2 + 3196*A^2a^4b^6c^5d^2e^3f^4z^2 - 3184*A^2a^4b^6c^5d^2e^5f^5z^2 + 1568*A^2a^3b^6c^6d^3e^4f^4z^2 + 1504*A^2a^3b^6c^6d^2e^5f^2z^2 - 656*A^2a^4b^3c^3d^2e^6f^6z^2 - 400*A^2a^6b^6c^3d^2e^4f^3z^2 + 314*A^2a^6b^5c^4d^2e^5f^2z^2 - 264*A^2a^3b^5c^2d^2e^6f^6z^2 + 240*A^2a^2b^2c^6c^6
\end{aligned}$$

$$\begin{aligned}
& d^6 e^6 f^2 z^2 - 224 A^2 a^2 b^7 c^7 d^4 e^5 f^3 z^2 + 216 A^2 a^2 b^5 c^4 d^3 e^5 f^4 z^2 - 192 A^2 a^2 b^7 c^7 d^2 e^5 f^3 z^2 + 178 A^2 a^2 b^7 c^2 d^4 e^3 f^4 z^2 - \\
& 154 A^2 a^2 b^7 c^2 d^2 e^5 f^5 z^2 + 128 A^2 a^2 b^3 c^6 d^4 e^5 f^3 z^2 + 106 A^2 a^2 b^3 c^6 d^2 e^5 f^3 z^2 - 12 A^2 a^2 b^2 c^7 d^3 e^4 f^3 z^2 - 58 A^2 a^2 b^8 c^2 d^2 e^3 f^3 z^2 + 40 A^2 a^2 b^7 c^3 d^2 e^4 f^2 z^2 - 28 A^2 a^2 b^7 c^3 d^3 e^2 f^3 z^2 - \\
& 24 A^2 a^2 b^5 c^5 d^4 e^2 f^2 z^2 - 20 A^2 a^2 b^6 c^4 d^3 e^3 f^2 z^2 + 2768 A^2 a^4 c^6 d^2 e^3 f^3 z^2 - 1712 A^2 a^3 c^7 d^3 e^3 f^2 z^2 - 156 A^2 a^4 b^2 c^4 e^5 f^3 z^2 + 146 A^2 a^4 b^3 c^3 e^4 f^4 z^2 - 106 A^2 a^5 b^2 c^3 e^3 f^5 z^2 + 90 A^2 a^5 b^3 c^2 e^2 f^6 z^2 + 38 A^2 a^3 b^3 c^4 e^6 f^2 z^2 - \\
& 36 A^2 a^3 b^5 c^2 e^4 f^4 z^2 + 16 A^2 a^3 b^4 c^3 e^5 f^3 z^2 - 9 A^2 a^4 b^4 c^2 e^3 f^5 z^2 - 8 A^2 a^2 b^5 c^3 e^6 f^2 z^2 + 2 A^2 a^2 b^6 c^2 e^5 f^3 z^2 + 920 A^2 a^4 b^3 c^3 d^2 f^6 z^2 - 480 A^2 a^2 b^5 c^3 d^3 f^5 z^2 - 336 A^2 a^2 b^3 c^5 d^4 f^4 z^2 - 272 A^2 a^3 b^3 c^4 d^3 f^5 z^2 + \\
& 240 A^2 a^3 b^5 c^2 d^2 f^6 z^2 - 32 A^2 a^3 c^9 d^6 e^5 f^3 z^2 - 792 A^2 a^2 b^3 c^5 d^3 e^3 f^2 z^2 + 714 A^2 a^2 b^4 c^4 d^3 e^2 f^3 z^2 - 572 A^2 a^3 b^2 c^5 d^3 e^2 f^3 z^2 - 475 A^2 a^2 b^2 c^6 d^4 e^2 f^2 z^2 + 265 A^2 a^4 b^2 c^4 d^2 e^2 f^4 z^2 + 260 A^2 a^3 b^3 c^4 d^2 e^3 f^3 z^2 - 212 A^2 a^3 b^4 c^3 d^2 e^2 f^4 z^2 + 180 A^2 a^3 b^2 c^5 d^2 e^4 f^2 z^2 - \\
& 158 A^2 a^2 b^4 c^4 d^2 e^4 f^2 z^2 + 47 A^2 a^2 b^6 c^2 d^2 e^2 f^4 z^2 + 16 A^2 a^2 b^5 c^3 d^2 e^3 f^3 z^2 + 2752 A^2 a^3 b^2 c^5 d^2 e^2 f^4 z^2 - 2148 A^2 a^2 b^4 c^4 d^2 e^2 f^4 z^2 + 2064 A^2 a^2 b^3 c^5 d^2 e^3 f^3 z^2 - 424 A^2 a^2 b^2 c^6 d^3 e^2 f^3 z^2 - 198 A^2 a^2 b^2 c^6 d^2 e^4 f^2 z^2 - 272 A^2 a^6 b^3 c^3 d^2 e^5 f^6 z^2 - 24 A^2 a^4 b^5 c^3 d^2 e^5 f^6 z^2 + 1808 A^2 a^5 b^3 c^4 d^2 e^5 f^6 z^2 - 244 A^2 a^2 b^7 c^8 d^4 e^3 f^3 z^2 + 208 A^2 a^2 b^7 c^8 d^5 e^5 f^2 z^2 + 134 A^2 a^2 b^7 c^8 d^4 e^3 f^3 z^2 - 76 A^2 a^2 b^4 c^5 d^6 e^6 f^3 z^2 + 4 A^2 a^2 b^8 c^5 d^2 e^2 f^5 z^2 + 148 A^2 a^2 b^4 c^6 d^5 e^5 f^2 z^2 + 65 A^2 a^2 b^6 c^4 d^4 e^5 f^3 z^2 + 46 A^2 a^2 b^8 c^2 d^3 e^5 f^4 z^2 - 38 A^2 a^2 b^3 c^7 d^5 e^2 f^3 z^2 + 34 A^2 a^2 b^9 c^6 d^2 e^2 f^4 z^2 - 29 A^2 a^2 b^4 c^6 d^4 e^3 f^3 z^2 + 20 A^2 a^2 b^5 c^5 d^3 e^4 f^3 z^2 + 12 A^2 a^2 b^8 c^2 d^2 e^5 f^2 z^2 - 7 A^2 a^2 b^6 c^4 d^2 e^5 f^3 z^2 - 2880 A^2 a^4 c^6 d^3 e^5 f^4 z^2 + 2784 A^2 a^5 c^5 d^2 e^5 f^5 z^2 - 1112 A^2 a^5 b^3 c^5 d^2 e^3 f^4 z^2 + 896 A^2 a^3 c^7 d^4 e^5 f^3 z^2 + 848 A^2 a^3 c^7 d^2 e^5 f^3 z^2 - 560 A^2 a^4 c^6 d^2 e^5 f^2 z^2 + 96 A^2 a^2 c^8 d^5 e^5 f^2 z^2 - 88 A^2 a^2 c^8 d^4 e^3 f^3 z^2 - 100 A^2 a^6 b^3 c^3 e^2 f^6 z^2 - 76 A^2 a^5 b^3 c^4 e^4 f^4 z^2 + 48 A^2 a^6 b^2 c^2 e^7 f^3 z^2 - 42 A^2 a^3 b^2 c^5 e^7 f^3 z^2 + 36 A^2 a^4 b^3 c^5 e^6 f^2 z^2 - 24 A^2 a^4 b^5 c^2 e^2 f^6 z^2 + 10 A^2 a^3 b^6 c^3 e^3 f^5 z^2 + 7 A^2 a^2 b^4 c^4 e^7 f^3 z^2 + 2 A^2 a^2 b^7 c^4 e^4 f^4 z^2 - 2496 A^2 a^5 b^3 c^4 d^2 f^6 z^2 + 1872 A^2 a^4 b^3 c^5 d^3 f^5 z^2 - 744 A^2 a^5 b^3 c^2 d^2 f^7 z^2 - 720 A^2 a^2 b^3 c^7 d^5 f^3 z^2 + 504 A^2 a^2 b^3 c^6 d^5 f^3 z^2 + 256 A^2 a^3 b^3 c^6 d^4 f^4 z^2 + 168 A^2 a^2 b^7 c^2 d^3 f^5 z^2 - 144 A^2 a^2 b^7 c^2 d^2 f^6 z^2 + 144 A^2 a^2 b^5 c^4 d^4 f^4 z^2 + 66 A^2 a^2 b^2 c^6 d^2 e^7 z^2 - 36 A^2 a^2 b^2 c^7 d^3 e^5 z^2 + 20 A^2 a^2 b^3 c^6 d^2 e^6 z^2 + 12 A^2 a^2 b^3 c^7 d^2 e^6 z^2 + 1208 B^2 a^3 b^3 c^6 d^3 e^3 f^2 z^2 - 848 B^2 a^3 b^3 c^4 d^3 e^4 f^4 z^2 + 672 B^2 a^2 b^3 c^5 d^4 e^3 f^3 z^2 - 632 B^2 a^4 b^3 c^5 d^2 e^3 f^3 z^2 + 432 B^2 a^4 b^3 c^3 d^2 e^5 f^5 z^2 + 276 B^2 a^2 b^2 c^6 d^3 e^4 f^3 z^2 - 196 B^2 a^2 b^6 c^3 d^3
\end{aligned}$$

$$\begin{aligned}
& *e^2*f^3*z^2 - 168*B^2*a^2*b^5*c^3*d^3*e*f^4*z^2 + 154*B^2*a^2*b^3*c^5*d^2* \\
& e^5*f*z^2 + 148*B^2*a*b^5*c^4*d^3*e^3*f^2*z^2 + 96*B^2*a*b^4*c^5*d^4*e^2*f^ \\
& 2*z^2 - 72*B^2*a^3*b^5*c^2*d^2*e*f^5*z^2 + 70*B^2*a^5*b^2*c^3*d*e^2*f^5*z^2 \\
& - 60*B^2*a^4*b^3*c^3*d*e^3*f^4*z^2 + 52*B^2*a*b^6*c^3*d^2*e^4*f^2*z^2 + 36 \\
& *B^2*a^4*b^2*c^4*d*e^4*f^3*z^2 - 32*B^2*a*b^7*c^2*d^2*e^3*f^3*z^2 + 24*B^2* \\
& a^3*b^5*c^2*d*e^3*f^4*z^2 + 15*B^2*a^4*b^4*c^2*d*e^2*f^5*z^2 - 8*B^2*a^3*b^ \\
& 4*c^3*d*e^4*f^3*z^2 + 8*B^2*a^2*b^5*c^3*d*e^5*f^2*z^2 - 2*B^2*a^3*b^3*c^4*d \\
& *e^5*f^2*z^2 - 2*B^2*a^2*b^6*c^2*d*e^4*f^3*z^2 - 3176*A^2*a^3*b*c^6*d^2*e^3 \\
& *f^3*z^2 - 2252*A^2*a^4*b^2*c^4*d*e^2*f^5*z^2 + 1952*A^2*a^3*b^4*c^3*d*e^2* \\
& f^5*z^2 - 1496*A^2*a^3*b^3*c^4*d*e^3*f^4*z^2 + 1378*A^2*a^2*b^4*c^4*d*e^4*f \\
& ^3*z^2 + 1184*A^2*a^3*b^3*c^4*d^2*e*f^5*z^2 - 1166*A^2*a^2*b^3*c^5*d*e^5*f^ \\
& 2*z^2 - 1164*A^2*a^3*b^2*c^5*d*e^4*f^3*z^2 - 1152*A^2*a^2*b^3*c^5*d^3*e*f^4 \\
& *z^2 + 578*A^2*a*b^6*c^3*d^2*e^2*f^4*z^2 - 548*A^2*a*b^5*c^4*d^2*e^3*f^3*z^ \\
& 2 + 440*A^2*a*b^2*c^7*d^4*e^2*f^2*z^2 - 412*A^2*a^2*b^6*c^2*d*e^2*f^5*z^2 - \\
& 360*A^2*a*b^3*c^6*d^3*e^3*f^2*z^2 + 312*A^2*a*b^4*c^5*d^3*e^2*f^3*z^2 + 24 \\
& 8*A^2*a^2*b*c^7*d^3*e^3*f^2*z^2 - 224*A^2*a^2*b^5*c^3*d*e^3*f^4*z^2 + 216*A \\
& ^2*a^2*b^5*c^3*d^2*e*f^5*z^2 + 52*A^2*a*b^4*c^5*d^2*e^4*f^2*z^2 - 16*B^2*b^ \\
& 3*c^7*d^6*e*f*z^2 - 14*B^2*b^9*c*d^3*e*f^4*z^2 + 32*B^2*a^4*c^6*d*e^6*f*z^2 \\
& - 20*A^2*b^9*c*d*e^3*f^4*z^2 + 18*A^2*b^9*c*d^2*e*f^5*z^2 + 8*A^2*b^6*c^4* \\
& d*e^6*f*z^2 - 360*A^2*a^3*c^7*d*e^6*f*z^2 + 136*A^2*a*c^9*d^5*e^2*f*z^2 + 2 \\
& *B^2*a^3*b^7*d*e*f^6*z^2 + 2*B^2*a*b^9*d^2*e*f^5*z^2 + 12*B^2*a^4*b*c^5*e^7 \\
& *f*z^2 - 204*A^2*a^3*b*c^6*e^7*f*z^2 - 128*A^2*a^6*b*c^3*e*f^7*z^2 - 48*A^2 \\
& *a*b^5*c^4*e^7*f*z^2 - 36*B^2*a^5*b^4*c*d*f^7*z^2 - 24*A^2*a^4*b^5*c*e*f^7* \\
& z^2 - 16*B^2*a*b^8*c*d^3*f^5*z^2 - 164*A^2*a^3*b^6*c*d*f^7*z^2 - 16*A^2*a*b \\
& ^8*c*d^2*f^6*z^2 + 4*B^2*a^3*b*c^6*d*e^7*z^2 - 4*B^2*a*b*c^8*d^5*e^3*z^2 + \\
& 48*A^2*a*b*c^8*d^3*e^5*z^2 + 36*A^2*a^2*b*c^7*d*e^7*z^2 - 6*A^2*a*b^3*c^6*d \\
& *e^7*z^2 + 136*A*B*a^6*c^4*e^3*f^5*z^2 - 96*A*B*b^5*c^5*d^5*f^3*z^2 + 80*A* \\
& B*a^5*c^5*e^5*f^3*z^2 - 72*A*B*b^3*c^7*d^4*e^4*z^2 - 24*A*B*b^7*c^3*d^4*f^4 \\
& *z^2 + 14*A*B*b^3*c^7*d^4*e^4*z^2 - 14*A*B*b^2*c^8*d^5*e^3*z^2 - 2*A*B*b^5* \\
& c^5*d^2*e^6*z^2 - 2*A*B*b^4*c^6*d^3*e^5*z^2 + 2*A*B*a^3*b^7*e^2*f^6*z^2 - A \\
& *B*a^2*b^8*e^3*f^5*z^2 + 16*A*B*a^2*c^8*d^3*e^5*z^2 - 2*A*B*a^2*b^3*c^5*e^8 \\
& *z^2 + 22*B^2*b^8*c^2*d^3*e^2*f^3*z^2 - 12*B^2*b^7*c^3*d^3*e^3*f^2*z^2 + 12 \\
& *B^2*b^6*c^4*d^4*e^2*f^2*z^2 - 6*B^2*b^8*c^2*d^2*e^4*f^2*z^2 - 864*B^2*a^4* \\
& c^6*d^3*e^2*f^3*z^2 + 496*B^2*a^3*c^7*d^4*e^2*f^2*z^2 + 224*B^2*a^5*c^5*d^2 \\
& *e^2*f^4*z^2 + 136*B^2*a^4*c^6*d^2*e^4*f^2*z^2 - 53*A^2*b^8*c^2*d^2*e^2*f^4 \\
& *z^2 + 52*A^2*b^7*c^3*d^2*e^3*f^3*z^2 + 52*A^2*b^5*c^5*d^3*e^3*f^2*z^2 - 36 \\
& *A^2*b^6*c^4*d^3*e^2*f^3*z^2 - 12*A^2*b^4*c^6*d^4*e^2*f^2*z^2 - 9*A^2*b^6*c \\
& ^4*d^2*e^4*f^2*z^2 + 836*A^2*a^4*c^6*d^2*e^2*f^4*z^2 - 668*A^2*a^2*c^8*d^4* \\
& e^2*f^2*z^2 + 656*A^2*a^3*c^7*d^2*e^4*f^2*z^2 + 368*A^2*a^3*c^7*d^3*e^2*f^3 \\
& *z^2 - 45*B^2*a^6*b^2*c^2*e^2*f^6*z^2 - 18*B^2*a^5*b^2*c^3*e^4*f^4*z^2 - 9* \\
& B^2*a^4*b^2*c^4*e^6*f^2*z^2 - 6*B^2*a^5*b^3*c^2*e^3*f^5*z^2 + 3*B^2*a^4*b^4 \\
& *c^2*e^4*f^4*z^2 - 2*B^2*a^4*b^3*c^3*e^5*f^3*z^2 - 580*B^2*a^4*b^2*c^4*d^3* \\
& f^5*z^2 + 536*B^2*a^3*b^4*c^3*d^3*f^5*z^2 + 471*A^2*a^4*b^2*c^4*e^4*f^4*z^2 \\
& - 436*A^2*a^3*b^4*c^3*e^4*f^4*z^2 - 348*B^2*a^4*b^4*c^2*d^2*f^6*z^2 + 316* \\
& B^2*a^2*b^2*c^6*d^5*f^3*z^2 + 310*A^2*a^3*b^3*c^4*e^5*f^3*z^2 + 232*A^2*a^5
\end{aligned}$$

$$\begin{aligned}
& *b^2*c^3*e^2*f^6*z^2 - 229*A^2*a^2*b^4*c^4*e^6*f^2*z^2 - 216*A^2*a^4*b^4*c^2*e^2*f^6*z^2 + 204*A^2*a^4*b^3*c^3*e^3*f^5*z^2 + 200*B^2*a^5*b^2*c^3*d^2*f^6*z^2 + 150*A^2*a^3*b^2*c^5*e^6*f^2*z^2 - 120*B^2*a^2*b^4*c^4*d^4*f^4*z^2 \\
& + 91*A^2*a^2*b^6*c^2*e^4*f^4*z^2 + 72*A^2*a^3*b^5*c^2*e^3*f^5*z^2 - 66*B^2*a^2*b^6*c^2*d^3*f^5*z^2 + 44*A^2*a^2*b^5*c^3*e^5*f^3*z^2 - 16*B^2*a^3*b^2*c^5*d^4*f^4*z^2 + 1952*A^2*a^4*b^2*c^4*d^2*f^6*z^2 - 1792*A^2*a^3*b^2*c^5*d^3*f^5*z^2 - 1272*A^2*a^3*b^4*c^3*d^2*f^6*z^2 + 976*A^2*a^2*b^2*c^6*d^4*f^4*z^2 + 960*A^2*a^2*b^4*c^4*d^3*f^5*z^2 + 282*A^2*a^2*b^6*c^2*d^2*f^6*z^2 - 45*B^2*a^2*b^2*c^6*d^2*e^6*z^2 - 48*A^2*b*c^9*d^6*e*f*z^2 - 14*A^2*a*b^9*d*e*f^6*z^2 - 7*A*B*b^10*d^2*e*f^5*z^2 + 2*A*B*b^10*d*e^3*f^4*z^2 - 64*A*B*a^7*c^3*e*f^7*z^2 - 16*A*B*b^9*c*d^3*f^5*z^2 + 8*A*B*a^4*c^6*e^7*f*z^2 + 4*A*B*b*c^9*d^6*e^2*z^2 + 2*A*B*b^6*c^4*d*e^7*z^2 - 120*A*B*a^3*c^7*d*e^7*z^2 - 16*A*B*a^3*b^7*d*f^7*z^2 + 16*A*B*a*b^9*d^2*f^6*z^2 + 8*A*B*a*c^9*d^5*e^3*z^2 + 12*A*B*a^3*b*c^6*e^8*z^2 - 48*B^2*b^5*c^5*d^5*e*f^2*z^2 + 15*B^2*b^4*c^6*d^5*e^2*f*z^2 - 14*B^2*b^7*c^3*d^4*e*f^3*z^2 + 4*B^2*b^9*c*d^2*e^3*f^3*z^2 + 4*B^2*b^7*c^3*d^2*e^5*f*z^2 + 4*B^2*b^5*c^5*d^4*e^3*f*z^2 - B^2*b^6*c^4*d^3*e^4*f*z^2 - 336*B^2*a^3*c^7*d^3*e^4*f*z^2 + 112*B^2*a^5*c^5*d*e^4*f^3*z^2 - 112*A^2*b^3*c^7*d^5*e*f^2*z^2 + 80*B^2*a^6*c^4*d*e^2*f^5*z^2 - 48*A^2*b^5*c^5*d^4*e*f^3*z^2 + 36*A^2*b^8*c^2*d*e^4*f^3*z^2 + 36*A^2*b^3*c^7*d^4*e^3*f*z^2 - 28*A^2*b^7*c^3*d*e^5*f^2*z^2 + 20*A^2*b^2*c^8*d^5*e^2*f*z^2 + 16*B^2*a^2*c^8*d^5*e^2*f*z^2 - 14*A^2*b^7*c^3*d^3*e*f^4*z^2 - 14*A^2*b^4*c^6*d^3*e^4*f*z^2 - 10*A^2*b^5*c^5*d^2*e^5*f*z^2 - 1008*A^2*a^4*c^6*d*e^4*f^3*z^2 - 760*A^2*a^5*c^5*d*e^2*f^5*z^2 + 272*A^2*a^2*c^8*d^3*e^4*f*z^2 + 48*B^2*a^5*b*c^4*e^5*f^3*z^2 + 36*B^2*a^6*b*c^3*e^3*f^5*z^2 + 12*B^2*a^5*b^4*c*e^2*f^6*z^2 - 624*A^2*a^4*b*c^5*e^5*f^3*z^2 - 548*A^2*a^5*b*c^4*e^3*f^5*z^2 + 182*A^2*a^2*b^3*c^5*e^7*f*z^2 - 180*B^2*a*b^4*c^5*d^5*f^3*z^2 + 132*B^2*a^6*b^2*c^2*d*f^7*z^2 + 108*B^2*a^3*b^6*c*d^2*f^6*z^2 + 96*A^2*a^5*b^3*c^2*e*f^7*z^2 + 68*A^2*a*b^6*c^3*e^6*f^2*z^2 + 58*A^2*a^3*b^6*c*e^2*f^6*z^2 - 56*B^2*a*b^2*c^7*d^6*f^2*z^2 - 38*A^2*a^2*b^7*c*e^3*f^5*z^2 - 36*A^2*a*b^7*c^2*e^5*f^3*z^2 + 20*B^2*a*b^6*c^3*d^4*f^4*z^2 - 736*A^2*a^5*b^2*c^3*d*f^7*z^2 + 624*A^2*a^4*b^4*c^2*d*f^7*z^2 - 416*A^2*a*b^2*c^7*d^5*f^3*z^2 - 276*A^2*a*b^4*c^5*d^4*f^4*z^2 - 196*A^2*a*b^6*c^3*d^3*f^5*z^2 + 8*B^2*a*b^4*c^5*d^2*e^6*z^2 + 6*B^2*a*b^2*c^7*d^4*e^4*z^2 + 2*B^2*a^2*b^3*c^5*d*e^7*z^2 + 2*B^2*a*b^3*c^6*d^3*e^5*z^2 - 18*A^2*a*b^2*c^7*d^2*e^6*z^2 - 16*A*B*b*c^9*d^7*f*z^2 - B^2*b^10*d^2*e^2*f^4*z^2 + 48*B^2*a^7*c^3*e^2*f^6*z^2 - 36*B^2*a^6*c^4*e^4*f^4*z^2 + 31*B^2*b^6*c^4*d^5*f^3*z^2 - 24*B^2*a^5*c^5*e^6*f^2*z^2 + 20*B^2*b^4*c^6*d^6*f^2*z^2 - 6*A^2*b^8*c^2*e^6*f^2*z^2 + 2*B^2*b^8*c^2*d^4*f^4*z^2 - 768*B^2*a^5*c^5*d^3*f^5*z^2 + 512*B^2*a^6*c^4*d^2*f^6*z^2 + 512*B^2*a^4*c^6*d^4*f^4*z^2 + 232*A^2*a^5*c^5*e^4*f^4*z^2 + 188*A^2*a^4*c^6*e^6*f^2*z^2 - 128*B^2*a^3*c^7*d^5*f^3*z^2 + 92*A^2*a^6*c^4*e^2*f^6*z^2 + 80*A^2*b^4*c^6*d^5*f^3*z^2 + 64*A^2*b^2*c^8*d^6*f^2*z^2 + 31*A^2*b^6*c^4*d^4*f^4*z^2 + 14*A^2*b^8*c^2*d^3*f^5*z^2 - 5*B^2*b^4*c^6*d^4*e^4*z^2 + 4*B^2*b^3*c^7*d^5*e^3*z^2 + 2*B^2*b^5*c^5*d^3*e^5*z^2 - B^2*b^6*c^4*d^2*e^6*z^2 - B^2*b^2*c^8*d^6*e^2*z^2 - B^2*a^4*b^6*e^2*f^6*z^2 - 1152*A^2*a^3*c^7*d^4*f^4*z^2 + 1008*A^2*a^4*c^6*d^3*f^5*z^2 + 624*A^2*a^2*c^8*d^5*f^3*z^2 - 288*A^2*a
\end{aligned}$$

$$\begin{aligned}
& ^5c^5d^2f^6z^2 + 56B^2a^3c^7d^2e^6z^2 - 10B^2a^2b^8d^2f^6z^2 \\
& - 9A^2b^2c^8d^4e^4z^2 - 5A^2a^2b^8e^2f^6z^2 - 4B^2a^2c^8d^4e^4z^2 + 3A^2b^4c^6d^2e^6z^2 - 2A^2b^3c^7d^3e^5z^2 - 36A^2 \\
& a^2c^8d^2e^6z^2 - 48A^2a^6b^2c^2f^8z^2 - 45A^2a^2b^2c^6e^8z^2 + 4A^2b^10d^2e^2f^5z^2 + 4B^2b^2c^8d^7f^7z^2 + 4A^2b^9c^2e^5f^3z^2 \\
& + 4A^2b^7c^3e^7f^7z^2 - 128B^2a^7c^3d^2f^7z^2 - 160A^2a^9d^6f^2z^2 - 112A^2a^6c^4d^2f^7z^2 + 12A^2b^9c^9d^5e^3z^2 + 4A^2 \\
& a^2b^9e^3f^5z^2 + 3B^2a^4b^6d^2f^7z^2 + 2A^2a^3b^7e^7f^7z^2 - 24A^2a^9d^4e^4z^2 + 14A^2a^2b^8d^2f^7z^2 + 12A^2a^5b^4c^2f^8z^2 \\
& + 12A^2a^2b^4c^5e^8z^2 + A^2B^2a^4b^6e^7f^7z^2 + B^2a^2b^8d^2e^2f^5z^2 + 16A^2c^10d^7f^7z^2 + 3B^2b^10d^3f^5z^2 - A^2b^10e^4f^4z^2 \\
& - 4A^2c^10d^6e^2z^2 - A^2b^10d^2f^6z^2 + 64A^2a^7c^3f^8z^2 - 4B^2a^4c^6e^8z^2 - A^2b^6c^4e^8z^2 + 48A^2a^3c^7e^8z^2 - A^2a^4b^6f^8z^2 \\
& + 720A^2B^2a^2b^2c^5d^2e^2f^3z - 600A^2B^2a^2b^2c^4d^2e^2f^4z + 576A^2B^2a^2b^2c^4d^2e^2f^4z + 348A^2B^2a^2b^2c^5d^2e^3f^2z \\
& - 336A^2B^2a^2b^2c^5d^2e^2f^3z - 260A^2B^2a^2b^3c^4d^2e^2f^3z - 240A^2B^2a^2b^2c^4d^2e^3f^3z + 196A^2B^2a^2b^3c^3d^2e^2f^4z \\
& + 172A^2B^2a^2b^3c^6d^2e^5f^5z + 20A^2B^2a^2b^6c^2d^2e^5f^5z - 912A^2B^2a^2b^3c^5d^2e^2f^4z - 644A^2B^2a^2b^3c^6d^2e^3f^2z \\
& - 432A^2B^2a^2b^2c^5d^3e^2f^3z + 372A^2B^2a^2b^3c^5d^2e^3f^3z - 330A^2B^2a^2b^2c^5d^2e^4f^2z + 312A^2B^2a^2b^3c^6d^3e^2f^2z \\
& - 208A^2B^2a^3b^2c^3d^2e^2f^5z + 192A^2B^2a^2b^3c^3d^2e^2f^5z + 172A^2B^2a^2b^3c^4d^2e^3f^3z + 108A^2B^2a^2b^3c^5d^2e^4f^2z \\
& + 104A^2B^2a^3b^3c^4d^2e^2f^4z - 80A^2B^2a^2b^3c^4d^2e^2f^4z + 68A^2B^2a^2b^4c^3d^2e^2f^4z - 60A^2B^2a^2b^5c^2d^2e^2f^4z \\
& + 58A^2B^2a^2b^3c^4d^2e^4f^2z - 36A^2B^2a^2b^4c^3d^2e^2f^4z - 24A^2B^2a^2b^4c^2d^2e^2f^5z + 24A^2B^2a^2b^4c^3d^2e^3f^3z \\
& + 592A^2B^2a^2b^3c^6d^3e^2f^3z + 240A^2B^2a^3b^3c^4d^2e^2f^5z - 132A^2B^2a^2b^3c^6d^2e^4f^2z - 60A^2B^2a^2b^2c^5d^2e^5f^2z \\
& - 48A^2B^2a^2b^5c^2d^2e^2f^5z + 20B^3a^2b^3c^6d^3e^3f^2z + 16B^3a^4b^3c^3d^2e^2f^5z - 16B^3a^2b^3c^6d^4e^2f^2z \\
& + 12B^3a^2b^2c^5d^2e^5f^2z + 320A^3a^2b^3c^6d^2e^4f^2z + 40A^3a^2b^4c^3d^2e^2f^5z - 48A^2B^2b^3c^7d^4e^2f^2z \\
& - 44A^2B^2b^3c^5d^2e^5f^2z - 20A^2B^2b^3c^7d^4e^2f^2z + 14A^2B^2b^4c^4d^2e^5f^2z + 12A^2B^2b^3c^7d^3e^3f^2z \\
& + 4A^2B^2b^7c^2d^2e^2f^4z + 160A^2B^2a^4c^4d^2e^2f^5z + 152A^2B^2a^2c^7d^3e^3f^2z + 32A^2B^2a^2c^7d^4e^2f^2z \\
& - 16A^2B^2a^2c^6d^2e^5f^2z + 128A^2B^2a^4b^3c^3e^2f^6z + 42A^2B^2a^2b^2c^5e^6f^2z + 24A^2B^2a^2b^5c^2e^2f^6z - 12A^2B^2a^3b^4c^2e^2f^6z \\
& - 12A^2B^2a^2b^3c^5e^6f^2z - 10A^2B^2a^2b^6c^2e^2f^5z - 160A^2B^2a^2b^3c^6d^4f^3z + 112A^2B^2a^4b^3c^3d^2f^6z - 24A^2B^2a^2b^5c^2d^2f^6z \\
& - 84B^3a^2b^2c^5d^3e^2f^2z - 80B^3a^2b^3c^3d^2e^2f^4z - 60B^3a^2b^3c^5d^2e^3f^2z - 20B^3a^3b^2c^3d^2e^2f^4z - 20B^3a^3b^3c^4d^2e^3f^2z \\
& - 9B^3a^2b^2c^4d^2e^4f^2z - 8B^3a^2b^4c^3d^2e^2f^3z + 6B^3a^2b^4c^2d^2e^2f^4z - 4B^3a^2b^3c^3d^2e^3f^3z - 216A^2B^2b^4c^4d^2e^2f^3z \\
& + 196A^2B^2b^3c^5d^2e^3f^2z - 108A^2B^2b^3c^5d^3e^2f^2z - 94A^2B^2b^4c^4d^2e^3f^2z + 88A^2B^2b^2c^6d^3e^2f^2z + 80A^2B^2b^5c^3d^2e^2f^3z \\
& + 360A^2B^2a^2b^2c^6d^3e^2f^2z + 80A^2B^2b^5c^3d^2e^2f^3z + 360A^2B^2a^2b^2c^6d^3e^2f^2z + 80A^2B^2b^5c^3d^2e^2f^3z + 360A^2B^2a^2b^2c^6d^3e^2f^2z
\end{aligned}$$

$$\begin{aligned}
& c^6 d^2 e^2 f^3 z + 8 A^2 B^2 a^2 c^6 d^2 e^3 f^2 z + 153 A^2 B a^2 b^2 c^4 e^4 f^3 z - 144 A^2 B a^2 b^3 c^3 e^3 f^4 z + 80 A^2 B a^3 b^2 c^3 e^2 f^5 z \\
& + 36 A^2 B^2 a^3 b^2 c^3 e^3 f^4 z + 12 A^2 B a^2 b^4 c^2 e^2 f^5 z + 12 A^2 B^2 a^3 b^3 c^2 e^2 f^5 z + 9 A^2 B^2 a^2 b^2 c^4 e^5 f^2 z - 6 A^2 B^2 a^2 b^4 c^2 e^3 f^4 z + 4 A^2 B^2 a^2 b^3 c^3 e^4 f^3 z + 480 A^2 B a^2 b^2 c^4 d^2 f^5 z - 176 A^2 B^2 a^2 b^3 c^3 d^2 f^5 z - 10 A^2 B a^2 b^6 c^3 d^2 f^6 z + 16 A^2 B^2 a^2 b^3 c^6 d^2 e^6 z + 80 B^3 a^2 b^3 c^4 d^3 e^3 f^3 z - 48 B^3 a^3 b^3 c^4 d^2 e^3 f^4 z + 48 B^3 a^2 b^3 c^5 d^3 e^3 f^3 z + 44 B^3 a^3 b^3 c^4 d^3 e^3 f^3 z + 24 B^3 a^2 b^5 c^2 d^2 e^3 f^4 z + 18 B^3 a^2 b^2 c^5 d^2 e^4 f^3 z + 696 A^3 a^2 b^3 c^5 d^2 e^2 f^4 z - 504 A^3 a^2 b^3 c^6 d^2 e^2 f^3 z - 192 A^3 a^2 b^2 c^5 d^2 e^3 f^3 z - 144 A^3 a^2 b^2 c^4 d^2 e^3 f^5 z + 96 A^3 a^2 b^2 c^5 d^2 e^3 f^4 z - 72 A^3 a^2 b^3 c^4 d^2 e^2 f^4 z - 208 A^2 B^2 b^3 c^5 d^3 e^3 f^3 z + 152 A^2 B^2 b^4 c^4 d^3 e^3 f^3 z + 80 A^2 B^2 b^5 c^3 d^2 e^3 f^4 z + 75 A^2 B^2 b^4 c^4 d^2 e^4 f^2 z - 59 A^2 B^2 b^2 c^6 d^2 e^4 f^3 z - 52 A^2 B^2 b^5 c^3 d^2 e^3 f^3 z + 42 A^2 B^2 b^3 c^5 d^2 e^4 f^3 z - 21 A^2 B^2 b^6 c^2 d^2 e^3 f^4 z - 16 A^2 B^2 b^5 c^3 d^2 e^4 f^2 z + 16 A^2 B^2 b^2 c^6 d^4 e^3 f^2 z + 16 A^2 B^2 b^2 c^6 d^3 e^3 f^3 z + 11 A^2 B^2 b^6 c^2 d^2 e^2 f^4 z + 4 A^2 B^2 b^6 c^2 d^2 e^3 f^3 z - 256 A^2 B a^2 c^7 d^3 e^2 f^2 z - 96 A^2 B^2 a^3 c^5 d^2 e^3 f^4 z - 36 A^2 B a^2 c^6 d^2 e^4 f^2 z - 32 A^2 B a^3 c^5 d^2 e^2 f^4 z - 32 A^2 B^2 a^2 c^6 d^3 e^3 f^3 z + 8 A^2 B^2 a^3 c^5 d^2 e^3 f^3 z - 96 A^2 B a^3 b^3 c^2 e^3 f^6 z + 68 A^2 B a^3 b^3 c^4 e^3 f^4 z - 60 A^2 B^2 a^4 b^3 c^3 e^2 f^5 z - 60 A^2 B^2 a^3 b^3 c^4 e^4 f^3 z + 48 A^2 B^2 a^4 b^2 c^2 e^3 f^6 z - 38 A^2 B a^2 b^3 c^4 e^5 f^2 z - 36 A^2 B a^2 b^3 c^5 e^5 f^2 z + 36 A^2 B a^2 b^5 c^2 e^3 f^4 z - 16 A^2 B a^2 b^4 c^3 e^4 f^3 z + 384 A^2 B a^2 b^3 c^5 d^3 f^4 z - 352 A^2 B^2 a^3 b^3 c^4 d^2 f^5 z - 288 A^2 B a^2 b^2 c^5 d^3 f^4 z - 160 A^2 B a^3 b^2 c^3 d^2 f^6 z - 148 A^2 B a^2 b^4 c^3 d^2 f^5 z + 112 A^2 B^2 a^2 b^3 c^4 d^3 f^4 z + 72 A^2 B a^2 b^4 c^2 d^2 f^6 z + 72 A^2 B^2 a^2 b^5 c^2 d^2 f^5 z + 48 A^2 B^2 a^3 b^3 c^2 d^2 f^6 z + 102 B^3 a^2 b^2 c^4 d^2 e^2 f^3 z - 32 B^3 b^5 c^3 d^3 e^3 f^3 z - 8 B^3 b^3 c^5 d^3 e^3 f^3 z - 7 B^3 b^4 c^4 d^2 e^4 f^3 z + 5 B^3 b^2 c^6 d^4 e^2 f^3 z + 80 A^3 b^2 c^6 d^3 e^3 f^3 z - 74 A^3 b^3 c^5 d^2 e^4 f^2 z - 64 A^3 b^4 c^4 d^2 e^3 f^4 z + 60 A^3 b^4 c^4 d^2 e^3 f^3 z - 48 B^3 a^4 c^4 d^2 e^2 f^4 z - 24 B^3 a^3 c^5 d^2 e^4 f^2 z + 20 B^3 a^2 c^6 d^2 e^4 f^3 z - 16 A^3 b^5 c^3 d^2 e^2 f^4 z + 8 A^3 b^3 c^7 d^3 e^2 f^2 z + 480 A^3 a^2 c^6 d^2 e^3 f^4 z - 392 A^3 a^2 c^6 d^2 e^3 f^3 z + 280 A^3 a^2 c^7 d^2 e^3 f^2 z - 4 B^3 a^4 b^3 c^3 e^3 f^4 z - 200 A^3 a^3 b^3 c^4 e^2 f^5 z - 144 A^3 a^2 b^3 c^5 e^4 f^3 z + 48 B^3 a^2 b^2 c^5 d^4 f^3 z + 42 A^3 a^2 b^2 c^5 e^5 f^2 z - 36 B^3 a^4 b^2 c^2 d^2 f^6 z - 32 A^3 a^3 b^2 c^3 e^3 f^6 z - 24 A^3 a^2 b^4 c^2 e^3 f^6 z - 24 A^3 a^2 b^5 c^2 e^2 f^5 z + 10 A^3 a^2 b^3 c^4 e^4 f^3 z - 4 B^3 a^2 b^4 c^3 d^3 f^4 z - 4 A^3 a^2 b^4 c^3 e^3 f^4 z - 480 A^3 a^2 b^3 c^5 d^2 f^5 z - 160 A^3 a^2 b^3 c^3 d^2 f^6 z + 128 A^3 a^2 b^3 c^4 d^2 f^5 z + 8 A^2 B^2 b^5 c^3 e^5 f^2 z - 2 A^2 B^2 b^6 c^2 e^4 f^3 z + 112 A^2 B^2 b^4 c^4 d^3 f^4 z - 92 A^2 B^2 a^4 c^4 e^2 f^5 z - 64 A^2 B^2 a^3 c^5 e^4 f^3 z - 64 A^2 B^2 b^5 c^3 d^3 f^4 z + 24 A^2 B^2 a^4 c^4 e^3 f^4 z + 24 A^2 B^2 a^3 c^5 e^5 f^2 z + 16 A^2 B^2 b^2 c^6 d^4 f^3 z + 16 A^2 B^2 b^3 c^5 d^4 f^3 z - A^2 B^2 b^6 c^2 d^2 f^5 z + 448 A^2 B a^3 c^5 d^2 f^5 z - 352 A^2 B a^2 c^6 d^3 f^4 z - 5 A^2 B^2 b^2 c^6 d^2 e^5 z - 48 A^2 B a^4 b^2 c^2 f^7 z - 2 B^3 b
\end{aligned}$$

$$\begin{aligned}
& ^7c*d^2*e*f^4*z + 34*A^3*b^2*c^6*d*e^5*f*z + 16*A^3*b*c^7*d^2*e^4*f*z + 2* \\
& A^3*b^6*c^2*d*e*f^5*z - 416*A^3*a^3*c^5*d*e*f^5*z - 224*A^3*a*c^7*d^3*e*f^3 \\
& *z + 12*B^3*a^3*b^4*c*d*f^6*z - 10*B^3*a*b^6*c*d^2*f^5*z + 416*A^3*a^3*b*c^ \\
& 4*d*f^6*z + 224*A^3*a*b*c^6*d^3*f^4*z + 24*A^3*a*b^5*c^2*d*f^6*z - 4*B^3*a* \\
& b*c^6*d^2*e^5*z + 20*A^2*B*c^8*d^4*e^2*f*z - 7*A^2*B*b^4*c^4*e^6*f*z - 2*A^ \\
& 2*B*b^7*c*e^3*f^4*z - 64*A*B^2*a^5*c^3*e*f^6*z + 16*A*B^2*b*c^7*d^5*f^2*z - \\
& 8*A^2*B*a^2*c^6*e^6*f*z - 2*A*B^2*b^7*c*d^2*f^5*z - 272*A^2*B*a^4*c^4*d*f^ \\
& 6*z + 128*A^2*B*a*c^7*d^4*f^3*z + 9*A^2*B*b^2*c^6*d*e^6*z - 4*A*B^2*b^3*c^5 \\
& *d*e^6*z + 4*A*B^2*b*c^7*d^3*e^4*z + 8*A*B^2*a*c^7*d^2*e^5*z + 12*A^2*B*a^3 \\
& *b^4*c*f^7*z + 30*B^3*b^4*c^4*d^3*e^2*f^2*z + 8*B^3*b^5*c^3*d^2*e^3*f^2*z - \\
& 2*B^3*b^6*c^2*d^2*e^2*f^3*z + 152*A^3*b^3*c^5*d^2*e^2*f^3*z - 108*A^3*b^2* \\
& c^6*d^2*e^3*f^2*z + 48*B^3*a^3*c^5*d^2*e^2*f^3*z - 16*B^3*a^2*c^6*d^3*e^2*f \\
& ^2*z - 3*B^3*a^4*b^2*c^2*e^2*f^5*z - 120*B^3*a^2*b^2*c^4*d^3*f^4*z + 112*B^ \\
& 3*a^3*b^2*c^3*d^2*f^5*z + 112*A^3*a^2*b^3*c^3*e^2*f^5*z + 12*A^3*a^2*b^2*c^ \\
& 4*e^3*f^4*z - 120*A^3*a*c^7*d*e^5*f*z - 52*A^3*a*b*c^6*e^6*f*z + 10*A^3*a*b \\
& ^6*c*e*f^6*z - 2*A*B^2*b^8*d*e*f^5*z - 2*A^2*B*a*b^7*e*f^6*z - 24*A^2*B*a*c \\
& ^7*d*e^6*z + 2*A*B^2*a*b^7*d*f^6*z - 12*A^2*B*a*b*c^6*e^7*z - 2*A^3*b^7*c*d \\
& *f^6*z - 4*A^3*b*c^7*d*e^6*z + 16*B^3*a^5*c^3*e^2*f^5*z + 11*B^3*b^6*c^2*d^ \\
& 3*f^4*z - 11*A^3*b^4*c^4*e^5*f^2*z - 8*B^3*b^4*c^4*d^4*f^3*z - 4*B^3*b^2*c^ \\
& 6*d^5*f^2*z + 4*B^3*a^4*c^4*e^4*f^3*z + 4*A^3*b^5*c^3*e^4*f^3*z - A^3*b^6*c \\
& ^2*e^3*f^4*z + 136*A^3*a^3*c^5*e^3*f^4*z + 68*A^3*a^2*c^6*e^5*f^2*z - 64*A^ \\
& 3*b^3*c^5*d^3*f^4*z + 2*B^3*b^3*c^5*d^2*e^5*z - B^3*b^2*c^6*d^3*e^4*z + 96* \\
& A^3*a^3*b^3*c^2*f^7*z + A*B^2*a^2*b^6*e*f^6*z + 32*A^3*c^8*d^4*e*f^2*z - 24 \\
& *A^3*c^8*d^3*e^3*f*z + 10*A^3*b^3*c^5*e^6*f*z + 2*A^3*b^7*c*e^2*f^5*z + 128 \\
& *A^3*a^4*c^4*e*f^6*z - 32*A^3*b*c^7*d^4*f^3*z - 4*B^3*a^2*c^6*d*e^6*z - B^3 \\
& *a^2*b^6*d*f^6*z - 128*A^3*a^4*b*c^3*f^7*z - 24*A^3*a^2*b^5*c*f^7*z - 16*A^ \\
& 2*B*c^8*d^5*f^2*z - 4*A^2*B*c^8*d^3*e^4*z + 64*A^2*B*a^5*c^3*f^7*z + 2*A^2* \\
& B*b^3*c^5*e^7*z + 4*A*B^2*a^2*c^6*e^7*z - A^2*B*a^2*b^6*f^7*z + 4*A^3*c^8*d \\
& ^2*e^5*z - 3*A^3*b^2*c^6*e^7*z + A^2*B*b^8*d*f^6*z - A^3*b^8*e*f^6*z + 16*A \\
& ^3*a*c^7*e^7*z + 2*A^3*a*b^7*f^7*z + A^2*B*b^8*e^2*f^5*z + B^3*b^8*d^2*f^5* \\
& z - 48*A^2*B^2*a*b*c^4*d*e*f^4 + 28*A*B^3*a*b^2*c^3*d*e*f^4 - 16*A*B^3*a*b* \\
& c^4*d*e^2*f^3 + 16*A^3*B*a*c^5*d*e*f^4 + 32*A^3*B*a*b*c^4*d*f^5 + 12*A^2*B^ \\
& 2*b^3*c^3*d*e*f^4 + 5*A*B^3*b^2*c^4*d^2*e*f^3 + 4*A*B^3*b^3*c^3*d*e^2*f^3 + \\
& 24*A^2*B^2*a*c^5*d*e^2*f^3 + 24*A^2*B^2*a^2*b*c^3*e*f^5 + 12*A^2*B^2*a*b*c \\
& ^4*e^3*f^3 - 6*A^2*B^2*a*b^3*c^2*e*f^5 + 4*A*B^3*a^2*b*c^3*e^2*f^4 + 3*A*B^ \\
& 3*a^2*b^2*c^2*e*f^5 - 18*A^2*B^2*a*b^2*c^3*d*f^5 - 4*B^4*a^2*b*c^3*d*e*f^4 \\
& + 4*B^4*a*b*c^4*d^2*e*f^3 - 6*A*B^3*b^4*c^2*d*e*f^4 + 4*A^3*B*b*c^5*d*e^2*f \\
& ^3 - 2*A^3*B*b^2*c^4*d*e*f^4 - 8*A*B^3*a^2*c^4*d*e*f^4 - 8*A*B^3*a*c^5*d^2* \\
& e*f^3 + 26*A^3*B*a*b^2*c^3*e*f^5 + 8*A^3*B*a*b*c^4*e^2*f^4 + 32*A*B^3*a*b*c \\
& ^4*d^2*f^4 - 28*A*B^3*a^2*b*c^3*d*f^5 + 6*A*B^3*a*b^3*c^2*d*f^5 - 9*A^2*B^2 \\
& *b^2*c^4*d*e^2*f^3 - 18*A^2*B^2*a*b^2*c^3*e^2*f^4 - 4*A^3*B*c^6*d^2*e*f^3 - \\
& 3*A^3*B*b^4*c^2*e*f^5 - 44*A^3*B*a^2*c^4*e*f^5 - 16*A^3*B*a*c^5*e^3*f^3 - \\
& 16*A*B^3*a^3*c^3*e*f^5 - 10*A^3*B*b^3*c^3*d*f^5 - 4*A^3*B*b*c^5*d^2*f^4 - 4 \\
& *A*B^3*b*c^5*d^3*f^3 - 28*A^3*B*a^2*b*c^3*f^6 + 6*A^3*B*a*b^3*c^2*f^6 - 4*A \\
& ^4*b*c^5*d*e*f^4 - 20*A^4*a*b*c^4*e*f^5 + 3*A^2*B^2*b^4*c^2*e^2*f^4 - 2*A^2
\end{aligned}$$

$$\begin{aligned}
& *B^2*b^3*c^3*e^3*f^3 + 12*A^2*B^2*a^2*c^4*e^2*f^4 + 9*A^2*B^2*b^2*c^4*d^2*f^4 \\
& - 3*A^2*B^2*a^2*b^2*c^2*f^6 - 2*B^4*b^3*c^3*d^2*e*f^3 + 4*B^4*a^2*c^4*d* \\
& e^2*f^3 - 10*B^4*a*b^2*c^3*d^2*f^4 - 3*B^4*a^2*b^2*c^2*d*f^5 + 3*A^3*B*b^2* \\
& c^4*e^3*f^3 - 2*A^3*B*b^3*c^3*e^2*f^4 - 10*A*B^3*b^3*c^3*d^2*f^4 - 4*A*B^3* \\
& a^2*c^4*e^3*f^3 + 3*A^2*B^2*b^4*c^2*d*f^5 + 36*A^2*B^2*a^2*c^4*d*f^5 - 24*A \\
& ^2*B^2*a*c^5*d^2*f^4 + 4*A^2*B^2*c^6*d^3*f^3 + 16*A^2*B^2*a^3*c^3*f^6 + 4*A \\
& ^4*b^3*c^3*e*f^5 + 16*B^4*a^3*c^3*d*f^5 + 16*A^4*a*c^5*e^2*f^4 + 8*A^4*b^2*c^4*d*f^5 \\
& - 8*A^4*a*b^2*c^3*f^6 - 24*A^4*a*c^5*d*f^5 + 3*B^4*b^4*c^2*d^2*f^4 - 3*A^4*b^2*c^4*e^2*f^4 \\
& + 4*A^4*c^6*d^2*f^4 + 36*A^4*a^2*c^4*f^6 + B^4*b^2*c^4*d^3*f^3, z, k)*((64*a^9*c^4*e*f^8 - 64*a^6*c^7*e^7*f^2 - 64*a^7*c^6*e^5*f^4 \\
& + 64*a^8*c^5*e^3*f^6 + 4*b^5*c^8*d^7*f^2 + 4*b^7*c^6*d^6*f^3 - 4*b^9*c^4*d^5*f^4 - 4*b^11*c^2*d^4*f^5 \\
& - 612*a^2*b^5*c^6*d^5*f^4 - 712*a^2*b^7*c^4*d^4*f^5 - 132*a^2*b^9*c^2*d^3*f^6 + 1696*a^3*b^3*c^7*d^5*f^4 \\
& + 2736*a^3*b^5*c^5*d^4*f^5 + 896*a^3*b^7*c^3*d^3*f^6 - 5120*a^4*b^3*c^6*d^4*f^5 - 3140*a^4*b^5*c^4*d^3*f^6 \\
& - 220*a^4*b^7*c^2*d^2*f^7 + 5664*a^5*b^3*c^5*d^3*f^6 + 1128*a^5*b^5*c^3*d^2*f^7 - 2560*a^6*b^3*c^4*d^2*f^7 \\
& + 4*a^3*b^6*c^4*e^7*f^2 - 6*a^3*b^7*c^3*e^6*f^3 + 4*a^3*b^8*c^2*e^5*f^4 - 36*a^4*b^4*c^5*e^7*f^2 \\
& + 57*a^4*b^5*c^4*e^6*f^3 - 37*a^4*b^6*c^3*e^5*f^4 + 7*a^4*b^7*c^2*e^4*f^5 + 96*a^5*b^2*c^6*e^7*f^2 \\
& - 168*a^5*b^3*c^5*e^6*f^3 + 100*a^5*b^4*c^4*e^5*f^4 - 3*a^5*b^5*c^3*e^4*f^5 - 10*a^5*b^6*c^2*e^3*f^6 \\
& - 48*a^6*b^2*c^5*e^5*f^4 - 56*a^6*b^3*c^4*e^4*f^5 + 36*a^6*b^4*c^3*e^3*f^6 - 13*a^6*b^5*c^2*e^2*f^7 \\
& - 64*a^7*b^2*c^4*e^3*f^6 + 56*a^7*b^3*c^3*e^2*f^7 - 1472*a^4*c^9*d^4*e^3*f^2 - 1088*a^5*c^8*d^2*e^5*f^2 \\
& + 3584*a^5*c^8*d^3*e^3*f^3 - 3200*a^6*c^7*d^2*e^3*f^4 - 2*b^7*c^6*d^5*e^2*f^2 - b^8*c^5*d^4*e^3*f^2 \\
& + 4*b^9*c^4*d^3*e^4*f^2 - 5*b^9*c^4*d^4*e^2*f^3 - 6*b^10*c^3*d^3*e^3*f^3 + 4*b^11*c^2*d^3*e^2*f^4 \\
& + 8*a*b^11*c*d^3*f^6 + 8*a^5*b^7*c*d*f^8 - 448*a^8*b*c^4*d*f^8 - 16*a^5*b*c^7*e^8*f - a^6*b^6*c*e*f^8 \\
& + 128*a^5*c^8*d*e^7*f + 128*a^8*c^5*d*e*f^7 - b^12*c*d^3*e*f^5 - 32*a*b^3*c^9*d^7*f^2 - 24*a*b^5*c^7*d^6*f^3 \\
& + 88*a*b^7*c^5*d^5*f^4 + 88*a*b^9*c^3*d^4*f^5 + 64*a^2*b*c^10*d^7*f^2 + 128*a^3*b*c^9*d^6*f^3 \\
& + 16*a^3*b^9*c*d^2*f^7 - 1600*a^4*b*c^8*d^5*f^4 + 3840*a^5*b*c^7*d^4*f^5 - 4160*a^6*b*c^6*d^3*f^6 \\
& - 92*a^6*b^5*c^2*d*f^8 + 2176*a^7*b*c^5*d^2*f^7 + 352*a^7*b^3*c^3*d*f^8 - a^3*b^5*c^5*e^8*f \\
& - a^3*b^9*c*e^4*f^5 + 8*a^4*b^3*c^6*e^8*f + a^4*b^8*c*e^3*f^6 + a^5*b^7*c*e^2*f^7 + 144*a^6*b*c^6*e^6*f^3 \\
& + 80*a^7*b*c^5*e^4*f^5 + 12*a^7*b^4*c^2*e*f^8 - 80*a^8*b*c^4*e^2*f^7 - 48*a^8*b^2*c^3*e*f^8 \\
& + 128*a^3*c^10*d^5*e^3*f - 448*a^3*c^10*d^6*e*f^2 + 256*a^4*c^9*d^3*e^5*f + 2176*a^4*c^9*d^5*e*f^3 \\
& - 4160*a^5*c^8*d^4*e*f^4 + 896*a^6*c^7*d^5*f^3 + 3840*a^6*c^7*d^3*e*f^5 + 896*a^7*c^6*d^5*f^5 \\
& - 1600*a^7*c^6*d^2*e*f^6 - b^5*c^8*d^6*e^2*f + b^6*c^7*d^5*e^3*f - 5*b^6*c^7*d^6*e*f^2 \\
& + b^7*c^6*d^4*e^4*f - b^8*c^5*d^3*e^5*f + 5*b^8*c^5*d^5*e*f^3 + 9*b^10*c^3*d^4*e*f^4 \\
& + 8*a*b^3*c^9*d^6*e^2*f + 12*a*b^4*c^8*d^6*e*f^2 - 27*a*b^5*c^7*d^4*e^4*f + 18*a*b^6*c^6*d^3*e^5*f \\
& - 154*a*b^6*c^6*d^5*e*f^3 + a*b^7*c^5*d^2*e^6*f - 200*a*b^8*c^4*d^4*e*f^4 - 2*a*b^10*c^2*d^3*e*f^5 \\
& + a*b^11*c*d^2*e^2*f^5 - 16*a^2*b*c^10*d^6*e^2*f + a^2*b^6*c^5*d*e^7*f + a^2*b^10*c*d^3*f^5 \\
& - 19*a^2*b^10*c*d^2*e*f^6 - 304*a^3*b*c^9*d^4*e^4*f + 10*a^3*b^9*c*d*e^2*f^6 - 304*a^4*b*c^8*d^2*e^6*f \\
& - 48*a^4*b^2*c^7*d*e^7*f - 160*a^5*b*c^7*
\end{aligned}$$

$$\begin{aligned}
& d^6e^6f^2 + 214a^5b^6c^2d^2e^6f^7 - 2048a^6b^6c^6d^2e^4f^4 - 792a^6b^4c^3d^2e^6f^7 - 1824a^7b^6c^5d^2e^2f^6 + 928a^7b^2c^4d^2e^6f^7 + 78a^8b^5c^7d^5e^2f^2 + 10a^8b^6c^6d^4e^3f^2 - 68a^8b^7c^5d^3e^4f^2 + 129a^8b^7c^5d^4e^2f^3 - 4a^8b^8c^4d^2e^5f^2 + 96a^8b^8c^4d^3e^3f^3 + 6a^8b^9c^3d^2e^4f^3 - 52a^8b^9c^3d^3e^2f^4 - 4a^8b^{10}c^2d^2e^3f^4 - 48a^2b^2c^9d^5e^3f + 144a^2b^2c^9d^6e^6f^2 + 168a^2b^3c^8d^4e^4f - 80a^2b^4c^7d^3e^5f + 1128a^2b^4c^7d^5e^6f^3 - 27a^2b^5c^6d^2e^6f + 1481a^2b^6c^5d^4e^6f^4 - 4a^2b^7c^4d^2e^6f^2 + 6a^2b^8c^3d^2e^5f^3 + 126a^2b^8c^3d^3e^6f^5 - 4a^2b^9c^2d^2e^4f^4 + 992a^3b^6c^9d^5e^2f^2 + 32a^3b^2c^8d^3e^5f - 2912a^3b^2c^8d^5e^6f^3 + 168a^3b^3c^7d^2e^6f - 4812a^3b^4c^6d^4e^6f^4 + 22a^3b^5c^5d^2e^6f^2 - 68a^3b^6c^4d^2e^5f^3 - 860a^3b^6c^4d^3e^6f^5 + 80a^3b^7c^3d^2e^4f^4 - 44a^3b^8c^2d^2e^3f^5 + 232a^3b^8c^2d^2e^2f^6 + 1280a^4b^6c^8d^3e^4f^2 - 816a^4b^6c^8d^4e^2f^3 + 7088a^4b^2c^7d^4e^6f^4 + 16a^4b^3c^6d^2e^6f^2 + 312a^4b^4c^5d^2e^5f^3 + 2864a^4b^4c^5d^3e^6f^5 - 576a^4b^5c^4d^2e^4f^4 + 393a^4b^6c^3d^2e^3f^5 - 995a^4b^6c^3d^2e^6f^6 - 78a^4b^7c^2d^2e^2f^6 + 992a^5b^6c^7d^2e^4f^3 - 3264a^5b^6c^7d^3e^2f^4 - 768a^5b^2c^6d^2e^5f^3 - 5184a^5b^2c^6d^3e^6f^5 + 1792a^5b^3c^5d^2e^4f^4 - 1232a^5b^4c^4d^2e^3f^5 + 1588a^5b^4c^4d^2e^6f^6 + 30a^5b^5c^3d^2e^2f^6 + 5008a^6b^6c^6d^2e^2f^5 + 976a^6b^2c^5d^2e^3f^5 - 16a^6b^2c^5d^2e^6f^6 + 944a^6b^3c^4d^2e^2f^6 - 19a^4b^8c^2d^2e^6f^7 - 528a^2b^3c^8d^5e^2f^2 - 124a^2b^4c^7d^4e^3f^2 + 432a^2b^5c^6d^3e^4f^2 - 843a^2b^5c^6d^4e^2f^3 + 83a^2b^6c^5d^2e^5f^2 - 722a^2b^6c^5d^3e^3f^3 - 80a^2b^7c^4d^2e^4f^3 + 376a^2b^7c^4d^3e^2f^4 + 43a^2b^9c^2d^2e^2f^5 + 768a^3b^2c^8d^4e^3f^2 - 1216a^3b^3c^7d^3e^4f^2 + 1832a^3b^3c^7d^4e^2f^3 - 540a^3b^4c^6d^2e^5f^2 + 2928a^3b^4c^6d^3e^3f^3 + 414a^3b^5c^5d^2e^4f^3 - 1740a^3b^5c^5d^3e^2f^4 + 344a^3b^6c^4d^2e^3f^4 - 634a^3b^7c^3d^2e^2f^5 + 1360a^4b^2c^7d^2e^5f^2 - 5664a^4b^2c^7d^3e^3f^3 - 1008a^4b^3c^6d^2e^4f^3 + 4064a^4b^3c^6d^3e^2f^4 - 1928a^4b^4c^5d^2e^3f^4 + 3065a^4b^5c^4d^2e^2f^5 + 4032a^5b^2c^6d^2e^3f^4 - 6376a^5b^3c^5d^2e^2f^5)/(16a^2c^6d^4 + a^4b^4f^4 + 16a^4c^4e^4 + b^4c^4d^4 + 16a^6c^2f^4 + b^8d^2f^2 - 8a^2b^2c^5d^4 - 8a^5b^2c^4f^4 + 2a^2b^6d^2f^3 - 2a^3b^5e^6f^3 - 64a^3c^5d^3f - 64a^5c^3d^2f^3 - 2b^5c^3d^3e + 2b^6c^2d^3f + a^2b^4c^2e^4 - 8a^3b^2c^3e^4 + 32a^3c^5d^2e^2 + a^2b^6e^2f^2 + 96a^4c^4d^2f^2 + b^6c^2d^2e^2 + 32a^5c^3e^2f^2 - 2a^2b^7d^2e^6f^2 - 2b^7c^2d^2e^6f + 54a^2b^4c^2d^2f^2 - 112a^3b^2c^3d^2f^2 + 16a^2b^3c^4d^3e - 2a^2b^5c^2d^2e^3 - 32a^2b^6c^5d^3e - 32a^3b^6c^4d^2e^3 - 20a^2b^4c^3d^3f - 12a^2b^6c^3d^2f^2 - 20a^3b^4c^3d^2f^3 - 2a^2b^5c^3e^3f - 32a^4b^6c^3e^3f + 16a^4b^3c^2e^6f^3 - 32a^5b^6c^2e^6f^3 - 64a^4c^4d^2e^2f - 6a^2b^4c^3d^2e^2 + 16a^2b^3c^3d^2e^3 + 64a^2b^2c^4d^3f + 64a^4b^2c^2d^2f^3 + 16a^3b^3c^2e^3f - 6a^3b^4c^2e^2f^2 - 48a^2b^3c^3d^2e^6f - 36a^2b^4c^2d^2e^2f + 96a^3b^2c^3d^2e^2f - 48a^3b^3c^2d^2e^6f^2 + 4
\end{aligned}$$

$$\begin{aligned}
& *a*b^6*c*d*e^2*f + 18*a*b^5*c^2*d^2*e*f + 18*a^2*b^5*c*d*e*f^2 + 32*a^3*b*c \\
& ^4*d^2*e*f + 32*a^4*b*c^3*d*e*f^2) + (x*(128*a^9*c^4*f^9 - 2*a^6*b^6*c*f^9 \\
& - 640*a^8*c^5*d*f^8 + 96*a^5*c^8*e^8*f + 6*b^12*c*d^3*f^6 + 24*a^7*b^4*c^2* \\
& f^9 - 96*a^8*b^2*c^3*f^9 + 128*a^2*c^11*d^7*f^2 - 640*a^3*c^10*d^6*f^3 + 11 \\
& 52*a^4*c^9*d^5*f^4 - 640*a^5*c^8*d^4*f^5 - 640*a^6*c^7*d^3*f^6 + 1152*a^7*c \\
& ^6*d^2*f^7 + 288*a^6*c^7*e^6*f^3 + 416*a^7*c^6*e^4*f^5 + 352*a^8*c^5*e^2*f^ \\
& 7 + 8*b^4*c^9*d^7*f^2 + 22*b^6*c^7*d^6*f^3 + 26*b^8*c^5*d^5*f^4 + 18*b^10*c \\
& ^3*d^4*f^5 + 672*a^2*b^2*c^9*d^6*f^3 + 1224*a^2*b^4*c^7*d^5*f^4 + 1202*a^2* \\
& b^6*c^5*d^4*f^5 + 564*a^2*b^8*c^3*d^3*f^6 - 2048*a^3*b^2*c^8*d^5*f^4 - 2744 \\
& *a^3*b^4*c^6*d^4*f^5 - 1736*a^3*b^6*c^4*d^3*f^6 - 128*a^3*b^8*c^2*d^2*f^7 + \\
& 2656*a^4*b^2*c^7*d^4*f^5 + 2648*a^4*b^4*c^5*d^3*f^6 + 570*a^4*b^6*c^3*d^2* \\
& f^7 - 1344*a^5*b^2*c^6*d^3*f^6 - 904*a^5*b^4*c^4*d^2*f^7 - 160*a^6*b^2*c^5* \\
& d^2*f^7 + 8*a^2*b^7*c^4*e^7*f^2 - 12*a^2*b^8*c^3*e^6*f^3 + 8*a^2*b^9*c^2*e^ \\
& 5*f^4 - 90*a^3*b^5*c^5*e^7*f^2 + 132*a^3*b^6*c^4*e^6*f^3 - 76*a^3*b^7*c^3*e^ \\
& ^5*f^4 + 6*a^3*b^8*c^2*e^4*f^5 + 336*a^4*b^3*c^6*e^7*f^2 - 462*a^4*b^4*c^5* \\
& e^6*f^3 + 164*a^4*b^5*c^4*e^5*f^4 + 106*a^4*b^6*c^3*e^4*f^5 - 56*a^4*b^7*c^ \\
& 2*e^3*f^6 + 432*a^5*b^2*c^6*e^6*f^3 + 288*a^5*b^3*c^5*e^5*f^4 - 598*a^5*b^4 \\
& *c^4*e^4*f^5 + 102*a^5*b^5*c^3*e^3*f^6 + 90*a^5*b^6*c^2*e^2*f^7 + 720*a^6*b \\
& ^2*c^5*e^4*f^5 + 336*a^6*b^3*c^4*e^3*f^6 - 314*a^6*b^4*c^3*e^2*f^7 + 240*a^ \\
& 7*b^2*c^4*e^2*f^7 + 64*a^3*c^10*d^5*e^2*f^2 - 768*a^4*c^9*d^3*e^4*f^2 + 416 \\
& *a^4*c^9*d^4*e^2*f^3 + 1856*a^5*c^8*d^2*e^4*f^3 - 1664*a^5*c^8*d^3*e^2*f^4 \\
& + 2336*a^6*c^7*d^2*e^2*f^5 + 26*b^6*c^7*d^5*e^2*f^2 - 8*b^7*c^6*d^4*e^3*f^2 \\
& - 10*b^8*c^5*d^3*e^4*f^2 + 58*b^8*c^5*d^4*e^2*f^3 + 8*b^9*c^4*d^2*e^5*f^2 \\
& - 12*b^9*c^4*d^3*e^3*f^3 - 12*b^10*c^3*d^2*e^4*f^3 + 36*b^10*c^3*d^3*e^2*f^ \\
& 4 + 8*b^11*c^2*d^2*e^3*f^4 + 2*a^4*b^8*c*d*f^8 + 6*a^5*b^7*c*e*f^8 - 384*a^ \\
& 8*b*c^4*e*f^8 - 64*a*b^2*c^10*d^7*f^2 - 216*a*b^4*c^8*d^6*f^3 - 300*a*b^6*c \\
& ^6*d^5*f^4 - 240*a*b^8*c^4*d^4*f^5 - 92*a*b^10*c^2*d^3*f^6 + 10*a^2*b^10*c* \\
& d^2*f^7 - 12*a^5*b^6*c^2*d*f^8 - 40*a^6*b^4*c^3*d*f^8 + 384*a^7*b^2*c^4*d*f \\
& ^8 - 2*a^2*b^6*c^5*e^8*f - 2*a^2*b^10*c*e^4*f^5 + 22*a^3*b^4*c^6*e^8*f + 6* \\
& a^3*b^9*c*e^3*f^6 - 80*a^4*b^2*c^7*e^8*f - 8*a^4*b^8*c*e^2*f^7 - 416*a^5*b* \\
& c^7*e^7*f^2 - 960*a^6*b*c^6*e^5*f^4 - 72*a^6*b^5*c^2*e*f^8 - 928*a^7*b*c^5* \\
& e^3*f^6 + 288*a^7*b^3*c^3*e*f^8 - 32*a^2*c^11*d^6*e^2*f + 32*a^3*c^10*d^4*e \\
& ^4*f + 160*a^4*c^9*d^2*e^6*f - 704*a^5*c^8*d*e^6*f^2 - 1536*a^6*c^7*d*e^4*f \\
& ^4 - 1472*a^7*c^6*d*e^2*f^6 - 2*b^4*c^9*d^6*e^2*f + 6*b^5*c^8*d^5*e^3*f - 2 \\
& 4*b^5*c^8*d^6*e*f^2 - 8*b^6*c^7*d^4*e^4*f + 6*b^7*c^6*d^3*e^5*f - 58*b^7*c^ \\
& 6*d^5*e*f^3 - 2*b^8*c^5*d^2*e^6*f - 60*b^9*c^4*d^4*e*f^4 - 26*b^11*c^2*d^3* \\
& e*f^5 - 2*b^12*c*d^2*e^2*f^5 + 16*a*b^2*c^10*d^6*e^2*f - 48*a*b^3*c^9*d^5*e \\
& ^3*f + 192*a*b^3*c^9*d^6*e*f^2 + 66*a*b^4*c^8*d^4*e^4*f - 52*a*b^5*c^7*d^3* \\
& e^5*f + 576*a*b^5*c^7*d^5*e*f^3 + 14*a*b^6*c^6*d^2*e^6*f + 718*a*b^7*c^5*d^ \\
& 4*e*f^4 - 16*a*b^8*c^4*d*e^6*f^2 + 24*a*b^9*c^3*d*e^5*f^3 + 356*a*b^9*c^3*d \\
& ^3*e*f^5 - 16*a*b^10*c^2*d*e^4*f^4 + 96*a^2*b*c^10*d^5*e^3*f - 384*a^2*b*c^ \\
& 10*d^6*e*f^2 - 42*a^2*b^5*c^6*d*e^7*f - 2*a^2*b^10*c*d*e^2*f^6 - 64*a^3*b*c \\
& ^9*d^3*e^5*f + 1792*a^3*b*c^9*d^5*e*f^3 + 144*a^3*b^3*c^7*d*e^7*f - 3712*a^ \\
& 4*b*c^8*d^4*e*f^4 + 14*a^4*b^7*c^2*d*e*f^7 + 2368*a^5*b*c^7*d*e^5*f^3 + 460 \\
& 8*a^5*b*c^7*d^3*e*f^5 + 192*a^5*b^5*c^3*d*e*f^7 + 3808*a^6*b*c^6*d*e^3*f^5
\end{aligned}$$

$$\begin{aligned}
& - 3712*a^6*b*c^6*d^2*e*f^6 - 1184*a^6*b^3*c^4*d*e*f^7 - 204*a*b^4*c^8*d^5*e \\
& ^2*f^2 + 46*a*b^5*c^7*d^4*e^3*f^2 + 132*a*b^6*c^6*d^3*e^4*f^2 - 590*a*b^6*c \\
& ^6*d^4*e^2*f^3 - 90*a*b^7*c^5*d^2*e^5*f^2 + 64*a*b^7*c^5*d^3*e^3*f^3 + 196* \\
& a*b^8*c^4*d^2*e^4*f^3 - 408*a*b^8*c^4*d^3*e^2*f^4 - 188*a*b^9*c^3*d^2*e^3*f \\
& ^4 + 78*a*b^10*c^2*d^2*e^2*f^5 - 144*a^2*b^2*c^9*d^4*e^4*f + 128*a^2*b^3*c^ \\
& 8*d^3*e^5*f - 1824*a^2*b^3*c^8*d^5*e*f^3 - 6*a^2*b^4*c^7*d^2*e^6*f - 3096*a \\
& ^2*b^5*c^6*d^4*e*f^4 + 190*a^2*b^6*c^5*d*e^6*f^2 - 316*a^2*b^7*c^4*d*e^5*f^ \\
& 3 - 1908*a^2*b^7*c^4*d^3*e*f^5 + 228*a^2*b^8*c^3*d*e^4*f^4 - 58*a^2*b^9*c^2 \\
& *d*e^3*f^5 + 92*a^2*b^9*c^2*d^2*e*f^6 - 288*a^3*b*c^9*d^4*e^3*f^2 - 112*a^3 \\
& *b^2*c^8*d^2*e^6*f + 5664*a^3*b^3*c^7*d^4*e*f^4 - 796*a^3*b^4*c^6*d*e^6*f^2 \\
& + 1524*a^3*b^5*c^5*d*e^5*f^3 + 5120*a^3*b^5*c^5*d^3*e*f^5 - 1176*a^3*b^6*c \\
& ^4*d*e^4*f^4 + 240*a^3*b^7*c^3*d*e^3*f^5 - 116*a^3*b^7*c^3*d^2*e*f^6 + 68*a \\
& ^3*b^8*c^2*d*e^2*f^6 + 192*a^4*b*c^8*d^2*e^5*f^2 + 2240*a^4*b*c^8*d^3*e^3*f \\
& ^3 + 1344*a^4*b^2*c^7*d*e^6*f^2 - 3168*a^4*b^3*c^6*d*e^5*f^3 - 7232*a^4*b^3 \\
& *c^6*d^3*e*f^5 + 2464*a^4*b^4*c^5*d*e^4*f^4 + 78*a^4*b^5*c^4*d*e^3*f^5 - 11 \\
& 60*a^4*b^5*c^4*d^2*e*f^6 - 574*a^4*b^6*c^3*d*e^2*f^6 - 4928*a^5*b*c^7*d^2*e \\
& ^3*f^4 - 1152*a^5*b^2*c^6*d*e^4*f^4 - 2416*a^5*b^3*c^5*d*e^3*f^5 + 4096*a^5 \\
& *b^3*c^5*d^2*e*f^6 + 1748*a^5*b^4*c^4*d*e^2*f^6 - 1280*a^6*b^2*c^5*d*e^2*f^ \\
& 6 + 4*a*b^7*c^5*d*e^7*f + 4*a*b^11*c*d*e^3*f^5 - 10*a*b^11*c*d^2*e*f^6 - 4* \\
& a^3*b^9*c*d*e*f^7 - 160*a^4*b*c^8*d*e^7*f + 1792*a^7*b*c^5*d*e*f^7 + 384*a^ \\
& 2*b^2*c^9*d^5*e^2*f^2 + 16*a^2*b^3*c^8*d^4*e^3*f^2 - 624*a^2*b^4*c^7*d^3*e^ \\
& 4*f^2 + 1962*a^2*b^4*c^7*d^4*e^2*f^3 + 348*a^2*b^5*c^6*d^2*e^5*f^2 + 204*a^ \\
& 2*b^5*c^6*d^3*e^3*f^3 - 1214*a^2*b^6*c^5*d^2*e^4*f^3 + 1636*a^2*b^6*c^5*d^3 \\
& *e^2*f^4 + 1520*a^2*b^7*c^4*d^2*e^3*f^4 - 750*a^2*b^8*c^3*d^2*e^2*f^5 + 121 \\
& 6*a^3*b^2*c^8*d^3*e^4*f^2 - 2224*a^3*b^2*c^8*d^4*e^2*f^3 - 512*a^3*b^3*c^7* \\
& d^2*e^5*f^2 - 1632*a^3*b^3*c^7*d^3*e^3*f^3 + 3492*a^3*b^4*c^6*d^2*e^4*f^3 - \\
& 2824*a^3*b^4*c^6*d^3*e^2*f^4 - 5492*a^3*b^5*c^5*d^2*e^3*f^4 + 2868*a^3*b^6 \\
& *c^4*d^2*e^2*f^5 - 4480*a^4*b^2*c^7*d^2*e^4*f^3 + 2432*a^4*b^2*c^7*d^3*e^2* \\
& f^4 + 8864*a^4*b^3*c^6*d^2*e^3*f^4 - 4206*a^4*b^4*c^5*d^2*e^2*f^5 + 432*a^5 \\
& *b^2*c^6*d^2*e^2*f^5)/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + 16*a^4*c^4*e^4 + b^4 \\
& *c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 \\
& + 2*a^2*b^6*d*f^3 - 2*a^3*b^5*e*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 \\
& - 2*b^5*c^3*d^3*e + 2*b^6*c^2*d^3*f + a^2*b^4*c^2*e^4 - 8*a^3*b^2*c^3*e^4 + \\
& 32*a^3*c^5*d^2*e^2 + a^2*b^6*e^2*f^2 + 96*a^4*c^4*d^2*f^2 + b^6*c^2*d^2*e^ \\
& 2 + 32*a^5*c^3*e^2*f^2 - 2*a*b^7*d*e*f^2 - 2*b^7*c*d^2*e*f + 54*a^2*b^4*c^2 \\
& *d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 + 16*a*b^3*c^4*d^3*e - 2*a*b^5*c^2*d*e^3 \\
& - 32*a^2*b*c^5*d^3*e - 32*a^3*b*c^4*d*e^3 - 20*a*b^4*c^3*d^3*f - 12*a*b^6* \\
& c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 - 2*a^2*b^5*c*e^3*f - 32*a^4*b*c^3*e^3*f + 1 \\
& 6*a^4*b^3*c*e*f^3 - 32*a^5*b*c^2*e*f^3 - 64*a^4*c^4*d*e^2*f - 6*a*b^4*c^3*d \\
& ^2*e^2 + 16*a^2*b^3*c^3*d*e^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3 \\
& + 16*a^3*b^3*c^2*e^3*f - 6*a^3*b^4*c*e^2*f^2 - 48*a^2*b^3*c^3*d^2*e*f - 36 \\
& *a^2*b^4*c^2*d*e^2*f + 96*a^3*b^2*c^3*d*e^2*f - 48*a^3*b^3*c^2*d*e*f^2 + 4* \\
& a*b^6*c*d*e^2*f + 18*a*b^5*c^2*d^2*e*f + 18*a^2*b^5*c*d*e*f^2 + 32*a^3*b*c^ \\
& 4*d^2*e*f + 32*a^4*b*c^3*d*e*f^2) - (64*A*a^7*c^4*f^8 - A*a^4*b^6*c*f^8 + \\
& 32*A*a*c^10*d^6*f^2 - 352*A*a^6*c^5*d*f^7 - A*b^10*c*d^2*f^6 + 8*B*a^4*c^7*
\end{aligned}$$

$$\begin{aligned}
& e^7 f - 64 B a^7 c^4 e f^7 + 12 A a^5 b^4 c^2 f^8 - 48 A a^6 b^2 c^3 f^8 - \\
& 224 A a^2 c^9 d^5 f^3 + 640 A a^3 c^8 d^4 f^4 - 960 A a^4 c^7 d^3 f^5 + 800 \\
& A a^5 c^6 d^2 f^6 - 40 A a^4 c^7 e^6 f^2 - 80 A a^5 c^6 e^4 f^4 + 24 A a^6 \\
& c^5 e^2 f^6 - 8 A b^2 c^9 d^6 f^2 - 16 A b^4 c^7 d^5 f^3 - A b^6 c^5 d^4 f^4 \\
& + 6 A b^8 c^3 d^3 f^5 + 16 B a^5 c^6 e^5 f^3 - 56 B a^6 c^5 e^3 f^5 + 4 B \\
& b^3 c^8 d^6 f^2 + 12 B b^5 c^6 d^5 f^3 + 4 B b^7 c^4 d^4 f^4 - 4 B b^9 c^2 \\
& d^3 f^5 + 120 A a b^2 c^8 d^5 f^3 + 60 A a b^4 c^6 d^4 f^4 - 36 A a b^6 c^4 \\
& d^3 f^5 + 8 A a b^8 c^2 d^2 f^6 + 20 A a^3 b^6 c^2 d f^7 - 80 A a^4 b^4 c^3 \\
& d f^7 + 216 A a^5 b^2 c^4 d f^7 + 4 A a a b^6 c^4 e^6 f^2 - 6 A a a b^7 c^3 \\
& e^5 f^3 + 4 A a a b^8 c^2 e^4 f^4 + 9 A a^2 b^3 c^6 e^7 f + 2 A a^2 b^8 c e^2 \\
& f^6 + 88 A a^4 b c^6 e^5 f^3 + 172 A a^5 b c^5 e^3 f^5 - 92 B a a b^3 c^7 d \\
& ^5 f^3 - 72 B a a b^5 c^5 d^4 f^4 + 20 B a a b^7 c^3 d^3 f^5 + 176 B a^2 b c^8 d \\
& ^5 f^3 - 544 B a^3 b c^7 d^4 f^4 + 736 B a^4 b c^6 d^3 f^5 + 4 B a^4 b^5 c^2 \\
& d f^7 - 464 B a^5 b c^5 d^2 f^6 - 44 B a^5 b^3 c^3 d f^7 - 2 B a^3 b^2 c^6 \\
& e^7 f - B a^3 b^7 c e^2 f^6 - 28 B a^4 b c^6 e^6 f^2 - 56 B a^5 b c^5 e^4 \\
& f^4 - 12 B a^5 b^4 c^2 e f^7 + 36 B a^6 b c^4 e^2 f^6 + 48 B a^6 b^2 c^3 e \\
& f^7 - 16 A a^2 c^9 d^3 e^4 f + 48 A a^4 c^7 d e^4 f^3 - 168 A a^5 c^6 d e^2 \\
& f^5 + 2 A b^2 c^9 d^5 e^2 f - 3 A b^3 c^8 d^4 e^3 f + 12 A b^3 c^8 d^5 e \\
& f^2 - 4 A b^7 c^4 d e^5 f^2 - 12 A b^7 c^4 d^3 e f^4 + 6 A b^8 c^3 d e^4 f^3 \\
& - 4 A b^9 c^2 d e^3 f^4 + 8 A b^9 c^2 d^2 e f^5 + 8 B a^2 c^9 d^4 e^3 f \\
& - 32 B a^2 c^9 d^5 e f^2 + 16 B a^3 c^8 d^2 e^5 f + 64 B a^3 c^8 d^4 e f^3 \\
& + 64 B a^4 c^7 d^3 e f^4 + 96 B a^5 c^6 d e^3 f^4 - 256 B a^5 c^6 d^2 e f^5 \\
& - B b^3 c^8 d^5 e^2 f + 2 B b^4 c^7 d^4 e^3 f - 8 B b^4 c^7 d^5 e f^2 - B \\
& b^6 c^5 d^2 e^5 f - 3 B b^6 c^5 d^4 e f^3 + 12 B b^8 c^3 d^3 e f^4 - 384 A a^2 \\
& b^2 c^7 d^4 f^4 - 32 A a^2 b^4 c^5 d^3 f^5 - 14 A a^2 b^6 c^3 d^2 f^6 + \\
& 560 A a^3 b^2 c^6 d^3 f^5 + 56 A a^3 b^4 c^4 d^2 f^6 - 456 A a^4 b^2 c^5 d^2 \\
& f^6 - 38 A a^2 b^4 c^5 e^6 f^2 + 58 A a^2 b^5 c^4 e^5 f^3 - 36 A a^2 b^6 c^3 \\
& e^4 f^4 + 5 A a^2 b^7 c^2 e^3 f^5 + 98 A a^3 b^2 c^6 e^6 f^2 - 158 A a^3 \\
& b^3 c^5 e^5 f^3 + 80 A a^3 b^4 c^4 e^4 f^4 + 22 A a^3 b^5 c^3 e^3 f^5 - \\
& 22 A a^3 b^6 c^2 e^2 f^6 + 20 A a^4 b^2 c^5 e^4 f^4 - 147 A a^4 b^3 c^4 e^3 \\
& f^5 + 80 A a^4 b^4 c^3 e^2 f^6 - 102 A a^5 b^2 c^4 e^2 f^6 + 360 B a^2 b^3 \\
& c^6 d^4 f^4 + 64 B a^2 b^5 c^4 d^3 f^5 - 504 B a^3 b^3 c^5 d^3 f^5 - 40 B a^3 \\
& b^5 c^3 d^2 f^6 + 276 B a^4 b^3 c^4 d^2 f^6 + 7 B a^3 b^3 c^5 e^6 f^2 - \\
& 8 B a^3 b^4 c^4 e^5 f^3 + 2 B a^3 b^5 c^3 e^4 f^4 + 2 B a^3 b^6 c^2 e^3 f^5 \\
& + 28 B a^4 b^2 c^5 e^5 f^3 + 6 B a^4 b^3 c^4 e^4 f^4 - 26 B a^4 b^4 c^3 e^3 \\
& f^5 + 11 B a^4 b^5 c^2 e^2 f^6 + 86 B a^5 b^2 c^4 e^3 f^5 - 37 B a^5 b^3 \\
& c^3 e^2 f^6 + 120 A a^2 c^9 d^4 e^2 f^2 + 48 A a^3 c^8 d^2 e^4 f^2 - 336 A \\
& a^3 c^8 d^3 e^2 f^3 + 368 A a^4 c^7 d^2 e^2 f^4 + 4 A b^4 c^7 d^4 e^2 f^2 \\
& - 5 A b^6 c^5 d^2 e^4 f^2 + 6 A b^6 c^5 d^3 e^2 f^3 + 16 A b^7 c^4 d^2 e^3 f^3 \\
& - 18 A b^8 c^3 d^2 e^2 f^4 - 32 B a^3 c^8 d^3 e^3 f^2 - 16 B a^4 c^7 d^2 \\
& e^3 f^3 - 3 B b^5 c^6 d^4 e^2 f^2 + 4 B b^6 c^5 d^3 e^3 f^2 + 4 B b^7 c^4 \\
& d^2 e^4 f^2 - 12 B b^7 c^4 d^3 e^2 f^3 - 6 B b^8 c^3 d^2 e^3 f^3 + 4 B b^9 \\
& c^2 d^2 e^2 f^4 - 2 A a^2 b^8 c d f^7 - A a b^5 c^5 e^7 f - A a b^9 c e^3 \\
& f^5 - 20 A a^3 b c^7 e^7 f - 16 B a a b c^9 d^6 f^2 + 112 B a^6 b c^4 d f^7 + \\
& B a^4 b^6 c e f^7 - 8 A a c^10 d^5 e^2 f - 8 A a^3 c^8 d e^6 f + A b^6 c^5
\end{aligned}$$

$$\begin{aligned}
& *d^6e^6f + A^10b^10c^10d^10e^10f^10 + 224A^6B^6c^5d^5e^5f^6 - B^10c^10d^10e^10f^10 \\
& + 12A^9a^9b^9c^9d^9e^9f^9 - 48A^9a^9b^9c^9d^9e^9f^9 - 10A^9a^9b^9c^9d^9e^9f^9 \\
& + 80A^9a^9b^9c^9d^9e^9f^9 + 4B^9a^9b^9c^9d^9e^9f^9 + B^9a^9b^9c^9d^9e^9f^9 + B^9a^9b^9c^9d^9e^9f^9 \\
& + 4B^9a^9b^9c^9d^9e^9f^9 - 92A^9a^9b^9c^9d^9e^9f^9 + 132A^9a^9b^9c^9d^9e^9f^9 \\
& + 446A^9a^9b^9c^9d^9e^9f^9 - 604A^9a^9b^9c^9d^9e^9f^9 + 340A^9a^9b^9c^9d^9e^9f^9 \\
& + 72B^9a^9b^9c^9d^9e^9f^9 + 134B^9a^9b^9c^9d^9e^9f^9 - 306B^9a^9b^9c^9d^9e^9f^9 \\
& - 264B^9a^9b^9c^9d^9e^9f^9 + 188B^9a^9b^9c^9d^9e^9f^9 + 292B^9a^9b^9c^9d^9e^9f^9 \\
& + 6B^9a^9b^9c^9d^9e^9f^9 + 4A^9a^9b^9c^9d^9e^9f^9 + 2A^9a^9b^9c^9d^9e^9f^9 \\
& - 16A^9a^9b^9c^9d^9e^9f^9 + 48A^9a^9b^9c^9d^9e^9f^9 + 72A^9a^9b^9c^9d^9e^9f^9 \\
& - 84A^9a^9b^9c^9d^9e^9f^9 + 64A^9a^9b^9c^9d^9e^9f^9 - 88A^9a^9b^9c^9d^9e^9f^9 \\
& - 18A^9a^9b^9c^9d^9e^9f^9 - 8A^9a^9b^9c^9d^9e^9f^9 + 64A^9a^9b^9c^9d^9e^9f^9 \\
& + 26A^9a^9b^9c^9d^9e^9f^9 + 16A^9a^9b^9c^9d^9e^9f^9 + 192A^9a^9b^9c^9d^9e^9f^9 \\
& + 96A^9a^9b^9c^9d^9e^9f^9 - 136A^9a^9b^9c^9d^9e^9f^9 + 80A^9a^9b^9c^9d^9e^9f^9 \\
& - 192A^9a^9b^9c^9d^9e^9f^9 + 268A^9a^9b^9c^9d^9e^9f^9 - 10B^9a^9b^9c^9d^9e^9f^9 \\
& + 40B^9a^9b^9c^9d^9e^9f^9 - 2B^9a^9b^9c^9d^9e^9f^9 + 8B^9a^9b^9c^9d^9e^9f^9 \\
& + 36B^9a^9b^9c^9d^9e^9f^9 - 4B^9a^9b^9c^9d^9e^9f^9 - 88B^9a^9b^9c^9d^9e^9f^9 + 6 \\
& *B^9a^9b^9c^9d^9e^9f^9 - 4B^9a^9b^9c^9d^9e^9f^9 + 16B^9a^9b^9c^9d^9e^9f^9 \\
& + 8B^9a^9b^9c^9d^9e^9f^9 - 5B^9a^9b^9c^9d^9e^9f^9 - 8B^9a^9b^9c^9d^9e^9f^9 \\
& + 40B^9a^9b^9c^9d^9e^9f^9 + 104B^9a^9b^9c^9d^9e^9f^9 + 148B^9a^9b^9c^9d^9e^9f^9 \\
& - 344B^9a^9b^9c^9d^9e^9f^9 - 46A^9a^9b^9c^9d^9e^9f^9 - 4A^9a^9b^9c^9d^9e^9f^9 \\
& + 40A^9a^9b^9c^9d^9e^9f^9 - 36A^9a^9b^9c^9d^9e^9f^9 - 158A^9a^9b^9c^9d^9e^9f^9 \\
& + 196A^9a^9b^9c^9d^9e^9f^9 + 16A^9a^9b^9c^9d^9e^9f^9 - 176A^9a^9b^9c^9d^9e^9f^9 \\
& - 120A^9a^9b^9c^9d^9e^9f^9 + 380A^9a^9b^9c^9d^9e^9f^9 - 324A^9a^9b^9c^9d^9e^9f^9 \\
& + 272A^9a^9b^9c^9d^9e^9f^9 + 80A^9a^9b^9c^9d^9e^9f^9 - 280A^9a^9b^9c^9d^9e^9f^9 \\
& - 572A^9a^9b^9c^9d^9e^9f^9 + 508A^9a^9b^9c^9d^9e^9f^9 - 144A^9a^9b^9c^9d^9e^9f^9 \\
& - 4A^9a^9b^9c^9d^9e^9f^9 - 326A^9a^9b^9c^9d^9e^9f^9 + 31B^9a^9b^9c^9d^9e^9f^9 \\
& - 32B^9a^9b^9c^9d^9e^9f^9 - 40B^9a^9b^9c^9d^9e^9f^9 + 102B^9a^9b^9c^9d^9e^9f^9 \\
& + 72B^9a^9b^9c^9d^9e^9f^9 - 56B^9a^9b^9c^9d^9e^9f^9 - 76B^9a^9b^9c^9d^9e^9f^9 \\
& - 20B^9a^9b^9c^9d^9e^9f^9 + 24B^9a^9b^9c^9d^9e^9f^9 + 192B^9a^9b^9c^9d^9e^9f^9 \\
& - 42B^9a^9b^9c^9d^9e^9f^9 + 32B^9a^9b^9c^9d^9e^9f^9 + 32B^9a^9b^9c^9d^9e^9f^9 \\
& - 38B^9a^9b^9c^9d^9e^9f^9 - 9B^9a^9b^9c^9d^9e^9f^9 - 152B^9a^9b^9c^9d^9e^9f^9 \\
& + 360B^9a^9b^9c^9d^9e^9f^9 - 32B^9a^9b^9c^9d^9e^9f^9 - 144B^9a^9b^9c^9d^9e^9f^9 \\
& + 62B^9a^9b^9c^9d^9e^9f^9 - 40B^9a^9b^9c^9d^9e^9f^9 - 152B^9a^9b^9c^9d^9e^9f^9 \\
& + 14B^9a^9b^9c^9d^9e^9f^9 - 472B^9a^9b^9c^9d^9e^9f^9 - 120B^9a^9b^9c^9d^9e^9f^9 \\
& + 512B^9a^9b^9c^9d^9e^9f^9 - 13B^9a^9b^9c^9d^9e^9f^9 + 16a^9b^9c^9d^9e^9f^9 \\
& + b^9c^9d^9e^9f^9 + 16a^9b^9c^9d^9e^9f^9 + b^9c^9d^9e^9f^9 - 8a^9b^9c^9d^9e^9f^9 \\
& - 8a^9b^9c^9d^9e^9f^9 + 2a^9b^9c^9d^9e^9f^9 - 2a^9b^9c^9d^9e^9f^9 - 64a^9b^9c^9d^9e^9f^9 \\
& - 64a^9b^9c^9d^9e^9f^9 - 2b^9c^9d^9e^9f^9 + 2b^9c^9d^9e^9f^9 + a^9b^9c^9d^9e^9f^9 \\
& - 8a^9b^9c^9d^9e^9f^9 + 32a^9b^9c^9d^9e^9f^9 + a^9b^9c^9d^9e^9f^9 + 96a^9b^9c^9d^9e^9f^9 \\
& + b^9c^9d^9e^9f^9 + 32a^9b^9c^9d^9e^9f^9 - 2a^9b^9c^9d^9e^9f^9 - 2b^9c^9d^9e^9f^9
\end{aligned}$$

$$\begin{aligned}
& e^f + 54a^2b^4c^2d^2f^2 - 112a^3b^2c^3d^2f^2 + 16ab^3c^4d^3 \\
& e - 2ab^5c^2d^3e^3 - 32a^2b^3c^5d^3e - 32a^3b^3c^4d^3e^3 - 20ab^4c^3d^3f \\
& - 12ab^6c^3d^2f^2 - 20a^3b^4c^3d^3f^3 - 2a^2b^5c^3e^3f - 3 \\
& 2a^4b^3c^3e^3f + 16a^4b^3c^3e^3f^3 - 32a^5b^3c^2e^3f^3 - 64a^4c^4d^3 \\
& e^2f - 6ab^4c^3d^2e^2 + 16a^2b^3c^3d^3e^3 + 64a^2b^2c^4d^3f + \\
& 64a^4b^2c^2d^3f^3 + 16a^3b^3c^2e^3f - 6a^3b^4c^3e^2f^2 - 48a^2 \\
& b^3c^3d^2e^3f - 36a^2b^4c^2d^3e^2f + 96a^3b^2c^3d^3e^2f - 48a^3 \\
& b^3c^2d^3e^3f^2 + 4ab^6c^3d^3e^2f + 18ab^5c^2d^2e^3f + 18a^2b^5c^3 \\
& d^3e^3f^2 + 32a^3b^3c^4d^2e^3f + 32a^4b^3c^3d^3e^3f^2) + (x(64B^7c^4f \\
& ^8 + 4A^3b^7c^3f^8 - 256A^6b^3c^4f^8 - B^4b^6c^3f^8 + 48A^3c^8 \\
& e^7f + 256A^6c^5e^3f^7 - 320B^6c^5d^3f^7 + 3B^10c^3d^2f^6 - \\
& 48A^4b^5c^2f^8 + 192A^5b^3c^3f^8 + 12B^5b^4c^2f^8 - 48B^6 \\
& b^2c^3f^8 + 256A^4c^7e^5f^3 + 464A^5c^6e^3f^5 - 16A^3b^3 \\
& c^8d^5f^3 - 48A^3b^5c^6d^4f^4 - 36A^3b^7c^4d^3f^5 - 4A^3b^9c^2d^2 \\
& f^6 - 64B^2c^9d^5f^3 + 320B^3c^8d^4f^4 - 640B^4c^7d^3f^5 + \\
& 640B^5c^6d^2f^6 - 16B^4c^7e^6f^2 - 64B^5c^6e^4f^4 + 1 \\
& 6B^6c^5e^2f^6 + 4B^4c^7d^5f^3 + 23B^6c^5d^4f^4 + 22B^8 \\
& c^3d^3f^5 + 320A^3b^3c^7d^4f^4 + 352A^3b^5c^5d^3f^5 + 76A^3b^7 \\
& c^3d^2f^6 - 512A^2b^3c^8d^4f^4 - 60A^2b^7c^2d^3f^7 + 1408A^3 \\
& b^3c^7d^3f^5 + 352A^3b^5c^3d^3f^7 - 1792A^4b^3c^6d^2f^6 - 97 \\
& 6A^4b^3c^4d^3f^7 - 6A^3b^5c^5e^6f^2 + 4A^3b^6c^4e^5f^3 + 4A^3 \\
& b^7c^3e^4f^4 - 6A^3b^8c^2e^3f^5 - 20A^2b^2c^7e^7f - 144A^3 \\
& b^3c^7e^6f^2 + 68A^3b^6c^2e^3f^7 - 640A^4b^3c^6e^4f^4 - 240 \\
& A^4b^4c^3e^3f^7 - 848A^5b^3c^5e^2f^6 + 192A^5b^2c^4e^3f^7 - \\
& 132B^4b^4c^6d^4f^4 - 196B^4b^6c^4d^3f^5 - 40B^4b^8c^2d^2f^6 - \\
& 20B^3b^6c^2d^3f^7 + 52B^4b^4c^3d^3f^7 + 64B^5b^2c^4d^3f^7 \\
& + 2B^2b^3c^6e^7f + B^2b^8c^2e^2f^6 + 16B^4b^3c^6e^5f^3 + 12 \\
& 0B^5b^3c^5e^3f^5 + 64A^2c^9d^2e^5f + 512A^2c^9d^4e^3f^3 - \\
& 384A^3c^8d^3e^5f^2 - 1408A^3c^8d^3e^3f^4 - 1280A^4c^7d^3e^3f^4 \\
& + 1792A^4c^7d^2e^3f^5 - 4A^3b^2c^9d^4e^3f + 16A^3b^2c^9d^5e^3 \\
& f^2 + 8A^3b^3c^8d^3e^4f - 2A^3b^4c^7d^2e^5f + 80A^3b^4c^7d^4e^3f^3 \\
& + 6A^3b^6c^5d^3e^5f^2 + 76A^3b^6c^5d^3e^3f^4 - 4A^3b^7c^4d^3e^4f^3 \\
& - 4A^3b^8c^3d^2e^3f^4 - 2A^3b^8c^3d^2e^3f^5 + 6A^3b^9c^2d^2e^2f^5 - 3 \\
& 2B^2c^9d^3e^4f - 96B^4c^7d^3e^4f^3 - 192B^5c^6d^3e^2f^5 + \\
& 2B^3c^8d^4e^3f - 8B^3c^8d^5e^3f^2 - 6B^4c^7d^3e^4f + 4B^5 \\
& c^6d^2e^5f - 48B^5c^6d^4e^3f^3 - 60B^6b^7c^4d^3e^3f^4 - 8B^7 \\
& b^9c^2d^2e^3f^5 + 4A^3b^9c^3d^3f^7 - 2A^3b^10c^3d^3e^3f^6 - 1184A^2b^3 \\
& c^6d^3f^5 - 544A^2b^5c^4d^2f^6 + 1664A^3b^3c^5d^2f^6 + 60A^2 \\
& b^3c^6e^6f^2 - 30A^2b^4c^5e^5f^3 - 64A^2b^5c^4e^4f^4 + 72A^2b^6 \\
& c^3e^3f^5 - 12A^2b^7c^2e^2f^6 - 8A^3b^2c^6e^5f^3 + 352A^3b^3c^5 \\
& e^4f^4 - 268A^3b^4c^4e^3f^5 - 52A^3b^5c^3e^2f^6 + 188A^4b^2c^5 \\
& e^3f^5 + 484A^4b^3c^4e^2f^6 + 80B^2b^2c^7d^4f^4 + 520B^2b^4c^5 \\
& d^3f^5 + 210B^2b^6c^3d^2f^6 - 192B^3b^2c^6d^3f^5 - 456B^3b^4c^4 \\
& d^2f^6 + 96B^4b^4b^2c^5d^2f^6 - 7B^2b^4c^5e^6f^2 + 8B^2b^5c^4 \\
& e^5f^3 - 2B^2b^5c^4e^5f^3 - 2B^2b^5c^4e^5f^3 - 2B^2b^5c^4e^5f^3
\end{aligned}$$

$$\begin{aligned}
& 6*c^3*e^4*f^4 - 2*B*a^2*b^7*c^2*e^3*f^5 + 32*B*a^3*b^2*c^6*e^6*f^2 - 36*B*a^3*b^3*c^5*e^5*f^3 - 4*B*a^3*b^4*c^4*e^4*f^4 + 28*B*a^3*b^5*c^3*e^3*f^5 - 1 \\
& 2*B*a^3*b^6*c^2*e^2*f^6 + 64*B*a^4*b^2*c^5*e^4*f^4 - 110*B*a^4*b^3*c^4*e^3*f^5 + 47*B*a^4*b^4*c^3*e^2*f^6 - 64*B*a^5*b^2*c^4*e^2*f^6 - 384*A*a^2*c^9*d^3*e^3*f^2 + 1184*A*a^3*c^8*d^2*e^3*f^3 - 28*A*b^3*c^8*d^4*e^2*f^2 - 12*A*b^4*c^7*d^3*e^3*f^2 + 20*A*b^5*c^6*d^2*e^4*f^2 - 36*A*b^5*c^6*d^3*e^2*f^3 - \\
& 44*A*b^6*c^5*d^2*e^3*f^3 + 32*A*b^7*c^4*d^2*e^2*f^4 + 144*B*a^2*c^9*d^4*e^2*f^2 + 192*B*a^3*c^8*d^2*e^4*f^2 - 448*B*a^3*c^8*d^3*e^2*f^3 + 480*B*a^4*c^7*d^2*e^2*f^4 + 23*B*b^4*c^7*d^4*e^2*f^2 - 4*B*b^5*c^6*d^3*e^3*f^2 - 13*B*b^6*c^5*d^2*e^4*f^2 + 48*B*b^6*c^5*d^3*e^2*f^3 + 12*B*b^7*c^4*d^2*e^3*f^3 + \\
& 2*B*b^8*c^3*d^2*e^2*f^4 + 64*A*a*b*c^9*d^5*f^3 + 1088*A*a^5*b*c^5*d*f^7 + 2*A*a*b^4*c^6*e^7*f + 2*A*a*b^9*c*e^2*f^6 - 6*A*a^2*b^8*c*e*f^7 + 2*B*a^2*b^8*c*d*f^7 - 8*B*a^3*b*c^7*e^7*f + 16*A*a*c^10*d^4*e^3*f - 64*A*a*c^10*d^5*e*f^2 - 1088*A*a^5*c^6*d*e*f^6 - 2*A*b^5*c^6*d*e^6*f - 32*B*a^3*c^8*d*e^6*f - \\
& 32*A*a*b*c^9*d^3*e^4*f + 24*A*a*b^3*c^7*d*e^6*f + 24*A*a*b^8*c^2*d*e*f^6 - 64*A*a^2*b*c^8*d*e^6*f - 8*B*a*b*c^9*d^4*e^3*f + 32*B*a*b*c^9*d^5*e*f^2 - 6*B*a*b^4*c^6*d*e^6*f + 288*B*a^5*b*c^5*d*e*f^6 - 1496*A*a^2*b^2*c^7*d^2*e^3*f^3 + 1496*A*a^2*b^3*c^6*d^2*e^2*f^4 - 272*B*a^2*b^2*c^7*d^2*e^4*f^2 + 6 \\
& 40*B*a^2*b^2*c^7*d^3*e^2*f^3 + 636*B*a^2*b^3*c^6*d^2*e^3*f^3 - 286*B*a^2*b^4*c^5*d^2*e^2*f^4 + 192*B*a^3*b^2*c^6*d^2*e^2*f^4 + 112*A*a*b*c^9*d^4*e^2*f^2 - 8*A*a*b^2*c^8*d^2*e^5*f - 448*A*a*b^2*c^8*d^4*e*f^3 - 88*A*a*b^4*c^6*d*e^5*f^2 - 576*A*a*b^4*c^6*d^3*e*f^4 + 84*A*a*b^5*c^5*d*e^4*f^3 + 32*A*a*b^6*c^4*d*e^3*f^4 + 12*A*a*b^6*c^4*d^2*e*f^5 - 80*A*a*b^7*c^3*d*e^2*f^5 - 108 \\
& *A*a^2*b^6*c^3*d*e*f^6 + 736*A*a^3*b*c^7*d*e^4*f^3 + 192*A*a^3*b^4*c^4*d*e*f^6 + 2048*A*a^4*b*c^6*d*e^2*f^5 + 208*A*a^4*b^2*c^5*d*e*f^6 + 32*B*a*b^2*c^8*d^3*e^4*f - 20*B*a*b^3*c^7*d^2*e^5*f + 288*B*a*b^3*c^7*d^4*e*f^3 + 20*B*a*b^5*c^5*d*e^5*f^2 + 480*B*a*b^5*c^5*d^3*e*f^4 - 20*B*a*b^6*c^4*d*e^4*f^3 + 72*B*a*b^7*c^3*d^2*e*f^5 + 10*B*a*b^8*c^2*d*e^2*f^5 + 16*B*a^2*b*c^8*d^2*e^5*f - 384*B*a^2*b*c^8*d^4*e*f^3 + 32*B*a^2*b^2*c^7*d*e^6*f + 44*B*a^2*b^7*c^2*d*e*f^6 + 192*B*a^3*b*c^7*d*e^5*f^2 + 960*B*a^3*b*c^7*d^3*e*f^4 - 160*B*a^3*b^5*c^3*d*e*f^6 + 544*B*a^4*b*c^6*d*e^3*f^4 - 896*B*a^4*b*c^6*d^2*e*f^5 + 120*B*a^4*b^3*c^4*d*e*f^6 - 4*B*a*b^9*c*d*e*f^6 + 144*A*a*b^2*c^8*d^3*e^3*f^2 - 144*A*a*b^3*c^7*d^2*e^4*f^2 + 112*A*a*b^3*c^7*d^3*e^2*f^3 + 476*A*a*b^4*c^6*d^2*e^3*f^3 - 412*A*a*b^5*c^5*d^2*e^2*f^4 + 256*A*a^2*b*c^8*d^2*e^4*f^2 + 128*A*a^2*b*c^8*d^3*e^2*f^3 + 352*A*a^2*b^2*c^7*d*e^5*f^2 + 1440*A*a^2*b^2*c^7*d^3*e*f^4 - 456*A*a^2*b^3*c^6*d*e^4*f^3 - 116*A*a^2*b^4*c^5*d*e^3*f^4 + 224*A*a^2*b^4*c^5*d^2*e*f^5 + 452*A*a^2*b^5*c^4*d*e^2*f^5 - 1440*A*a^3*b*c^7*d^2*e^2*f^4 + 528*A*a^3*b^2*c^6*d*e^3*f^4 - 1408*A*a^3*b^2*c^6*d^2*e*f^5 - 1424*A*a^3*b^3*c^5*d*e^2*f^5 - 128*B*a*b^2*c^8*d^4*e^2*f^2 + 8*B*a*b^3*c^7*d^3*e^3*f^2 + 108*B*a*b^4*c^6*d^2*e^4*f^2 - 324*B*a*b^4*c^6*d^3*e^2*f^3 - 164*B*a*b^5*c^5*d^2*e^3*f^3 + 44*B*a*b^6*c^4*d^2*e^2*f^4 + 32*B*a^2*b*c^8*d^3*e^3*f^2 - 128*B*a^2*b^3*c^6*d*e^5*f^2 - 1200*B*a^2*b^3*c^6*d^3*e*f^4 + 142*B*a^2*b^4*c^5*d*e^4*f^3 + 20*B*a^2*b^5*c^4*d*e^3*f^4 - 304*B*a^2*b^5*c^4*d^2*e*f^5 - 112*B*a^2*b^6*c^3*d*e^2*f^5 - 688*B*a^3*b*c^7*d^2*e^3*f^3 - 224*B*a^3*b^2*c^6*d*e^4*f^3 - 216*B*a^3*b^3*c^5*d*e^3*f^4 + 800*B
\end{aligned}$$

$$\begin{aligned}
& a^3 b^3 c^5 d^2 e f^5 + 460 B a^3 b^4 c^4 d e^2 f^5 - 640 B a^4 b^2 c^5 d e^2 f^5) / (16 a^2 c^6 d^4 + a^4 b^4 f^4 + 16 a^4 c^4 e^4 + b^4 c^4 d^4 + 16 \\
& a^6 c^2 f^4 + b^8 d^2 f^2 - 8 a b^2 c^5 d^4 - 8 a^5 b^2 c f^4 + 2 a^2 b^6 d f^3 - 2 a^3 b^5 e f^3 - 64 a^3 c^5 d^3 f - 64 a^5 c^3 d f^3 - 2 b^5 c^3 d \\
& ^3 e + 2 b^6 c^2 d^3 f + a^2 b^4 c^2 e^4 - 8 a^3 b^2 c^3 e^4 + 32 a^3 c^5 d^2 e^2 + a^2 b^6 e^2 f^2 + 96 a^4 c^4 d^2 f^2 + b^6 c^2 d^2 e^2 + 32 a^5 c^3 \\
& e^2 f^2 - 2 a b^7 d e f^2 - 2 b^7 c d^2 e f + 54 a^2 b^4 c^2 d^2 f^2 - 11 \\
& 2 a^3 b^2 c^3 d^2 f^2 + 16 a b^3 c^4 d^3 e - 2 a b^5 c^2 d e^3 - 32 a^2 b c^5 d^3 e - 32 a^3 b c^4 d e^3 - 20 a b^4 c^3 d^3 f - 12 a b^6 c d^2 f^2 - 2 \\
& 0 a^3 b^4 c d f^3 - 2 a^2 b^5 c e^3 f - 32 a^4 b c^3 e^3 f + 16 a^4 b^3 c e \\
& * f^3 - 32 a^5 b c^2 e f^3 - 64 a^4 c^4 d e^2 f - 6 a b^4 c^3 d^2 e^2 + 16 a \\
& ^2 b^3 c^3 d e^3 + 64 a^2 b^2 c^4 d^3 f + 64 a^4 b^2 c^2 d f^3 + 16 a^3 b^3 \\
& * c^2 e^3 f - 6 a^3 b^4 c e^2 f^2 - 48 a^2 b^3 c^3 d^2 e f - 36 a^2 b^4 c^2 \\
& d e^2 f + 96 a^3 b^2 c^3 d e^2 f - 48 a^3 b^3 c^2 d e f^2 + 4 a b^6 c d e^2 \\
& * f + 18 a b^5 c^2 d^2 e f + 18 a^2 b^5 c d e f^2 + 32 a^3 b c^4 d^2 e f + 3 \\
& 2 a^4 b c^3 d e f^2) - (56 A^2 a^3 b^3 c^3 f^7 - 13 A^2 a^2 b^5 c^2 f^7 - \\
& 24 A^2 a^2 c^7 e^5 f^2 - 16 A^2 b^3 c^6 d^3 f^4 - A^2 b^5 c^4 d^2 f^5 + 2 A \\
& ^2 b^4 c^5 e^5 f^2 - A^2 b^5 c^4 e^4 f^3 - A^2 b^6 c^3 e^3 f^4 + 2 A^2 b^7 c^2 \\
& e^2 f^5 - 12 B^2 a^4 c^5 e^3 f^4 - 8 B^2 b^3 c^6 d^4 f^3 - 9 B^2 b^5 c^4 \\
& d^3 f^4 + 64 A B a^5 c^4 f^7 + A^2 a b^7 c f^7 - A^2 b^8 c e f^6 - 80 A^2 \\
& a^4 b c^4 f^7 - 16 A^2 b c^8 d^4 f^3 + 28 A^2 a^4 c^5 e f^6 - 2 A^2 b^7 c^4 \\
& 2 d f^6 - A^2 b^3 c^6 e^6 f - 16 B^2 a^5 c^4 e f^6 - 4 A^2 c^9 d^3 e^3 f + \\
& 12 A^2 c^9 d^4 e f^2 + 48 A^2 a b c^7 d^3 f^4 + 22 A^2 a b^5 c^3 d f^6 + 48 \\
& * A^2 a^3 b c^5 d f^6 + 12 A^2 a b^6 c^2 e f^6 + 16 B^2 a b c^7 d^4 f^3 + 64 \\
& * B^2 a^4 b c^4 d f^6 - 16 A^2 a c^8 d^3 e f^3 + 80 A^2 a^3 c^6 d e f^5 + 4 * \\
& A^2 b c^8 d^2 e^4 f + A^2 b^2 c^7 d e^5 f + 2 A^2 b^6 c^3 d e f^5 + 4 B^2 a \\
& ^2 c^7 d e^5 f - 64 B^2 a^4 c^5 d e f^5 + A B b^8 c d f^6 + 8 A^2 a b^3 c^5 \\
& * d^2 f^5 - 64 A^2 a^2 b^3 c^4 d f^6 - 2 A^2 a b^2 c^6 e^5 f^2 - 10 A^2 a b^3 \\
& c^5 e^4 f^3 + 20 A^2 a b^4 c^4 e^3 f^4 - 25 A^2 a b^5 c^3 e^2 f^5 + 56 A^2 \\
& a^2 b c^6 e^4 f^3 - 44 A^2 a^2 b^4 c^3 e f^6 - 76 A^2 a^3 b c^5 e^2 f^5 + \\
& 40 A^2 a^3 b^2 c^4 e f^6 + 56 B^2 a b^3 c^5 d^3 f^4 - 2 B^2 a b^5 c^3 d^2 \\
& f^5 - 96 B^2 a^2 b c^6 d^3 f^4 + 11 B^2 a^2 b^5 c^2 d f^6 + 16 B^2 a^3 b c^5 \\
& d^2 f^5 - 40 B^2 a^3 b^3 c^3 d f^6 + 16 B^2 a^4 b c^4 e^2 f^5 + 3 B^2 a^4 \\
& * b^2 c^3 e f^6 + 24 A^2 a c^8 d^2 e^3 f^2 + 92 A^2 a^2 c^7 d e^3 f^3 - 104 * \\
& A^2 a^2 c^7 d^2 e f^4 - 4 A^2 b c^8 d^3 e^2 f^2 + 20 A^2 b^2 c^7 d^3 e f^3 \\
& - A^2 b^4 c^5 d e^3 f^3 - 8 A^2 b^4 c^5 d^2 e f^4 + 32 B^2 a^2 c^7 d^3 e f^3 \\
& - 8 B^2 a^3 c^6 d e^3 f^3 + 48 B^2 a^3 c^6 d^2 e f^4 - B^2 b^2 c^7 d^3 e^3 \\
& * f + 3 B^2 b^2 c^7 d^4 e f^2 + B^2 b^3 c^6 d^2 e^4 f + 8 B^2 b^4 c^5 d^3 e \\
& * f^3 - 3 B^2 b^6 c^3 d^2 e f^4 - A B a^2 b^6 c f^7 - 32 A B a c^8 d^4 f^3 - \\
& 32 A B a^4 c^5 d f^6 - 8 A B a^2 c^7 e^6 f + 4 A^2 a b c^7 e^6 f - B^2 a b \\
& ^7 c d f^6 - 8 A^2 a c^8 d e^5 f - 65 A^2 a^2 b^2 c^5 e^3 f^4 + 88 A^2 a^2 * \\
& b^3 c^4 e^2 f^5 + 8 B^2 a^2 b^3 c^4 d^2 f^5 + 2 B^2 a^3 b^2 c^4 e^3 f^4 - 3 \\
& * B^2 a^3 b^3 c^3 e^2 f^5 - 13 A^2 b^2 c^7 d^2 e^3 f^2 + 16 A^2 b^3 c^6 d^2 * \\
& e^2 f^3 - 28 B^2 a^2 c^7 d^2 e^3 f^2 + B^2 b^3 c^6 d^3 e^2 f^2 - 5 B^2 b^4 c^5 \\
& d^2 e^3 f^2 + 7 B^2 b^5 c^4 d^2 e^2 f^3 + 12 A B a^3 b^4 c^2 f^7 - 48 A
\end{aligned}$$

$$\begin{aligned}
& *B*a^4*b^2*c^3*f^7 + 160*A*B*a^2*c^7*d^3*f^4 - 160*A*B*a^3*c^6*d^2*f^5 - 16 \\
& *A*B*a^3*c^6*e^4*f^3 + 48*A*B*a^4*c^5*e^2*f^5 + 24*A*B*b^2*c^7*d^4*f^3 + 24 \\
& *A*B*b^4*c^5*d^3*f^4 - A*B*b^6*c^3*d^2*f^5 + 20*B^2*a*b^2*c^6*d^2*e^3*f^2 - \\
& 49*B^2*a*b^3*c^5*d^2*e^2*f^3 + 96*B^2*a^2*b*c^6*d^2*e^2*f^3 + 19*B^2*a^2*b \\
& ^2*c^5*d*e^3*f^3 - 102*B^2*a^2*b^2*c^5*d^2*e*f^4 + 3*B^2*a^2*b^3*c^4*d*e^2* \\
& f^4 - 120*A*B*a*b^2*c^6*d^3*f^4 + 4*A*B*a*b^4*c^4*d^2*f^5 + 24*A*B*a^2*b^4* \\
& c^3*d*f^6 - 8*A*B*a^3*b^2*c^4*d*f^6 - 7*A*B*a*b^3*c^5*e^5*f^2 + 8*A*B*a*b^4 \\
& *c^4*e^4*f^3 - 2*A*B*a*b^5*c^3*e^3*f^4 - 2*A*B*a*b^6*c^2*e^2*f^5 + 28*A*B*a \\
& ^2*b*c^6*e^5*f^2 - 11*A*B*a^2*b^5*c^2*e*f^6 + 72*A*B*a^3*b*c^5*e^3*f^4 + 34 \\
& *A*B*a^3*b^3*c^3*e*f^6 + 16*A*B*a*c^8*d^3*e^2*f^2 + 80*A*B*a^2*c^7*d*e^4*f^ \\
& 2 + 16*A*B*a^3*c^6*d*e^2*f^4 - 4*A*B*b^2*c^7*d^2*e^4*f - 22*A*B*b^3*c^6*d^3 \\
& *e*f^3 + 3*A*B*b^4*c^5*d*e^4*f^2 - 6*A*B*b^5*c^4*d*e^3*f^3 + 15*A*B*b^5*c^4 \\
& *d^2*e*f^4 + 4*A*B*b^6*c^3*d*e^2*f^4 + 12*A^2*a*b*c^7*d*e^4*f^2 - 28*A^2*a* \\
& b^4*c^4*d*e*f^5 - 2*B^2*a*b^2*c^6*d*e^5*f + 2*B^2*a*b^6*c^2*d*e*f^5 + A*B*a \\
& *b^7*c*e*f^6 + 24*A*B*a^2*b^2*c^5*d^2*f^5 - 28*A*B*a^2*b^2*c^5*e^4*f^3 - 9* \\
& A*B*a^2*b^3*c^4*e^3*f^4 + 29*A*B*a^2*b^4*c^3*e^2*f^5 - 100*A*B*a^3*b^2*c^4* \\
& e^2*f^5 - 208*A*B*a^2*c^7*d^2*e^2*f^3 + 13*A*B*b^3*c^6*d^2*e^3*f^2 - 23*A*B \\
& *b^4*c^5*d^2*e^2*f^3 - 52*A^2*a*b*c^7*d^2*e^2*f^3 - 34*A^2*a*b^2*c^6*d*e^3* \\
& f^3 + 48*A^2*a*b^2*c^6*d^2*e*f^4 + 40*A^2*a*b^3*c^5*d*e^2*f^4 - 108*A^2*a^2 \\
& *b*c^6*d*e^2*f^4 + 36*A^2*a^2*b^2*c^5*d*e*f^5 - 8*B^2*a*b*c^7*d^3*e^2*f^2 - \\
& 24*B^2*a*b^2*c^6*d^3*e*f^3 + 7*B^2*a*b^3*c^5*d*e^4*f^2 - 8*B^2*a*b^4*c^4*d \\
& *e^3*f^3 + 32*B^2*a*b^4*c^4*d^2*e*f^4 + 2*B^2*a*b^5*c^3*d*e^2*f^4 - 16*B^2* \\
& a^2*b*c^6*d*e^4*f^2 - 20*B^2*a^2*b^4*c^3*d*e*f^5 + 8*B^2*a^3*b*c^5*d*e^2*f^ \\
& 4 + 40*B^2*a^3*b^2*c^4*d*e*f^5 - 10*A*B*a*b^6*c^2*d*f^6 + 2*A*B*a*b^2*c^6*e \\
& ^6*f - 20*A*B*a^4*b*c^4*e*f^6 + 4*A*B*b*c^8*d^3*e^3*f - 12*A*B*b*c^8*d^4*e* \\
& f^2 - 2*A*B*b^7*c^2*d*e*f^5 + 132*A*B*a*b^2*c^6*d^2*e^2*f^3 + 96*A*B*a^2*b^ \\
& 2*c^5*d*e^2*f^4 + 24*A*B*a*b*c^7*d^3*e*f^3 + 20*A*B*a*b^5*c^3*d*e*f^5 - 24* \\
& A*B*a^3*b*c^5*d*e*f^5 - 24*A*B*a*b*c^7*d^2*e^3*f^2 - 44*A*B*a*b^2*c^6*d*e^4 \\
& *f^2 + 84*A*B*a*b^3*c^5*d*e^3*f^3 - 122*A*B*a*b^3*c^5*d^2*e*f^4 - 54*A*B*a* \\
& b^4*c^4*d*e^2*f^4 - 180*A*B*a^2*b*c^6*d*e^3*f^3 + 288*A*B*a^2*b*c^6*d^2*e*f \\
& ^4 - 18*A*B*a^2*b^3*c^4*d*e*f^5 + 4*A*B*a*b*c^7*d*e^5*f)/(16*a^2*c^6*d^4 + \\
& a^4*b^4*f^4 + 16*a^4*c^4*e^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - \\
& 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 2*a^3*b^5*e*f^3 - 64 \\
& *a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 - 2*b^5*c^3*d^3*e + 2*b^6*c^2*d^3*f + a^2 \\
& *b^4*c^2*e^4 - 8*a^3*b^2*c^3*e^4 + 32*a^3*c^5*d^2*e^2 + a^2*b^6*e^2*f^2 + 9 \\
& 6*a^4*c^4*d^2*f^2 + b^6*c^2*d^2*e^2 + 32*a^5*c^3*e^2*f^2 - 2*a*b^7*d*e*f^2 \\
& - 2*b^7*c*d^2*e*f + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 + 16*a \\
& *b^3*c^4*d^3*e - 2*a*b^5*c^2*d*e^3 - 32*a^2*b*c^5*d^3*e - 32*a^3*b*c^4*d*e^ \\
& 3 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 - 2*a^2*b^ \\
& 5*c*e^3*f - 32*a^4*b*c^3*e^3*f + 16*a^4*b^3*c*e*f^3 - 32*a^5*b*c^2*e*f^3 - \\
& 64*a^4*c^4*d*e^2*f - 6*a*b^4*c^3*d^2*e^2 + 16*a^2*b^3*c^3*d*e^3 + 64*a^2*b^ \\
& 2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3 + 16*a^3*b^3*c^2*e^3*f - 6*a^3*b^4*c*e^2 \\
& *f^2 - 48*a^2*b^3*c^3*d^2*e*f - 36*a^2*b^4*c^2*d*e^2*f + 96*a^3*b^2*c^3*d*e \\
& ^2*f - 48*a^3*b^3*c^2*d*e*f^2 + 4*a*b^6*c*d*e^2*f + 18*a*b^5*c^2*d^2*e*f + \\
& 18*a^2*b^5*c*d*e*f^2 + 32*a^3*b*c^4*d^2*e*f + 32*a^4*b*c^3*d*e*f^2) + (x*(1
\end{aligned}$$

$$\begin{aligned}
& 04*A^2*a^4*c^5*f^7 - 32*B^2*a^5*c^4*f^7 + 8*A^2*c^9*d^4*f^3 + A^2*b^8*c*f^7 \\
& + 50*A^2*a^2*b^4*c^3*f^7 - 96*A^2*a^3*b^2*c^4*f^7 - 12*B^2*a^3*b^4*c^2*f^7 \\
& + 42*B^2*a^4*b^2*c^3*f^7 + 208*A^2*a^2*c^7*d^2*f^5 + 36*A^2*a^2*c^7*e^4*f^3 \\
& + 72*A^2*a^3*c^6*e^2*f^5 + 8*A^2*b^2*c^7*d^3*f^4 + 18*A^2*b^4*c^5*d^2*f^5 \\
& - 32*B^2*a^2*c^7*d^3*f^4 + 32*B^2*a^3*c^6*d^2*f^5 - 2*A^2*b^3*c^6*e^5*f^2 \\
& + A^2*b^4*c^5*e^4*f^3 + A^2*b^6*c^3*e^2*f^5 + 24*B^2*a^3*c^6*e^4*f^3 + 56*B^2 \\
& ^2*a^4*c^5*e^2*f^5 + 2*B^2*b^2*c^7*d^4*f^3 - 6*B^2*b^4*c^5*d^3*f^4 + 9*B^2*b^6 \\
& ^6*c^3*d^2*f^5 - 16*A^2*c^9*d^3*e^2*f^2 - 12*A^2*a*b^6*c^2*f^7 + B^2*a^2*b^6 \\
& ^6*c*f^7 - 64*A^2*a*c^8*d^3*f^4 - 256*A^2*a^3*c^6*d*f^6 + 2*A^2*b^6*c^3*d*f^6 \\
& + 32*B^2*a^4*c^5*d*f^6 + A^2*b^2*c^7*e^6*f - 2*A^2*b^7*c^2*e*f^6 + 4*B^2 \\
& ^2*a^2*c^7*e^6*f + 4*A^2*c^9*d^2*e^4*f - 36*A^2*a*b^4*c^4*d*f^6 - 4*A^2*a*b*c^7 \\
& ^7*e^5*f^2 + 22*A^2*a*b^5*c^3*e*f^6 - 16*A^2*a^3*b*c^5*e*f^6 - 2*B^2*a*b^6*c^2 \\
& ^2*d*f^6 - 40*B^2*a^4*b*c^4*e*f^6 + 8*A^2*a*c^8*d*e^4*f^2 + 16*A^2*b*c^8*d^3 \\
& ^3*e*f^3 + 6*A^2*b^5*c^4*d*e*f^5 - 2*A*B*a*b^7*c*f^7 - 144*A^2*a*b^2*c^6*d^2 \\
& ^2*f^5 + 168*A^2*a^2*b^2*c^5*d*f^6 + 2*A^2*a*b^2*c^6*e^4*f^3 + 10*A^2*a*b^3*c^5 \\
& ^5*e^3*f^4 - 18*A^2*a*b^4*c^4*e^2*f^5 - 80*A^2*a^2*b*c^6*e^3*f^4 - 56*A^2*a^2 \\
& ^2*b^3*c^4*e*f^6 + 24*B^2*a*b^2*c^6*d^3*f^4 - 64*B^2*a*b^4*c^4*d^2*f^5 + 2 \\
& ^6*B^2*a^2*b^4*c^3*d*f^6 - 88*B^2*a^3*b^2*c^4*d*f^6 - 12*B^2*a^2*b*c^6*e^5*f^2 \\
& ^2 - 40*B^2*a^3*b*c^5*e^3*f^4 + 6*B^2*a^3*b^3*c^3*e*f^6 + 8*A^2*a*c^8*d^2*e^2 \\
& ^2*f^3 - 128*A^2*a^2*c^7*d*e^2*f^4 + 8*A^2*b*c^8*d^2*e^3*f^2 + 4*A^2*b^2*c^7 \\
& ^7*d*e^4*f^2 + 10*A^2*b^3*c^6*d*e^3*f^3 - 18*A^2*b^4*c^5*d*e^2*f^4 - 32*B^2*a^2 \\
& ^2*c^7*d*e^4*f^2 - 80*B^2*a^3*c^6*d*e^2*f^4 + B^2*b^2*c^7*d^2*e^4*f + 10*B^2 \\
& ^2*b^3*c^6*d^3*e*f^3 - 12*B^2*b^5*c^4*d^2*e*f^4 + 72*A*B*a^4*b*c^4*f^7 - 8*A \\
& ^4*B*b*c^8*d^4*f^3 - 176*A*B*a^4*c^5*e*f^6 + 2*A*B*b^7*c^2*d*f^6 - 4*A^2*b*c^8 \\
& ^8*d*e^5*f + 54*A^2*a^2*b^2*c^5*e^2*f^5 + 84*B^2*a^2*b^2*c^5*d^2*f^5 + 9*B^2 \\
& ^2*a^2*b^2*c^5*e^4*f^3 + 2*B^2*a^3*b^2*c^4*e^2*f^5 - 26*A^2*b^2*c^7*d^2*e^2*f^3 \\
& + 88*B^2*a^2*c^7*d^2*e^2*f^3 - 4*B^2*b^2*c^7*d^3*e^2*f^2 - 4*B^2*b^3*c^6 \\
& ^6*d^2*e^3*f^2 + 8*B^2*b^4*c^5*d^2*e^2*f^3 + 24*A*B*a^2*b^5*c^2*f^7 - 84*A*B \\
& ^4*a^3*b^3*c^3*f^7 + 8*A*B*a^2*c^7*e^5*f^2 - 32*A*B*a^3*c^6*e^3*f^4 + 4*A*B*b^3 \\
& ^3*c^6*d^3*f^4 - 20*A*B*b^5*c^4*d^2*f^5 - 4*B^2*a*b*c^7*d*e^5*f - 78*B^2*a \\
& ^2*b^2*c^6*d^2*e^2*f^3 - 48*B^2*a^2*b^2*c^5*d*e^2*f^4 + 148*A*B*a*b^3*c^5*d^2 \\
& ^2*f^5 - 192*A*B*a^2*b*c^6*d^2*f^5 - 4*A*B*a^2*b^3*c^4*d*f^6 + 10*A*B*a*b^2*c^6 \\
& ^6*e^5*f^2 - 2*A*B*a*b^3*c^5*e^4*f^3 - 12*A*B*a*b^4*c^4*e^3*f^4 + 8*A*B*a*b^5 \\
& ^5*c^3*e^2*f^5 - 52*A*B*a^2*b*c^6*e^4*f^3 - 44*A*B*a^2*b^4*c^3*e*f^6 - 48*A \\
& ^4*B*a^3*b*c^5*e^2*f^5 + 204*A*B*a^3*b^2*c^4*e*f^6 - 48*A*B*a*c^8*d^2*e^3*f^2 \\
& - 48*A*B*a^2*c^7*d*e^3*f^3 - 16*A*B*a^2*c^7*d^2*e*f^4 + 16*A*B*b*c^8*d^3*e^2 \\
& ^2*f^2 - 28*A*B*b^2*c^7*d^3*e*f^3 - 6*A*B*b^3*c^6*d*e^4*f^2 + 8*A*B*b^4*c^5 \\
& ^5*d^2*e*f^4 + 12*A*B*b^5*c^4*d*e^2*f^4 - 40*A^2*a*b*c^7*d*e^3*f^3 + 80*A^2*a \\
& ^2*b*c^7*d^2*e*f^4 - 24*A^2*a*b^3*c^5*d*e*f^5 + 48*A^2*a^2*b*c^6*d*e*f^5 - 24 \\
& ^4*B^2*a*b*c^7*d^3*e*f^3 - 8*B^2*a*b^5*c^3*d*e*f^5 + 56*B^2*a^3*b*c^5*d*e*f^5 \\
& - 4*A*B*a*b*c^7*e^6*f + 8*A*B*a*c^8*d*e^5*f + 96*A*B*a^2*b^2*c^5*e^3*f^4 - \\
& ^4*36*A*B*a^2*b^3*c^4*e^2*f^5 + 4*A*B*b^2*c^7*d^2*e^3*f^2 + 8*A*B*b^3*c^6*d^2 \\
& ^2*e^2*f^3 + 84*A^2*a*b^2*c^6*d*e^2*f^4 + 24*B^2*a*b*c^7*d^2*e^3*f^2 + 14*B^2 \\
& ^2*a*b^2*c^6*d*e^4*f^2 - 16*B^2*a*b^3*c^5*d*e^3*f^3 + 98*B^2*a*b^3*c^5*d^2*e*f^4 \\
& ^4 + 12*B^2*a*b^4*c^4*d*e^2*f^4 + 64*B^2*a^2*b*c^6*d*e^3*f^3 - 120*B^2*a^2*
\end{aligned}$$

$$\begin{aligned}
& b^6c^6d^2ef^4 + 30B^2a^2b^3c^4d^2ef^5 + 16AB^2a^2b^3c^4d^2ef^5 + 16A^2B^2a^2b^3c^4d^2ef^5 - 12A^2B^2a^2b^3c^4d^2ef^5 \\
& + 112A^2B^2a^2b^3c^4d^2ef^5 + 2A^2B^2a^2b^3c^4d^2ef^5 + 48A^2B^2a^2b^3c^4d^2ef^5 + 144A^2B^2a^2b^3c^4d^2ef^5 - 4A^2B^2a^2b^3c^4d^2ef^5 + 2A^2B^2a^2b^3c^4d^2ef^5 \\
& + 64A^2B^2a^2b^3c^4d^2ef^5 - 10A^2B^2a^2b^3c^4d^2ef^5 + 100A^2B^2a^2b^3c^4d^2ef^5 + 64A^2B^2a^2b^3c^4d^2ef^5 + 12A^2B^2a^2b^3c^4d^2ef^5 \\
& - 108A^2B^2a^2b^3c^4d^2ef^5 - 100A^2B^2a^2b^3c^4d^2ef^5 + 288A^2B^2a^2b^3c^4d^2ef^5 - 324A^2B^2a^2b^3c^4d^2ef^5 \\
& + 16A^2B^2a^2b^3c^4d^2ef^5) / (16a^2c^6d^4 + a^4b^4f^4 + 16a^4c^4e^4 + b^4c^4d^4 + 16a^6c^2f^4 + b^8d^2f^2 - 8a^2b^2c^5d^4 - 8a^5b^2c^2f^4 \\
& + 2a^2b^6d^2f^3 - 2a^3b^5ef^3 - 64a^3c^5d^3f - 64a^5c^3d^2f^3 - 2b^5c^3d^3e + 2b^6c^2d^3f + a^2b^4c^2e^4 - 8a^3b^2c^3e^4 \\
& + 32a^3c^5d^2e^2 + a^2b^6e^2f^2 + 96a^4c^4d^2f^2 + b^6c^2d^2e^2 + 32a^5c^3e^2f^2 - 2a^2b^7d^2ef^2 - 2b^7c^2d^2ef + 54a^2b^4c^2d^2f^2 \\
& - 112a^3b^2c^3d^2f^2 + 16a^2b^3c^4d^3e - 2a^2b^5c^2d^2e^3 - 32a^2b^3c^5d^3e - 32a^3b^3c^4d^2ef^3 - 20a^2b^4c^3d^3f - 12a^2b^6c^2d^2f^2 \\
& - 20a^3b^4c^2d^2f^3 - 2a^2b^5c^2e^3f - 32a^4b^3c^3e^3f + 16a^4b^3c^2ef^3 - 32a^5b^3c^2ef^3 - 64a^4c^4d^2ef^2 - 6a^2b^4c^3d^2e^2 \\
& + 16a^2b^3c^3d^2e^3 + 64a^2b^2c^4d^3f + 64a^4b^2c^2d^2f^3 + 16a^3b^3c^2e^3f - 6a^3b^4c^2ef^2 - 48a^2b^3c^3d^2ef - 36a^2b^4c^2d^2ef \\
& + 96a^3b^2c^3d^2ef - 48a^3b^3c^2d^2ef^2 + 4a^2b^6c^2d^2ef + 18a^2b^5c^2d^2ef + 18a^2b^5c^2d^2ef + 32a^3b^3c^4d^2ef + 32a^4b^3c^3d^2ef^2) \\
& - (3A^3b^2c^5e^2f^4 - 4A^3c^7d^2f^4 - 12A^3a^2c^5f^6 - B^3b^3c^4d^2f^4 + 16A^3a^2c^6d^2f^5 - 16A^3B^2a^3c^4f^6 + 2A^3a^2b^2c^4f^6 \\
& - 16A^3a^2c^6e^2f^4 - 6A^3b^2c^5d^2f^5 - 3A^3b^3c^4ef^5 + 4B^3a^2b^3c^5d^2f^4 + 3B^3a^2b^3c^3d^2f^5 - 12B^3a^2b^3c^4d^2f^5 \\
& + 8B^3a^2c^5d^2ef^4 + 3A^2B^2a^2b^2c^3f^6 - 12A^2B^2a^2c^5e^2f^4 + A^2B^2b^2c^5d^2f^4 - 3A^2B^2b^3c^4e^2f^4 + 16A^3a^2b^3c^5ef^5 \\
& + 4A^3b^3c^6d^2ef^4 - 3A^2B^2a^2b^3c^3f^6 + 16A^2B^2a^2b^3c^4f^6 - 8A^2B^2a^2c^6d^2f^4 + 24A^2B^2a^2c^5d^2f^5 - 3A^2B^2b^4c^3d^2f^5 \\
& + 4A^2B^2b^3c^6d^2f^4 - 8A^2B^2a^2c^5ef^5 + 9A^2B^2b^3c^4d^2f^5 + 3A^2B^2b^4c^3ef^5 + 4A^2B^2a^2b^2c^4d^2f^5 - 3A^2B^2a^2b^3c^3ef^5 \\
& + 16A^2B^2a^2b^3c^4ef^5 + 16A^2B^2a^2b^3c^4ef^5 + 16A^2B^2a^2b^3c^5e^2f^4 - 14A^2B^2a^2b^2c^4ef^5 + 4A^2B^2b^3c^4d^2ef^4 - 8A^2B^2b^2c^5d^2ef^4 \\
& - 2B^3a^2b^2c^4d^2ef^4 + 2A^2B^2a^2b^2c^4e^2f^4 - 28A^2B^2a^2b^3c^5d^2f^5 + 16A^2B^2a^2b^3c^6d^2ef^4 - 12A^2B^2a^2b^3c^5d^2ef^4) / (16a^2c^6d^4 \\
& + a^4b^4f^4 + 16a^4c^4e^4 + b^4c^4d^4 + 16a^6c^2f^4 + b^8d^2f^2 - 8a^2b^2c^5d^4 - 8a^5b^2c^2f^4 + 2a^2b^6d^2f^3 - 2a^3b^5ef^3 - 64a^3c^5d^3f \\
& - 64a^5c^3d^2f^3 - 2b^5c^3d^3e + 2b^6c^2d^3f + a^2b^4c^2e^4 - 8a^3b^2c^3e^4 + 32a^3c^5d^2e^2 + a^2b^6e^2f^2 + 96a^4c^4d^2f^2 + b^6c^2d^2e^2 \\
& + 32a^5c^3e^2f^2 - 2a^2b^7d^2ef^2 - 2b^7c^2d^2ef + 54a^2b^4c^2d^2f^2 - 112a^3b^2c^3d^2f^2 + 16a^2b^3c^4d^3e - 2a^2b^5c^2d^2e^3 - 32a^2b^3c^5d^3e \\
& - 32a^3b^3c^4d^2ef^3 - 20a^2b^4c^3d^3f - 12a^2b^6c^2d^2f^2 - 20a^3b^4c^2d^2f^3 - 2a^2b^5c^2e^3f - 32a^4b^3c^3e^3f + 16a^4b^3c^2ef^3 - 32a^5b^3c^2ef^3 \\
& - 64a^4c^4d^2ef^2 - 6a^2b^4c^3d^2e^2 + 16a^2b^3c^3d^2e^3 + 64a^2b^2c^4d^3f + 64a^4b^2c^2d^2f^3 + 16a^3b^3c^2e^3f - 6a^3b^3c^2e^3f - 6a^3b^3c^2e^3f)
\end{aligned}$$

$$\begin{aligned}
&^4*c*e^2*f^2 - 48*a^2*b^3*c^3*d^2*e*f - 36*a^2*b^4*c^2*d*e^2*f + 96*a^3*b^2 \\
&*c^3*d*e^2*f - 48*a^3*b^3*c^2*d*e*f^2 + 4*a*b^6*c*d*e^2*f + 18*a*b^5*c^2*d^ \\
&2*e*f + 18*a^2*b^5*c*d*e*f^2 + 32*a^3*b*c^4*d^2*e*f + 32*a^4*b*c^3*d*e*f^2) \\
&)*\text{root}(48416*a^6*b^2*c^6*d^4*e^2*f^4*z^4 - 41544*a^5*b^4*c^5*d^4*e^2*f^4*z^ \\
&4 - 31872*a^7*b^2*c^5*d^3*e^2*f^5*z^4 - 31872*a^5*b^2*c^7*d^5*e^2*f^3*z^4 - \\
&29184*a^6*b^2*c^6*d^3*e^4*f^3*z^4 + 28800*a^5*b^4*c^5*d^3*e^4*f^3*z^4 + 21 \\
&510*a^4*b^6*c^4*d^4*e^2*f^4*z^4 + 21408*a^6*b^4*c^4*d^3*e^2*f^5*z^4 + 21408 \\
&*a^4*b^4*c^6*d^5*e^2*f^3*z^4 - 18112*a^7*b^3*c^4*d^2*e^3*f^5*z^4 - 18112*a^ \\
&4*b^3*c^7*d^5*e^3*f^2*z^4 - 15600*a^5*b^5*c^4*d^3*e^3*f^4*z^4 - 15600*a^4*b \\
&^5*c^5*d^4*e^3*f^3*z^4 + 15296*a^6*b^3*c^5*d^3*e^3*f^4*z^4 + 15296*a^5*b^3* \\
&c^6*d^4*e^3*f^3*z^4 + 14016*a^7*b^2*c^5*d^2*e^4*f^4*z^4 + 14016*a^5*b^2*c^7 \\
&*d^4*e^4*f^2*z^4 - 13920*a^4*b^6*c^4*d^3*e^4*f^3*z^4 - 11648*a^6*b^3*c^5*d^ \\
&2*e^5*f^3*z^4 - 11648*a^5*b^3*c^6*d^3*e^5*f^2*z^4 + 10432*a^6*b^2*c^6*d^2*e \\
&^6*f^2*z^4 + 9008*a^6*b^5*c^3*d^2*e^3*f^5*z^4 + 9008*a^3*b^5*c^6*d^5*e^3*f^ \\
&2*z^4 + 8544*a^5*b^5*c^4*d^2*e^5*f^3*z^4 + 8544*a^4*b^5*c^5*d^3*e^5*f^2*z^4 \\
&- 8496*a^5*b^4*c^5*d^2*e^6*f^2*z^4 + 7488*a^8*b^2*c^4*d^2*e^2*f^6*z^4 + 74 \\
&88*a^4*b^2*c^8*d^6*e^2*f^2*z^4 + 7380*a^4*b^7*c^3*d^3*e^3*f^4*z^4 + 7380*a^ \\
&3*b^7*c^4*d^4*e^3*f^3*z^4 - 6720*a^3*b^8*c^3*d^4*e^2*f^4*z^4 - 5784*a^5*b^6 \\
&*c^3*d^3*e^2*f^5*z^4 - 5784*a^3*b^6*c^5*d^5*e^2*f^3*z^4 - 3440*a^6*b^4*c^4* \\
&d^2*e^4*f^4*z^4 - 3440*a^4*b^4*c^6*d^4*e^4*f^2*z^4 + 3360*a^3*b^8*c^3*d^3*e \\
&^4*f^3*z^4 + 3140*a^4*b^6*c^4*d^2*e^6*f^2*z^4 - 2760*a^4*b^7*c^3*d^2*e^5*f^ \\
&3*z^4 - 2760*a^3*b^7*c^4*d^3*e^5*f^2*z^4 - 1764*a^5*b^7*c^2*d^2*e^3*f^5*z^4 \\
&- 1764*a^2*b^7*c^5*d^5*e^3*f^2*z^4 - 1640*a^3*b^9*c^2*d^3*e^3*f^4*z^4 - 16 \\
&40*a^2*b^9*c^3*d^4*e^3*f^3*z^4 - 1604*a^6*b^6*c^2*d^2*e^2*f^6*z^4 - 1604*a^ \\
&2*b^6*c^6*d^6*e^2*f^2*z^4 - 1500*a^5*b^6*c^3*d^2*e^4*f^4*z^4 - 1500*a^3*b^6 \\
&*c^5*d^4*e^4*f^2*z^4 + 1140*a^2*b^10*c^2*d^4*e^2*f^4*z^4 + 810*a^4*b^8*c^2* \\
&d^2*e^4*f^4*z^4 + 810*a^2*b^8*c^4*d^4*e^4*f^2*z^4 - 544*a^3*b^8*c^3*d^2*e^6 \\
&*f^2*z^4 + 416*a^3*b^9*c^2*d^2*e^5*f^3*z^4 + 416*a^2*b^9*c^3*d^3*e^5*f^2*z^ \\
&4 - 384*a^2*b^10*c^2*d^3*e^4*f^3*z^4 + 180*a^4*b^8*c^2*d^3*e^2*f^5*z^4 + 18 \\
&0*a^2*b^8*c^4*d^5*e^2*f^3*z^4 + 48*a^7*b^4*c^3*d^2*e^2*f^6*z^4 + 48*a^3*b^4 \\
&*c^7*d^6*e^2*f^2*z^4 + 36*a^2*b^10*c^2*d^2*e^6*f^2*z^4 - 1024*a^10*b*c^3*d* \\
&e*f^8*z^4 - 1024*a^3*b*c^10*d^8*e*f*z^4 - 192*a^8*b^5*c*d*e*f^8*z^4 - 192*a \\
&*b^5*c^8*d^8*e*f*z^4 + 16128*a^7*b^3*c^4*d^3*e*f^6*z^4 + 16128*a^4*b^3*c^7* \\
&d^6*e*f^3*z^4 - 11712*a^6*b^5*c^3*d^3*e*f^6*z^4 - 11712*a^3*b^5*c^6*d^6*e*f \\
&^3*z^4 + 11520*a^8*b*c^5*d^2*e^3*f^5*z^4 + 11520*a^5*b*c^8*d^5*e^3*f^2*z^4 \\
&- 9984*a^6*b^3*c^5*d^4*e*f^5*z^4 - 9984*a^5*b^3*c^6*d^5*e*f^4*z^4 + 8640*a^ \\
&5*b^5*c^4*d^4*e*f^5*z^4 + 8640*a^4*b^5*c^5*d^5*e*f^4*z^4 - 7424*a^7*b*c^6*d \\
&^3*e^3*f^4*z^4 - 7424*a^6*b*c^7*d^4*e^3*f^3*z^4 - 6912*a^8*b^3*c^3*d^2*e*f^ \\
&7*z^4 - 6912*a^3*b^3*c^8*d^7*e*f^2*z^4 + 4800*a^7*b^3*c^4*d*e^5*f^4*z^4 + 4 \\
&800*a^4*b^3*c^7*d^4*e^5*f*z^4 + 4608*a^7*b*c^6*d^2*e^5*f^3*z^4 + 4608*a^6*b \\
&*c^7*d^3*e^5*f^2*z^4 - 4560*a^4*b^7*c^3*d^4*e*f^5*z^4 - 4560*a^3*b^7*c^4*d^ \\
&5*e*f^4*z^4 + 4176*a^5*b^7*c^2*d^3*e*f^6*z^4 + 4176*a^2*b^7*c^5*d^6*e*f^3*z \\
&^4 + 3264*a^7*b^5*c^2*d^2*e*f^7*z^4 + 3264*a^2*b^5*c^7*d^7*e*f^2*z^4 + 3008 \\
&*a^8*b^3*c^3*d*e^3*f^6*z^4 + 3008*a^3*b^3*c^8*d^6*e^3*f*z^4 + 2880*a^6*b^3* \\
&c^5*d*e^7*f^2*z^4 + 2880*a^5*b^3*c^6*d^2*e^7*f*z^4 - 2240*a^7*b^4*c^3*d*e^4
\end{aligned}$$

$$\begin{aligned}
& *f^5z^4 - 2240a^3b^4c^7d^5e^4f^2z^4 - 1488a^5b^5c^4d^6e^7f^2z^4 \\
& - 1488a^4b^5c^5d^2e^7f^2z^4 + 1440a^3b^9c^2d^4e^5f^5z^4 + 1440a^2b^9c^3d^5e^5f^4z^4 - 1328a^6b^5c^3d^6e^5f^4z^4 - 1328a^3b^5c^6d^4e^5f^2z^4 - 1152a^7b^2c^5d^6e^6f^3z^4 - 1152a^5b^2c^7d^3e^6f^2z^4 - 1120a^6b^4c^4d^6e^6f^3z^4 - 1120a^4b^4c^6d^3e^6f^2z^4 + 912a^6b^6c^2d^6e^4f^5z^4 + 912a^2b^6c^6d^5e^4f^2z^4 + 872a^5b^6c^3d^6e^6f^3z^4 + 872a^3b^6c^5d^3e^6f^2z^4 + 768a^8b^2c^4d^6e^4f^5z^4 + 768a^4b^2c^8d^5e^4f^2z^4 - 672a^8b^4c^2d^6e^2f^7z^4 - 672a^2b^4c^8d^7e^2f^2z^4 - 624a^7b^5c^2d^6e^3f^6z^4 - 624a^2b^5c^7d^6e^3f^2z^4 + 480a^5b^8c^4d^2e^2f^6z^4 + 480a^4b^8c^5d^6e^2f^2z^4 + 316a^4b^7c^3d^6e^7f^2z^4 + 316a^3b^7c^4d^2e^7f^2z^4 - 204a^4b^8c^2d^6e^6f^3z^4 - 204a^2b^8c^4d^3e^6f^2z^4 + 168a^3b^10c^3d^3e^2f^5z^4 + 168a^4b^10c^3d^5e^2f^3z^4 + 156a^2b^11c^3d^3e^3f^4z^4 + 156a^4b^11c^2d^4e^3f^3z^4 + 128a^9b^2c^3d^6e^2f^7z^4 + 128a^3b^2c^9d^7e^2f^2z^4 - 124a^3b^10c^3d^2e^4f^4z^4 - 124a^4b^10c^3d^4e^4f^2z^4 + 100a^4b^9c^3d^2e^3f^5z^4 + 100a^4b^9c^4d^5e^3f^2z^4 + 36a^5b^7c^2d^6e^5f^4z^4 + 36a^2b^7c^5d^4e^5f^2z^4 - 24a^3b^9c^2d^6e^7f^2z^4 - 24a^2b^11c^2d^2e^5f^3z^4 - 24a^2b^9c^3d^2e^7f^2z^4 - 24a^4b^11c^2d^3e^5f^2z^4 - 9216a^8b^3c^5d^3e^6f^6z^4 - 9216a^5b^3c^8d^6e^6f^3z^4 - 5376a^8b^3c^5d^6e^5f^4z^4 - 5376a^5b^3c^8d^4e^5f^2z^4 + 5120a^9b^3c^4d^2e^6f^7z^4 + 5120a^7b^3c^6d^4e^6f^5z^4 + 5120a^6b^3c^7d^5e^6f^4z^4 + 5120a^4b^3c^9d^7e^6f^2z^4 - 4352a^9b^3c^4d^6e^3f^6z^4 - 4352a^4b^3c^9d^6e^3f^2z^4 - 1792a^7b^3c^6d^6e^7f^2z^4 - 1792a^6b^3c^7d^2e^7f^2z^4 - 1600a^6b^2c^6d^6e^8f^2z^4 + 912a^5b^4c^5d^6e^8f^2z^4 + 768a^9b^3c^2d^6e^8f^2z^4 + 768a^2b^3c^9d^8e^8f^2z^4 - 720a^4b^9c^3d^3e^6f^6z^4 - 720a^4b^9c^4d^6e^6f^3z^4 - 656a^6b^7c^3d^2e^6f^7z^4 - 656a^4b^7c^6d^7e^6f^2z^4 - 240a^2b^11c^3d^4e^6f^5z^4 - 240a^4b^11c^2d^5e^6f^4z^4 + 216a^7b^6c^3d^6e^2f^7z^4 + 216a^4b^6c^7d^7e^2f^2z^4 - 204a^4b^6c^4d^6e^8f^2z^4 - 144a^5b^8c^3d^6e^4f^5z^4 - 144a^4b^8c^5d^5e^4f^2z^4 - 84a^4b^12c^3d^4e^2f^4z^4 + 36a^4b^9c^3d^6e^5f^4z^4 + 36a^4b^9c^4d^4e^5f^2z^4 + 20a^6b^7c^3d^6e^3f^6z^4 + 20a^4b^7c^6d^6e^3f^2z^4 + 16a^3b^10c^3d^6e^6f^3z^4 + 16a^3b^8c^3d^6e^8f^2z^4 + 16a^4b^12c^3d^3e^4f^3z^4 + 16a^4b^10c^3d^3e^6f^2z^4 + 48b^11c^3d^6e^6f^3z^4 + 48b^9c^5d^7e^6f^2z^4 - 20b^8c^6d^7e^2f^2z^4 + 8b^10c^4d^5e^4f^2z^4 - 4b^13c^3d^4e^3f^3z^4 - 4b^11c^3d^4e^5f^2z^4 + 4b^9c^5d^6e^3f^2z^4 + 3072a^9c^5d^6e^4f^5z^4 + 3072a^5c^9d^5e^4f^2z^4 + 2560a^8c^6d^6e^6f^3z^4 + 2560a^6c^8d^3e^6f^2z^4 + 1536a^10c^4d^6e^2f^7z^4 + 1536a^4c^10d^7e^2f^2z^4 + 48a^5b^9d^2e^6f^7z^4 + 48a^3b^11d^3e^6f^6z^4 - 20a^6b^8d^6e^2f^7z^4 + 8a^4b^10d^6e^4f^5z^4 + 4a^5b^9d^6e^3f^6z^4 - 4a^3b^11d^6e^5f^4z^4 - 4a^4b^13d^3e^3f^4z^4 + 768a^9b^3c^4e^5f^5z^4 + 768a^8b^3c^5e^7f^3z^4 + 256a^10b^3c^3e^3f^7z^4 - 192a^6b^3c^5e^9f^2z^4 - 68a^7b^6c^4e^4f^6z^4 + 48a^8b^5c^3e^3f^7z^4 + 48a^5b^5c^4e^9f^2z^4 + 36a^6b^7c^4e^5f^5z^4 - 12a^9b^4c^2e^2f^8z^4 - 4a^4b^9c^7e^7f^3z^4 - 4a^4b^7c^3e^9f^2z^4 + 384a^5b^8c^3d^3f^7z^4 + 384
\end{aligned}$$

$$\begin{aligned}
& *a^8b^8c^5d^7f^3z^4 + 288a^3b^{10}c^4d^4f^6z^4 + 288a^8b^{10}c^3d^6f^4z^4 + 224a^7b^6c^4d^2f^8z^4 + 224a^8b^6c^7d^8f^2z^4 - 192a^{10}b^2c^2d^2f^9z^4 - 192a^2b^2c^{10}d^9f^3z^4 + 768a^5b^3c^8d^3e^7z^4 + 768a^4b^3c^9d^5e^5z^4 + 256a^3b^3c^{10}d^7e^3z^4 - 192a^5b^3c^6d^8e^9z^4 - 68a^8b^6c^7d^6e^4z^4 + 48a^4b^5c^5d^8e^9z^4 + 48a^8b^5c^8d^7e^3z^4 + 36a^8b^7c^6d^5e^5z^4 - 12a^8b^4c^9d^8e^2z^4 - 4a^3b^7c^4d^8e^9z^4 - 4a^8b^9c^4d^3e^7z^4 + 16b^{13}c^5d^5e^4f^4z^4 + 16b^7c^7d^8e^4f^8z^4 + 768a^7c^7d^8e^8f^8z^4 + 16a^7b^7d^8e^8f^8z^4 + 16a^8b^13d^4e^8f^5z^4 + 256a^7b^6c^6e^9f^8z^4 + 80a^8b^12c^5d^5f^5z^4 + 48a^9b^4c^4d^8f^9z^4 + 48a^8b^4c^9d^9f^8z^4 + 256a^6b^6c^7d^8e^9z^4 - 42b^{10}c^4d^6e^2f^2z^4 - 20b^{12}c^2d^5e^2f^3z^4 + 6b^{12}c^2d^4e^4f^2z^4 + 4b^{11}c^3d^5e^3f^2z^4 - 24960a^7c^7d^4e^2f^4z^4 + 18944a^8c^6d^3e^2f^5z^4 + 18944a^6c^8d^5e^2f^3z^4 + 14336a^7c^7d^3e^4f^3z^4 - 9984a^8c^6d^2e^4f^4z^4 - 9984a^6c^8d^4e^4f^2z^4 - 7936a^9c^5d^2e^2f^6z^4 - 7936a^5c^9d^6e^2f^2z^4 - 4352a^7c^7d^2e^6f^2z^4 - 42a^4b^{10}d^2e^2f^6z^4 - 20a^2b^{12}d^3e^2f^5z^4 + 6a^2b^{12}d^2e^4f^4z^4 + 4a^3b^{11}d^2e^3f^5z^4 - 480a^8b^2c^4e^6f^4z^4 + 440a^7b^4c^3e^6f^4z^4 - 320a^8b^3c^3e^5f^5z^4 - 320a^7b^3c^4e^7f^3z^4 + 240a^8b^4c^2e^4f^6z^4 + 240a^6b^4c^4e^8f^2z^4 - 192a^9b^3c^2e^3f^7z^4 - 192a^9b^2c^3e^4f^6z^4 - 192a^7b^2c^5e^8f^2z^4 - 90a^6b^6c^2e^6f^4z^4 - 68a^5b^6c^3e^8f^2z^4 + 48a^{10}b^2c^2e^2f^8z^4 - 48a^7b^5c^2e^5f^5z^4 - 48a^6b^5c^3e^7f^3z^4 + 36a^5b^7c^2e^7f^3z^4 + 6a^4b^8c^2e^8f^2z^4 - 33920a^6b^2c^6d^5f^5z^4 + 27936a^5b^4c^5d^5f^5z^4 + 26112a^7b^2c^5d^4f^6z^4 + 26112a^5b^2c^7d^6f^4z^4 - 20352a^6b^4c^4d^4f^6z^4 - 20352a^4b^4c^6d^6f^4z^4 - 13080a^4b^6c^4d^5f^5z^4 - 11520a^8b^2c^4d^3f^7z^4 - 11520a^4b^2c^8d^7f^3z^4 + 8736a^5b^6c^3d^4f^6z^4 + 8736a^3b^6c^5d^6f^4z^4 + 7488a^7b^4c^3d^3f^7z^4 + 7488a^3b^4c^7d^7f^3z^4 + 3840a^3b^8c^3d^5f^5z^4 + 2560a^9b^2c^3d^2f^8z^4 + 2560a^3b^2c^9d^8f^2z^4 - 2416a^6b^6c^2d^3f^7z^4 - 2416a^2b^6c^6d^7f^3z^4 - 2160a^4b^8c^2d^4f^6z^4 - 2160a^2b^8c^4d^6f^4z^4 - 1152a^8b^4c^2d^2f^8z^4 - 1152a^2b^4c^8d^8f^2z^4 - 720a^2b^{10}c^2d^5f^5z^4 - 480a^4b^2c^8d^4e^6z^4 + 440a^3b^4c^7d^4e^6z^4 - 320a^4b^3c^7d^3e^7z^4 - 320a^3b^3c^8d^5e^5z^4 + 240a^4b^4c^6d^2e^8z^4 + 240a^2b^4c^8d^6e^4z^4 - 192a^5b^2c^7d^2e^8z^4 - 192a^3b^2c^9d^6e^4z^4 - 192a^2b^3c^9d^7e^3z^4 - 90a^2b^6c^6d^4e^6z^4 - 68a^3b^6c^5d^2e^8z^4 - 48a^3b^5c^6d^3e^7z^4 - 48a^2b^5c^7d^5e^5z^4 + 48a^2b^2c^{10}d^8e^2z^4 + 36a^2b^7c^5d^3e^7z^4 + 6a^2b^8c^4d^2e^8z^4 - 4b^6c^8d^9f^8z^4 + 256a^{11}c^3d^8f^9z^4 + 256a^3c^{11}d^9f^8z^4 - 4a^8b^6d^8f^9z^4 - 384a^9c^5e^6f^4z^4 - 256a^{10}c^4e^4f^6z^4 - 256a^8c^6e^8f^2z^4 - 64a^{11}c^3e^2f^8z^4 - 24b^{10}c^4d^7f^3z^4 - 16b^{12}c^2d^6f^4z^4 - 16b^8c^6d^8f^2z^4 + 17920a^7c^7d^5f^5z^4 - 14336a^8c^6d^4f^6z^4 - 14336a^6c^8d^6f^4z^4 + 7168a^9c^5d^3f^7z^4 + 7168a^5c^9d^7f^3z^4 - 2048a^{10}c^4d^2f^8
\end{aligned}$$

$$\begin{aligned}
& *z^4 - 2048*a^4*c^{10}*d^8*f^2*z^4 + 6*b^8*c^6*d^6*e^4*z^4 + 6*a^6*b^8*e^4*f^6*z^4 - 4*b^9*c^5*d^5*e^5*z^4 - 4*b^7*c^7*d^7*e^3*z^4 - 4*a^7*b^7*e^3*f^7*z^4 \\
& - 4*a^5*b^9*e^5*f^5*z^4 - 384*a^5*c^9*d^4*e^6*z^4 - 256*a^6*c^8*d^2*e^8*z^4 - 256*a^4*c^{10}*d^6*e^4*z^4 - 64*a^3*c^{11}*d^8*e^2*z^4 - 24*a^4*b^{10}*d^3*f^7*z^4 \\
& - 16*a^6*b^8*d^2*f^8*z^4 - 16*a^2*b^{12}*d^4*f^6*z^4 + 48*a^6*b^2*c^6*e^{10}*z^4 - 12*a^5*b^4*c^5*e^{10}*z^4 - 4*b^{14}*d^5*f^5*z^4 - 64*a^7*c^7*e^{10}*z^4 \\
& + b^{14}*d^4*e^2*f^4*z^4 + b^{10}*c^4*d^4*e^6*z^4 + b^6*c^8*d^8*e^2*z^4 + a^8*b^6*e^2*f^8*z^4 + a^4*b^{10}*e^6*f^4*z^4 + a^4*b^6*c^4*e^{10}*z^4 - 4820*A*B \\
& *a^4*b*c^5*d^2*e^2*f^4*z^2 + 2976*A*B*a^3*b*c^6*d^3*e^2*f^3*z^2 - 2328*A*B*a^3*b*c^6*d^2*e^4*f^2*z^2 + 1848*A*B*a^2*b^4*c^4*d^3*e*f^4*z^2 - 1768*A*B*a^3*b^4*c^3*d^2*e*f^5*z^2 \\
& + 1528*A*B*a^4*b^2*c^4*d^2*e*f^5*z^2 - 1136*A*B*a^3*b^2*c^5*d^3*e*f^4*z^2 - 974*A*B*a^4*b^3*c^3*d*e^2*f^5*z^2 + 692*A*B*a^2*b*c^7*d^4*e^2*f^2*z^2 + 588*A*B*a*b^6*c^3*d^2*e^3*f^3*z^2 - 580*A*B*a^3*b^3*c^4*d*e^4*f^3*z^2 \\
& + 488*A*B*a^3*b^4*c^3*d*e^3*f^4*z^2 - 444*A*B*a^2*b^2*c^6*d^2*e^5*f*z^2 - 412*A*B*a*b^5*c^4*d^2*e^4*f^2*z^2 + 366*A*B*a^2*b^6*c^2*d^2*e*f^5*z^2 - 352*A*B*a^2*b^2*c^6*d^4*e*f^3*z^2 + 326*A*B*a^2*b^4*c^4*d*e^5*f^2*z^2 \\
& + 324*A*B*a*b^5*c^4*d^3*e^2*f^3*z^2 - 302*A*B*a*b^3*c^6*d^4*e^2*f^2*z^2 - 296*A*B*a*b^7*c^2*d^2*e^2*f^4*z^2 + 122*A*B*a^4*b^2*c^4*d*e^3*f^4*z^2 - 122*A*B*a^2*b^6*c^2*d*e^3*f^4*z^2 - 84*A*B*a^3*b^2*c^5*d*e^5*f^2*z^2 + 72*A*B*a*b^4*c^5*d^3*e^3*f^2*z^2 \\
& - 64*A*B*a^2*b^5*c^3*d*e^4*f^3*z^2 + 60*A*B*a^3*b^5*c^2*d*e^2*f^5*z^2 + 1312*A*B*a^5*b*c^4*d*e^2*f^5*z^2 + 1040*A*B*a^4*b*c^5*d*e^4*f^3*z^2 - 500*A*B*a*b^6*c^3*d^3*e*f^4*z^2 - 376*A*B*a*b^2*c^7*d^5*e*f^2*z^2 + 276*A*B*a^4*b^4*c^2*d*e*f^6*z^2 - 262*A*B*a^2*b^3*c^5*d*e^6*f*z^2 \\
& + 238*A*B*a*b^2*c^7*d^4*e^3*f*z^2 + 232*A*B*a^5*b^2*c^3*d*e*f^6*z^2 - 176*A*B*a^2*b*c^7*d^3*e^4*f*z^2 - 120*A*B*a*b^6*c^3*d*e^5*f^2*z^2 - 108*A*B*a*b^4*c^5*d^4*e*f^3*z^2 + 68*A*B*a*b^7*c^2*d*e^4*f^3*z^2 + 68*A*B*a*b^4*c^5*d^2*e^5*f*z^2 \\
& + 46*A*B*a^2*b^7*c*d*e^2*f^5*z^2 - 36*A*B*a*b^3*c^6*d^3*e^4*f*z^2 - 1932*A*B*a^2*b^3*c^5*d^3*e^2*f^3*z^2 - 1818*A*B*a^2*b^4*c^4*d^2*e^3*f^3*z^2 + 1620*A*B*a^3*b^3*c^4*d^2*e^2*f^4*z^2 + 1560*A*B*a^2*b^3*c^5*d^2*e^4*f^2*z^2 \\
& + 1244*A*B*a^3*b^2*c^5*d^2*e^3*f^3*z^2 + 820*A*B*a^2*b^2*c^6*d^3*e^3*f^2*z^2 + 480*A*B*a^2*b^5*c^3*d^2*e^2*f^4*z^2 + 352*A*B*a^3*b*c^6*d*e^6*f*z^2 - 108*A*B*a^3*b^6*c*d*e*f^6*z^2 + 82*A*B*a*b^5*c^4*d*e^6*f*z^2 - 64*A*B*a*b*c^8*d^5*e^2*f*z^2 \\
& + 16*A*B*a*b^8*c*d^2*e*f^5*z^2 - 4*A*B*a*b^8*c*d*e^3*f^4*z^2 + 16*B^2*a*b*c^8*d^6*e*f*z^2 + 56*A*B*b^2*c^8*d^6*e*f*z^2 - 8*A*B*b^9*c*d*e^4*f^3*z^2 - 8*A*B*b^7*c^3*d*e^6*f*z^2 - 800*A*B*a^6*c^4*d*e*f^6*z^2 \\
& + 10*A*B*a^2*b^8*d*e*f^6*z^2 - 6*A*B*a*b^9*d*e^2*f^5*z^2 - 12*A*B*a^5*b^4*c*e*f^7*z^2 + 912*A*B*a^6*b*c^3*d*f^7*z^2 + 192*A*B*a^4*b^5*c*d*f^7*z^2 + 192*A*B*a*b*c^8*d^6*f^2*z^2 - 20*A*B*a*b^4*c^5*d*e^7*z^2 + 4*A*B*a*b*c^8*d^4*e^4*z^2 \\
& + 2144*B^2*a^4*b*c^5*d^3*e*f^4*z^2 - 1120*B^2*a^3*b*c^6*d^4*e*f^3*z^2 - 688*B^2*a^5*b*c^4*d^2*e*f^5*z^2 - 256*B^2*a^3*b*c^6*d^2*e^5*f*z^2 + 152*B^2*a*b^3*c^6*d^5*e*f^2*z^2 + 120*B^2*a^5*b^3*c^2*d*e*f^6*z^2 - 116*B^2*a^5*b*c^4*d*e^3*f^4*z^2 + 110*B^2*a*b^7*c^2*d^3*e*f^4*z^2 - 80*B^2*a^2*b*c^7*d^5*e*f^2*z^2 - 72*B^2*a*b^5*c^4*d^4*e*f^3*z^2 - 48*B^2*a^4*b*c^5*d*e^5*f^2*z^2 - 46*B^2*a*b^3*c^6*d^4*e^3*f*z^2 - 44*B^2*a*b^4*c^5*d^3*e^4*f*z^2 - 34*B^2*a*b^5*c^4*d^2*e^5*f*z^2 + 20*B^2*a^2*b*c^7*d^4*e^3*f*z^2
\end{aligned}$$

$$\begin{aligned}
& 2 - 10*B^2*a^3*b^6*c*d*e^2*f^5*z^2 - 10*B^2*a^2*b^7*c*d^2*e*f^5*z^2 - 10*B^2*a*b^2*c^7*d^5*e^2*f*z^2 - 7*B^2*a^2*b^4*c^4*d*e^6*f*z^2 - 6*B^2*a^3*b^2*c^5*d*e^6*f*z^2 + 4*B^2*a*b^8*c*d^2*e^2*f^4*z^2 - 2*B^2*a^2*b^7*c*d*e^3*f^4*z^2 + 3196*A^2*a^4*b*c^5*d*e^3*f^4*z^2 - 3184*A^2*a^4*b*c^5*d^2*e*f^5*z^2 + 1568*A^2*a^3*b*c^6*d^3*e*f^4*z^2 + 1504*A^2*a^3*b*c^6*d*e^5*f^2*z^2 - 656*A^2*a^4*b^3*c^3*d*e*f^6*z^2 - 400*A^2*a*b^6*c^3*d*e^4*f^3*z^2 + 314*A^2*a*b^5*c^4*d*e^5*f^2*z^2 - 264*A^2*a^3*b^5*c^2*d*e*f^6*z^2 + 240*A^2*a^2*b^2*c^6*d*e^6*f*z^2 - 224*A^2*a^2*b*c^7*d^4*e*f^3*z^2 + 216*A^2*a*b^5*c^4*d^3*e*f^4*z^2 - 192*A^2*a^2*b*c^7*d^2*e^5*f*z^2 + 178*A^2*a*b^7*c^2*d*e^3*f^4*z^2 - 154*A^2*a*b^7*c^2*d^2*e*f^5*z^2 + 128*A^2*a*b^3*c^6*d^4*e*f^3*z^2 + 106*A^2*a*b^3*c^6*d^2*e^5*f*z^2 - 12*A^2*a*b^2*c^7*d^3*e^4*f*z^2 - 58*A*B*b^8*c^2*d^2*e^3*f^3*z^2 + 40*A*B*b^7*c^3*d^2*e^4*f^2*z^2 - 28*A*B*b^7*c^3*d^3*e^2*f^3*z^2 - 24*A*B*b^5*c^5*d^4*e^2*f^2*z^2 - 20*A*B*b^6*c^4*d^3*e^3*f^2*z^2 + 2768*A*B*a^4*c^6*d^2*e^3*f^3*z^2 - 1712*A*B*a^3*c^7*d^3*e^3*f^2*z^2 - 156*A*B*a^4*b^2*c^4*e^5*f^3*z^2 + 146*A*B*a^4*b^3*c^3*e^4*f^4*z^2 - 106*A*B*a^5*b^2*c^3*e^3*f^5*z^2 + 90*A*B*a^5*b^3*c^2*e^2*f^6*z^2 + 38*A*B*a^3*b^3*c^4*e^6*f^2*z^2 - 36*A*B*a^3*b^5*c^2*e^4*f^4*z^2 + 16*A*B*a^3*b^4*c^3*e^5*f^3*z^2 - 9*A*B*a^4*b^4*c^2*e^3*f^5*z^2 - 8*A*B*a^2*b^5*c^3*e^6*f^2*z^2 + 2*A*B*a^2*b^6*c^2*e^5*f^3*z^2 + 920*A*B*a^4*b^3*c^3*d^2*f^6*z^2 - 480*A*B*a^2*b^5*c^3*d^3*f^5*z^2 - 336*A*B*a^2*b^3*c^5*d^4*f^4*z^2 - 272*A*B*a^3*b^3*c^4*d^3*f^5*z^2 + 240*A*B*a^3*b^5*c^2*d^2*f^6*z^2 - 32*A*B*a*c^9*d^6*e*f*z^2 - 792*B^2*a^2*b^3*c^5*d^3*e^3*f^2*z^2 + 714*B^2*a^2*b^4*c^4*d^3*e^2*f^3*z^2 - 572*B^2*a^3*b^2*c^5*d^3*e^2*f^3*z^2 - 475*B^2*a^2*b^2*c^6*d^4*e^2*f^2*z^2 + 265*B^2*a^4*b^2*c^4*d^2*e^2*f^4*z^2 + 260*B^2*a^3*b^3*c^4*d^2*e^3*f^3*z^2 - 212*B^2*a^3*b^4*c^3*d^2*e^2*f^4*z^2 + 180*B^2*a^3*b^2*c^5*d^2*e^4*f^2*z^2 - 158*B^2*a^2*b^4*c^4*d^2*e^4*f^2*z^2 + 47*B^2*a^2*b^6*c^2*d^2*e^2*f^4*z^2 + 16*B^2*a^2*b^5*c^3*d^2*e^3*f^3*z^2 + 2752*A^2*a^3*b^2*c^5*d^2*e^2*f^4*z^2 - 2148*A^2*a^2*b^4*c^4*d^2*e^2*f^4*z^2 + 2064*A^2*a^2*b^3*c^5*d^2*e^3*f^3*z^2 - 424*A^2*a^2*b^2*c^6*d^3*e^2*f^3*z^2 - 198*A^2*a^2*b^2*c^6*d^2*e^4*f^2*z^2 - 272*B^2*a^6*b*c^3*d*e*f^6*z^2 - 24*B^2*a^4*b^5*c*d*e*f^6*z^2 + 1808*A^2*a^5*b*c^4*d*e*f^6*z^2 - 244*A^2*a*b*c^8*d^4*e^3*f*z^2 + 208*A^2*a*b*c^8*d^5*e*f^2*z^2 + 134*A^2*a^2*b^7*c*d*e*f^6*z^2 - 76*A^2*a*b^4*c^5*d*e^6*f*z^2 + 4*A^2*a*b^8*c*d*e^2*f^5*z^2 + 148*A*B*b^4*c^6*d^5*e*f^2*z^2 + 65*A*B*b^6*c^4*d^4*e*f^3*z^2 + 46*A*B*b^8*c^2*d^3*e*f^4*z^2 - 38*A*B*b^3*c^7*d^5*e^2*f*z^2 + 34*A*B*b^9*c*d^2*e^2*f^4*z^2 - 29*A*B*b^4*c^6*d^4*e^3*f*z^2 + 20*A*B*b^5*c^5*d^3*e^4*f*z^2 + 12*A*B*b^8*c^2*d*e^5*f^2*z^2 - 7*A*B*b^6*c^4*d^2*e^5*f*z^2 - 2880*A*B*a^4*c^6*d^3*e*f^4*z^2 + 2784*A*B*a^5*c^5*d^2*e*f^5*z^2 - 1112*A*B*a^5*c^5*d*e^3*f^4*z^2 + 896*A*B*a^3*c^7*d^4*e*f^3*z^2 + 848*A*B*a^3*c^7*d^2*e^5*f*z^2 - 560*A*B*a^4*c^6*d*e^5*f^2*z^2 + 96*A*B*a^2*c^8*d^5*e*f^2*z^2 - 88*A*B*a^2*c^8*d^4*e^3*f*z^2 - 100*A*B*a^6*b*c^3*e^2*f^6*z^2 - 76*A*B*a^5*b*c^4*e^4*f^4*z^2 + 48*A*B*a^6*b^2*c^2*e*f^7*z^2 - 42*A*B*a^3*b^2*c^5*e^7*f*z^2 + 36*A*B*a^4*b*c^5*e^6*f^2*z^2 - 24*A*B*a^4*b^5*c*e^2*f^6*z^2 + 10*A*B*a^3*b^6*c*e^3*f^5*z^2 + 7*A*B*a^2*b^4*c^4*e^7*f*z^2 + 2*A*B*a^2*b^7*c*e^4*f^4*z^2 - 2496*A*B*a^5*b*c^4*d^2*f^6*z^2 + 1872*A*B*a^4*b*c^5*d^3*f^5*z^2 - 744*A*B*a^5*b^3*c^2*d*f^7*z^2 - 720*A*B*a^2*b*c^7*d^5*f^3*z^2
\end{aligned}$$

$$\begin{aligned}
&^2 + 504*A*B*a*b^3*c^6*d^5*f^3*z^2 + 256*A*B*a^3*b*c^6*d^4*f^4*z^2 + 168*A* \\
&B*a*b^7*c^2*d^3*f^5*z^2 - 144*A*B*a^2*b^7*c*d^2*f^6*z^2 + 144*A*B*a*b^5*c^4 \\
&*d^4*f^4*z^2 + 66*A*B*a^2*b^2*c^6*d*e^7*z^2 - 36*A*B*a*b^2*c^7*d^3*e^5*z^2 \\
&+ 20*A*B*a*b^3*c^6*d^2*e^6*z^2 + 12*A*B*a^2*b*c^7*d^2*e^6*z^2 + 1208*B^2*a^ \\
&3*b*c^6*d^3*e^3*f^2*z^2 - 848*B^2*a^3*b^3*c^4*d^3*e*f^4*z^2 + 672*B^2*a^2*b \\
&^3*c^5*d^4*e*f^3*z^2 - 632*B^2*a^4*b^3*c^3*d^2*e^3*f^3*z^2 + 432*B^2*a^4*b^3* \\
&c^3*d^2*e*f^5*z^2 + 276*B^2*a^2*b^2*c^6*d^3*e^4*f*z^2 - 196*B^2*a*b^6*c^3*d \\
&^3*e^2*f^3*z^2 - 168*B^2*a^2*b^5*c^3*d^3*e*f^4*z^2 + 154*B^2*a^2*b^3*c^5*d^ \\
&2*e^5*f*z^2 + 148*B^2*a*b^5*c^4*d^3*e^3*f^2*z^2 + 96*B^2*a*b^4*c^5*d^4*e^2* \\
&f^2*z^2 - 72*B^2*a^3*b^5*c^2*d^2*e*f^5*z^2 + 70*B^2*a^5*b^2*c^3*d*e^2*f^5*z \\
&^2 - 60*B^2*a^4*b^3*c^3*d*e^3*f^4*z^2 + 52*B^2*a*b^6*c^3*d^2*e^4*f^2*z^2 + \\
&36*B^2*a^4*b^2*c^4*d*e^4*f^3*z^2 - 32*B^2*a*b^7*c^2*d^2*e^3*f^3*z^2 + 24*B^ \\
&2*a^3*b^5*c^2*d*e^3*f^4*z^2 + 15*B^2*a^4*b^4*c^2*d*e^2*f^5*z^2 - 8*B^2*a^3* \\
&b^4*c^3*d*e^4*f^3*z^2 + 8*B^2*a^2*b^5*c^3*d*e^5*f^2*z^2 - 2*B^2*a^3*b^3*c^4 \\
&*d*e^5*f^2*z^2 - 2*B^2*a^2*b^6*c^2*d*e^4*f^3*z^2 - 3176*A^2*a^3*b*c^6*d^2*e \\
&^3*f^3*z^2 - 2252*A^2*a^4*b^2*c^4*d*e^2*f^5*z^2 + 1952*A^2*a^3*b^4*c^3*d*e^ \\
&2*f^5*z^2 - 1496*A^2*a^3*b^3*c^4*d*e^3*f^4*z^2 + 1378*A^2*a^2*b^4*c^4*d*e^4 \\
&*f^3*z^2 + 1184*A^2*a^3*b^3*c^4*d^2*e*f^5*z^2 - 1166*A^2*a^2*b^3*c^5*d*e^5* \\
&f^2*z^2 - 1164*A^2*a^3*b^2*c^5*d*e^4*f^3*z^2 - 1152*A^2*a^2*b^3*c^5*d^3*e*f \\
&^4*z^2 + 578*A^2*a*b^6*c^3*d^2*e^2*f^4*z^2 - 548*A^2*a*b^5*c^4*d^2*e^3*f^3* \\
&z^2 + 440*A^2*a*b^2*c^7*d^4*e^2*f^2*z^2 - 412*A^2*a^2*b^6*c^2*d*e^2*f^5*z^2 \\
&- 360*A^2*a*b^3*c^6*d^3*e^3*f^2*z^2 + 312*A^2*a*b^4*c^5*d^3*e^2*f^3*z^2 + \\
&248*A^2*a^2*b^5*c^3*d^2*e*f^5*z^2 + 52*A^2*a*b^4*c^5*d^2*e^4*f^2*z^2 - 16*B^2* \\
&b^3*c^7*d^6*e*f*z^2 - 14*B^2*b^9*c*d^3*e*f^4*z^2 + 32*B^2*a^4*c^6*d*e^6*f*z \\
&^2 - 20*A^2*b^9*c*d*e^3*f^4*z^2 + 18*A^2*b^9*c*d^2*e*f^5*z^2 + 8*A^2*b^6*c^ \\
&4*d*e^6*f*z^2 - 360*A^2*a^3*c^7*d*e^6*f*z^2 + 136*A^2*a*c^9*d^5*e^2*f*z^2 + \\
&2*B^2*a^3*b^7*d*e*f^6*z^2 + 2*B^2*a*b^9*d^2*e*f^5*z^2 + 12*B^2*a^4*b*c^5*e \\
&^7*f*z^2 - 204*A^2*a^3*b*c^6*e^7*f*z^2 - 128*A^2*a^6*b*c^3*e*f^7*z^2 - 48*A \\
&^2*a*b^5*c^4*e^7*f*z^2 - 36*B^2*a^5*b^4*c*d*f^7*z^2 - 24*A^2*a^4*b^5*c*e*f^ \\
&7*z^2 - 16*B^2*a*b^8*c*d^3*f^5*z^2 - 164*A^2*a^3*b^6*c*d*f^7*z^2 - 16*A^2*a \\
&*b^8*c*d^2*f^6*z^2 + 4*B^2*a^3*b*c^6*d*e^7*z^2 - 4*B^2*a*b*c^8*d^5*e^3*z^2 \\
&+ 48*A^2*a*b*c^8*d^3*e^5*z^2 + 36*A^2*a^2*b*c^7*d*e^7*z^2 - 6*A^2*a*b^3*c^6 \\
&*d*e^7*z^2 + 136*A*B*a^6*c^4*e^3*f^5*z^2 - 96*A*B*b^5*c^5*d^5*f^3*z^2 + 80* \\
&A*B*a^5*c^5*e^5*f^3*z^2 - 72*A*B*b^3*c^7*d^6*f^2*z^2 - 24*A*B*b^7*c^3*d^4*f \\
&^4*z^2 + 14*A*B*b^3*c^7*d^4*e^4*z^2 - 14*A*B*b^2*c^8*d^5*e^3*z^2 - 2*A*B*b^ \\
&5*c^5*d^2*e^6*z^2 - 2*A*B*b^4*c^6*d^3*e^5*z^2 + 2*A*B*a^3*b^7*e^2*f^6*z^2 - \\
&A*B*a^2*b^8*e^3*f^5*z^2 + 16*A*B*a^2*c^8*d^3*e^5*z^2 - 2*A*B*a^2*b^3*c^5*e \\
&^8*z^2 + 22*B^2*b^8*c^2*d^3*e^2*f^3*z^2 - 12*B^2*b^7*c^3*d^3*e^3*f^2*z^2 + \\
&12*B^2*b^6*c^4*d^4*e^2*f^2*z^2 - 6*B^2*b^8*c^2*d^2*e^4*f^2*z^2 - 864*B^2*a^ \\
&4*c^6*d^3*e^2*f^3*z^2 + 496*B^2*a^3*c^7*d^4*e^2*f^2*z^2 + 224*B^2*a^5*c^5*d \\
&^2*e^2*f^4*z^2 + 136*B^2*a^4*c^6*d^2*e^4*f^2*z^2 - 53*A^2*b^8*c^2*d^2*e^2*f \\
&^4*z^2 + 52*A^2*b^7*c^3*d^2*e^3*f^3*z^2 + 52*A^2*b^5*c^5*d^3*e^3*f^2*z^2 - \\
&36*A^2*b^6*c^4*d^3*e^2*f^3*z^2 - 12*A^2*b^4*c^6*d^4*e^2*f^2*z^2 - 9*A^2*b^6 \\
&*c^4*d^2*e^4*f^2*z^2 + 836*A^2*a^4*c^6*d^2*e^2*f^4*z^2 - 668*A^2*a^2*c^8*d^
\end{aligned}$$

$$\begin{aligned}
& 4e^2f^2z^2 + 656A^2a^3c^7d^2e^4f^2z^2 + 368A^2a^3c^7d^3e^2f^3z^2 - 45B^2a^6b^2c^2e^2f^6z^2 - 18B^2a^5b^2c^3e^4f^4z^2 - \\
& 9B^2a^4b^2c^4e^6f^2z^2 - 6B^2a^5b^3c^2e^3f^5z^2 + 3B^2a^4b^4c^2e^4f^4z^2 - 2B^2a^4b^3c^3e^5f^3z^2 - 580B^2a^4b^2c^4d^3f^5z^2 + 536B^2a^3b^4c^3d^3f^5z^2 + 471A^2a^4b^2c^4e^4f^4z^2 - \\
& 436A^2a^3b^4c^3e^4f^4z^2 - 348B^2a^4b^4c^2d^2f^6z^2 + 316B^2a^2b^2c^6d^5f^3z^2 + 310A^2a^3b^3c^4e^5f^3z^2 + 232A^2a^5b^2c^3e^2f^6z^2 - 229A^2a^2b^4c^4e^6f^2z^2 - 216A^2a^4b^4c^2e^2f^6z^2 + 204A^2a^4b^3c^3e^3f^5z^2 + 200B^2a^5b^2c^3d^2f^6z^2 + 150A^2a^3b^2c^5e^6f^2z^2 - 120B^2a^2b^4c^4d^4f^4z^2 + 91A^2a^2b^6c^2e^4f^4z^2 + 72A^2a^3b^5c^2e^3f^5z^2 - 66B^2a^2b^6c^2d^3f^5z^2 + 44A^2a^2b^5c^3e^5f^3z^2 - 16B^2a^3b^2c^5d^4f^4z^2 + 1952A^2a^4b^2c^4d^2f^6z^2 - 1792A^2a^3b^2c^5d^3f^5z^2 - 1272A^2a^3b^4c^3d^2f^6z^2 + 976A^2a^2b^2c^6d^4f^4z^2 + 960A^2a^2b^4c^4d^3f^5z^2 + 282A^2a^2b^6c^2d^2f^6z^2 - 45B^2a^2b^2c^6d^2e^6z^2 - 48A^2b^9c^9d^6e^6f^6z^2 - 14A^2a^9b^9d^6e^6f^6z^2 - 7A^2B^2b^10d^2e^6f^5z^2 + 2A^2B^2b^10d^2e^3f^4z^2 - 64A^2B^2a^7c^3e^6f^7z^2 - 16A^2B^2b^9c^4d^3f^5z^2 + 8A^2B^2a^4c^6e^7f^6z^2 + 4A^2B^2b^9c^9d^6e^2z^2 + 2A^2B^2b^6c^4d^6e^7z^2 - 120A^2B^2a^3c^7d^6e^7z^2 - 16A^2B^2a^3b^7d^6f^7z^2 + 16A^2B^2a^9d^2f^6z^2 + 8A^2B^2a^9c^9d^5e^3z^2 + 12A^2B^2a^3b^6c^6e^8z^2 - 48B^2b^5c^5d^5e^6f^2z^2 + 15B^2b^4c^6d^5e^2f^3z^2 - 14B^2b^7c^3d^4e^6f^3z^2 + 4B^2b^9c^4d^2e^3f^3z^2 + 4B^2b^7c^3d^2e^5f^4z^2 + 4B^2b^5c^5d^4e^3f^4z^2 - B^2b^6c^4d^3e^4f^4z^2 - 336B^2a^3c^7d^3e^4f^3z^2 + 112B^2a^5c^5d^6e^4f^3z^2 - 112A^2b^3c^7d^5e^6f^2z^2 + 80B^2a^6c^4d^6e^2f^5z^2 - 48A^2b^5c^5d^4e^6f^3z^2 + 36A^2b^8c^2d^6e^4f^3z^2 + 36A^2b^3c^7d^4e^3f^3z^2 - 28A^2b^7c^3d^6e^5f^2z^2 + 20A^2b^2c^8d^5e^2f^3z^2 + 16B^2a^2c^8d^5e^2f^3z^2 - 14A^2b^7c^3d^3e^6f^4z^2 - 14A^2b^4c^6d^3e^4f^3z^2 - 10A^2b^5c^5d^2e^5f^4z^2 - 1008A^2a^4c^6d^6e^4f^3z^2 - 760A^2a^5c^5d^6e^2f^5z^2 + 272A^2a^2c^8d^3e^4f^3z^2 + 48B^2a^5b^6c^4e^5f^3z^2 + 36B^2a^6b^6c^3e^3f^5z^2 + 12B^2a^5b^4c^6e^2f^6z^2 - 624A^2a^4b^6c^5e^5f^3z^2 - 548A^2a^5b^6c^4e^3f^5z^2 + 182A^2a^2b^3c^5e^7f^6z^2 - 180B^2a^6b^4c^5d^5f^3z^2 + 132B^2a^6b^2c^2d^6f^7z^2 + 108B^2a^3b^6c^4d^2f^6z^2 + 96A^2a^5b^3c^2e^6f^7z^2 + 68A^2a^6b^6c^3e^6f^2z^2 + 58A^2a^3b^6c^6e^2f^6z^2 - 56B^2a^6b^2c^7d^6f^2z^2 - 38A^2a^2b^7c^6e^3f^5z^2 - 36A^2a^6b^7c^2e^5f^3z^2 + 20B^2a^6b^6c^3d^4f^4z^2 - 736A^2a^5b^2c^3d^6f^7z^2 + 624A^2a^4b^4c^2d^6f^7z^2 - 416A^2a^6b^2c^7d^5f^3z^2 - 276A^2a^6b^4c^5d^4f^4z^2 - 196A^2a^6b^6c^3d^3f^5z^2 + 8B^2a^6b^4c^5d^2e^6z^2 + 6B^2a^6b^2c^7d^4e^4z^2 + 2B^2a^2b^3c^5d^6e^7z^2 + 2B^2a^6b^3c^6d^3e^5z^2 - 18A^2a^6b^2c^7d^2e^6z^2 - 16A^2B^2b^9c^9d^7f^6z^2 - B^2b^10d^2e^2f^4z^2 + 48B^2a^7c^3e^2f^6z^2 - 36B^2a^6c^4e^4f^4z^2 + 31B^2b^6c^4d^5f^3z^2 - 24B^2a^5c^5e^6f^2z^2 + 20B^2b^4c^6d^6f^2z^2 - 6A^2b^8c^2e^6f^2z^2 + 2B^2b^8c^2d^4f^4z^2 - 768B^2a^5c^5d^3f^5z^2 + 512B^2a^6c^4d^2f^6z^2 + 5
\end{aligned}$$

$$\begin{aligned}
& 12*B^2*a^4*c^6*d^4*f^4*z^2 + 232*A^2*a^5*c^5*e^4*f^4*z^2 + 188*A^2*a^4*c^6* \\
& e^6*f^2*z^2 - 128*B^2*a^3*c^7*d^5*f^3*z^2 + 92*A^2*a^6*c^4*e^2*f^6*z^2 + 80 \\
& *A^2*b^4*c^6*d^5*f^3*z^2 + 64*A^2*b^2*c^8*d^6*f^2*z^2 + 31*A^2*b^6*c^4*d^4* \\
& f^4*z^2 + 14*A^2*b^8*c^2*d^3*f^5*z^2 - 5*B^2*b^4*c^6*d^4*e^4*z^2 + 4*B^2*b^ \\
& 3*c^7*d^5*e^3*z^2 + 2*B^2*b^5*c^5*d^3*e^5*z^2 - B^2*b^6*c^4*d^2*e^6*z^2 - B \\
& ^2*b^2*c^8*d^6*e^2*z^2 - B^2*a^4*b^6*e^2*f^6*z^2 - 1152*A^2*a^3*c^7*d^4*f^4 \\
& *z^2 + 1008*A^2*a^4*c^6*d^3*f^5*z^2 + 624*A^2*a^2*c^8*d^5*f^3*z^2 - 288*A^2 \\
& *a^5*c^5*d^2*f^6*z^2 + 56*B^2*a^3*c^7*d^2*e^6*z^2 - 10*B^2*a^2*b^8*d^2*f^6* \\
& z^2 - 9*A^2*b^2*c^8*d^4*e^4*z^2 - 5*A^2*a^2*b^8*e^2*f^6*z^2 - 4*B^2*a^2*c^8 \\
& *d^4*e^4*z^2 + 3*A^2*b^4*c^6*d^2*e^6*z^2 - 2*A^2*b^3*c^7*d^3*e^5*z^2 - 36*A \\
& ^2*a^2*c^8*d^2*e^6*z^2 - 48*A^2*a^6*b^2*c^2*f^8*z^2 - 45*A^2*a^2*b^2*c^6*e^ \\
& 8*z^2 + 4*A^2*b^10*d*e^2*f^5*z^2 + 4*B^2*b^2*c^8*d^7*f*z^2 + 4*A^2*b^9*c*e^ \\
& 5*f^3*z^2 + 4*A^2*b^7*c^3*e^7*f*z^2 - 128*B^2*a^7*c^3*d*f^7*z^2 - 160*A^2*a \\
& *c^9*d^6*f^2*z^2 - 112*A^2*a^6*c^4*d*f^7*z^2 + 12*A^2*b*c^9*d^5*e^3*z^2 + 4 \\
& *A^2*a*b^9*e^3*f^5*z^2 + 3*B^2*a^4*b^6*d*f^7*z^2 + 2*A^2*a^3*b^7*e*f^7*z^2 \\
& - 24*A^2*a*c^9*d^4*e^4*z^2 + 14*A^2*a^2*b^8*d*f^7*z^2 + 12*A^2*a^5*b^4*c*f^ \\
& 8*z^2 + 12*A^2*a*b^4*c^5*e^8*z^2 + A*B*a^4*b^6*e*f^7*z^2 + B^2*a^2*b^8*d*e^ \\
& 2*f^5*z^2 + 16*A^2*c^10*d^7*f*z^2 + 3*B^2*b^10*d^3*f^5*z^2 - A^2*b^10*e^4*f \\
& ^4*z^2 - 4*A^2*c^10*d^6*e^2*z^2 - A^2*b^10*d^2*f^6*z^2 + 64*A^2*a^7*c^3*f^8 \\
& *z^2 - 4*B^2*a^4*c^6*e^8*z^2 - A^2*b^6*c^4*e^8*z^2 + 48*A^2*a^3*c^7*e^8*z^2 \\
& - A^2*a^4*b^6*f^8*z^2 + 720*A^2*B*a*b^2*c^5*d^2*e^2*f^3*z - 600*A^2*B*a^2* \\
& b^2*c^4*d*e^2*f^4*z + 576*A*B^2*a^2*b^2*c^4*d^2*e*f^4*z + 348*A*B^2*a*b^2*c \\
& ^5*d^2*e^3*f^2*z - 336*A*B^2*a^2*b*c^5*d^2*e^2*f^3*z - 260*A*B^2*a*b^3*c^4* \\
& d^2*e^2*f^3*z - 240*A*B^2*a^2*b^2*c^4*d*e^3*f^3*z + 196*A*B^2*a^2*b^3*c^3*d \\
& *e^2*f^4*z + 172*A^2*B*a*b*c^6*d*e^5*f*z + 20*A*B^2*a*b^6*c*d*e*f^5*z - 912 \\
& *A^2*B*a^2*b*c^5*d^2*e*f^4*z - 644*A^2*B*a*b*c^6*d^2*e^3*f^2*z - 432*A*B^2* \\
& a*b^2*c^5*d^3*e*f^3*z + 372*A^2*B*a^2*b*c^5*d*e^3*f^3*z - 330*A^2*B*a*b^2*c \\
& ^5*d*e^4*f^2*z + 312*A*B^2*a*b*c^6*d^3*e^2*f^2*z - 208*A*B^2*a^3*b^2*c^3*d* \\
& e*f^5*z + 192*A^2*B*a^2*b^3*c^3*d*e*f^5*z + 172*A^2*B*a*b^3*c^4*d*e^3*f^3*z \\
& + 108*A*B^2*a^2*b*c^5*d*e^4*f^2*z + 104*A*B^2*a^3*b*c^4*d*e^2*f^4*z - 80*A \\
& ^2*B*a*b^3*c^4*d^2*e*f^4*z + 68*A^2*B*a*b^4*c^3*d*e^2*f^4*z - 60*A*B^2*a*b^ \\
& 5*c^2*d*e^2*f^4*z + 58*A*B^2*a*b^3*c^4*d*e^4*f^2*z - 36*A*B^2*a*b^4*c^3*d^2 \\
& *e*f^4*z - 24*A*B^2*a^2*b^4*c^2*d*e*f^5*z + 24*A*B^2*a*b^4*c^3*d*e^3*f^3*z \\
& + 592*A^2*B*a*b*c^6*d^3*e*f^3*z + 240*A^2*B*a^3*b*c^4*d*e*f^5*z - 132*A*B^2 \\
& *a*b*c^6*d^2*e^4*f*z - 60*A*B^2*a*b^2*c^5*d*e^5*f*z - 48*A^2*B*a*b^5*c^2*d* \\
& e*f^5*z + 20*B^3*a*b*c^6*d^3*e^3*f*z + 16*B^3*a^4*b*c^3*d*e*f^5*z - 16*B^3* \\
& a*b*c^6*d^4*e*f^2*z + 12*B^3*a^2*b*c^5*d*e^5*f*z + 320*A^3*a*b*c^6*d*e^4*f^ \\
& 2*z + 40*A^3*a*b^4*c^3*d*e*f^5*z - 48*A^2*B*b*c^7*d^4*e*f^2*z - 44*A^2*B*b^ \\
& 3*c^5*d*e^5*f*z - 20*A*B^2*b*c^7*d^4*e^2*f*z + 14*A*B^2*b^4*c^4*d*e^5*f*z + \\
& 12*A^2*B*b*c^7*d^3*e^3*f*z + 4*A*B^2*b^7*c*d*e^2*f^4*z + 160*A*B^2*a^4*c^4 \\
& *d*e*f^5*z + 152*A^2*B*a*c^7*d^2*e^4*f*z - 40*A*B^2*a*c^7*d^3*e^3*f*z + 32* \\
& A*B^2*a*c^7*d^4*e*f^2*z - 16*A*B^2*a^2*c^6*d*e^5*f*z + 128*A^2*B*a^4*b*c^3* \\
& e*f^6*z + 42*A^2*B*a*b^2*c^5*e^6*f*z + 24*A^2*B*a^2*b^5*c*e*f^6*z - 12*A*B^ \\
& 2*a^3*b^4*c*e*f^6*z - 12*A*B^2*a^2*b*c^5*e^6*f*z - 10*A^2*B*a*b^6*c*e^2*f^5 \\
& *z - 160*A*B^2*a*b*c^6*d^4*f^3*z + 112*A*B^2*a^4*b*c^3*d*f^6*z - 24*A*B^2*a
\end{aligned}$$

$$\begin{aligned}
&^2*b^5*c*d*f^6*z - 84*B^3*a*b^2*c^5*d^3*e^2*f^2*z - 80*B^3*a^2*b^3*c^3*d^2* \\
&e*f^4*z - 60*B^3*a^2*b*c^5*d^2*e^3*f^2*z - 20*B^3*a^3*b^2*c^3*d*e^2*f^4*z - \\
&20*B^3*a*b^3*c^4*d^2*e^3*f^2*z - 9*B^3*a^2*b^2*c^4*d*e^4*f^2*z - 8*B^3*a*b \\
&^4*c^3*d^2*e^2*f^3*z + 6*B^3*a^2*b^4*c^2*d*e^2*f^4*z - 4*B^3*a^2*b^3*c^3*d* \\
&e^3*f^3*z - 216*A^2*B*b^4*c^4*d^2*e^2*f^3*z + 196*A^2*B*b^3*c^5*d^2*e^3*f^2 \\
&*z - 108*A*B^2*b^3*c^5*d^3*e^2*f^2*z - 94*A*B^2*b^4*c^4*d^2*e^3*f^2*z + 88* \\
&A^2*B*b^2*c^6*d^3*e^2*f^2*z + 80*A*B^2*b^5*c^3*d^2*e^2*f^3*z + 360*A^2*B*a^ \\
&2*c^6*d^2*e^2*f^3*z + 8*A*B^2*a^2*c^6*d^2*e^3*f^2*z + 153*A^2*B*a^2*b^2*c^4 \\
&*e^4*f^3*z - 144*A^2*B*a^2*b^3*c^3*e^3*f^4*z + 80*A^2*B*a^3*b^2*c^3*e^2*f^5 \\
&*z + 36*A*B^2*a^3*b^2*c^3*e^3*f^4*z + 12*A^2*B*a^2*b^4*c^2*e^2*f^5*z + 12*A \\
&*B^2*a^3*b^3*c^2*e^2*f^5*z + 9*A*B^2*a^2*b^2*c^4*e^5*f^2*z - 6*A*B^2*a^2*b^ \\
&4*c^2*e^3*f^4*z + 4*A*B^2*a^2*b^3*c^3*e^4*f^3*z + 480*A^2*B*a^2*b^2*c^4*d^2 \\
&*f^5*z - 176*A*B^2*a^2*b^3*c^3*d^2*f^5*z - 10*A^2*B*a*b^6*c*d*f^6*z + 16*A* \\
&B^2*a*b*c^6*d*e^6*z + 80*B^3*a*b^3*c^4*d^3*e*f^3*z - 48*B^3*a^3*b*c^4*d^2*e \\
&*f^4*z + 48*B^3*a^2*b*c^5*d^3*e*f^3*z + 44*B^3*a^3*b*c^4*d*e^3*f^3*z + 24*B \\
&^3*a*b^5*c^2*d^2*e*f^4*z + 18*B^3*a*b^2*c^5*d^2*e^4*f*z + 696*A^3*a^2*b*c^5 \\
&*d*e^2*f^4*z - 504*A^3*a*b*c^6*d^2*e^2*f^3*z - 192*A^3*a*b^2*c^5*d*e^3*f^3* \\
&z - 144*A^3*a^2*b^2*c^4*d*e*f^5*z + 96*A^3*a*b^2*c^5*d^2*e*f^4*z - 72*A^3*a \\
&*b^3*c^4*d*e^2*f^4*z - 208*A^2*B*b^3*c^5*d^3*e*f^3*z + 152*A*B^2*b^4*c^4*d^ \\
&3*e*f^3*z + 80*A^2*B*b^5*c^3*d^2*e*f^4*z + 75*A^2*B*b^4*c^4*d*e^4*f^2*z - 5 \\
&9*A^2*B*b^2*c^6*d^2*e^4*f*z - 52*A^2*B*b^5*c^3*d*e^3*f^3*z + 42*A*B^2*b^3*c \\
&^5*d^2*e^4*f*z - 21*A*B^2*b^6*c^2*d^2*e*f^4*z - 16*A*B^2*b^5*c^3*d*e^4*f^2* \\
&z + 16*A*B^2*b^2*c^6*d^4*e*f^2*z + 16*A*B^2*b^2*c^6*d^3*e^3*f*z + 11*A^2*B* \\
&b^6*c^2*d*e^2*f^4*z + 4*A*B^2*b^6*c^2*d*e^3*f^3*z - 256*A^2*B*a*c^7*d^3*e^2 \\
&*f^2*z - 96*A*B^2*a^3*c^5*d^2*e*f^4*z - 36*A^2*B*a^2*c^6*d*e^4*f^2*z - 32*A \\
&^2*B*a^3*c^5*d*e^2*f^4*z - 32*A*B^2*a^2*c^6*d^3*e*f^3*z + 8*A*B^2*a^3*c^5*d \\
&*e^3*f^3*z - 96*A^2*B*a^3*b^3*c^2*e*f^6*z + 68*A^2*B*a^3*b*c^4*e^3*f^4*z - \\
&60*A*B^2*a^4*b*c^3*e^2*f^5*z - 60*A*B^2*a^3*b*c^4*e^4*f^3*z + 48*A*B^2*a^4* \\
&b^2*c^2*e*f^6*z - 38*A^2*B*a*b^3*c^4*e^5*f^2*z - 36*A^2*B*a^2*b*c^5*e^5*f^2 \\
&*z + 36*A^2*B*a*b^5*c^2*e^3*f^4*z - 16*A^2*B*a*b^4*c^3*e^4*f^3*z + 384*A*B^ \\
&2*a^2*b*c^5*d^3*f^4*z - 352*A*B^2*a^3*b*c^4*d^2*f^5*z - 288*A^2*B*a*b^2*c^5 \\
&*d^3*f^4*z - 160*A^2*B*a^3*b^2*c^3*d*f^6*z - 148*A^2*B*a*b^4*c^3*d^2*f^5*z \\
&+ 112*A*B^2*a*b^3*c^4*d^3*f^4*z + 72*A^2*B*a^2*b^4*c^2*d*f^6*z + 72*A*B^2*a \\
&*b^5*c^2*d^2*f^5*z + 48*A*B^2*a^3*b^3*c^2*d*f^6*z + 102*B^3*a^2*b^2*c^4*d^2 \\
&*e^2*f^3*z - 32*B^3*b^5*c^3*d^3*e*f^3*z - 8*B^3*b^3*c^5*d^3*e^3*f*z - 7*B^3 \\
&*b^4*c^4*d^2*e^4*f*z + 5*B^3*b^2*c^6*d^4*e^2*f*z + 80*A^3*b^2*c^6*d^3*e*f^3 \\
&*z - 74*A^3*b^3*c^5*d*e^4*f^2*z - 64*A^3*b^4*c^4*d^2*e*f^4*z + 60*A^3*b^4*c \\
&^4*d*e^3*f^3*z - 48*B^3*a^4*c^4*d*e^2*f^4*z - 24*B^3*a^3*c^5*d*e^4*f^2*z + \\
&20*B^3*a^2*c^6*d^2*e^4*f*z - 16*A^3*b^5*c^3*d*e^2*f^4*z + 8*A^3*b*c^7*d^3*e \\
&^2*f^2*z + 480*A^3*a^2*c^6*d^2*e*f^4*z - 392*A^3*a^2*c^6*d*e^3*f^3*z + 280* \\
&A^3*a*c^7*d^2*e^3*f^2*z - 4*B^3*a^4*b*c^3*e^3*f^4*z - 200*A^3*a^3*b*c^4*e^2 \\
&*f^5*z - 144*A^3*a^2*b*c^5*e^4*f^3*z + 48*B^3*a*b^2*c^5*d^4*f^3*z + 42*A^3* \\
&a*b^2*c^5*e^5*f^2*z - 36*B^3*a^4*b^2*c^2*d*f^6*z - 32*A^3*a^3*b^2*c^3*e*f^6 \\
&*z - 24*A^3*a^2*b^4*c^2*e*f^6*z - 24*A^3*a*b^5*c^2*e^2*f^5*z + 10*A^3*a*b^3 \\
&*c^4*e^4*f^3*z - 4*B^3*a*b^4*c^3*d^3*f^4*z - 4*A^3*a*b^4*c^3*e^3*f^4*z - 48
\end{aligned}$$

$$\begin{aligned}
& 0*A^3*a^2*b*c^5*d^2*f^5*z - 160*A^3*a^2*b^3*c^3*d*f^6*z + 128*A^3*a*b^3*c^4 \\
& *d^2*f^5*z + 8*A^2*B*b^5*c^3*e^5*f^2*z - 2*A^2*B*b^6*c^2*e^4*f^3*z + 112*A^ \\
& 2*B*b^4*c^4*d^3*f^4*z - 92*A^2*B*a^4*c^4*e^2*f^5*z - 64*A^2*B*a^3*c^5*e^4*f \\
& ^3*z - 64*A*B^2*b^5*c^3*d^3*f^4*z + 24*A*B^2*a^4*c^4*e^3*f^4*z + 24*A*B^2*a \\
& ^3*c^5*e^5*f^2*z + 16*A^2*B*b^2*c^6*d^4*f^3*z + 16*A*B^2*b^3*c^5*d^4*f^3*z \\
& - A^2*B*b^6*c^2*d^2*f^5*z + 448*A^2*B*a^3*c^5*d^2*f^5*z - 352*A^2*B*a^2*c^6 \\
& *d^3*f^4*z - 5*A*B^2*b^2*c^6*d^2*e^5*z - 48*A^2*B*a^4*b^2*c^2*f^7*z - 2*B^3 \\
& *b^7*c*d^2*e*f^4*z + 34*A^3*b^2*c^6*d*e^5*f*z + 16*A^3*b*c^7*d^2*e^4*f*z + \\
& 2*A^3*b^6*c^2*d*e*f^5*z - 416*A^3*a^3*c^5*d*e*f^5*z - 224*A^3*a*c^7*d^3*e*f \\
& ^3*z + 12*B^3*a^3*b^4*c*d*f^6*z - 10*B^3*a*b^6*c*d^2*f^5*z + 416*A^3*a^3*b* \\
& c^4*d*f^6*z + 224*A^3*a*b*c^6*d^3*f^4*z + 24*A^3*a*b^5*c^2*d*f^6*z - 4*B^3* \\
& a*b*c^6*d^2*e^5*z + 20*A^2*B*c^8*d^4*e^2*f*z - 7*A^2*B*b^4*c^4*e^6*f*z - 2* \\
& A^2*B*b^7*c*e^3*f^4*z - 64*A*B^2*a^5*c^3*e*f^6*z + 16*A*B^2*b*c^7*d^5*f^2*z \\
& - 8*A^2*B*a^2*c^6*e^6*f*z - 2*A*B^2*b^7*c*d^2*f^5*z - 272*A^2*B*a^4*c^4*d* \\
& f^6*z + 128*A^2*B*a*c^7*d^4*f^3*z + 9*A^2*B*b^2*c^6*d*e^6*z - 4*A*B^2*b^3*c \\
& ^5*d*e^6*z + 4*A*B^2*b*c^7*d^3*e^4*z + 8*A*B^2*a*c^7*d^2*e^5*z + 12*A^2*B*a \\
& ^3*b^4*c*f^7*z + 30*B^3*b^4*c^4*d^3*e^2*f^2*z + 8*B^3*b^5*c^3*d^2*e^3*f^2*z \\
& - 2*B^3*b^6*c^2*d^2*e^2*f^3*z + 152*A^3*b^3*c^5*d^2*e^2*f^3*z - 108*A^3*b^ \\
& 2*c^6*d^2*e^3*f^2*z + 48*B^3*a^3*c^5*d^2*e^2*f^3*z - 16*B^3*a^2*c^6*d^3*e^2 \\
& *f^2*z - 3*B^3*a^4*b^2*c^2*e^2*f^5*z - 120*B^3*a^2*b^2*c^4*d^3*f^4*z + 112* \\
& B^3*a^3*b^2*c^3*d^2*f^5*z + 112*A^3*a^2*b^3*c^3*e^2*f^5*z + 12*A^3*a^2*b^2* \\
& c^4*e^3*f^4*z - 120*A^3*a*c^7*d*e^5*f*z - 52*A^3*a*b*c^6*e^6*f*z + 10*A^3*a \\
& *b^6*c*e*f^6*z - 2*A*B^2*b^8*d*e*f^5*z - 2*A^2*B*a*b^7*e*f^6*z - 24*A^2*B*a \\
& *c^7*d*e^6*z + 2*A*B^2*a*b^7*d*f^6*z - 12*A^2*B*a*b*c^6*e^7*z - 2*A^3*b^7*c \\
& *d*f^6*z - 4*A^3*b*c^7*d*e^6*z + 16*B^3*a^5*c^3*e^2*f^5*z + 11*B^3*b^6*c^2* \\
& d^3*f^4*z - 11*A^3*b^4*c^4*e^5*f^2*z - 8*B^3*b^4*c^4*d^4*f^3*z - 4*B^3*b^2* \\
& c^6*d^5*f^2*z + 4*B^3*a^4*c^4*e^4*f^3*z + 4*A^3*b^5*c^3*e^4*f^3*z - A^3*b^6 \\
& *c^2*e^3*f^4*z + 136*A^3*a^3*c^5*e^3*f^4*z + 68*A^3*a^2*c^6*e^5*f^2*z - 64* \\
& A^3*b^3*c^5*d^3*f^4*z + 2*B^3*b^3*c^5*d^2*e^5*z - B^3*b^2*c^6*d^3*e^4*z + 9 \\
& 6*A^3*a^3*b^3*c^2*f^7*z + A*B^2*a^2*b^6*e*f^6*z + 32*A^3*c^8*d^4*e*f^2*z - \\
& 24*A^3*c^8*d^3*e^3*f*z + 10*A^3*b^3*c^5*e^6*f*z + 2*A^3*b^7*c*e^2*f^5*z + 1 \\
& 28*A^3*a^4*c^4*e*f^6*z - 32*A^3*b*c^7*d^4*f^3*z - 4*B^3*a^2*c^6*d*e^6*z - B \\
& ^3*a^2*b^6*d*f^6*z - 128*A^3*a^4*b*c^3*f^7*z - 24*A^3*a^2*b^5*c*f^7*z - 16* \\
& A^2*B*c^8*d^5*f^2*z - 4*A^2*B*c^8*d^3*e^4*z + 64*A^2*B*a^5*c^3*f^7*z + 2*A^ \\
& 2*B*b^3*c^5*e^7*z + 4*A*B^2*a^2*c^6*e^7*z - A^2*B*a^2*b^6*f^7*z + 4*A^3*c^8 \\
& *d^2*e^5*z - 3*A^3*b^2*c^6*e^7*z + A^2*B*b^8*d*f^6*z - A^3*b^8*e*f^6*z + 16 \\
& *A^3*a*c^7*e^7*z + 2*A^3*a*b^7*f^7*z + A^2*B*b^8*e^2*f^5*z + B^3*b^8*d^2*f^ \\
& 5*z - 48*A^2*B^2*a*b*c^4*d*e*f^4 + 28*A*B^3*a*b^2*c^3*d*e*f^4 - 16*A*B^3*a* \\
& b*c^4*d*e^2*f^3 + 16*A^3*B*a*c^5*d*e*f^4 + 32*A^3*B*a*b*c^4*d*f^5 + 12*A^2* \\
& B^2*b^3*c^3*d*e*f^4 + 5*A*B^3*b^2*c^4*d^2*e*f^3 + 4*A*B^3*b^3*c^3*d*e^2*f^3 \\
& + 24*A^2*B^2*a*c^5*d*e^2*f^3 + 24*A^2*B^2*a^2*b*c^3*e*f^5 + 12*A^2*B^2*a*b \\
& *c^4*e^3*f^3 - 6*A^2*B^2*a*b^3*c^2*e*f^5 + 4*A*B^3*a^2*b*c^3*e^2*f^4 + 3*A* \\
& B^3*a^2*b^2*c^2*e*f^5 - 18*A^2*B^2*a*b^2*c^3*d*f^5 - 4*B^4*a^2*b*c^3*d*e*f^ \\
& 4 + 4*B^4*a*b*c^4*d^2*e*f^3 - 6*A*B^3*b^4*c^2*d*e*f^4 + 4*A^3*B*b*c^5*d*e^2 \\
& *f^3 - 2*A^3*B*b^2*c^4*d*e*f^4 - 8*A*B^3*a^2*c^4*d*e*f^4 - 8*A*B^3*a*c^5*d^
\end{aligned}$$

$$\begin{aligned}
& 2*e*f^3 + 26*A^3*B*a*b^2*c^3*e*f^5 + 8*A^3*B*a*b*c^4*e^2*f^4 + 32*A*B^3*a*b \\
& *c^4*d^2*f^4 - 28*A*B^3*a^2*b*c^3*d*f^5 + 6*A*B^3*a*b^3*c^2*d*f^5 - 9*A^2*B \\
& ^2*b^2*c^4*d*e^2*f^3 - 18*A^2*B^2*a*b^2*c^3*e^2*f^4 - 4*A^3*B*c^6*d^2*e*f^3 \\
& - 3*A^3*B*b^4*c^2*e*f^5 - 44*A^3*B*a^2*c^4*e*f^5 - 16*A^3*B*a*c^5*e^3*f^3 \\
& - 16*A*B^3*a^3*c^3*e*f^5 - 10*A^3*B*b^3*c^3*d*f^5 - 4*A^3*B*b*c^5*d^2*f^4 - \\
& 4*A*B^3*b*c^5*d^3*f^3 - 28*A^3*B*a^2*b*c^3*f^6 + 6*A^3*B*a*b^3*c^2*f^6 - 4 \\
& *A^4*b*c^5*d*e*f^4 - 20*A^4*a*b*c^4*e*f^5 + 3*A^2*B^2*b^4*c^2*e^2*f^4 - 2*A \\
& ^2*B^2*b^3*c^3*e^3*f^3 + 12*A^2*B^2*a^2*c^4*e^2*f^4 + 9*A^2*B^2*b^2*c^4*d^2 \\
& *f^4 - 3*A^2*B^2*a^2*b^2*c^2*f^6 - 2*B^4*b^3*c^3*d^2*e*f^3 + 4*B^4*a^2*c^4* \\
& d*e^2*f^3 - 10*B^4*a*b^2*c^3*d^2*f^4 - 3*B^4*a^2*b^2*c^2*d*f^5 + 3*A^3*B*b^ \\
& 2*c^4*e^3*f^3 - 2*A^3*B*b^3*c^3*e^2*f^4 - 10*A*B^3*b^3*c^3*d^2*f^4 - 4*A*B^ \\
& 3*a^2*c^4*e^3*f^3 + 3*A^2*B^2*b^4*c^2*d*f^5 + 36*A^2*B^2*a^2*c^4*d*f^5 - 24 \\
& *A^2*B^2*a*c^5*d^2*f^4 + 4*A^2*B^2*c^6*d^3*f^3 + 16*A^2*B^2*a^3*c^3*f^6 + 4 \\
& *A^4*b^3*c^3*e*f^5 + 16*B^4*a^3*c^3*d*f^5 + 16*A^4*a*c^5*e^2*f^4 + 8*A^4*b^ \\
& 2*c^4*d*f^5 - 8*A^4*a*b^2*c^3*f^6 - 24*A^4*a*c^5*d*f^5 + 3*B^4*b^4*c^2*d^2* \\
& f^4 - 3*A^4*b^2*c^4*e^2*f^4 + 4*A^4*c^6*d^2*f^4 + 36*A^4*a^2*c^4*f^6 + B^4* \\
& b^2*c^4*d^3*f^3, z, k), k, 1, 4) - ((A*b^3*f + 2*A*a*c^2*e + A*b*c^2*d - 2* \\
& B*a*c^2*d - A*b^2*c*e - B*a*b^2*f + 2*B*a^2*c*f - 3*A*a*b*c*f + B*a*b*c*e)/ \\
& (a^2*b^2*f^2 - 4*a^3*c*f^2 - 4*a*c^3*d^2 - 4*a^2*c^2*e^2 + b^2*c^2*d^2 + b^ \\
& 4*d*f - a*b^3*e*f - b^3*c*d*e + a*b^2*c*e^2 + 8*a^2*c^2*d*f + 4*a*b*c^2*d*e \\
& - 6*a*b^2*c*d*f + 4*a^2*b*c*e*f) - (x*(2*A*a*c^2*f - 2*A*c^3*d + A*b*c^2*e \\
& - 2*B*a*c^2*e + B*b*c^2*d - A*b^2*c*f + B*a*b*c*f))/(a^2*b^2*f^2 - 4*a^3*c \\
& *f^2 - 4*a*c^3*d^2 - 4*a^2*c^2*e^2 + b^2*c^2*d^2 + b^4*d*f - a*b^3*e*f - b^ \\
& 3*c*d*e + a*b^2*c*e^2 + 8*a^2*c^2*d*f + 4*a*b*c^2*d*e - 6*a*b^2*c*d*f + 4*a \\
& ^2*b*c*e*f))/(a + b*x + c*x^2)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x**2+b*x+a)**2/(f*x**2+e*x+d),x)

[Out] Timed out

$$3.17 \quad \int \frac{g+hx}{(a+bx+cx^2)(ad+bdx+cdx^2)^2} dx$$

Optimal. Leaf size=140

$$\frac{3(b+2cx)(2cg-bh)}{2d^2(b^2-4ac)^2(a+bx+cx^2)} - \frac{-2ah+x(2cg-bh)+bg}{2d^2(b^2-4ac)(a+bx+cx^2)^2} - \frac{6c(2cg-bh)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{d^2(b^2-4ac)^{5/2}}$$

[Out] $1/2*(-b*g+2*a*h-(-b*h+2*c*g)*x)/(-4*a*c+b^2)/d^2/(c*x^2+b*x+a)^2+3/2*(-b*h+2*c*g)*(2*c*x+b)/(-4*a*c+b^2)^2/d^2/(c*x^2+b*x+a)-6*c*(-b*h+2*c*g)*\arctanh((2*c*x+b)/(-4*a*c+b^2)^{(1/2)})/(-4*a*c+b^2)^{(5/2)}/d^2$

Rubi [A] time = 0.13, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {998, 638, 614, 618, 206}

$$\frac{3(b+2cx)(2cg-bh)}{2d^2(b^2-4ac)^2(a+bx+cx^2)} - \frac{-2ah+x(2cg-bh)+bg}{2d^2(b^2-4ac)(a+bx+cx^2)^2} - \frac{6c(2cg-bh)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{d^2(b^2-4ac)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(g + h*x)/((a + b*x + c*x^2)*(a*d + b*d*x + c*d*x^2)^2), x]

[Out] $-(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*(b^2 - 4*a*c)*d^2*(a + b*x + c*x^2)^2) + (3*(2*c*g - b*h)*(b + 2*c*x))/(2*(b^2 - 4*a*c)^2*d^2*(a + b*x + c*x^2)) - (6*c*(2*c*g - b*h)*\text{ArcTanh}[(b + 2*c*x)/\text{Sqrt}[b^2 - 4*a*c]])/((b^2 - 4*a*c)^{(5/2)*d^2)}$

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 614

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 998

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Dist[(c/f)^p, Int[(g + h*x)^m*(d + e*x + f*x^2)^(p + q), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q}, x] && EqQ[c*d - a*f, 0] && EqQ[b*d - a*e, 0] && (IntegerQ[p] || GtQ[c/f, 0]) && (!IntegerQ[q] || LeafCount[d + e*x + f*x^2] <= LeafCount[a + b*x + c*x^2])
```

Rubi steps

$$\begin{aligned} \int \frac{g + hx}{(a + bx + cx^2)(ad + bdx + cdx^2)^2} dx &= \frac{\int \frac{g+hx}{(a+bx+cx^2)^3} dx}{d^2} \\ &= -\frac{bg - 2ah + (2cg - bh)x}{2(b^2 - 4ac)d^2(a + bx + cx^2)^2} - \frac{(3(2cg - bh)) \int \frac{1}{(a+bx+cx^2)^2} dx}{2(b^2 - 4ac)d^2} \\ &= -\frac{bg - 2ah + (2cg - bh)x}{2(b^2 - 4ac)d^2(a + bx + cx^2)^2} + \frac{3(2cg - bh)(b + 2cx)}{2(b^2 - 4ac)^2 d^2 (a + bx + cx^2)} + \\ &= -\frac{bg - 2ah + (2cg - bh)x}{2(b^2 - 4ac)d^2(a + bx + cx^2)^2} + \frac{3(2cg - bh)(b + 2cx)}{2(b^2 - 4ac)^2 d^2 (a + bx + cx^2)} - \\ &= -\frac{bg - 2ah + (2cg - bh)x}{2(b^2 - 4ac)d^2(a + bx + cx^2)^2} + \frac{3(2cg - bh)(b + 2cx)}{2(b^2 - 4ac)^2 d^2 (a + bx + cx^2)} \end{aligned}$$

Mathematica [A] time = 0.14, size = 131, normalized size = 0.94

$$\frac{\frac{(b^2-4ac)(2ah-bg+bx-2cgx)}{(a+x(b+cx))^2} - \frac{12c(bh-2cg) \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{3(b+2cx)(2cg-bh)}{a+x(b+cx)}}{2d^2(b^2-4ac)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(g + h*x)/((a + b*x + c*x^2)*(a*d + b*d*x + c*d*x^2)^2), x]

[Out] (((b^2 - 4*a*c)*(-b*g) + 2*a*h - 2*c*g*x + b*h*x)/(a + x*(b + c*x))^2 + (3*(2*c*g - b*h)*(b + 2*c*x)/(a + x*(b + c*x)) - (12*c*(-2*c*g + b*h)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2*(b^2 - 4*a*c)^2*d^2)

fricas [B] time = 1.42, size = 1150, normalized size = 8.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)/(c*x^2+b*x+a)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="fricas")

[Out] [1/2*(6*(2*(b^2*c^3 - 4*a*c^4)*g - (b^3*c^2 - 4*a*b*c^3)*h)*x^3 + 9*(2*(b^3*c^2 - 4*a*b*c^3)*g - (b^4*c - 4*a*b^2*c^2)*h)*x^2 - 6*(2*a^2*c^2*g - a^2*b*c*h + (2*c^4*g - b*c^3*h)*x^4 + 2*(2*b*c^3*g - b^2*c^2*h)*x^3 + (2*(b^2*c^2 + 2*a*c^3)*g - (b^3*c + 2*a*b*c^2)*h)*x^2 + 2*(2*a*b*c^2*g - a*b^2*c*h)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c))*(2*c*x + b))/(c*x^2 + b*x + a) - (b^5 - 14*a*b^3*c + 40*a^2*b*c^2)*g - (a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2)*h + 2*(2*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*g - (b^5 + a*b^3*c - 20*a^2*b*c^2)*h)*x)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*d^2), 1/2*(6*(2*(b^2*c^3 - 4*a*c^4)*g - (b^3*c^2 - 4*a*b*c^3)*h)*x^3 + 9*(2*(b^3*c^2 - 4*a*b*c^3)*g - (b^4*c - 4*a*b^2*c^2)*h)*x^2 - 12*(2*a^2*c^2*g - a^2*b*c*h + (2*c^4*g - b*c^3*h)*x^4 + 2*(2*b*c^3*g - b^2*c^2*h)*x^3 + (2*(b^2*c^2 + 2*a*c^3)*g - (b^3*c + 2*a*b*c^2)*h)*x^2 + 2*(2*a*b*c^2*g - a*b^2*c*h)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (b^5 - 14*a*b^3*c + 40*a^2*b*c^2)*g - (a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2)*h + 2*(2*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*g - (b^5 + a*b^3*c - 20*a^2*b*c^2)*h)*x)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2*x^3 + (b^8

$$- 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*d^2)]$$

giac [A] time = 0.17, size = 219, normalized size = 1.56

$$\frac{6(2c^2g - bch) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(b^4d^2 - 8ab^2cd^2 + 16a^2c^2d^2)\sqrt{-b^2+4ac}} + \frac{12c^3gx^3 - 6bc^2hx^3 + 18bc^2gx^2 - 9b^2chx^2 + 4b^2cgx + 20ac^2gx - 2(b^4d^2 - 8ab^2cd^2 + 16a^2c^2d^2)}{2(b^4d^2 - 8ab^2cd^2 + 16a^2c^2d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)/(c*x^2+b*x+a)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="giac")

[Out] 6*(2*c^2*g - b*c*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4*d^2 - 8*a*b^2*c*d^2 + 16*a^2*c^2*d^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(12*c^3*g*x^3 - 6*b*c^2*h*x^3 + 18*b*c^2*g*x^2 - 9*b^2*c*h*x^2 + 4*b^2*c*g*x + 20*a*c^2*g*x - 2*b^3*h*x - 10*a*b*c*h*x - b^3*g + 10*a*b*c*g - a*b^2*h - 8*a^2*c*h)/(b^4*d^2 - 8*a*b^2*c*d^2 + 16*a^2*c^2*d^2)*(c*x^2 + b*x + a)^2)

maple [B] time = 0.01, size = 340, normalized size = 2.43

$$\frac{3bchx}{(4ac - b^2)^2 (cx^2 + bx + a)d^2} - \frac{6bch \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac - b^2)^{\frac{5}{2}} d^2} + \frac{6c^2gx}{(4ac - b^2)^2 (cx^2 + bx + a)d^2} + \frac{12c^2g \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac - b^2)^{\frac{5}{2}} d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h*x+g)/(c*x^2+b*x+a)/(c*d*x^2+b*d*x+a*d)^2,x)

[Out] -1/2/d^2/(4*a*c-b^2)/(c*x^2+b*x+a)^2*x*b*h+1/d^2/(4*a*c-b^2)/(c*x^2+b*x+a)^2*x*c*g-1/d^2/(4*a*c-b^2)/(c*x^2+b*x+a)^2*a*h+1/2/d^2/(4*a*c-b^2)/(c*x^2+b*x+a)^2*b*g-3/d^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)*c*x*b*h+6/d^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)*c^2*x*g-3/2/d^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)*b^2*h+3/d^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)*b*c*g-6/d^2/(4*a*c-b^2)^(5/2)*c*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*h+12/d^2/(4*a*c-b^2)^(5/2)*c^2*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*g

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)/(c*x^2+b*x+a)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see 'assume?' for more details) Is 4*a*c-b^2 positive or negative?

mupad [B] time = 0.42, size = 395, normalized size = 2.82

$$\frac{6c \operatorname{atan} \left(\frac{d^2 \left(\frac{6c^2 x (bh-2cg)}{d^2 (4ac-b^2)^{5/2}} + \frac{3c (bh-2cg) (16a^2 b c^2 d^2 - 8ab^3 c d^2 + b^5 d^2)}{d^4 (4ac-b^2)^{5/2} (16a^2 c^2 - 8ab^2 c + b^4)} \right) (16a^2 c^2 - 8ab^2 c + b^4)}{6c^2 g - 3bch} \right) (bh-2cg)}{d^2 (4ac-b^2)^{5/2}} - \frac{\frac{8cha^2 + hab^2 - 10cga + gb^3}{2(16a^2 c^2 - 8ab^2 c + b^4)} + \dots}{x^2 (b^2 d^2 + 2ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g + h*x)/((a*d + b*d*x + c*d*x^2)^2*(a + b*x + c*x^2)),x)`

[Out] $(6*c*\operatorname{atan}((d^2*((6*c^2*x*(b*h - 2*c*g))/(d^2*(4*a*c - b^2)^{(5/2)})) + (3*c*(b*h - 2*c*g)*(b^5*d^2 + 16*a^2*b*c^2*d^2 - 8*a*b^3*c*d^2))/(d^4*(4*a*c - b^2)^{(5/2)}*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(6*c^2*g - 3*b*c*h))*(b*h - 2*c*g)/(d^2*(4*a*c - b^2)^{(5/2)}) - ((b^3*g + a*b^2*h + 8*a^2*c*h - 10*a*b*c*g)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x*(5*a*c + b^2)*(b*h - 2*c*g))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (3*c^2*x^3*(b*h - 2*c*g))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (9*b*c*x^2*(b*h - 2*c*g))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(b^2*d^2 + 2*a*c*d^2) + a^2*d^2 + c^2*d^2*x^4 + 2*a*b*d^2*x + 2*b*c*d^2*x^3)$

sympy [B] time = 2.44, size = 709, normalized size = 5.06

$$\frac{3c \sqrt{-\frac{1}{(4ac-b^2)^5}} (bh-2cg) \log \left(x + \frac{-192a^3c^4 \sqrt{-\frac{1}{(4ac-b^2)^5}} (bh-2cg) + 144a^2b^2c^3 \sqrt{-\frac{1}{(4ac-b^2)^5}} (bh-2cg) - 36ab^4c^2 \sqrt{-\frac{1}{(4ac-b^2)^5}} (bh-2cg) + 3b^5c}{6bc^2h - 12c^3g} \right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((h*x+g)/(c*x**2+b*x+a)/(c*d*x**2+b*d*x+a*d)**2,x)`

[Out] $3*c*\sqrt{-1/(4*a*c - b**2)**5}*(b*h - 2*c*g)*\log(x + (-192*a**3*c**4*\sqrt{-1/(4*a*c - b**2)**5}*(b*h - 2*c*g) + 144*a**2*b**2*c**3*\sqrt{-1/(4*a*c - b**2)**5}*(b*h - 2*c*g) - 36*a*b**4*c**2*\sqrt{-1/(4*a*c - b**2)**5}*(b*h - 2*c*g) + 3*b**6*c*\sqrt{-1/(4*a*c - b**2)**5}*(b*h - 2*c*g) + 3*b**2*c*h - 6*b*c**2*g)/(6*b*c**2*h - 12*c**3*g))/d**2 - 3*c*\sqrt{-1/(4*a*c - b**2)**5}*(b*h - 2*c*g)*\log(x + (192*a**3*c**4*\sqrt{-1/(4*a*c - b**2)**5}*(b*h - 2*c*g)$

$$\begin{aligned}
& - 144a^{**2}b^{**2}c^{**3}\sqrt{-1/(4ac - b^{**2})^{**5}}(bh - 2c^*g) + 36ab^{**4}c^{**2}\sqrt{-1/(4ac - b^{**2})^{**5}}(bh - 2c^*g) - 3b^{**6}c\sqrt{-1/(4ac - b^{**2})^{**5}}(bh - 2c^*g) + 3b^{**2}c^*h - 6b^*c^{**2}g)/(6b^*c^{**2}h - 12c^{**3}g)) \\
& /d^{**2} + (-8a^{**2}c^*h - ab^{**2}h + 10ab^*c^*g - b^{**3}g + x^{**3}(-6b^*c^{**2}h + 12c^{**3}g) + x^{**2}(-9b^{**2}c^*h + 18b^*c^{**2}g) + x(-10ab^*c^*h + 20a^*c^{**2}g - 2b^{**3}h + 4b^{**2}c^*g))/(32a^{**4}c^{**2}d^{**2} - 16a^{**3}b^{**2}c^*d^{**2} + 2a^{**2}b^{**4}d^{**2} + x^{**4}(32a^{**2}c^{**4}d^{**2} - 16ab^{**2}c^{**3}d^{**2} + 2b^{**4}c^{**2}d^{**2}) + x^{**3}(64a^{**2}b^*c^{**3}d^{**2} - 32ab^{**3}c^{**2}d^{**2} + 4b^{**5}c^*d^{**2}) + x^{**2}(64a^{**3}c^{**3}d^{**2} - 12ab^{**4}c^*d^{**2} + 2b^{**6}d^{**2}) + x(64a^{**3}b^*c^{**2}d^{**2} - 32a^{**2}b^{**3}c^*d^{**2} + 4ab^{**5}d^{**2}))
\end{aligned}$$

$$3.18 \quad \int \frac{g+hx}{(a+bx+cx^2)^2(ad+bdx+cdx^2)} dx$$

Optimal. Leaf size=140

$$\frac{3(b+2cx)(2cg-bh)}{2d(b^2-4ac)^2(a+bx+cx^2)} - \frac{-2ah+x(2cg-bh)+bg}{2d(b^2-4ac)(a+bx+cx^2)^2} - \frac{6c(2cg-bh)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{d(b^2-4ac)^{5/2}}$$

[Out] 1/2*(-b*g+2*a*h-(-b*h+2*c*g)*x)/(-4*a*c+b^2)/d/(c*x^2+b*x+a)^2+3/2*(-b*h+2*c*g)*(2*c*x+b)/(-4*a*c+b^2)^2/d/(c*x^2+b*x+a)-6*c*(-b*h+2*c*g)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)/d

Rubi [A] time = 0.10, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {998, 638, 614, 618, 206}

$$\frac{3(b+2cx)(2cg-bh)}{2d(b^2-4ac)^2(a+bx+cx^2)} - \frac{-2ah+x(2cg-bh)+bg}{2d(b^2-4ac)(a+bx+cx^2)^2} - \frac{6c(2cg-bh)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{d(b^2-4ac)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(g + h*x)/((a + b*x + c*x^2)^2*(a*d + b*d*x + c*d*x^2)), x]

[Out] -(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*(b^2 - 4*a*c)*d*(a + b*x + c*x^2)^2) + (3*(2*c*g - b*h)*(b + 2*c*x))/(2*(b^2 - 4*a*c)^2*d*(a + b*x + c*x^2)) - (6*c*(2*c*g - b*h)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(5/2)*d)

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 614

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 998

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Dist[(c/f)^p, Int[(g + h*x)^m*(d + e*x + f*x^2)^(p + q), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q}, x] && EqQ[c*d - a*f, 0] && EqQ[b*d - a*e, 0] && (IntegerQ[p] || GtQ[c/f, 0]) && (!IntegerQ[q] || LeafCount[d + e*x + f*x^2] <= LeafCount[a + b*x + c*x^2])
```

Rubi steps

$$\begin{aligned} \int \frac{g + hx}{(a + bx + cx^2)^2 (ad + bdx + cd x^2)} dx &= \frac{\int \frac{g+hx}{(a+bx+cx^2)^3} dx}{d} \\ &= -\frac{bg - 2ah + (2cg - bh)x}{2(b^2 - 4ac)d(a + bx + cx^2)^2} - \frac{(3(2cg - bh)) \int \frac{1}{(a+bx+cx^2)^2} dx}{2(b^2 - 4ac)d} \\ &= -\frac{bg - 2ah + (2cg - bh)x}{2(b^2 - 4ac)d(a + bx + cx^2)^2} + \frac{3(2cg - bh)(b + 2cx)}{2(b^2 - 4ac)^2 d(a + bx + cx^2)} + \dots \\ &= -\frac{bg - 2ah + (2cg - bh)x}{2(b^2 - 4ac)d(a + bx + cx^2)^2} + \frac{3(2cg - bh)(b + 2cx)}{2(b^2 - 4ac)^2 d(a + bx + cx^2)} - \dots \\ &= -\frac{bg - 2ah + (2cg - bh)x}{2(b^2 - 4ac)d(a + bx + cx^2)^2} + \frac{3(2cg - bh)(b + 2cx)}{2(b^2 - 4ac)^2 d(a + bx + cx^2)} - \dots \end{aligned}$$

Mathematica [A] time = 0.03, size = 131, normalized size = 0.94

$$\frac{\frac{(b^2-4ac)(2ah-bg+bhx-2cgx)}{(a+x(b+cx))^2} - \frac{12c(bh-2cg) \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{3(b+2cx)(2cg-bh)}{a+x(b+cx)}}{2d(b^2-4ac)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(g + h*x)/((a + b*x + c*x^2)^2*(a*d + b*d*x + c*d*x^2)),x]

[Out] (((b^2 - 4*a*c)*(-b*g) + 2*a*h - 2*c*g*x + b*h*x)/(a + x*(b + c*x))^2 + (3*(2*c*g - b*h)*(b + 2*c*x)/(a + x*(b + c*x)) - (12*c*(-2*c*g + b*h)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2*(b^2 - 4*a*c)^2*d)

fricas [B] time = 0.89, size = 1130, normalized size = 8.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)/(c*x^2+b*x+a)^2/(c*d*x^2+b*d*x+a*d),x, algorithm="fricas")

[Out] [1/2*(6*(2*(b^2*c^3 - 4*a*c^4)*g - (b^3*c^2 - 4*a*b*c^3)*h)*x^3 + 9*(2*(b^3*c^2 - 4*a*b*c^3)*g - (b^4*c - 4*a*b^2*c^2)*h)*x^2 - 6*(2*a^2*c^2*g - a^2*b*c*h + (2*c^4*g - b*c^3*h)*x^4 + 2*(2*b*c^3*g - b^2*c^2*h)*x^3 + (2*(b^2*c^2 + 2*a*c^3)*g - (b^3*c + 2*a*b*c^2)*h)*x^2 + 2*(2*a*b*c^2*g - a*b^2*c*h)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c))*(2*c*x + b))/(c*x^2 + b*x + a) - (b^5 - 14*a*b^3*c + 40*a^2*b*c^2)*g - (a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2)*h + 2*(2*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*g - (b^5 + a*b^3*c - 20*a^2*b*c^2)*h)*x)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*d), 1/2*(6*(2*(b^2*c^3 - 4*a*c^4)*g - (b^3*c^2 - 4*a*b*c^3)*h)*x^3 + 9*(2*(b^3*c^2 - 4*a*b*c^3)*g - (b^4*c - 4*a*b^2*c^2)*h)*x^2 - 12*(2*a^2*c^2*g - a^2*b*c*h + (2*c^4*g - b*c^3*h)*x^4 + 2*(2*b*c^3*g - b^2*c^2*h)*x^3 + (2*(b^2*c^2 + 2*a*c^3)*g - (b^3*c + 2*a*b*c^2)*h)*x^2 + 2*(2*a*b*c^2*g - a*b^2*c*h)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (b^5 - 14*a*b^3*c + 40*a^2*b*c^2)*g - (a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2)*h + 2*(2*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*g - (b^5 + a*b^3*c - 20*a^2*b*c^2)*h)*x)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d*x^3 + (b^8 - 10*a*b^6*c

+ 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*d)]

giac [A] time = 0.17, size = 207, normalized size = 1.48

$$\frac{6(2c^2g - bch) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(b^4d - 8ab^2cd + 16a^2c^2d)\sqrt{-b^2+4ac}} + \frac{12c^3gx^3 - 6bc^2hx^3 + 18bc^2gx^2 - 9b^2chx^2 + 4b^2cgx + 20ac^2gx - 2b^3}{2(b^4d - 8ab^2cd + 16a^2c^2d)(cx^2 -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)/(c*x^2+b*x+a)^2/(c*d*x^2+b*d*x+a*d),x, algorithm="giac")

[Out] 6*(2*c^2*g - b*c*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4*d - 8*a*b^2*c*d + 16*a^2*c^2*d)*sqrt(-b^2 + 4*a*c)) + 1/2*(12*c^3*g*x^3 - 6*b*c^2*h*x^3 + 18*b*c^2*g*x^2 - 9*b^2*c*h*x^2 + 4*b^2*c*g*x + 20*a*c^2*g*x - 2*b^3*h*x - 10*a*b*c*h*x - b^3*g + 10*a*b*c*g - a*b^2*h - 8*a^2*c*h)/(b^4*d - 8*a*b^2*c*d + 16*a^2*c^2*d)*(c*x^2 + b*x + a)^2)

maple [B] time = 0.01, size = 340, normalized size = 2.43

$$\frac{3bchx}{(4ac - b^2)^2 (cx^2 + bx + a)d} - \frac{6bch \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac - b^2)^{\frac{5}{2}} d} + \frac{6c^2gx}{(4ac - b^2)^2 (cx^2 + bx + a)d} + \frac{12c^2g \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac - b^2)^{\frac{5}{2}} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h*x+g)/(c*x^2+b*x+a)^2/(c*d*x^2+b*d*x+a*d),x)

[Out] -1/2/d/(4*a*c-b^2)/(c*x^2+b*x+a)^2*x*b*h+1/d/(4*a*c-b^2)/(c*x^2+b*x+a)^2*x*c*g-1/d/(4*a*c-b^2)/(c*x^2+b*x+a)^2*a*h+1/2/d/(4*a*c-b^2)/(c*x^2+b*x+a)^2*b*g-3/d/(4*a*c-b^2)^2/(c*x^2+b*x+a)*c*x*b*h+6/d/(4*a*c-b^2)^2/(c*x^2+b*x+a)*c^2*x*g-3/2/d/(4*a*c-b^2)^2/(c*x^2+b*x+a)*b^2*h+3/d/(4*a*c-b^2)^2/(c*x^2+b*x+a)*b*c*g-6/d/(4*a*c-b^2)^(5/2)*c*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*h+12/d/(4*a*c-b^2)^(5/2)*c^2*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*g

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)/(c*x^2+b*x+a)^2/(c*d*x^2+b*d*x+a*d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see 'assume?' for more details) Is 4*a*c-b^2 positive or negative?

mupad [B] time = 4.02, size = 375, normalized size = 2.68

$$6c \operatorname{atan} \left(\frac{d \left(\frac{6c^2 x (bh-2cg)}{d(4ac-b^2)^{5/2}} + \frac{3c(bh-2cg)(16da^2bc^2-8dab^3c+db^5)}{d^2(4ac-b^2)^{5/2}(16a^2c^2-8ab^2c+b^4)} \right) (16a^2c^2-8ab^2c+b^4)}{6c^2g-3bch} \right) (bh-2cg) \frac{8cha^2+h ab^2-10cga b+g b^3}{2(16a^2c^2-8ab^2c+b^4)} + \frac{x(b^2}{16a} \\ \frac{d(4ac-b^2)^{5/2}}{a^2d+x^2(d b^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g + h*x)/((a*d + b*d*x + c*d*x^2)*(a + b*x + c*x^2)^2), x)`

[Out] $(6c \operatorname{atan}((d((6c^2*x*(b*h - 2c*g))/(d*(4a*c - b^2)^{(5/2)})) + (3c*(b*h - 2c*g)*(b^5*d - 8a*b^3*c*d + 16a^2*b*c^2*d))/(d^2*(4a*c - b^2)^{(5/2)}*(b^4 + 16a^2*c^2 - 8a*b^2*c)))*(b^4 + 16a^2*c^2 - 8a*b^2*c))/(6c^2*g - 3*b*c*h))*(b*h - 2c*g)/(d*(4a*c - b^2)^{(5/2)}) - ((b^3*g + a*b^2*h + 8a^2*c*h - 10a*b*c*g)/(2*(b^4 + 16a^2*c^2 - 8a*b^2*c)) + (x*(5a*c + b^2)*(b*h - 2c*g))/(b^4 + 16a^2*c^2 - 8a*b^2*c) + (3c^2*x^3*(b*h - 2c*g))/(b^4 + 16a^2*c^2 - 8a*b^2*c) + (9b*c*x^2*(b*h - 2c*g))/(2*(b^4 + 16a^2*c^2 - 8a*b^2*c)))/(a^2*d + x^2*(b^2*d + 2a*c*d) + c^2*d*x^4 + 2b*c*d*x^3 + 2a*b*d*x)$

sympy [B] time = 2.26, size = 680, normalized size = 4.86

$$3c \sqrt{\frac{1}{(4ac-b^2)^5}} (bh-2cg) \log \left(x + \frac{-192a^3c^4 \sqrt{\frac{1}{(4ac-b^2)^5}} (bh-2cg) + 144a^2b^2c^3 \sqrt{\frac{1}{(4ac-b^2)^5}} (bh-2cg) - 36ab^4c^2 \sqrt{\frac{1}{(4ac-b^2)^5}} (bh-2cg) + 3b^5}{6bc^2h-12c^3g} \right) \\ d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((h*x+g)/(c*x**2+b*x+a)**2/(c*d*x**2+b*d*x+a*d), x)`

[Out] $3c \operatorname{sqrt}(-1/(4a*c - b**2)**5)*(b*h - 2c*g)*\log(x + (-192a**3c**4*\operatorname{sqrt}(-1/(4a*c - b**2)**5)*(b*h - 2c*g) + 144a**2b**2c**3*\operatorname{sqrt}(-1/(4a*c - b**2)**5)*(b*h - 2c*g) - 36a*b**4c**2*\operatorname{sqrt}(-1/(4a*c - b**2)**5)*(b*h - 2c*g) + 3b**6c*\operatorname{sqrt}(-1/(4a*c - b**2)**5)*(b*h - 2c*g) + 3b**2c*h - 6b*c**2g)/(6b*c**2h - 12c**3g))/d - 3c \operatorname{sqrt}(-1/(4a*c - b**2)**5)*(b*h - 2c*g)*\log(x + (192a**3c**4*\operatorname{sqrt}(-1/(4a*c - b**2)**5)*(b*h - 2c*g) -$

$$\begin{aligned}
& 144*a**2*b**2*c**3*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 36*a*b**4*c** \\
& 2*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) - 3*b**6*c*sqrt(-1/(4*a*c - b**2) \\
&)**5)*(b*h - 2*c*g) + 3*b**2*c*h - 6*b*c**2*g)/(6*b*c**2*h - 12*c**3*g))/d \\
& + (-8*a**2*c*h - a*b**2*h + 10*a*b*c*g - b**3*g + x**3*(-6*b*c**2*h + 12*c* \\
& *3*g) + x**2*(-9*b**2*c*h + 18*b*c**2*g) + x*(-10*a*b*c*h + 20*a*c**2*g - 2 \\
& *b**3*h + 4*b**2*c*g))/(32*a**4*c**2*d - 16*a**3*b**2*c*d + 2*a**2*b**4*d + \\
& x**4*(32*a**2*c**4*d - 16*a*b**2*c**3*d + 2*b**4*c**2*d) + x**3*(64*a**2*b \\
& *c**3*d - 32*a*b**3*c**2*d + 4*b**5*c*d) + x**2*(64*a**3*c**3*d - 12*a*b**4 \\
& *c*d + 2*b**6*d) + x*(64*a**3*b*c**2*d - 32*a**2*b**3*c*d + 4*a*b**5*d))
\end{aligned}$$

$$3.19 \quad \int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d+ex+fx^2} dx$$

Optimal. Leaf size=617

$$\frac{(2f(Af(cd - af) - Bd(ce - bf)) - (e - \sqrt{e^2 - 4df})(B(f(be - af) - c(e^2 - df)) + Af(ce - bf))) \tanh^{-1}\left(\frac{4a}{2\sqrt{2}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2} f^2 \sqrt{e^2 - 4df} \sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}}$$

[Out] $-1/2*(-2*A*c*f-B*b*f+2*B*c*e)*\operatorname{arctanh}(1/2*(2*c*x+b)/c^{(1/2)})/(c*x^2+b*x+a)^{(1/2)}/f^2/c^{(1/2)}+B*(c*x^2+b*x+a)^{(1/2)}/f+1/2*\operatorname{arctanh}(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^{(1/2)}))-b*(e-(-4*d*f+e^2)^{(1/2)}))*2^{(1/2)})/(c*x^2+b*x+a)^{(1/2)}/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}*(2*f*(A*f*(-a*f+c*d)-B*d*(-b*f+c*e))-(A*f*(-b*f+c*e)+B*(f*(-a*f+b*e)-c*(-d*f+e^2)))*(e-(-4*d*f+e^2)^{(1/2)}))/f^2*2^{(1/2)}/(-4*d*f+e^2)^{(1/2)}/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}-1/2*\operatorname{arctanh}(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^{(1/2)}))+2*x*(b*f-c*(e+(-4*d*f+e^2)^{(1/2)}))) * 2^{(1/2)}/(c*x^2+b*x+a)^{(1/2)}/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}*(2*f*(A*f*(-a*f+c*d)-B*d*(-b*f+c*e))-(A*f*(-b*f+c*e)+B*(f*(-a*f+b*e)-c*(-d*f+e^2)))*(e+(-4*d*f+e^2)^{(1/2)}))/f^2*2^{(1/2)}/(-4*d*f+e^2)^{(1/2)}/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}$

Rubi [A] time = 9.00, antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1019, 1076, 621, 206, 1032, 724}

$$\frac{(2f(Af(cd - af) - Bd(ce - bf)) - (e - \sqrt{e^2 - 4df})(Bf(be - af) + Af(ce - bf) - Bc(e^2 - df))) \tanh^{-1}\left(\frac{4a}{2\sqrt{2}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2} f^2 \sqrt{e^2 - 4df} \sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}}$$

Antiderivative was successfully verified.

[In] Int[((A + B*x)*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2), x]

[Out] $(B*\operatorname{Sqrt}[a + b*x + c*x^2])/f - ((2*B*c*e - b*B*f - 2*A*c*f)*\operatorname{ArcTanh}[(b + 2*c*x)/(2*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*\operatorname{Sqrt}[c]*f^2) + ((2*f*(A*f*(c*d - a*f) - B*d*(c*e - b*f)) - (e - \operatorname{Sqrt}[e^2 - 4*d*f])*(B*f*(b*e - a*f) + A*f*(c*e - b*f) - B*c*(e^2 - d*f)))*\operatorname{ArcTanh}[(4*a*f - b*(e - \operatorname{Sqrt}[e^2 - 4*d*f]) + 2*(b*f - c*(e - \operatorname{Sqrt}[e^2 - 4*d*f]))*x)/(2*\operatorname{Sqrt}[2]*\operatorname{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*\operatorname{Sqrt}[e^2 - 4*d*f]]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*\operatorname{Sqrt}[2]*f^2*\operatorname{Sqrt}[e^2 - 4*d*f]*\operatorname{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*\operatorname{Sqrt}[e^2 - 4*d*f]]) - ((2*f*(A*f*(c*d - a*f) - B*d*(c*e - b*f)) - (e$

$$\begin{aligned} & + \text{Sqrt}[e^2 - 4*d*f])*(B*f*(b*e - a*f) + A*f*(c*e - b*f) - B*c*(e^2 - d*f)) \\ &)*\text{ArcTanh}[(4*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]) + 2*(b*f - c*(e + \text{Sqrt}[e^2 - 4 \\ & *d*f]))*x)/(2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)* \\ & \text{Sqrt}[e^2 - 4*d*f]]*\text{Sqrt}[a + b*x + c*x^2]])]/(\text{Sqrt}[2]*f^2*\text{Sqrt}[e^2 - 4*d*f]* \\ & \text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*\text{Sqrt}[e^2 - 4*d*f]]) \end{aligned}$$

Rule 206

$$\text{Int}[\frac{(a_ + (b_)*(x_)^2)^{-1}}{x}, x_Symbol] \rightarrow \text{Simp}[\frac{1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]}{\text{Rt}[a, 2]}]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] \text{ /; FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$$

Rule 621

$$\text{Int}[1/\text{Sqrt}[(a_ + (b_)*(x_) + (c_)*(x_)^2)], x_Symbol] \rightarrow \text{Dist}[2, \text{Subst}[\text{Int}[1/(4*c - x^2), x], x, (b + 2*c*x)/\text{Sqrt}[a + b*x + c*x^2]], x] \text{ /; FreeQ}\{a, b, c\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$$

Rule 724

$$\text{Int}[1/(((d_ + (e_)*(x_))*\text{Sqrt}[(a_ + (b_)*(x_) + (c_)*(x_)^2])), x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/\text{Sqrt}[a + b*x + c*x^2]], x] \text{ /; FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0]$$

Rule 1019

$$\text{Int}[\frac{((g_ + (h_)*(x_))*((a_ + (b_)*(x_) + (c_)*(x_)^2)^{p_})*((d_ + (e_)*(x_) + (f_)*(x_)^2)^{q_}))}{x}, x_Symbol] \rightarrow \text{Simp}[\frac{h*(a + b*x + c*x^2)^p*(d + e*x + f*x^2)^{q+1}}{(2*f*(p + q + 1))}, x] - \text{Dist}[1/(2*f*(p + q + 1)), \text{Int}[(a + b*x + c*x^2)^{p-1}*(d + e*x + f*x^2)^q*\text{Simp}[h*p*(b*d - a*e) + a*(h*e - 2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(h*e - 2*g*f)*(p + q + 1))*x + (h*p*(c*e - b*f) + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x] \text{ /; FreeQ}\{a, b, c, d, e, f, g, h, q\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[e^2 - 4*d*f, 0] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{NeQ}[p + q + 1, 0]$$

Rule 1032

$$\text{Int}[\frac{((g_ + (h_)*(x_)))/(((a_ + (b_)*(x_) + (c_)*(x_)^2)*\text{Sqrt}[(d_ + (e_)*(x_) + (f_)*(x_)^2]))}{x}, x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[\frac{(2*c*g - h*(b - q))/q}{\text{Int}[1/((b - q + 2*c*x)*\text{Sqrt}[d + e*x + f*x^2]), x], x] - \text{Dist}[\frac{(2*c*g - h*(b + q))/q}{\text{Int}[1/((b + q + 2*c*x)*\text{Sqrt}[d + e*x + f*x^2]), x], x]} \text{ /; FreeQ}\{a, b, c, d, e, f, g, h\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[e^2 - 4*d*f, 0] \ \&\& \ \text{PosQ}[b^2 - 4*a*c]$$

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(A + Bx)\sqrt{a + bx + cx^2}}{d + ex + fx^2} dx &= \frac{B\sqrt{a + bx + cx^2}}{f} - \frac{\int \frac{\frac{1}{2}(bBd - 2aAf) - \frac{1}{2}(2Abf - B(2cd + be - 2af))x + \frac{1}{2}(2Bce - bBf - 2Acf)x^2}{\sqrt{a + bx + cx^2}(d + ex + fx^2)} dx}{f} \\ &= \frac{B\sqrt{a + bx + cx^2}}{f} - \frac{\int \frac{\frac{1}{2}f(bBd - 2aAf) - \frac{1}{2}d(2Bce - bBf - 2Acf) + \left(-\frac{1}{2}e(2Bce - bBf - 2Acf) + \frac{1}{2}f(-2Abf - 2aAc)\right)x + \frac{1}{2}(2Bce - bBf - 2Acf)x^2}{\sqrt{a + bx + cx^2}(d + ex + fx^2)} dx}{f^2} \\ &= \frac{B\sqrt{a + bx + cx^2}}{f} - \frac{(2Bce - bBf - 2Acf) \operatorname{Subst}\left(\int \frac{1}{4c - x^2} dx, x, \frac{b + 2cx}{\sqrt{a + bx + cx^2}}\right)}{f^2} + \frac{(2f(Af(c d - b^2) + c^2) - (2Bce - bBf - 2Acf)e)}{2\sqrt{c} f^2} \\ &= \frac{B\sqrt{a + bx + cx^2}}{f} - \frac{(2Bce - bBf - 2Acf) \tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c} \sqrt{a + bx + cx^2}}\right)}{2\sqrt{c} f^2} - \frac{(2f(Af(c d - b^2) + c^2) - (2Bce - bBf - 2Acf)e)}{2\sqrt{c} f^2} \\ &= \frac{B\sqrt{a + bx + cx^2}}{f} - \frac{(2Bce - bBf - 2Acf) \tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c} \sqrt{a + bx + cx^2}}\right)}{2\sqrt{c} f^2} + \frac{(2f(Af(c d - b^2) + c^2) - (2Bce - bBf - 2Acf)e)}{2\sqrt{c} f^2} \end{aligned}$$

Mathematica [A] time = 2.09, size = 517, normalized size = 0.84

$$\frac{-\sqrt{2} \left(B \left(\sqrt{e^2 - 4df} + e \right) - 2Af \right) \sqrt{f \left(2af - b \left(\sqrt{e^2 - 4df} + e \right) \right) + c \left(e \sqrt{e^2 - 4df} - 2df + e^2 \right)} \tanh^{-1} \left(\frac{4}{2\sqrt{2} \sqrt{a + x(b + cx)}} \right)}{2\sqrt{c} f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2), x]
```

```
[Out] ((-2*B*c*e + b*B*f + 2*A*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(2*Sqrt[c]*f^2) + (4*B*f*Sqrt[e^2 - 4*d*f]*Sqrt[a + x*(b + c*x)])/(2*Sqrt[c]*f^2)
```

$$\begin{aligned}
& - \text{Sqrt}[2] * (-2 * A * f + B * (e + \text{Sqrt}[e^2 - 4 * d * f])) * \text{Sqrt}[c * (e^2 - 2 * d * f + e * \text{Sqrt}[e^2 - 4 * d * f]) + f * (2 * a * f - b * (e + \text{Sqrt}[e^2 - 4 * d * f]))] * \text{ArcTanh}[(4 * a * f - 2 * c * (e + \text{Sqrt}[e^2 - 4 * d * f])) * x - b * (e + \text{Sqrt}[e^2 - 4 * d * f]) - 2 * f * x] / (2 * \text{Sqrt}[2] * \text{Sqrt}[c * (e^2 - 2 * d * f + e * \text{Sqrt}[e^2 - 4 * d * f]) + f * (2 * a * f - b * (e + \text{Sqrt}[e^2 - 4 * d * f]))] * \text{Sqrt}[a + x * (b + c * x)]) \\
& - \text{Sqrt}[2] * (2 * A * f + B * (-e + \text{Sqrt}[e^2 - 4 * d * f])) * \text{Sqrt}[c * (e^2 - 2 * d * f - e * \text{Sqrt}[e^2 - 4 * d * f]) + f * (2 * a * f + b * (-e + \text{Sqrt}[e^2 - 4 * d * f]))] * \text{ArcTanh}[(4 * a * f + 2 * c * (-e + \text{Sqrt}[e^2 - 4 * d * f])) * x + b * (-e + \text{Sqrt}[e^2 - 4 * d * f]) + 2 * f * x] / (2 * \text{Sqrt}[2] * \text{Sqrt}[c * (e^2 - 2 * d * f - e * \text{Sqrt}[e^2 - 4 * d * f]) + f * (2 * a * f + b * (-e + \text{Sqrt}[e^2 - 4 * d * f]))] * \text{Sqrt}[a + x * (b + c * x)]) \\
& / (4 * f^2 * \text{Sqrt}[e^2 - 4 * d * f])
\end{aligned}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
i,const vecteur & l) Error: Bad Argument Value

maple [B] time = 0.05, size = 16209, normalized size = 26.27

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x+A)*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)

[Out] result too large to display

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details)Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + Bx) \sqrt{cx^2 + bx + a}}{fx^2 + ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/(d + e*x + f*x^2),x)

[Out] int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/(d + e*x + f*x^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx) \sqrt{a + bx + cx^2}}{d + ex + fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)

[Out] Integral((A + B*x)*sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)

$$3.20 \quad \int \frac{(A+Bx)(a+bx+cx^2)^{3/2}}{d+ex+fx^2} dx$$

Optimal. Leaf size=1092

$$\frac{B(cx^2 + bx + a)^{3/2}}{3f} \frac{(2Acf(4ce - 5bf) - B(8(e^2 - df)c^2 - 2f(5be - 4af)c + b^2f^2) + 2cf(2Bce - bBf - 2Acf))}{8cf^3}$$

[Out] $1/3*B*(c*x^2+b*x+a)^{(3/2)}/f+1/16*(2*A*c*f*(3*b^2*f^2-12*c*f*(-a*f+b*e)+8*c^2*(-d*f+e^2))-B*(b^3*f^3+6*b*c*f^2*(-2*a*f+b*e)-24*c^2*f*(-a*e*f-b*d*f+b*e^2)+16*c^3*(-2*d*e*f+e^3))$
 $*\operatorname{arctanh}(1/2*(2*c*x+b)/c^{(1/2)})/(c*x^2+b*x+a)^{(1/2)}/c^{(3/2)}/f^4-1/8*(2*A*c*f*(-5*b*f+4*c*e)-B*(b^2*f^2-2*c*f*(-4*a*f+5*b*e)+8*c^2*(-d*f+e^2))+2*c*f*(-2*A*c*f-B*b*f+2*B*c*e)*x*(c*x^2+b*x+a)^{(1/2)}/c/f^3-1/2*\operatorname{arctanh}(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^{(1/2)}))-b*(e-(-4*d*f+e^2)^{(1/2)}))$
 $*2^{(1/2)}/(c*x^2+b*x+a)^{(1/2)}/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}*(2*c*f*(B*d*(-b*f+c*e)*(2*a*f^2-b*e*f-2*c*d*f+c*e^2)+A*f*(2*c*d*f*(-a*f+b*e)-f^2*(-a^2*f+b^2*d)-c^2*d*(-d*f+e^2)))-c*(A*f*(-b*f+c*e)*(f*(-2*a*f+b*e)-c*(-2*d*f+e^2))+B*(c^2*(d^2*f^2-3*d*e^2*f+e^4)-f^2*(2*a*b*e*f-a^2*f^2-b^2*(-d*f+e^2))+2*c*f*(a*f*(-d*f+e^2)-b*(-2*d*e*f+e^3))))$
 $*e^{(-4*d*f+e^2)^{(1/2)})}/c/f^4*2^{(1/2)}/(-4*d*f+e^2)^{(1/2)}/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}+1/2*\operatorname{arctanh}(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^{(1/2)}))+2*x*(b*f-c*(e+(-4*d*f+e^2)^{(1/2)})))$
 $*2^{(1/2)}/(c*x^2+b*x+a)^{(1/2)}/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}*(2*f*(B*d*(-b*f+c*e)*(2*a*f^2-b*e*f-2*c*d*f+c*e^2)+A*f*(2*c*d*f*(-a*f+b*e)-f^2*(-a^2*f+b^2*d)-c^2*d*(-d*f+e^2)))-$
 $(A*f*(-b*f+c*e)*(f*(-2*a*f+b*e)-c*(-2*d*f+e^2))+B*(c^2*(d^2*f^2-3*d*e^2*f+e^4)-f^2*(2*a*b*e*f-a^2*f^2-b^2*(-d*f+e^2))+2*c*f*(a*f*(-d*f+e^2)-b*(-2*d*e*f+e^3))))$
 $*e^{(-4*d*f+e^2)^{(1/2)})}/f^4*2^{(1/2)}/(-4*d*f+e^2)^{(1/2)}/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}$

Rubi [A] time = 18.87, antiderivative size = 1092, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {1019, 1066, 1076, 621, 206, 1032, 724}

$$\frac{B(cx^2 + bx + a)^{3/2}}{3f} \frac{(2Acf(4ce - 5bf) - B(8(e^2 - df)c^2 - 2f(5be - 4af)c + b^2f^2) + 2cf(2Bce - bBf - 2Acf))}{8cf^3}$$

Antiderivative was successfully verified.

[In] Int[((A + B*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x + f*x^2),x]

[Out]
$$-\left(\left(2Acf(4ce - 5bf) - B(b^2f^2 - 2cf(5be - 4af)) + 8c^2(e^2 - df)\right) + 2cf(2Bce - bBf - 2Acf)x\right)\sqrt{a + bx + cx^2} / \left(8c^3f + B(a + bx + cx^2)^{3/2} / (3f) + \left(2Acf(3b^2f^2 - 12cf(be - af)) + 8c^2(e^2 - df) - B(b^3f^3 + 6b^2cf^2(be - 2af) - 4c^2f^2(be^2 - bdf - aef) + 16c^3(e^3 - 2d^2ef))\right)\text{ArcTanh}\left[\frac{b + 2cx}{2\sqrt{c}\sqrt{a + bx + cx^2}}\right]\right) / \left(16c^{3/2}f^4 - \left(2cf(Bd(ce - bf)(ce^2 - 2cdf - b^2ef + 2af^2) + Af(2cdf(be - af) - f^2(b^2d - a^2f) - c^2d(e^2 - df))) - c(e - \sqrt{e^2 - 4df})(Af(ce - bf)(f(be - 2af) - c(e^2 - 2df)) + B(c^2(e^4 - 3d^2ef + d^2f^2) - f^2(2abef - a^2f^2 - b^2(e^2 - df)) + 2cf(af(e^2 - df) - b(e^3 - 2d^2ef))))\right)\text{ArcTanh}\left[\frac{4af - b(e - \sqrt{e^2 - 4df}) + 2(bf - c(e - \sqrt{e^2 - 4df}))x}{2\sqrt{2}\sqrt{ce^2 - 2cdf - b^2ef + 2af^2} - (ce - bf)\sqrt{e^2 - 4df}}\right]\right) / \left(\sqrt{2}cf^4\sqrt{e^2 - 4df}\sqrt{ce^2 - 2cdf - b^2ef + 2af^2} - (ce - bf)\sqrt{e^2 - 4df}\right) + \left(2f(Bd(ce - bf)(ce^2 - 2cdf - b^2ef + 2af^2) + Af(2cdf(be - af) - f^2(b^2d - a^2f) - c^2d(e^2 - df))) - (e + \sqrt{e^2 - 4df})(Af(ce - bf)(f(be - 2af) - c(e^2 - 2df)) + B(c^2(e^4 - 3d^2ef + d^2f^2) - f^2(2abef - a^2f^2 - b^2(e^2 - df)) + 2cf(af(e^2 - df) - b(e^3 - 2d^2ef))))\right)\text{ArcTanh}\left[\frac{4af - b(e + \sqrt{e^2 - 4df}) + 2(bf - c(e + \sqrt{e^2 - 4df}))x}{2\sqrt{2}\sqrt{ce^2 - 2cdf - b^2ef + 2af^2} + (ce - bf)\sqrt{e^2 - 4df}}\right]\right) / \left(\sqrt{2}f^4\sqrt{e^2 - 4df}\sqrt{ce^2 - 2cdf - b^2ef + 2af^2} + (ce - bf)\sqrt{e^2 - 4df}\right)$$

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1019

```

Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[(h*(a + b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[h*p*(b*d - a*e) + a*(h*e - 2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(h*e - 2*g*f)*(p + q + 1))*x + (h*p*(c*e - b*f) + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]

```

Rule 1032

```

Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]

```

Rule 1066

```

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((B*c*f*(2*p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3)))]*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3)))]*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]

```

Rule 1076

```

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{(A + Bx)(a + bx + cx^2)^{3/2}}{d + ex + fx^2} dx &= \frac{B(a + bx + cx^2)^{3/2}}{3f} - \frac{\int \frac{\sqrt{a+bx+cx^2} \left(\frac{3}{2}(bBd-2aAf) - \frac{3}{2}(2Abf-B(2cd+be-2af))x + \frac{3}{2}(2Bce-bBf) \right)}{d+ex+fx^2}}{3f} \\
&= -\frac{(2Acf(4ce - 5bf) - B(b^2f^2 - 2cf(5be - 4af) + 8c^2(e^2 - df)) + 2cf(2Bce)}{8cf^3} \\
&= -\frac{(2Acf(4ce - 5bf) - B(b^2f^2 - 2cf(5be - 4af) + 8c^2(e^2 - df)) + 2cf(2Bce)}{8cf^3} \\
&= -\frac{(2Acf(4ce - 5bf) - B(b^2f^2 - 2cf(5be - 4af) + 8c^2(e^2 - df)) + 2cf(2Bce)}{8cf^3} \\
&= -\frac{(2Acf(4ce - 5bf) - B(b^2f^2 - 2cf(5be - 4af) + 8c^2(e^2 - df)) + 2cf(2Bce)}{8cf^3} \\
&= -\frac{(2Acf(4ce - 5bf) - B(b^2f^2 - 2cf(5be - 4af) + 8c^2(e^2 - df)) + 2cf(2Bce)}{8cf^3}
\end{aligned}$$

Mathematica [A] time = 6.56, size = 1627, normalized size = 1.49

result too large to display

Antiderivative was successfully verified.

[In] Integrate[((A + B*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x + f*x^2), x]

[Out] ((B - (B*e - 2*A*f)/Sqrt[e^2 - 4*d*f])*(a + x*(b + c*x))^(3/2))/(6*f) + ((B + (B*e - 2*A*f)/Sqrt[e^2 - 4*d*f])*(a + x*(b + c*x))^(3/2))/(6*f) - ((B + (-B*e) + 2*A*f)/Sqrt[e^2 - 4*d*f])*(a + x*(b + c*x))^(3/2)*(((4*c*f*(-4*a*f + b*(e - Sqrt[e^2 - 4*d*f])) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*(-(b*f) + 2*c*(e - Sqrt[e^2 - 4*d*f])) - 4*c*f*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^2) - ((-2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) - 4*c*f*(3*a*f - b*(e - Sqrt[e^2 - 4*d*f])))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])

$$\frac{2)]]) / (\sqrt{c} * f) - (2 * \sqrt{2} * \sqrt{c * e^2 - 2 * c * d * f - b * e * f + 2 * a * f^2 - c * e * \sqrt{e^2 - 4 * d * f}} + b * f * \sqrt{e^2 - 4 * d * f}) * (4 * (e - \sqrt{e^2 - 4 * d * f}) * (b * f - c * (e - \sqrt{e^2 - 4 * d * f}))) * (b^2 * f^2 - 4 * c^2 * (e^2 - 2 * d * f - e * \sqrt{e^2 - 4 * d * f})) - 4 * c * f * (3 * a * f - b * (e - \sqrt{e^2 - 4 * d * f}))) + 4 * f * (2 * c * f * (4 * a * f - b * (e - \sqrt{e^2 - 4 * d * f})))^2 - (e - \sqrt{e^2 - 4 * d * f}) * (b * f - c * (e - \sqrt{e^2 - 4 * d * f})) * (b^2 * f + 4 * a * c * f - 2 * b * c * (e - \sqrt{e^2 - 4 * d * f}))) * \text{ArcTanh}[(4 * a * f - b * (e - \sqrt{e^2 - 4 * d * f})) - (-2 * b * f + 2 * c * (e - \sqrt{e^2 - 4 * d * f})) * x] / (2 * \sqrt{2} * \sqrt{c * e^2 - 2 * c * d * f - b * e * f + 2 * a * f^2 - c * e * \sqrt{e^2 - 4 * d * f}} + b * f * \sqrt{e^2 - 4 * d * f}) * \sqrt{a + b * x + c * x^2}) / (f * (16 * a * f^2 - 8 * b * f * (e - \sqrt{e^2 - 4 * d * f})) + 4 * c * (e - \sqrt{e^2 - 4 * d * f})^2)) / (16 * c * f^2)) / (4 * f * (a + b * x + c * x^2)^{(3/2)}) - ((B - (-B * e) + 2 * A * f) / \sqrt{e^2 - 4 * d * f}) * (a + x * (b + c * x))^{(3/2)} * (((4 * c * f * (-4 * a * f + b * (e + \sqrt{e^2 - 4 * d * f}))) + 2 * (b * f - c * (e + \sqrt{e^2 - 4 * d * f}))) * (-b * f) + 2 * c * (e + \sqrt{e^2 - 4 * d * f})) - 4 * c * f * (b * f - c * (e + \sqrt{e^2 - 4 * d * f})) * x) * \sqrt{a + b * x + c * x^2}) / (8 * c * f^2) - ((-2 * (b * f - c * (e + \sqrt{e^2 - 4 * d * f}))) * (b^2 * f^2 - 4 * c^2 * (e^2 - 2 * d * f + e * \sqrt{e^2 - 4 * d * f})) - 4 * c * f * (3 * a * f - b * (e + \sqrt{e^2 - 4 * d * f}))) * \text{ArcTanh}[(b + 2 * c * x) / (2 * \sqrt{c} * \sqrt{a + b * x + c * x^2})]) / (\sqrt{c} * f) - (2 * \sqrt{2} * \sqrt{c * e^2 - 2 * c * d * f - b * e * f + 2 * a * f^2 + c * e * \sqrt{e^2 - 4 * d * f}} - b * f * \sqrt{e^2 - 4 * d * f}) * (4 * (e + \sqrt{e^2 - 4 * d * f}) * (b * f - c * (e + \sqrt{e^2 - 4 * d * f}))) * (b^2 * f^2 - 4 * c^2 * (e^2 - 2 * d * f + e * \sqrt{e^2 - 4 * d * f})) - 4 * c * f * (3 * a * f - b * (e + \sqrt{e^2 - 4 * d * f}))) + 4 * f * (2 * c * f * (4 * a * f - b * (e + \sqrt{e^2 - 4 * d * f})))^2 - (e + \sqrt{e^2 - 4 * d * f}) * (b * f - c * (e + \sqrt{e^2 - 4 * d * f})) * (b^2 * f + 4 * a * c * f - 2 * b * c * (e + \sqrt{e^2 - 4 * d * f}))) * \text{ArcTanh}[(4 * a * f - b * (e + \sqrt{e^2 - 4 * d * f})) - (-2 * b * f + 2 * c * (e + \sqrt{e^2 - 4 * d * f})) * x] / (2 * \sqrt{2} * \sqrt{c * e^2 - 2 * c * d * f - b * e * f + 2 * a * f^2 + c * e * \sqrt{e^2 - 4 * d * f}} - b * f * \sqrt{e^2 - 4 * d * f}) * \sqrt{a + b * x + c * x^2}) / (f * (16 * a * f^2 - 8 * b * f * (e + \sqrt{e^2 - 4 * d * f})) + 4 * c * (e + \sqrt{e^2 - 4 * d * f})^2)) / (16 * c * f^2)) / (4 * f * (a + b * x + c * x^2)^{(3/2)})$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:Error: Bad Argument Type

maple [B] time = 0.03, size = 59465, normalized size = 54.46

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x+A)*(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)

[Out] result too large to display

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + Bx)(cx^2 + bx + a)^{3/2}}{fx^2 + ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x + f*x^2),x)

[Out] int(((A + B*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x + f*x^2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)

[Out] Timed out

$$3.21 \quad \int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+ex+fx^2}} dx$$

Optimal. Leaf size=416

$$\frac{\left(-B\sqrt{b^2-4ac}-2Ac+bB\right)\tanh^{-1}\left(\frac{2x\left(ce-f\left(b-\sqrt{b^2-4ac}\right)\right)-e\left(b-\sqrt{b^2-4ac}\right)+4cd}{2\sqrt{2}\sqrt{d+ex+fx^2}\sqrt{b^2-4ac}\left(ce-bf\right)-2acf+b^2f-bce+2c^2d}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}\left(ce-bf\right)-2acf+b^2f-bce+2c^2d}}+\frac{\left(2Ac-B\left(\sqrt{b^2-4ac}+b\right)\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}\left(ce-bf\right)-2acf+b^2f-bce+2c^2d}}$$

[Out] $1/2*\operatorname{arctanh}(1/4*(4*c*d-e*(b+(-4*a*c+b^2)^(1/2))+2*x*(c*e-f*(b+(-4*a*c+b^2)^(1/2))))*2^(1/2)/(f*x^2+e*x+d)^(1/2)/(2*c^2*d-b*c*e+b^2*f-2*a*c*f-(-b*f+c*e)*(-4*a*c+b^2)^(1/2))^(1/2))*(2*A*c-B*(b+(-4*a*c+b^2)^(1/2)))*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c^2*d-b*c*e+b^2*f-2*a*c*f-(-b*f+c*e)*(-4*a*c+b^2)^(1/2))^(1/2)+1/2*\operatorname{arctanh}(1/4*(4*c*d+2*x*(c*e-f*(b-(-4*a*c+b^2)^(1/2))))-e*(b-(-4*a*c+b^2)^(1/2)))*2^(1/2)/(f*x^2+e*x+d)^(1/2)/(2*c^2*d-b*c*e+b^2*f-2*a*c*f+(-b*f+c*e)*(-4*a*c+b^2)^(1/2))^(1/2))*(b*B-2*A*c-B*(-4*a*c+b^2)^(1/2))*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c^2*d-b*c*e+b^2*f-2*a*c*f+(-b*f+c*e)*(-4*a*c+b^2)^(1/2))^(1/2)$

Rubi [A] time = 2.70, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {1032, 724, 206}

$$\frac{\left(-B\sqrt{b^2-4ac}-2Ac+bB\right)\tanh^{-1}\left(\frac{2x\left(ce-f\left(b-\sqrt{b^2-4ac}\right)\right)-e\left(b-\sqrt{b^2-4ac}\right)+4cd}{2\sqrt{2}\sqrt{d+ex+fx^2}\sqrt{b^2-4ac}\left(ce-bf\right)-2acf+b^2f-bce+2c^2d}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}\left(ce-bf\right)-2acf+b^2f-bce+2c^2d}}+\frac{\left(2Ac-B\left(\sqrt{b^2-4ac}+b\right)\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}\left(ce-bf\right)-2acf+b^2f-bce+2c^2d}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]),x]

[Out] $((b*B-2*A*c-B*\operatorname{Sqrt}[b^2-4*a*c])*ArcTanh[(4*c*d-(b-\operatorname{Sqrt}[b^2-4*a*c])*e+2*(c*e-(b-\operatorname{Sqrt}[b^2-4*a*c])*f)*x)/(2*\operatorname{Sqrt}[2]*\operatorname{Sqrt}[2*c^2*d-b*c*e+b^2*f-2*a*c*f+\operatorname{Sqrt}[b^2-4*a*c]*(c*e-b*f)]*\operatorname{Sqrt}[d+e*x+f*x^2]])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b^2-4*a*c]*\operatorname{Sqrt}[2*c^2*d-b*c*e+b^2*f-2*a*c*f+\operatorname{Sqrt}[b^2-4*a*c]*(c*e-b*f)])+(2*A*c-B*(b+\operatorname{Sqrt}[b^2-4*a*c]))*ArcTanh[(4*c*d-(b+\operatorname{Sqrt}[b^2-4*a*c])*e+2*(c*e-(b+\operatorname{Sqrt}[b^2-4*a*c])*f)*x)/(2*\operatorname{Sqrt}[2]*\operatorname{Sqrt}[2*c^2*d-b*c*e+b^2*f-2*a*c*f-\operatorname{Sqrt}[b^2-4*a*c]*(c*e-b*f)]*\operatorname{Sqrt}[d+e*x+f*x^2]])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b^2-4*a*c]*\operatorname{Sqrt}[2*c^2*d-b*c*e+b^2*f-2*a*c*f-\operatorname{Sqrt}[b^2-4*a*c]*(c*e-b*f)])$

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1032

Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]

Rubi steps

$$\int \frac{A + Bx}{(a + bx + cx^2)\sqrt{d + ex + fx^2}} dx = \frac{(2Ac - B(b - \sqrt{b^2 - 4ac})) \int \frac{1}{(b - \sqrt{b^2 - 4ac} + 2cx)\sqrt{d + ex + fx^2}} dx}{\sqrt{b^2 - 4ac}} - \frac{(2Ac - B(b + \sqrt{b^2 - 4ac})) \int \frac{1}{(b + \sqrt{b^2 - 4ac} + 2cx)\sqrt{d + ex + fx^2}} dx}{\sqrt{b^2 - 4ac}}$$

$$= \frac{(2(bB - 2Ac - B\sqrt{b^2 - 4ac})) \text{Subst}\left(\int \frac{1}{16c^2d - 8c(b - \sqrt{b^2 - 4ac})e + 4(b - \sqrt{b^2 - 4ac})^2f}\right)}{\sqrt{b^2 - 4ac}}$$

$$= \frac{(bB - 2Ac - B\sqrt{b^2 - 4ac}) \tanh^{-1}\left(\frac{4cd - (b - \sqrt{b^2 - 4ac})e + 2(ce - (b - \sqrt{b^2 - 4ac})f)}{2\sqrt{2}\sqrt{2c^2d - bce + b^2f - 2acf + \sqrt{b^2 - 4ac}(ce - bf)}\sqrt{d + ex + fx^2}}\right)}{\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{2c^2d - bce + b^2f - 2acf + \sqrt{b^2 - 4ac}(ce - bf)}}$$

Mathematica [A] time = 4.18, size = 393, normalized size = 0.94

$$\frac{(B\sqrt{b^2-4ac}+2Ac-bB)\tanh^{-1}\left(\frac{(\sqrt{b^2-4ac}-b)(e+2fx)+2c(2d+ex)}{2\sqrt{2}\sqrt{d+xe+fx}\sqrt{c(e\sqrt{b^2-4ac}-2af-be)+bf(b-\sqrt{b^2-4ac})+2c^2d}}\right)}{\sqrt{c(e\sqrt{b^2-4ac}-2af-be)+bf(b-\sqrt{b^2-4ac})+2c^2d}} - \frac{(B\sqrt{b^2-4ac}-2Ac+bB)\tanh^{-1}\left(\frac{2c(2d+ex)}{2\sqrt{d+xe+fx}\sqrt{-2c(e\sqrt{b^2-4ac}+2af+be)+bf(b+\sqrt{b^2-4ac})+2c^2d}}\right)}{\sqrt{-c(e\sqrt{b^2-4ac}+2af+be)+bf(b+\sqrt{b^2-4ac})+2c^2d}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]),x]

[Out]
$$\frac{-(((-(b*B) + 2*A*c + B*\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTanh}[(2*c*(2*d + e*x) + (-b + \text{Sqrt}[b^2 - 4*a*c])*(e + 2*f*x))/(2*\text{Sqrt}[2]*\text{Sqrt}[2*c^2*d + b*(b - \text{Sqrt}[b^2 - 4*a*c])]*f + c*(-(b*e) + \text{Sqrt}[b^2 - 4*a*c]*e - 2*a*f)]*\text{Sqrt}[d + x*(e + f*x)])))/\text{Sqrt}[2*c^2*d + b*(b - \text{Sqrt}[b^2 - 4*a*c])]*f + c*(-(b*e) + \text{Sqrt}[b^2 - 4*a*c]*e - 2*a*f)) - ((b*B - 2*A*c + B*\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTanh}[(2*c*(2*d + e*x) - (b + \text{Sqrt}[b^2 - 4*a*c])*(e + 2*f*x))/(2*\text{Sqrt}[4*c^2*d + 2*b*(b + \text{Sqrt}[b^2 - 4*a*c])]*f - 2*c*(b*e + \text{Sqrt}[b^2 - 4*a*c]*e + 2*a*f)]*\text{Sqrt}[d + x*(e + f*x)])))/\text{Sqrt}[2*c^2*d + b*(b + \text{Sqrt}[b^2 - 4*a*c])]*f - c*(b*e + \text{Sqrt}[b^2 - 4*a*c]*e + 2*a*f)))/(\text{Sqrt}[2]*\text{Sqrt}[b^2 - 4*a*c])$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INP UT:sage2:=int(sage0,x);;OUTPUT:Unable to divide, perhaps due to rounding error

$$\frac{\text{poly1}[\text{poly}[-4, [3, 2, 0]] + \text{poly}[16, [1, 3, 1]] + \text{poly}[4, [4, 2, 0]] + \text{poly}[-24, [2, 3, 1]] + \text{poly}[32, [0, 4, 2]]}{[1, 0, \text{poly}[-1, [2, 0, 0]] + \text{poly}[4, [0, 1, 1]] + \text{poly}[2, 1, 0, 0, 0]] + \text{poly}[\text{poly}[-4, [5, 0, 0]] + \text{poly}[24, [3, 1, 1]] + \text{poly}[-32, [1, 2, 2]] + \text{poly}[4, [6, 0, 0]] + \text{poly}[-32, [4, 1, 1]] + \text{poly}[72, [2, 2, 2]] + \text{poly}[\dots]}$$

$\{-32, [0, 3, 3]\}$: $[1, 0, \{-1, [2, 0, 0]\}] + \{4, [0, 1, 1]\}$, $[2, 0, 0, 1, 0]$
 $\} + \{4, [4, 1, 0]\} + \{-20, [2, 2, 1]\} + \{16, [0, 3, 2]\}$, $\{-4,$
 $[5, 1, 0]\} + \{28, [3, 2, 1]\} + \{-48, [1, 3, 2]\}$: $[1, 0, \{-1, [2, 0, 0]$
 $\}] + \{4, [0, 1, 1]\}$, $[2, 0, 0, 0, 1]$ + $\{\text{poly1}[8, [2, 3, 0]] + \{-32,$
 $[0, 4, 1]\}$, $\{-8, [3, 3, 0]\} + \{32, [1, 4, 1]\}$: $[1, 0, \{-1, [2, 0, 0]$
 $\}] + \{4, [0, 1, 1]\}$, $[1, 1, 1, 0, 0]$ + $\{\{8, [4, 1, 0]\} + \{-4$
 $0, [2, 2, 1]\} + \{32, [0, 3, 2]\}$, $\{-8, [5, 1, 0]\} + \{56, [3, 2, 1]\} + \{-$
 $96, [1, 3, 2]\}$: $[1, 0, \{-1, [2, 0, 0]\}] + \{4, [0, 1, 1]\}$, $[1, 0, 1, 1, 0]$
 $\} + \{\{-8, [3, 2, 0]\} + \{32, [1, 3, 1]\}$, $\{8, [4, 2, 0]\} + \{-48$
 $, [2, 3, 1]\} + \{64, [0, 4, 2]\}$: $[1, 0, \{-1, [2, 0, 0]\}] + \{4, [0, 1, 1]\}$
 $\}$, $[1, 0, 1, 0, 1]$ + $\{\{8, [2, 4, 0]\} + \{-32, [0, 5, 1]\}$, $[0, 1, 2, 0, 0]$
 $\} + \{\text{poly1}[-4, [3, 2, 0]] + \{16, [1, 3, 1]\}$, $\{4, [4, 2, 0]\} + \{-$
 $24, [2, 3, 1]\} + \{32, [0, 4, 2]\}$: $[1, 0, \{-1, [2, 0, 0]\}] + \{4, [0, 1, 1]$
 $\}] + \{0, 0, 2, 1, 0\}$ + $\{\text{poly1}[4, [2, 3, 0]] + \{-16, [0, 4, 1]\}$,
 $\{-4, [3, 3, 0]\} + \{16, [1, 4, 1]\}$: $[1, 0, \{-1, [2, 0, 0]\}] + \{4, [0, 1, 1]$
 $\}] + \{0, 0, 2, 0, 1\} / \{\{4, [3, 2, 0]\} + \{-16, [1, 3, 1]\}$,
 $\{-4, [4, 2, 0]\} + \{24, [2, 3, 1]\} + \{-32, [0, 4, 2]\}$: $[1, 0, \{-1, [2, 0,$
 $0]\}] + \{4, [0, 1, 1]\}$, $[0, 1, 0, 0, 0]$ + $\{\{4, [5, 0, 0]\} + \{-$
 $24, [3, 1, 1]\} + \{32, [1, 2, 2]\}$, $\{-4, [6, 0, 0]\} + \{32, [4, 1, 1]\} + \{-$
 $72, [2, 2, 2]\} + \{32, [0, 3, 3]\}$: $[1, 0, \{-1, [2, 0, 0]\}] + \{4, [0, 1, 1]$
 $\}] + \{0, 0, 0, 1, 0\}$ + $\{\{-4, [4, 1, 0]\} + \{20, [2, 2, 1]\} + \{-$
 $16, [0, 3, 2]\}$, $\{4, [5, 1, 0]\} + \{-28, [3, 2, 1]\} + \{48, [1, 3, 2]\}$: $[$
 $1, 0, \{-1, [2, 0, 0]\}] + \{4, [0, 1, 1]\}$, $[0, 0, 0, 0, 1]$ Error: Bad Arg
 ument ValueEvaluation time: 0.92Unable to divide, perhaps due to rounding e
 rror $\{\{1, [3, 0, 0]\} + \{-4, [1, 1, 1]\}$, $\{1, [4, 0, 0]\} + \{-6, [$
 $2, 1, 1]\} + \{8, [0, 2, 2]\}$: $[1, 0, \{-1, [2, 0, 0]\}] + \{4, [0, 1, 1]\}$
 $\} / \{4, [0, 0, 2]\}$, $[2, 1, 0, 0, 0]$ + $\{\{1, [2, 0, 0]\} + \{-4, [0, 1, 1]$
 $\} / 2, [2, 0, 0, 1, 0]\} + \{\{-1, [2, 0, 0]\} + \{4, [0, 1, 1]\}$, $\{-1, [3, 0,$
 $0]\} + \{4, [1, 1, 1]\}$: $[1, 0, \{-1, [2, 0, 0]\}] + \{4, [0, 1, 1]\}$ /
 $\{4, [0, 0, 1]\}$, $[2, 0, 0, 0, 1]$ + $\{\{-1, [4, 0, 0]\} + \{5, [2, 1, 1]$
 $\} + \{-4, [0, 2, 2]\}$, $\{-1, [5, 0, 0]\} + \{7, [3, 1, 1]\} + \{-12, [1, 2, 2]$
 $\}$: $[1, 0, \{-1, [2, 0, 0]\}] + \{4, [0, 1, 1]\}$ / $\{2, [0, 0, 3]\}$, $[1, 1, 1,$
 $0, 0]$ + $\{\{-1, [2, 0, 0]\} + \{4, [0, 1, 1]\}$, $\{-1, [3, 0, 0]\} + \{$
 $4, [1, 1, 1]\}$: $[1, 0, \{-1, [2, 0, 0]\}] + \{4, [0, 1, 1]\}$ / $\{2, [0, 0, 1]$
 $\}$, $[1, 0, 1, 1, 0]$ + $\{\{1, [3, 0, 0]\} + \{-4, [1, 1, 1]\}$, $\{1, [4,$
 $0, 0]\} + \{-6, [2, 1, 1]\} + \{8, [0, 2, 2]\}$: $[1, 0, \{-1, [2, 0, 0]\}] + \{$
 $4, [0, 1, 1]\}$ / $\{2, [0, 0, 2]\}$, $[1, 0, 1, 0, 1]$ + $\{\{1, [5, 0, 0]$
 $\} + \{-6, [3, 1, 1]\} + \{8, [1, 2, 2]\}$, $\{1, [6, 0, 0]\} + \{-8, [4, 1, 1]$
 $\} + \{18, [2, 2, 2]\} + \{-8, [0, 3, 3]\}$: $[1, 0, \{-1, [2, 0, 0]\}] + \{4, [0,$
 $1, 1]\}$ / $\{4, [0, 0, 4]\}$, $[0, 1, 2, 0, 0]$ + $\{\{1, [3, 0, 0]\} + \{$
 $-4, [1, 1, 1]\}$, $\{1, [4, 0, 0]\} + \{-6, [2, 1, 1]\} + \{8, [0, 2, 2]\}$: $[1$
 $, 0, \{-1, [2, 0, 0]\}] + \{4, [0, 1, 1]\}$ / $\{4, [0, 0, 2]\}$, $[0, 0, 2, 1, 0]$
 $\} + \{\{-1, [4, 0, 0]\} + \{5, [2, 1, 1]\} + \{-4, [0, 2, 2]\}$, $\{-1, [$
 $5, 0, 0]\} + \{7, [3, 1, 1]\} + \{-12, [1, 2, 2]\}$: $[1, 0, \{-1, [2, 0, 0]\}] + \{$
 $4, [0, 1, 1]\}$ / $\{4, [0, 0, 3]\}$, $[0, 0, 2, 0, 1]$ / $\{\{-1, [3,$

```
0,0]%%}+%%{4,[1,1,1]%%},%%{-1,[4,0,0]%%}+%%{6,[2,1,1]%%}+%%{-8,[0,2,2]%%}: [1,0,%%{-1,[2,0,0]%%}+%%{4,[0,1,1]%%}]%%}/%%{4,[0,0,4]%%}, [0,1,0,0,0]%%}+%%{%%{-1,[2,0,0]%%}+%%{4,[0,1,1]%%}}/%%{2,[0,0,2]%%}, [0,0,0,1,0]%%}+%%{%%{1,[2,0,0]%%}+%%{-4,[0,1,1]%%},%%{1,[3,0,0]%%}+%%{-4,[1,1,1]%%}}: [1,0,%%{-1,[2,0,0]%%}+%%{4,[0,1,1]%%}]%%}/%%{4,[0,0,3]%%}, [0,0,0,0,1]%%} Error: Bad Argument Value
```

maple [B] time = 0.04, size = 2269, normalized size = 5.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d)^{(1/2)}, x)$

[Out]
$$\frac{2}{(-4ac+b^2)^{1/2}} \frac{1}{(-2(-(-4ac+b^2)^{1/2})bf+(-4ac+b^2)^{1/2})ce+2af-b^2f+bce-2c^2d)/c^2} \ln\left(\frac{(-(-4ac+b^2)^{1/2})bf+(-4ac+b^2)^{1/2})ce+2af-b^2f+bce-2c^2d}{c^2} - \frac{f(-4ac+b^2)^{1/2}+bf-ce}{c(x+1/2(b+(-4ac+b^2)^{1/2}))/c} + \frac{1}{2} \frac{(-2(-(-4ac+b^2)^{1/2})bf+(-4ac+b^2)^{1/2})ce+2af-b^2f+bce-2c^2d)/c^2}{(4(x+1/2(b+(-4ac+b^2)^{1/2}))/c)^2} - \frac{4(f(-4ac+b^2)^{1/2}+bf-ce)/c}{(x+1/2(b+(-4ac+b^2)^{1/2}))/c} - 2 \frac{(-(-4ac+b^2)^{1/2})bf+(-4ac+b^2)^{1/2})ce+2af-b^2f+bce-2c^2d}{c^2} \ln\left(\frac{(-(-4ac+b^2)^{1/2})bf+(-4ac+b^2)^{1/2})ce+2af-b^2f+bce-2c^2d}{c^2} - \frac{f(-4ac+b^2)^{1/2}+bf-ce}{c(x+1/2(b+(-4ac+b^2)^{1/2}))/c} + \frac{1}{2} \frac{(-2(-(-4ac+b^2)^{1/2})bf+(-4ac+b^2)^{1/2})ce+2af-b^2f+bce-2c^2d)/c^2}{(4(x+1/2(b+(-4ac+b^2)^{1/2}))/c)^2} - \frac{4(f(-4ac+b^2)^{1/2}+bf-ce)/c}{(x+1/2(b+(-4ac+b^2)^{1/2}))/c} - 2 \frac{(-(-4ac+b^2)^{1/2})bf+(-4ac+b^2)^{1/2})ce+2af-b^2f+bce-2c^2d}{c^2} \ln\left(\frac{(-(-4ac+b^2)^{1/2})bf+(-4ac+b^2)^{1/2})ce+2af-b^2f+bce-2c^2d}{c^2} - \frac{f(-4ac+b^2)^{1/2}+bf-ce}{c(x-1/2(-b+(-4ac+b^2)^{1/2}))/c} + \frac{1}{2} \frac{(-2(-(-4ac+b^2)^{1/2})bf+(-4ac+b^2)^{1/2})ce+2af-b^2f+bce-2c^2d)/c^2}{(4(x-1/2(-b+(-4ac+b^2)^{1/2}))/c)^2} - \frac{4(f(-4ac+b^2)^{1/2}+bf-ce)/c}{(x-1/2(-b+(-4ac+b^2)^{1/2}))/c} - 2 \frac{(-(-4ac+b^2)^{1/2})bf+(-4ac+b^2)^{1/2})ce+2af-b^2f+bce-2c^2d}{c^2} \ln\left(\frac{(-(-4ac+b^2)^{1/2})bf+(-4ac+b^2)^{1/2})ce+2af-b^2f+bce-2c^2d}{c^2} - \frac{f(-4ac+b^2)^{1/2}+bf-ce}{c(x-1/2(-b+(-4ac+b^2)^{1/2}))/c} + \frac{1}{2} \frac{(-2(-(-4ac+b^2)^{1/2})bf+(-4ac+b^2)^{1/2})ce+2af-b^2f+bce-2c^2d)/c^2}{(4(x-1/2(-b+(-4ac+b^2)^{1/2}))/c)^2} - \frac{4(f(-4ac+b^2)^{1/2}+bf-ce)/c}{(x-1/2(-b+(-4ac+b^2)^{1/2}))/c} - 2 \frac{(-(-4ac+b^2)^{1/2})bf+(-4ac+b^2)^{1/2})ce+2af-b^2f+bce-2c^2d}{c^2} \right)$$

$$\begin{aligned} & \left. \frac{(-2)^{(1/2)}}{(x-1/2*(-b+(-4*a*c+b^2)^{(1/2))}/c)} \right) * A - 1/c / (-2 * ((-4*a*c+b^2)^{(1/2)} \\ & * b * f - (-4*a*c+b^2)^{(1/2)} * c * e + 2*a*c*f - b^2*f + b*c*e - 2*c^2*d) / c^2)^{(1/2)} * \ln\left(\frac{-((-4*a*c+b^2)^{(1/2)} * b * f - (-4*a*c+b^2)^{(1/2)} * c * e + 2*a*c*f - b^2*f + b*c*e - 2*c^2*d) / c}{-2 - (-f * (-4*a*c+b^2)^{(1/2)} + b * f - c * e) / c * (x - 1/2 * (-b + (-4*a*c+b^2)^{(1/2))} / c) + 1/2 * (-2 * ((-4*a*c+b^2)^{(1/2)} * b * f - (-4*a*c+b^2)^{(1/2)} * c * e + 2*a*c*f - b^2*f + b*c*e - 2*c^2*d) / c^2)^{(1/2)} * (4 * (x - 1/2 * (-b + (-4*a*c+b^2)^{(1/2))} / c))^2 * f - 4 * (-f * (-4*a*c+b^2)^{(1/2)} + b * f - c * e) / c * (x - 1/2 * (-b + (-4*a*c+b^2)^{(1/2))} / c) - 2 * ((-4*a*c+b^2)^{(1/2)} * b * f - (-4*a*c+b^2)^{(1/2)} * c * e + 2*a*c*f - b^2*f + b*c*e - 2*c^2*d) / c^2)^{(1/2)}\right) / (x - 1/2 * (-b + (-4*a*c+b^2)^{(1/2))} / c) * B + 1 / (-4*a*c+b^2)^{(1/2)} / c / (-2 * ((-4*a*c+b^2)^{(1/2)} * b * f - (-4*a*c+b^2)^{(1/2)} * c * e + 2*a*c*f - b^2*f + b*c*e - 2*c^2*d) / c^2)^{(1/2)} * \ln\left(\frac{-((-4*a*c+b^2)^{(1/2)} * b * f - (-4*a*c+b^2)^{(1/2)} * c * e + 2*a*c*f - b^2*f + b*c*e - 2*c^2*d) / c}{-2 - (-f * (-4*a*c+b^2)^{(1/2)} + b * f - c * e) / c * (x - 1/2 * (-b + (-4*a*c+b^2)^{(1/2))} / c) + 1/2 * (-2 * ((-4*a*c+b^2)^{(1/2)} * b * f - (-4*a*c+b^2)^{(1/2)} * c * e + 2*a*c*f - b^2*f + b*c*e - 2*c^2*d) / c^2)^{(1/2)} * (4 * (x - 1/2 * (-b + (-4*a*c+b^2)^{(1/2))} / c))^2 * f - 4 * (-f * (-4*a*c+b^2)^{(1/2)} + b * f - c * e) / c * (x - 1/2 * (-b + (-4*a*c+b^2)^{(1/2))} / c) - 2 * ((-4*a*c+b^2)^{(1/2)} * b * f - (-4*a*c+b^2)^{(1/2)} * c * e + 2*a*c*f - b^2*f + b*c*e - 2*c^2*d) / c^2)^{(1/2)}\right) / (x - 1/2 * (-b + (-4*a*c+b^2)^{(1/2))} / c) * b * B \end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details) Is 4*a*c-b^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + Bx}{(cx^2 + bx + a) \sqrt{fx^2 + ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)^(1/2)),x)

[Out] int((A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx}{(a + bx + cx^2) \sqrt{d + ex + fx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)/(f*x**2+e*x+d)**(1/2),x)
```

```
[Out] Integral((A + B*x)/((a + b*x + c*x**2)*sqrt(d + e*x + f*x**2)), x)
```

$$3.22 \quad \int \frac{A+Bx}{(a+cx^2)\sqrt{d+ex+fx^2}} dx$$

Optimal. Leaf size=780

$$\sqrt{A\left(-\sqrt{a^2f^2+ac(e^2-2df)+c^2d^2}-af+cd\right)+aBe}\sqrt{B\left(\sqrt{a^2f^2+ac(e^2-2df)+c^2d^2}-af+cd\right)-Ace}\operatorname{tanh}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{e}\sqrt{a^2f^2+ac(e^2-2df)+c^2d^2}-af+cd}{\sqrt{a^2f^2+ac(e^2-2df)+c^2d^2}-af+cd}}\right)$$

[Out] $-1/2*\operatorname{arctanh}(1/2*e^{(1/2)}*(a*(A*c*e-B*(c*d-a*f-(c^2*d^2+a^2*f^2+a*c*(-2*d*f+e^2)))^{(1/2)}))-c*x*(a*B*e+A*(c*d-a*f+(c^2*d^2+a^2*f^2+a*c*(-2*d*f+e^2)))^{(1/2)}))^{(1/2)})/a^{(1/2)}/c^{(1/2)}/(f*x^2+e*x+d)^{(1/2)}/(-A*c*e+B*(c*d-a*f-(c^2*d^2+a^2*f^2+a*c*(-2*d*f+e^2)))^{(1/2)}))^{(1/2)})/(a*B*e+A*(c*d-a*f+(c^2*d^2+a^2*f^2+a*c*(-2*d*f+e^2)))^{(1/2)}))^{(1/2)})*(-A*c*e+B*(c*d-a*f-(c^2*d^2+a^2*f^2+a*c*(-2*d*f+e^2)))^{(1/2)}))^{(1/2)}*(a*B*e+A*(c*d-a*f+(c^2*d^2+a^2*f^2+a*c*(-2*d*f+e^2)))^{(1/2)}))^{(1/2)}*2^{(1/2)}/a^{(1/2)}/c^{(1/2)}/e^{(1/2)}/(c^2*d^2+a^2*f^2+a*c*(-2*d*f+e^2))^{(1/2)}+1/2*\operatorname{arctanh}(1/2*e^{(1/2)}*(-c*x*(a*B*e+A*(c*d-a*f-(c^2*d^2+a^2*f^2+a*c*(-2*d*f+e^2)))^{(1/2)}))^{(1/2)})+a*(A*c*e-B*(c*d-a*f+(c^2*d^2+a^2*f^2+a*c*(-2*d*f+e^2)))^{(1/2)}))^{(1/2)}))^{(1/2)})/(-A*c*e+B*(c*d-a*f+(c^2*d^2+a^2*f^2+a*c*(-2*d*f+e^2)))^{(1/2)}))^{(1/2)})*(a*B*e+A*(c*d-a*f-(c^2*d^2+a^2*f^2+a*c*(-2*d*f+e^2)))^{(1/2)}))^{(1/2)}*(-A*c*e+B*(c*d-a*f+(c^2*d^2+a^2*f^2+a*c*(-2*d*f+e^2)))^{(1/2)}))^{(1/2)}*2^{(1/2)}/a^{(1/2)}/c^{(1/2)}/e^{(1/2)}/(c^2*d^2+a^2*f^2+a*c*(-2*d*f+e^2))^{(1/2)}$

Rubi [A] time = 5.16, antiderivative size = 780, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1036, 1030, 208}

$$\sqrt{A\left(-\sqrt{a^2f^2+ac(e^2-2df)+c^2d^2}-af+cd\right)+aBe}\sqrt{B\left(\sqrt{a^2f^2+ac(e^2-2df)+c^2d^2}-af+cd\right)-Ace}\operatorname{tanh}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{e}\sqrt{a^2f^2+ac(e^2-2df)+c^2d^2}-af+cd}{\sqrt{a^2f^2+ac(e^2-2df)+c^2d^2}-af+cd}}\right)$$

Antiderivative was successfully verified.

[In] Int[(A + B*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x]

[Out] (Sqrt[a*B*e + A*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*Sqrt[-(A*c*e) + B*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*ArcTanh[(Sqrt[e]*(a*(A*c*e - B*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])) - c*(a*B*e + A*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])))]

$$\frac{e^{2-2*d*f}}{\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{a*B*e + A*(c*d - a*f - \sqrt{c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))}}\sqrt{-(A*c*e) + B*(c*d - a*f + \sqrt{c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))}}\sqrt{d + e*x + f*x^2}}{\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{e}\sqrt{c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)}} - \frac{(\sqrt{-(A*c*e) + B*(c*d - a*f - \sqrt{c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))}}\sqrt{a*B*e + A*(c*d - a*f + \sqrt{c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))})\text{ArcTanh}[\frac{\sqrt{e}(a*(A*c*e - B*(c*d - a*f - \sqrt{c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))}) - c*(a*B*e + A*(c*d - a*f + \sqrt{c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))})}{\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{-(A*c*e) + B*(c*d - a*f - \sqrt{c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))}}\sqrt{a*B*e + A*(c*d - a*f + \sqrt{c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))}}\sqrt{d + e*x + f*x^2}}]{\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{e}\sqrt{c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))}}$$

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1030

Int[((g_) + (h_)*(x_))/(((a_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2], x] /; FreeQ[{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]

Rule 1036

Int[((g_) + (h_)*(x_))/(((a_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist[1/(2*q), Int[Simp[-(a*h*e) - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[-(a*h*e) - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && NegQ[-(a*c)]

Rubi steps

$$\int \frac{A + Bx}{(a + cx^2)\sqrt{d + ex + fx^2}} dx = \frac{\int \frac{-aBe - A(cd - af + \sqrt{c^2d^2 + a^2f^2 + ac(e^2 - 2df)}) + (-Ace + B(cd - af - \sqrt{c^2d^2 + a^2f^2 + ac(e^2 - 2df)}))x}{(a + cx^2)\sqrt{d + ex + fx^2}} dx}{2\sqrt{c^2d^2 + a^2f^2 + ac(e^2 - 2df)}}$$

$$= \frac{\left(a\left(Ace - B\left(cd - af - \sqrt{c^2d^2 + a^2f^2 + ac(e^2 - 2df)}\right)\right)\right)\left(aBe + A\left(cd - af + \sqrt{c^2d^2 + a^2f^2 + ac(e^2 - 2df)}\right)\right)}{2\sqrt{c^2d^2 + a^2f^2 + ac(e^2 - 2df)}\sqrt{-Ace + B\left(cd - af + \sqrt{c^2d^2 + a^2f^2 + ac(e^2 - 2df)}\right)}}$$

Mathematica [A] time = 0.41, size = 254, normalized size = 0.33

$$\frac{(A\sqrt{c} - \sqrt{-a}B) \tanh^{-1}\left(\frac{\sqrt{c}(2d+ex) - \sqrt{-a}(e+2fx)}{2\sqrt{d+x(e+fx)}\sqrt{-\sqrt{-a}\sqrt{c}e-af+cd}}\right)}{\sqrt{-\sqrt{-a}\sqrt{c}e-af+cd}} - \frac{(\sqrt{-a}B + A\sqrt{c}) \tanh^{-1}\left(\frac{\sqrt{-a}(e+2fx) + \sqrt{c}(2d+ex)}{2\sqrt{d+x(e+fx)}\sqrt{\sqrt{-a}\sqrt{c}e-af+cd}}\right)}{\sqrt{\sqrt{-a}\sqrt{c}e-af+cd}}$$

$$2\sqrt{-a}\sqrt{c}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x]

[Out] ((((-Sqrt[-a]*B) + A*Sqrt[c])*ArcTanh[(Sqrt[c]*(2*d + e*x) - Sqrt[-a]*(e + 2*f*x))/(2*Sqrt[c*d - Sqrt[-a]*Sqrt[c]*e - a*f]*Sqrt[d + x*(e + f*x)])])/Sqrt[c*d - Sqrt[-a]*Sqrt[c]*e - a*f] - ((Sqrt[-a]*B + A*Sqrt[c])*ArcTanh[(Sqrt[c]*(2*d + e*x) + Sqrt[-a]*(e + 2*f*x))/(2*Sqrt[c*d + Sqrt[-a]*Sqrt[c]*e - a*f]*Sqrt[d + x*(e + f*x)])])/Sqrt[c*d + Sqrt[-a]*Sqrt[c]*e - a*f))/(2*Sqrt[-a]*Sqrt[c])

fricas [B] time = 134.78, size = 6861, normalized size = 8.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+a)/(f*x^2+e*x+d)^(1/2), x, algorithm="fricas")

```
[Out] -1/4*sqrt(-(2*A*B*a*c*e - (B^2*a*c - A^2*c^2)*d + (B^2*a^2 - A^2*a*c)*f + (
a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2)*sqrt(-(4*A^2*B^2*c^2*d
^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B
^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e)*f)/
(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c*f^4
+ 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2)*f))
)/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2))*log(-(2*(A*B^3*a*c
+ A^3*B*c^2)*d*e + (B^4*a^2 - A^4*c^2)*e^2 - 2*(A*B^3*a^2 + A^3*B*a*c)*e*f
- 2*(2*(A*B^3*a^2 + A^3*B*a*c)*f^2 - (2*(A*B^3*a*c + A^3*B*c^2)*d + (B^4*a
^2 - A^4*c^2)*e)*f)*x + 2*(2*A^2*B*c^3*d^2 + 2*A^2*B*a^2*c*f^2 + (3*A*B^2*a
*c^2 - A^3*c^3)*d*e + (B^3*a^2*c - A^2*B*a*c^2)*e^2 - (4*A^2*B*a*c^2*d + (3
*A*B^2*a^2*c - A^3*a*c^2)*e)*f - (B*a*c^4*d^3 - A*a*c^4*d^2*e + B*a^2*c^3*d
*e^2 - A*a^2*c^3*e^3 - B*a^4*c*f^3 + (3*B*a^3*c^2*d - A*a^3*c^2*e)*f^2 - (3
*B*a^2*c^3*d^2 - 2*A*a^2*c^3*d*e + B*a^3*c^2*e^2)*f)*sqrt(-(4*A^2*B^2*c^2*d
^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B
^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e)*f)/
(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c*f^4
+ 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2)*f))
)*sqrt(f*x^2 + e*x + d)*sqrt(-(2*A*B*a*c*e - (B^2*a*c - A^2*c^2)*d + (B^2*a
^2 - A^2*a*c)*f + (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2)*sqr
t(-(4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e +
(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2
- A^3*B*a*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2
*d*f^3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 +
a^3*c^3*d*e^2)*f)))/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2))
- (2*(B^2*a*c^3 + A^2*c^4)*d^3 + 2*(B^2*a^2*c^2 + A^2*a*c^3)*d*e^2 - 4*(B^
2*a^2*c^2 + A^2*a*c^3)*d^2*f + 2*(B^2*a^3*c + A^2*a^2*c^2)*d*f^2 + ((B^2*a*
c^3 + A^2*c^4)*d^2*e + (B^2*a^2*c^2 + A^2*a*c^3)*e^3 - 2*(B^2*a^2*c^2 + A^2
*a*c^3)*d*e*f + (B^2*a^3*c + A^2*a^2*c^2)*e*f^2)*x)*sqrt(-(4*A^2*B^2*c^2*d^
2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B
^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e)*f)/
(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c*f^4 +
2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2)*f))
)/x) + 1/4*sqrt(-(2*A*B*a*c*e - (B^2*a*c - A^2*c^2)*d + (B^2*a^2 - A^2*a*c)*
f + (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2)*sqrt(-(4*A^2*B^2*
c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*
A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e
)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c
*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2
)*f)))/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2))*log(-(2*(A*B^
3*a*c + A^3*B*c^2)*d*e + (B^4*a^2 - A^4*c^2)*e^2 - 2*(A*B^3*a^2 + A^3*B*a*c
)*e*f - 2*(2*(A*B^3*a^2 + A^3*B*a*c)*f^2 - (2*(A*B^3*a*c + A^3*B*c^2)*d + (
B^4*a^2 - A^4*c^2)*e)*f)*x - 2*(2*A^2*B*c^3*d^2 + 2*A^2*B*a^2*c*f^2 + (3*A*
B^2*a*c^2 - A^3*c^3)*d*e + (B^3*a^2*c - A^2*B*a*c^2)*e^2 - (4*A^2*B*a*c^2*d
+ (3*A*B^2*a^2*c - A^3*a*c^2)*e)*f - (B*a*c^4*d^3 - A*a*c^4*d^2*e + B*a^2*
```

$$\begin{aligned}
& c^3*d*e^2 - A*a^2*c^3*e^3 - B*a^4*c*f^3 + (3*B*a^3*c^2*d - A*a^3*c^2*e)*f^2 \\
& - (3*B*a^2*c^3*d^2 - 2*A*a^2*c^3*d*e + B*a^3*c^2*e^2)*f)*\sqrt{-(4*A^2*B^2* \\
& c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2* \\
& A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e \\
&)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c \\
& *f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2 \\
&)*f)))*\sqrt{f*x^2 + e*x + d)*\sqrt{-(2*A*B*a*c*e - (B^2*a*c - A^2*c^2)*d + (\\
& B^2*a^2 - A^2*a*c)*f + (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2 \\
&)*\sqrt{-(4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)* \\
& d*e + (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3 \\
& *a^2 - A^3*B*a*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4 \\
& c^2*d*f^3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3 \\
& c^3*d*e^2)*f)))/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2)) - (2*(B^2*a*c^3 + A^2*c^4)*d^3 + 2*(B^2*a^2*c^2 + A^2*a*c^3)*d*e^2 - \\
& 4*(B^2*a^2*c^2 + A^2*a*c^3)*d^2*f + 2*(B^2*a^3*c + A^2*a^2*c^2)*d*f^2 + ((B \\
& ^2*a*c^3 + A^2*c^4)*d^2*e + (B^2*a^2*c^2 + A^2*a*c^3)*e^3 - 2*(B^2*a^2*c^2 \\
& + A^2*a*c^3)*d*e*f + (B^2*a^3*c + A^2*a^2*c^2)*e*f^2)*x)*\sqrt{-(4*A^2*B^2*c \\
& ^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A \\
& ^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e) \\
& *f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c* \\
& f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2) \\
& *f)))/x) - 1/4*\sqrt{-(2*A*B*a*c*e - (B^2*a*c - A^2*c^2)*d + (B^2*a^2 - A^2* \\
& a*c)*f - (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2)*\sqrt{-(4*A^2 \\
& *B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 \\
& - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a \\
& *c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + \\
& a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3* \\
& d*e^2)*f)))/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2))*\log(-(2* \\
& (A*B^3*a*c + A^3*B*c^2)*d*e + (B^4*a^2 - A^4*c^2)*e^2 - 2*(A*B^3*a^2 + A^3* \\
& B*a*c)*e*f - 2*(2*(A*B^3*a^2 + A^3*B*a*c)*f^2 - (2*(A*B^3*a*c + A^3*B*c^2)* \\
& d + (B^4*a^2 - A^4*c^2)*e)*f)*x + 2*(2*A^2*B*c^3*d^2 + 2*A^2*B*a^2*c*f^2 + \\
& (3*A*B^2*a*c^2 - A^3*c^3)*d*e + (B^3*a^2*c - A^2*B*a*c^2)*e^2 - (4*A^2*B*a* \\
& c^2*d + (3*A*B^2*a^2*c - A^3*a*c^2)*e)*f + (B*a*c^4*d^3 - A*a*c^4*d^2*e + B \\
& *a^2*c^3*d*e^2 - A*a^2*c^3*e^3 - B*a^4*c*f^3 + (3*B*a^3*c^2*d - A*a^3*c^2*e) \\
&)*f^2 - (3*B*a^2*c^3*d^2 - 2*A*a^2*c^3*d*e + B*a^3*c^2*e^2)*f)*\sqrt{-(4*A^2 \\
& *B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 \\
& - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a \\
& *c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + \\
& a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3* \\
& d*e^2)*f)))*\sqrt{f*x^2 + e*x + d)*\sqrt{-(2*A*B*a*c*e - (B^2*a*c - A^2*c^2)* \\
& d + (B^2*a^2 - A^2*a*c)*f - (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3* \\
& c*f^2)*\sqrt{-(4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B* \\
& c^2)*d*e + (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (\\
& A*B^3*a^2 - A^3*B*a*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - \\
& 4*a^4*c^2*d*f^3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2
\end{aligned}$$

$$\begin{aligned} & *c^4*d^3 + a^3*c^3*d*e^2)*f))) / (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a \\ & ^3*c*f^2)) + (2*(B^2*a*c^3 + A^2*c^4)*d^3 + 2*(B^2*a^2*c^2 + A^2*a*c^3)*d*e \\ & ^2 - 4*(B^2*a^2*c^2 + A^2*a*c^3)*d^2*f + 2*(B^2*a^3*c + A^2*a^2*c^2)*d*f^2 \\ & + ((B^2*a*c^3 + A^2*c^4)*d^2*e + (B^2*a^2*c^2 + A^2*a*c^3)*e^3 - 2*(B^2*a^2 \\ & *c^2 + A^2*a*c^3)*d*e*f + (B^2*a^3*c + A^2*a^2*c^2)*e*f^2)*x)*\sqrt{-(4*A^2* \\ & B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 \\ & - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a \\ & c)*e)*f) / (a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a \\ & ^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d \\ & *e^2)*f)) / x) + 1/4*\sqrt{-(2*A*B*a*c*e - (B^2*a*c - A^2*c^2)*d + (B^2*a^2 - \\ & A^2*a*c)*f - (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2)*\sqrt{-(\\ & 4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^ \\ & 4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^ \\ & 3*B*a*c)*e)*f) / (a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f \\ & ^3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3 \\ & *c^3*d*e^2)*f)) / (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2))*\log \\ & (- (2*(A*B^3*a*c + A^3*B*c^2)*d*e + (B^4*a^2 - A^4*c^2)*e^2 - 2*(A*B^3*a^2 + \\ & A^3*B*a*c)*e*f - 2*(2*(A*B^3*a^2 + A^3*B*a*c)*f^2 - (2*(A*B^3*a*c + A^3*B \\ & c^2)*d + (B^4*a^2 - A^4*c^2)*e)*f)*x - 2*(2*A^2*B*c^3*d^2 + 2*A^2*B*a^2*c*f \\ & ^2 + (3*A*B^2*a*c^2 - A^3*c^3)*d*e + (B^3*a^2*c - A^2*B*a*c^2)*e^2 - (4*A^2 \\ & *B*a*c^2*d + (3*A*B^2*a^2*c - A^3*a*c^2)*e)*f + (B*a*c^4*d^3 - A*a*c^4*d^2* \\ & e + B*a^2*c^3*d*e^2 - A*a^2*c^3*e^3 - B*a^4*c*f^3 + (3*B*a^3*c^2*d - A*a^3* \\ & c^2*e)*f^2 - (3*B*a^2*c^3*d^2 - 2*A*a^2*c^3*d*e + B*a^3*c^2*e^2)*f)*\sqrt{-(\\ & 4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^ \\ & 4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^ \\ & 3*B*a*c)*e)*f) / (a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f \\ & ^3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3 \\ & *c^3*d*e^2)*f)))*\sqrt{f*x^2 + e*x + d)*\sqrt{-(2*A*B*a*c*e - (B^2*a*c - A^2* \\ & c^2)*d + (B^2*a^2 - A^2*a*c)*f - (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + \\ & a^3*c*f^2)*\sqrt{-(4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A \\ & ^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c* \\ & d + (A*B^3*a^2 - A^3*B*a*c)*e)*f) / (a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3* \\ & e^4 - 4*a^4*c^2*d*f^3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4 \\ & *(a^2*c^4*d^3 + a^3*c^3*d*e^2)*f)) / (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f \\ & + a^3*c*f^2)) + (2*(B^2*a*c^3 + A^2*c^4)*d^3 + 2*(B^2*a^2*c^2 + A^2*a*c^3) \\ &)*d*e^2 - 4*(B^2*a^2*c^2 + A^2*a*c^3)*d^2*f + 2*(B^2*a^3*c + A^2*a^2*c^2)*d \\ & *f^2 + ((B^2*a*c^3 + A^2*c^4)*d^2*e + (B^2*a^2*c^2 + A^2*a*c^3)*e^3 - 2*(B^ \\ & 2*a^2*c^2 + A^2*a*c^3)*d*e*f + (B^2*a^3*c + A^2*a^2*c^2)*e*f^2)*x)*\sqrt{-(4 \\ & *A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4 \\ & *a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3 \\ & *B*a*c)*e)*f) / (a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^ \\ & 3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3* \\ & c^3*d*e^2)*f)) / x) \end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
i,const vecteur & l) Error: Bad Argument ValueConj Error: Bad Argument Typ
e

maple [A] time = 0.04, size = 784, normalized size = 1.01

$$\frac{A \ln \left(\frac{-\frac{2(af-cd-\sqrt{-ac}e)}{c} + \frac{(ce+2\sqrt{-ac}f)\left(x-\frac{\sqrt{-ac}}{c}\right)}{c} + 2\sqrt{\frac{-af-cd-\sqrt{-ac}e}{c}} \sqrt{\left(x-\frac{\sqrt{-ac}}{c}\right)^2 f + \frac{(ce+2\sqrt{-ac}f)\left(x-\frac{\sqrt{-ac}}{c}\right) - af-cd-\sqrt{-ac}e}{c}}}{x-\frac{\sqrt{-ac}}{c}} \right)}{2\sqrt{-ac} \sqrt{\frac{-af-cd-\sqrt{-ac}e}{c}}} + A \ln \left(\frac{-\frac{2(af-cd+\sqrt{-ac}e)}{c}}{c} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x+A)/(c*x^2+a)/(f*x^2+e*x+d)^(1/2),x)

[Out]
$$\begin{aligned} & -1/2/(-a*c)^{(1/2)}/(-(-e*(-a*c)^{(1/2)}+a*f-c*d)/c)^{(1/2)}*\ln((-2*(-e*(-a*c)^{(1/2)}+a*f-c*d)/c+(2*f*(-a*c)^{(1/2)}+c*e)/c*(x-(-a*c)^{(1/2)}/c)+2*(-(-e*(-a*c)^{(1/2)}+a*f-c*d)/c)^{(1/2)}*((x-(-a*c)^{(1/2)}/c)^2*f+(2*f*(-a*c)^{(1/2)}+c*e)/c*(x-(-a*c)^{(1/2)}/c)-(-e*(-a*c)^{(1/2)}+a*f-c*d)/c)^{(1/2)})/(x-(-a*c)^{(1/2)}/c))*A-1/2/c/(-(-e*(-a*c)^{(1/2)}+a*f-c*d)/c)^{(1/2)}*\ln((-2*(-e*(-a*c)^{(1/2)}+a*f-c*d)/c+(2*f*(-a*c)^{(1/2)}+c*e)/c*(x-(-a*c)^{(1/2)}/c)+2*(-(-e*(-a*c)^{(1/2)}+a*f-c*d)/c)^{(1/2)}*((x-(-a*c)^{(1/2)}/c)^2*f+(2*f*(-a*c)^{(1/2)}+c*e)/c*(x-(-a*c)^{(1/2)}/c)-(-e*(-a*c)^{(1/2)}+a*f-c*d)/c)^{(1/2)})/(x-(-a*c)^{(1/2)}/c))*B+1/2/(-a*c)^{(1/2)}/(-(-e*(-a*c)^{(1/2)}+a*f-c*d)/c)^{(1/2)}*\ln((-2*(e*(-a*c)^{(1/2)}+a*f-c*d)/c+1/c*(-2*f*(-a*c)^{(1/2)}+c*e)*(x+(-a*c)^{(1/2)}/c)+2*(-(e*(-a*c)^{(1/2)}+a*f-c*d)/c)^{(1/2)}*((x+(-a*c)^{(1/2)}/c)^2*f+1/c*(-2*f*(-a*c)^{(1/2)}+c*e)*(x+(-a*c)^{(1/2)}/c)-(e*(-a*c)^{(1/2)}+a*f-c*d)/c)^{(1/2)})/(x+(-a*c)^{(1/2)}/c))*A-1/2/c/(-(-e*(-a*c)^{(1/2)}+a*f-c*d)/c)^{(1/2)}*\ln((-2*(e*(-a*c)^{(1/2)}+a*f-c*d)/c+1/c*(-2*f*(-a*c)^{(1/2)}+c*e)*(x+(-a*c)^{(1/2)}/c)+2*(-(e*(-a*c)^{(1/2)}+a*f-c*d)/c)^{(1/2)}*((x+(-a*c)^{(1/2)}/c)^2*f+1/c*(-2*f*(-a*c)^{(1/2)}+c*e)*(x+(-a*c)^{(1/2)}/c)-(e*(-a*c)^{(1/2)}+a*f-c*d)/c)^{(1/2)})/(x+(-a*c)^{(1/2)}/c))*B \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx + A}{(cx^2 + a)\sqrt{fx^2 + ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")

[Out] integrate((B*x + A)/((c*x^2 + a)*sqrt(f*x^2 + e*x + d)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + Bx}{(cx^2 + a)\sqrt{fx^2 + ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x)/((a + c*x^2)*(d + e*x + f*x^2)^(1/2)),x)

[Out] int((A + B*x)/((a + c*x^2)*(d + e*x + f*x^2)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx}{(a + cx^2)\sqrt{d + ex + fx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x**2+a)/(f*x**2+e*x+d)**(1/2),x)

[Out] Integral((A + B*x)/((a + c*x**2)*sqrt(d + e*x + f*x**2)), x)

$$3.23 \quad \int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+fx^2}} dx$$

Optimal. Leaf size=302

$$\frac{\left(-B\sqrt{b^2-4ac}-2Ac+bB\right)\tanh^{-1}\left(\frac{2cd-fx(b-\sqrt{b^2-4ac})}{\sqrt{2}\sqrt{d+fx^2}\sqrt{bf(b-\sqrt{b^2-4ac})-2acf+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{bf(b-\sqrt{b^2-4ac})-2acf+2c^2d}} + \frac{\left(2Ac-B(\sqrt{b^2-4ac}+b)\right)\tanh^{-1}\left(\frac{2cd-fx(b+\sqrt{b^2-4ac})}{\sqrt{2}\sqrt{d+fx^2}\sqrt{bf(b+\sqrt{b^2-4ac})-2acf+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{bf(b+\sqrt{b^2-4ac})-2acf+2c^2d}}$$

[Out] $1/2*\operatorname{arctanh}(1/2*(2*c*d-f*x*(b-(-4*a*c+b^2)^{(1/2)}))*2^{(1/2)}/(f*x^2+d)^{(1/2)}/(2*c^2*d-2*a*c*f+b*f*(b-(-4*a*c+b^2)^{(1/2)}))^((1/2)))*(b*B-2*A*c-B*(-4*a*c+b^2)^{(1/2}))*2^{(1/2)}/(-4*a*c+b^2)^{(1/2)}/(2*c^2*d-2*a*c*f+b*f*(b-(-4*a*c+b^2)^{(1/2)}))^((1/2)))+1/2*\operatorname{arctanh}(1/2*(2*c*d-f*x*(b+(-4*a*c+b^2)^{(1/2)}))*2^{(1/2)}/(f*x^2+d)^{(1/2)}/(2*c^2*d-2*a*c*f+b*f*(b+(-4*a*c+b^2)^{(1/2)}))^((1/2)))*(2*A*c-B*(b+(-4*a*c+b^2)^{(1/2}))*2^{(1/2)}/(-4*a*c+b^2)^{(1/2)}/(2*c^2*d-2*a*c*f+b*f*(b+(-4*a*c+b^2)^{(1/2)}))^((1/2))$

Rubi [A] time = 0.84, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1034, 725, 206}

$$\frac{\left(-B\sqrt{b^2-4ac}-2Ac+bB\right)\tanh^{-1}\left(\frac{2cd-fx(b-\sqrt{b^2-4ac})}{\sqrt{2}\sqrt{d+fx^2}\sqrt{bf(b-\sqrt{b^2-4ac})-2acf+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{bf(b-\sqrt{b^2-4ac})-2acf+2c^2d}} + \frac{\left(2Ac-B(\sqrt{b^2-4ac}+b)\right)\tanh^{-1}\left(\frac{2cd-fx(b+\sqrt{b^2-4ac})}{\sqrt{2}\sqrt{d+fx^2}\sqrt{bf(b+\sqrt{b^2-4ac})-2acf+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{bf(b+\sqrt{b^2-4ac})-2acf+2c^2d}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]), x]

[Out] $((b*B - 2*A*c - B*\operatorname{Sqrt}[b^2 - 4*a*c])*ArcTanh[(2*c*d - (b - \operatorname{Sqrt}[b^2 - 4*a*c])*f*x)/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[2*c^2*d - 2*a*c*f + b*(b - \operatorname{Sqrt}[b^2 - 4*a*c])*f]*\operatorname{Sqrt}[d + f*x^2])]) / (\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b^2 - 4*a*c]*\operatorname{Sqrt}[2*c^2*d - 2*a*c*f + b*(b - \operatorname{Sqrt}[b^2 - 4*a*c])*f]) + ((2*A*c - B*(b + \operatorname{Sqrt}[b^2 - 4*a*c]))*ArcTanh[(2*c*d - (b + \operatorname{Sqrt}[b^2 - 4*a*c])*f*x)/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[2*c^2*d - 2*a*c*f + b*(b + \operatorname{Sqrt}[b^2 - 4*a*c])*f]*\operatorname{Sqrt}[d + f*x^2])]) / (\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b^2 - 4*a*c]*\operatorname{Sqrt}[2*c^2*d - 2*a*c*f + b*(b + \operatorname{Sqrt}[b^2 - 4*a*c])*f])$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && Gt

Q[a, 0] || LtQ[b, 0])

Rule 725

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (c_)*(x_)^2]), x_Symbol] := -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]

Rule 1034

Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (f_)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rubi steps

$$\int \frac{A + Bx}{(a + bx + cx^2)\sqrt{d + fx^2}} dx = \frac{(2Ac - B(b - \sqrt{b^2 - 4ac})) \int \frac{1}{(b - \sqrt{b^2 - 4ac} + 2cx)\sqrt{d + fx^2}} dx}{\sqrt{b^2 - 4ac}} - \frac{(2Ac - B(b + \sqrt{b^2 - 4ac})) \int \frac{1}{(b + \sqrt{b^2 - 4ac} + 2cx)\sqrt{d + fx^2}} dx}{\sqrt{b^2 - 4ac}}$$

$$= \frac{(2Ac - B(b - \sqrt{b^2 - 4ac})) \operatorname{Subst}\left(\int \frac{1}{4c^2d + (b - \sqrt{b^2 - 4ac})^2 f - x^2} dx, x, \frac{2cd - (b - \sqrt{b^2 - 4ac})fx}{\sqrt{d + fx^2}}\right)}{\sqrt{b^2 - 4ac}} - \frac{(2Ac - B(b + \sqrt{b^2 - 4ac})) \operatorname{Subst}\left(\int \frac{1}{4c^2d + (b + \sqrt{b^2 - 4ac})^2 f - x^2} dx, x, \frac{2cd - (b + \sqrt{b^2 - 4ac})fx}{\sqrt{d + fx^2}}\right)}{\sqrt{b^2 - 4ac}}$$

$$= \frac{(bB - 2Ac - B\sqrt{b^2 - 4ac}) \tanh^{-1}\left(\frac{2cd - (b - \sqrt{b^2 - 4ac})fx}{\sqrt{2}\sqrt{2c^2d - 2acf + b(b - \sqrt{b^2 - 4ac})}f\sqrt{d + fx^2}}\right)}{\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{2c^2d - 2acf + b(b - \sqrt{b^2 - 4ac})}f} + \frac{(2Ac - B(b + \sqrt{b^2 - 4ac})) \tanh^{-1}\left(\frac{2cd - (b + \sqrt{b^2 - 4ac})fx}{\sqrt{2}\sqrt{2c^2d - 2acf + b(b + \sqrt{b^2 - 4ac})}f\sqrt{d + fx^2}}\right)}{\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{2c^2d - 2acf + b(b + \sqrt{b^2 - 4ac})}f}$$

Mathematica [A] time = 0.43, size = 283, normalized size = 0.94

$$\sqrt{2} \frac{\left((B\sqrt{b^2 - 4ac} + 2Ac - bB) \tanh^{-1}\left(\frac{fx(\sqrt{b^2 - 4ac} - b) + 2cd}{\sqrt{d + fx^2}\sqrt{2bf(b - \sqrt{b^2 - 4ac}) - 4acf + 4c^2d}}\right) - (B\sqrt{b^2 - 4ac} - 2Ac + bB) \tanh^{-1}\left(\frac{2cd - fx(\sqrt{b^2 - 4ac} + b)}{\sqrt{d + fx^2}\sqrt{2bf(\sqrt{b^2 - 4ac} + b) - 4acf + 4c^2d}}\right) \right)}{2\sqrt{bf(b - \sqrt{b^2 - 4ac}) - 2acf + 2c^2d} - 2\sqrt{bf(\sqrt{b^2 - 4ac} + b) - 2acf + 2c^2d}}$$

$$\frac{\sqrt{b^2 - 4ac}}{\sqrt{b^2 - 4ac}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]),x]

[Out] (Sqrt[2]*(-1/2*((-b*B) + 2*A*c + B*Sqrt[b^2 - 4*a*c])*ArcTanh[(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*f*x)/(Sqrt[4*c^2*d - 4*a*c*f + 2*b*(b - Sqrt[b^2 - 4*a*c])*f])*Sqrt[d + f*x^2]])/Sqrt[2*c^2*d - 2*a*c*f + b*(b - Sqrt[b^2 - 4*a*c])*f] - ((b*B - 2*A*c + B*Sqrt[b^2 - 4*a*c])*ArcTanh[(2*c*d - (b + Sqrt[b^2 - 4*a*c])*f*x)/(Sqrt[4*c^2*d - 4*a*c*f + 2*b*(b + Sqrt[b^2 - 4*a*c])*f])*Sqrt[d + f*x^2]])/(2*Sqrt[2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f]))/Sqrt[b^2 - 4*a*c]

fricas [B] time = 124.91, size = 8977, normalized size = 29.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d)^(1/2),x, algorithm="fricas")

[Out] 1/4*sqrt(2)*sqrt(((B^2*b^2 + 2*A^2*c^2 - 2*(B^2*a + A*B*b)*c)*d + (2*B^2*a^2 - 2*A*B*a*b + A^2*b^2 - 2*A^2*a*c)*f + ((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2))*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))/((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2))*log(((2*(B^4*a*b^2 - A*B^3*b^3 - 2*A^3*B*b*c^2 - (2*A*B^3*a*b - 3*A^2*B^2*b^2)*c)*d^2 + 2*(2*A*B^3*a^2*b - 3*A^2*B^2*a*b^2 + A^3*B*b^3 + (2*A^3*B*a*b - A^4*b^2)*c)*d*f + sqrt(2)*((B^3*b^4 - 8*A^2*B*a*c^3 + 2*(6*A*B^2*a*b + A^2*B*b^2)*c^2 - (4*B^3*a*b^2 + 3*A*B^2*b^3)*c)*d^2 + (3*A*B^2*a*b^3 - A^2*B*b^4 + 4*(4*A^2*B*a^2 - A^3*a*b)*c^2 - (12*A*B^2*a^2*b - A^3*b^3)*c)*d*f + (2*A^2*B*a^2*b^2 - A^3*a*b^3 - 4*(2*A^2*B*a^3 - A^3*a^2*b)*c)*f^2 - ((B*b^4*c^2 + 4*(2*B*a^2 + A*a*b)*c^4 - (6*B*a*b^2 + A*b^3)*c^3)*d^3 + (B*b^6 - 4*(6*B*a^3 + A*a^2*b)*c^3 + (22*B*a^2*b^2 + 5*A*a*b^3)*c^2 - (8*B*a*b^4 + A*b^5)*c)*d^2*f + (3*B*a^2*b^4 - A*a*b^5 + 4*(6*B*a^4 - A*a^3*b)*c^2 - (18*B*a^3*b^2 - 5*A*a^2*b^3)*c)*d*f^2 + (2*B*a^4*b^2 - A*a^3*b^3 - 4*(2*B*a^5 - A*a^4*b)*c)*f^3))*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))*sqrt(f*x^2 + d)*sqrt(((B^2*b^2 + 2*A^2*c^2 - 2*(B^2*a + A*B*b)*c)*d + (2*B^2*a^2 - 2*A*B*a*b + A^2*b^2 - 2*A^2*a*c)*f + ((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2)))/((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2))

$$\begin{aligned}
& B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - \\
& 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2) \\
& *f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3* \\
& f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - \\
& 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4))/((b^2*c^2 - 4*a \\
& *c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2)) - \\
& 4*((B^4*a^2*b - A*B^3*a*b^2 - 2*A^3*B*a*c^2 - (2*A*B^3*a^2 - 3*A^2*B^2*a*b) \\
&)*c)*d*f + (2*A*B^3*a^3 - 3*A^2*B^2*a^2*b + A^3*B*a*b^2 + (2*A^3*B*a^2 - A^ \\
& 4*a*b)*c)*f^2)*x + 2*((4*A^2*a*c^4 + (4*B^2*a^2 - 4*A*B*a*b - A^2*b^2)*c^3 \\
& - (B^2*a*b^2 - A*B*b^3)*c^2)*d^3 - (B^2*a*b^4 - A*B*b^5 + 8*A^2*a^2*c^3 + 2 \\
& *(4*B^2*a^3 - 4*A*B*a^2*b - 3*A^2*a*b^2)*c^2 - (6*B^2*a^2*b^2 - 6*A*B*a*b^3 \\
& - A^2*b^4)*c)*d^2*f - (B^2*a^3*b^2 - A*B*a^2*b^3 - 4*A^2*a^3*c^2 - (4*B^2* \\
& a^4 - 4*A*B*a^3*b - A^2*a^2*b^2)*c)*d*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4 \\
& *A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b) \\
&)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5) \\
&)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22 \\
& *a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)* \\
& d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4))/x) + 1/4*sqrt(2)*sqrt(((B^2*b^2 + 2*A^2* \\
& c^2 - 2*(B^2*a + A*B*b)*c)*d + (2*B^2*a^2 - 2*A*B*a*b + A^2*b^2 - 2*A^2*a*c) \\
&)*f - ((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b \\
& ^2 - 4*a^3*c)*f^2))*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2 \\
& *A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^ \\
& 2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a \\
& *b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^ \\
& 3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5 \\
& *c)*f^4))/((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (\\
& a^2*b^2 - 4*a^3*c)*f^2))*log((2*(B^4*a*b^2 - A*B^3*b^3 - 2*A^3*B*b*c^2 - (2 \\
& *A*B^3*a*b - 3*A^2*B^2*b^2)*c)*d^2 + 2*(2*A*B^3*a^2*b - 3*A^2*B^2*a*b^2 + A \\
& ^3*B*b^3 + (2*A^3*B*a*b - A^4*b^2)*c)*d*f + sqrt(2)*((B^3*b^4 - 8*A^2*B*a*c \\
& ^3 + 2*(6*A*B^2*a*b + A^2*B*b^2)*c^2 - (4*B^3*a*b^2 + 3*A*B^2*b^3)*c)*d^2 + \\
& (3*A*B^2*a*b^3 - A^2*B*b^4 + 4*(4*A^2*B*a^2 - A^3*a*b)*c^2 - (12*A*B^2*a^2 \\
& *b - A^3*b^3)*c)*d*f + (2*A^2*B*a^2*b^2 - A^3*a*b^3 - 4*(2*A^2*B*a^3 - A^3* \\
& a^2*b)*c)*f^2 + ((B*b^4*c^2 + 4*(2*B*a^2 + A*a*b)*c^4 - (6*B*a*b^2 + A*b^3) \\
&)*c^3)*d^3 + (B*b^6 - 4*(6*B*a^3 + A*a^2*b)*c^3 + (22*B*a^2*b^2 + 5*A*a*b^3) \\
&)*c^2 - (8*B*a*b^4 + A*b^5)*c)*d^2*f + (3*B*a^2*b^4 - A*a*b^5 + 4*(6*B*a^4 - \\
& A*a^3*b)*c^2 - (18*B*a^3*b^2 - 5*A*a^2*b^3)*c)*d*f^2 + (2*B*a^4*b^2 - A*a^ \\
& 3*b^3 - 4*(2*B*a^5 - A*a^4*b)*c)*f^3)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2* \\
& B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)* \\
& d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 \\
& + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2* \\
& b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 \\
& + (a^4*b^2 - 4*a^5*c)*f^4))*sqrt(f*x^2 + d)*sqrt(((B^2*b^2 + 2*A^2*c^2 - \\
& 2*(B^2*a + A*B*b)*c)*d + (2*B^2*a^2 - 2*A*B*a*b + A^2*b^2 - 2*A^2*a*c)*f - \\
& ((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4 \\
& *a^3*c)*f^2))*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3
\end{aligned}$$

$$\begin{aligned}
& *a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4* \\
& A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c \\
& ^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2 \\
& *f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4 \\
& 4)))/((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^ \\
& 2 - 4*a^3*c)*f^2)) - 4*((B^4*a^2*b - A*B^3*a*b^2 - 2*A^3*B*a*c^2 - (2*A*B^3 \\
& *a^2 - 3*A^2*B^2*a*b)*c)*d*f + (2*A*B^3*a^3 - 3*A^2*B^2*a^2*b + A^3*B*a*b^2 \\
& + (2*A^3*B*a^2 - A^4*a*b)*c)*f^2))*x - 2*((4*A^2*a*c^4 + (4*B^2*a^2 - 4*A*B \\
& *a*b - A^2*b^2)*c^3 - (B^2*a*b^2 - A*B*b^3)*c^2)*d^3 - (B^2*a*b^4 - A*B*b^5 \\
& + 8*A^2*a^2*c^3 + 2*(4*B^2*a^3 - 4*A*B*a^2*b - 3*A^2*a*b^2)*c^2 - (6*B^2*a \\
& ^2*b^2 - 6*A*B*a*b^3 - A^2*b^4)*c)*d^2*f - (B^2*a^3*b^2 - A*B*a^2*b^3 - 4*A \\
& ^2*a^3*c^2 - (4*B^2*a^4 - 4*A*B*a^3*b - A^2*a^2*b^2)*c)*d*f^2)*sqrt(((B^4*b \\
& ^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2 \\
& *A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2) \\
& /((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (\\
& b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3 \\
& *b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4))/x) - 1/4*sqrt(2)*sqrt \\
& t(((B^2*b^2 + 2*A^2*c^2 - 2*(B^2*a + A*B*b)*c)*d + (2*B^2*a^2 - 2*A*B*a*b + \\
& A^2*b^2 - 2*A^2*a*c)*f - ((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a \\
& ^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2))*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2 \\
& *B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c) \\
& *d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^ \\
& 4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2 \\
& *b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^ \\
& 3 + (a^4*b^2 - 4*a^5*c)*f^4)))/((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c \\
& + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2))*log((2*(B^4*a*b^2 - A*B^3*b^3 \\
& - 2*A^3*B*b*c^2 - (2*A*B^3*a*b - 3*A^2*B^2*b^2)*c)*d^2 + 2*(2*A*B^3*a^2*b - \\
& 3*A^2*B^2*a*b^2 + A^3*B*b^3 + (2*A^3*B*a*b - A^4*b^2)*c)*d*f - sqrt(2)*((B \\
& ^3*b^4 - 8*A^2*B*a*c^3 + 2*(6*A*B^2*a*b + A^2*B*b^2)*c^2 - (4*B^3*a*b^2 + 3 \\
& *A*B^2*b^3)*c)*d^2 + (3*A*B^2*a*b^3 - A^2*B*b^4 + 4*(4*A^2*B*a^2 - A^3*a*b) \\
& *c^2 - (12*A*B^2*a^2*b - A^3*b^3)*c)*d*f + (2*A^2*B*a^2*b^2 - A^3*a*b^3 - 4 \\
& *(2*A^2*B*a^3 - A^3*a^2*b)*c)*f^2 + ((B*b^4*c^2 + 4*(2*B*a^2 + A*a*b)*c^4 - \\
& (6*B*a*b^2 + A*b^3)*c^3)*d^3 + (B*b^6 - 4*(6*B*a^3 + A*a^2*b)*c^3 + (22*B \\
& a^2*b^2 + 5*A*a*b^3)*c^2 - (8*B*a*b^4 + A*b^5)*c)*d^2*f + (3*B*a^2*b^4 - A \\
& a*b^5 + 4*(6*B*a^4 - A*a^3*b)*c^2 - (18*B*a^3*b^2 - 5*A*a^2*b^3)*c)*d*f^2 + \\
& (2*B*a^4*b^2 - A*a^3*b^3 - 4*(2*B*a^5 - A*a^4*b)*c)*f^3)*sqrt(((B^4*b^2 - \\
& 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2* \\
& B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^ \\
& 2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - \\
& 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2* \\
& c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4))*sqrt(f*x^2 + d)*sqrt(((B^ \\
& 2*b^2 + 2*A^2*c^2 - 2*(B^2*a + A*B*b)*c)*d + (2*B^2*a^2 - 2*A*B*a*b + A^2*b \\
& ^2 - 2*A^2*a*c)*f - ((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2 \\
&)*d*f + (a^2*b^2 - 4*a^3*c)*f^2))*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c \\
& ^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f +
\end{aligned}$$

$$\frac{(4A^2B^2a^2 - 4A^3B^2ab + A^4b^2) f^2}{(b^2c^4 - 4a^2c^5) d^4 + 2(b^4c^2 - 6a^2b^2c^3 + 8a^4c^4) d^3 f + (b^6 - 8a^2b^4c + 22a^2b^2c^2 - 24a^3c^3) d^2 f^2 + 2(a^2b^4 - 6a^3b^2c + 8a^4c^2) d f^3 + (a^4b^2 - 4a^5c) f^4} \cdot \frac{1}{(b^2c^2 - 4a^2c^3) d^2 + (b^4 - 6a^2b^2c + 8a^2c^2) d f + (a^2b^2 - 4a^3c) f^2} - 4((B^4a^2b - AB^3a^2b^2 - 2A^3B^2a^2c - (2AB^3a^2 - 3A^2B^2ab) c) d f + (2AB^3a^3 - 3A^2B^2a^2b + A^3B^2ab^2 + (2A^3B^2a^2 - A^4ab) c) f^2) x - 2((4A^2a^2c^4 + (4B^2a^2 - 4AB^2ab - A^2b^2) c^3 - (B^2a^2b^2 - AB^2b^3) c^2) d^3 - (B^2a^2b^4 - AB^2b^5 + 8A^2a^2c^3 + 2(4B^2a^3 - 4AB^2a^2b - 3A^2a^2b^2) c^2 - (6B^2a^2b^2 - 6AB^2ab^3 - A^2b^4) c) d^2 f - (B^2a^3b^2 - AB^2a^2b^3 - 4A^2a^3c^2 - (4B^2a^4 - 4AB^2a^3b - A^2a^2b^2) c) d f^2) \sqrt{(B^4b^2 - 4AB^3b^2c + 4A^2B^2c^2) d^2 + 2(2AB^3ab - A^2B^2b^2 - 2(2A^2B^2a - A^3B^2b) c) d f + (4A^2B^2a^2 - 4A^3B^2ab + A^4b^2) f^2} / ((b^2c^4 - 4a^2c^5) d^4 + 2(b^4c^2 - 6a^2b^2c^3 + 8a^4c^4) d^3 f + (b^6 - 8a^2b^4c + 22a^2b^2c^2 - 24a^3c^3) d^2 f^2 + 2(a^2b^4 - 6a^3b^2c + 8a^4c^2) d f^3 + (a^4b^2 - 4a^5c) f^4) / x$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INP
 UT:sage2:=int(sage0,x); OUTPUT: Unable to divide, perhaps due to rounding error
 %poly1{poly1{poly1{-4, [3,2,0]}+{16, [1,3,1]}+{4, [4,2,0]}+{-24, [2,3,1]}+{32, [0,4,2]}}: [1,0,{-1, [2,0,0]}+{4, [0,1,1]}], [2,1,0,0]}+{poly1{-4, [5,0,0]}+{24, [3,1,1]}+{-32, [1,2,2]}+{4, [6,0,0]}+{-32, [4,1,1]}+{72, [2,2,2]}+{-32, [0,3,3]}}: [1,0,{-1, [2,0,0]}+{4, [0,1,1]}], [2,0,0,1]}+{poly1{poly1{poly1{8, [2,3,0]}+{-32, [0,4,1]}+{-8, [3,3,0]}+{32, [1,4,1]}}: [1,0,{-1, [2,0,0]}+{4, [0,1,1]}], [1,1,1,0]}+{poly1{poly1{8, [4,1,0]}+{-40, [2,2,1]}+{32, [0,3,2]}+{-8, [5,1,0]}+{56, [3,2,1]}+{-96, [1,3,2]}}: [1,0,{-1, [2,0,0]}+{4, [0,1,1]}], [1,0,1,1]}+{poly1{poly1{8, [2,4,0]}+{-32, [0,5,1]}}, [0,1,2,0]}+{poly1{poly1{-4, [3,2,0]}+{16, [1,3,1]}+{4, [4,2,0]}+{-24, [2,3,1]}+{32, [0,4,2]}}: [1,0,{-1, [2,0,0]}+{4, [0,1,1]}], [0,0,2,1]} / %poly1{poly1{-4, [3,2,0]}+{16, [1,3,1]}+{4, [4,2,0]}+{-24, [2,3,1]}+{32, [0,4,2]}}: [1,0,{-1, [2,0,0]}+{4, [0,1,1]}], [0,1,0,0]}+{poly1{poly1{-4, [5,0,0]}+{24, [3,1,1]}+{-32, [1,2,2]}+{4, [6,0,0]}+{-32, [4,1,1]}+{72, [2,2,2]}+{-32, [0,3,3]}}: [1,0,{-1, [2,0,0]}+{4, [0,1,1]}], [0,0,0,1]} Error: Bad Argument Value Unable to divide, pe

rhaps due to rounding error
 $\{1, [4, 0, 0]\} + \{-6, [2, 1, 1]\} + \{8, [0, 2, 2]\} : [1, 0, \{-1, [2, 0, 0]\} + \{4, [0, 1, 1]\}] / \{4, [0, 0, 2]\}, [2, 1, 0, 0]\} + \{1, [2, 0, 0]\} + \{-4, [0, 1, 1]\} / 2, [2, 0, 0, 1]\} + \{-1, [4, 0, 0]\} + \{5, [2, 1, 1]\} + \{-4, [0, 2, 2]\}, \{-1, [5, 0, 0]\} + \{7, [3, 1, 1]\} + \{-12, [1, 2, 2]\} : [1, 0, \{-1, [2, 0, 0]\} + \{4, [0, 1, 1]\}] / \{2, [0, 0, 3]\}, [1, 1, 1, 0]\} + \{-1, [2, 0, 0]\} + \{4, [0, 1, 1]\}, \{-1, [3, 0, 0]\} + \{4, [1, 1, 1]\} : [1, 0, \{-1, [2, 0, 0]\} + \{4, [0, 1, 1]\}] / \{2, [0, 0, 1]\}, [1, 0, 1, 1]\} + \{1, [5, 0, 0]\} + \{-6, [3, 1, 1]\} + \{8, [1, 2, 2]\}, \{1, [6, 0, 0]\} + \{-8, [4, 1, 1]\} + \{18, [2, 2, 2]\} + \{-8, [0, 3, 3]\} : [1, 0, \{-1, [2, 0, 0]\} + \{4, [0, 1, 1]\}] / \{4, [0, 0, 4]\}, [0, 1, 2, 0]\} + \{1, [3, 0, 0]\} + \{-4, [1, 1, 1]\}, \{1, [4, 0, 0]\} + \{-6, [2, 1, 1]\} + \{8, [0, 2, 2]\} : [1, 0, \{-1, [2, 0, 0]\} + \{4, [0, 1, 1]\}] / \{4, [0, 0, 2]\}, [0, 0, 2, 1]\} / \{1, [3, 0, 0]\} + \{-4, [1, 1, 1]\}, \{1, [4, 0, 0]\} + \{-6, [2, 1, 1]\} + \{8, [0, 2, 2]\} : [1, 0, \{-1, [2, 0, 0]\} + \{4, [0, 1, 1]\}] / \{4, [0, 0, 4]\}, [0, 1, 0, 0]\} + \{1, [2, 0, 0]\} + \{-4, [0, 1, 1]\} / \{2, [0, 0, 2]\}, [0, 0, 0, 1]\} Error: Bad Argument Value$

maple [B] time = 0.03, size = 1771, normalized size = 5.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d)^{(1/2)}, x)$

[Out] $2/(-4*a*c+b^2)^{(1/2)}/(-2*(-(-4*a*c+b^2)^{(1/2)}*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2-f*(b+(-4*a*c+b^2)^{(1/2)})/c*(x+1/2*(b+(-4*a*c+b^2)^{(1/2)})/c)+1/2*(-2*(-(-4*a*c+b^2)^{(1/2)}*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^{(1/2)}*(4*(x+1/2*(b+(-4*a*c+b^2)^{(1/2)})/c)^2*f-4*f*(b+(-4*a*c+b^2)^{(1/2)})/c*(x+1/2*(b+(-4*a*c+b^2)^{(1/2)})/c)-2*(-(-4*a*c+b^2)^{(1/2)}*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^{(1/2)})/(x+1/2*(b+(-4*a*c+b^2)^{(1/2)})/c)*A-1/c/(-2*(-(-4*a*c+b^2)^{(1/2)}*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^{(1/2)}*ln((-(-4*a*c+b^2)^{(1/2)}*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2-f*(b+(-4*a*c+b^2)^{(1/2)})/c*(x+1/2*(b+(-4*a*c+b^2)^{(1/2)})/c)+1/2*(-2*(-(-4*a*c+b^2)^{(1/2)}*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^{(1/2)}*(4*(x+1/2*(b+(-4*a*c+b^2)^{(1/2)})/c)^2*f-4*f*(b+(-4*a*c+b^2)^{(1/2)})/c*(x+1/2*(b+(-4*a*c+b^2)^{(1/2)})/c)-2*(-(-4*a*c+b^2)^{(1/2)}*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^{(1/2)})/(x+1/2*(b+(-4*a*c+b^2)^{(1/2)})/c)*B-1/(-4*a*c+b^2)^{(1/2)}/c/(-2*(-(-4*a*c+b^2)^{(1/2)}*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^{(1/2)}*ln((-(-4*a*c+b^2)^{(1/2)}*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2-f*(b+(-4*a*c+b^2)^{(1/2)})/c*(x+1/2*(b+(-4*a*c+b^2)^{(1/2)})/c)+1/2*(-2*(-(-4*a*c+b^2)^{(1/2)}*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^{(1/2)}*(4*(x+1/2*(b+(-4*a*c+b^2)^{(1/2)})/c)^2*f-4*f*(b+(-4*a*c+b^2)^{(1/2)})/c*(x+1/2*(b+(-4*a*c+b^2)^{(1/2)})/c)-2*(-(-4*a*c+b^2)^{(1/2)}*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^{(1/2)})/(x+1/2*(b+(-4*a*c+b^2)^{(1/2)})/c)*B-2/(-4*a*c+b^2)^{(1/2)}/(-2*(-(-4*a*c+b^2)^{(1/2)}*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^{(1/2)}$

$$b^2)^{(1/2)} * b * f + 2 * a * c * f - b^2 * f - 2 * c^2 * d) / c^2)^{(1/2)} * \ln\left(\frac{(-((-4 * a * c + b^2)^{(1/2)} * b * f + 2 * a * c * f - b^2 * f - 2 * c^2 * d) / c^2 - f * (b - (-4 * a * c + b^2)^{(1/2)}) / c * (x - 1/2 * (-b + (-4 * a * c + b^2)^{(1/2)}) / c) + 1/2 * (-2 * ((-4 * a * c + b^2)^{(1/2)} * b * f + 2 * a * c * f - b^2 * f - 2 * c^2 * d) / c^2)^{(1/2)} * (4 * (x - 1/2 * (-b + (-4 * a * c + b^2)^{(1/2)}) / c)^2 * f - 4 * f * (b - (-4 * a * c + b^2)^{(1/2)}) / c * (x - 1/2 * (-b + (-4 * a * c + b^2)^{(1/2)}) / c) - 2 * ((-4 * a * c + b^2)^{(1/2)} * b * f + 2 * a * c * f - b^2 * f - 2 * c^2 * d) / c^2)^{(1/2)}) / (x - 1/2 * (-b + (-4 * a * c + b^2)^{(1/2)}) / c)} * A - 1 / c / (-2 * ((-4 * a * c + b^2)^{(1/2)} * b * f + 2 * a * c * f - b^2 * f - 2 * c^2 * d) / c^2)^{(1/2)} * \ln\left(\frac{(-((-4 * a * c + b^2)^{(1/2)} * b * f + 2 * a * c * f - b^2 * f - 2 * c^2 * d) / c^2 - f * (b - (-4 * a * c + b^2)^{(1/2)}) / c * (x - 1/2 * (-b + (-4 * a * c + b^2)^{(1/2)}) / c) + 1/2 * (-2 * ((-4 * a * c + b^2)^{(1/2)} * b * f + 2 * a * c * f - b^2 * f - 2 * c^2 * d) / c^2)^{(1/2)} * (4 * (x - 1/2 * (-b + (-4 * a * c + b^2)^{(1/2)}) / c)^2 * f - 4 * f * (b - (-4 * a * c + b^2)^{(1/2)}) / c * (x - 1/2 * (-b + (-4 * a * c + b^2)^{(1/2)}) / c) - 2 * ((-4 * a * c + b^2)^{(1/2)} * b * f + 2 * a * c * f - b^2 * f - 2 * c^2 * d) / c^2)^{(1/2)}) / (x - 1/2 * (-b + (-4 * a * c + b^2)^{(1/2)}) / c)} * B + 1 / (-4 * a * c + b^2)^{(1/2)} / c / (-2 * ((-4 * a * c + b^2)^{(1/2)} * b * f + 2 * a * c * f - b^2 * f - 2 * c^2 * d) / c^2)^{(1/2)} * \ln\left(\frac{(-((-4 * a * c + b^2)^{(1/2)} * b * f + 2 * a * c * f - b^2 * f - 2 * c^2 * d) / c^2 - f * (b - (-4 * a * c + b^2)^{(1/2)}) / c * (x - 1/2 * (-b + (-4 * a * c + b^2)^{(1/2)}) / c) + 1/2 * (-2 * ((-4 * a * c + b^2)^{(1/2)} * b * f + 2 * a * c * f - b^2 * f - 2 * c^2 * d) / c^2)^{(1/2)} * (4 * (x - 1/2 * (-b + (-4 * a * c + b^2)^{(1/2)}) / c)^2 * f - 4 * f * (b - (-4 * a * c + b^2)^{(1/2)}) / c * (x - 1/2 * (-b + (-4 * a * c + b^2)^{(1/2)}) / c) - 2 * ((-4 * a * c + b^2)^{(1/2)} * b * f + 2 * a * c * f - b^2 * f - 2 * c^2 * d) / c^2)^{(1/2)}) / (x - 1/2 * (-b + (-4 * a * c + b^2)^{(1/2)}) / c)}\right) * b * B$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + Bx}{\sqrt{fx^2 + d} (cx^2 + bx + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x)/((d + f*x^2)^(1/2)*(a + b*x + c*x^2)),x)

[Out] int((A + B*x)/((d + f*x^2)^(1/2)*(a + b*x + c*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx}{\sqrt{d + fx^2} (a + bx + cx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)/(f*x**2+d)**(1/2),x)
```

```
[Out] Integral((A + B*x)/(sqrt(d + f*x**2)*(a + b*x + c*x**2)), x)
```

$$3.24 \quad \int \frac{A+Bx}{(a+cx^2)\sqrt{d+fx^2}} dx$$

Optimal. Leaf size=101

$$\frac{A \tan^{-1}\left(\frac{x\sqrt{cd-af}}{\sqrt{a}\sqrt{d+fx^2}}\right)}{\sqrt{a}\sqrt{cd-af}} - \frac{B \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+fx^2}}{\sqrt{cd-af}}\right)}{\sqrt{c}\sqrt{cd-af}}$$

[Out] A*arctan(x*(-a*f+c*d)^(1/2)/a^(1/2)/(f*x^2+d)^(1/2))/a^(1/2)/(-a*f+c*d)^(1/2)-B*arctanh(c^(1/2)*(f*x^2+d)^(1/2)/(-a*f+c*d)^(1/2))/c^(1/2)/(-a*f+c*d)^(1/2)

Rubi [A] time = 0.13, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1010, 377, 205, 444, 63, 208}

$$\frac{A \tan^{-1}\left(\frac{x\sqrt{cd-af}}{\sqrt{a}\sqrt{d+fx^2}}\right)}{\sqrt{a}\sqrt{cd-af}} - \frac{B \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+fx^2}}{\sqrt{cd-af}}\right)}{\sqrt{c}\sqrt{cd-af}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x)/((a + c*x^2)*Sqrt[d + f*x^2]),x]

[Out] (A*ArcTan[(Sqrt[c*d - a*f]*x)/(Sqrt[a]*Sqrt[d + f*x^2])]/(Sqrt[a]*Sqrt[c*d - a*f]) - (B*ArcTanh[(Sqrt[c]*Sqrt[d + f*x^2])/Sqrt[c*d - a*f]])/(Sqrt[c]*Sqrt[c*d - a*f])

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rule 1010

Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h, Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}, x]

Rubi steps

$$\begin{aligned} \int \frac{A + Bx}{(a + cx^2)\sqrt{d + fx^2}} dx &= A \int \frac{1}{(a + cx^2)\sqrt{d + fx^2}} dx + B \int \frac{x}{(a + cx^2)\sqrt{d + fx^2}} dx \\ &= A \operatorname{Subst}\left(\int \frac{1}{a - (-cd + af)x^2} dx, x, \frac{x}{\sqrt{d + fx^2}}\right) + \frac{1}{2} B \operatorname{Subst}\left(\int \frac{1}{(a + cx)\sqrt{d + fx}}\right) \\ &= \frac{A \tan^{-1}\left(\frac{\sqrt{cd-af}x}{\sqrt{a}\sqrt{d+fx^2}}\right)}{\sqrt{a}\sqrt{cd-af}} + \frac{B \operatorname{Subst}\left(\int \frac{1}{a - \frac{cd}{f} + \frac{cx^2}{f}} dx, x, \sqrt{d + fx^2}\right)}{f} \\ &= \frac{A \tan^{-1}\left(\frac{\sqrt{cd-af}x}{\sqrt{a}\sqrt{d+fx^2}}\right)}{\sqrt{a}\sqrt{cd-af}} - \frac{B \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+fx^2}}{\sqrt{cd-af}}\right)}{\sqrt{c}\sqrt{cd-af}} \end{aligned}$$

Mathematica [A] time = 0.17, size = 154, normalized size = 1.52

$$\frac{(A\sqrt{c} - \sqrt{-a}B) \tanh^{-1}\left(\frac{\sqrt{c}d - \sqrt{-a}fx}{\sqrt{d+fx^2}\sqrt{cd-af}}\right) - (\sqrt{-a}B + A\sqrt{c}) \tanh^{-1}\left(\frac{\sqrt{-a}fx + \sqrt{c}d}{\sqrt{d+fx^2}\sqrt{cd-af}}\right)}{2\sqrt{-a}\sqrt{c}\sqrt{cd-af}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x)/((a + c*x^2)*Sqrt[d + f*x^2]),x]

[Out] $((-\text{Sqrt}[-a]*B) + A*\text{Sqrt}[c])*ArcTanh[(\text{Sqrt}[c]*d - \text{Sqrt}[-a]*f*x)/(\text{Sqrt}[c*d - a*f]*\text{Sqrt}[d + f*x^2])] - (\text{Sqrt}[-a]*B + A*\text{Sqrt}[c])*ArcTanh[(\text{Sqrt}[c]*d + \text{Sqrt}[-a]*f*x)/(\text{Sqrt}[c*d - a*f]*\text{Sqrt}[d + f*x^2])]/(2*\text{Sqrt}[-a]*\text{Sqrt}[c]*\text{Sqrt}[c*d - a*f])$

fricas [B] time = 1.20, size = 1515, normalized size = 15.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+a)/(f*x^2+d)^(1/2),x, algorithm="fricas")

[Out] $-1/4*\text{sqrt}((B^2*a - A^2*c + 2*(a*c^2*d - a^2*c*f)*\text{sqrt}(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^2*d - a^2*c*f))*\log(((A*B^3*a + A^3*B*c)*f*x + (A^2*B*c^2*d - A^2*B*a*c*f + (B*a*c^3*d^2 - 2*B*a^2*c^2*d*f + B*a^3*c*f^2)*\text{sqrt}(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))*\text{sqrt}(f*x^2 + d)*\text{sqrt}((B^2*a - A^2*c + 2*(a*c^2*d - a^2*c*f)*\text{sqrt}(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^2*d - a^2*c*f)) + \text{sqrt}(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2))*((B^2*a*c^2 + A^2*c^3)*d^2 - (B^2*a^2*c + A^2*a*c^2)*d*f))/x) + 1/4*\text{sqrt}((B^2*a - A^2*c + 2*(a*c^2*d - a^2*c*f)*\text{sqrt}(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^2*d - a^2*c*f))*\log(((A*B^3*a + A^3*B*c)*f*x - (A^2*B*c^2*d - A^2*B*a*c*f + (B*a*c^3*d^2 - 2*B*a^2*c^2*d*f + B*a^3*c*f^2)*\text{sqrt}(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))*\text{sqrt}(f*x^2 + d)*\text{sqrt}((B^2*a - A^2*c + 2*(a*c^2*d - a^2*c*f)*\text{sqrt}(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^2*d - a^2*c*f)) + \text{sqrt}(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2))*((B^2*a*c^2 + A^2*c^3)*d^2 - (B^2*a^2*c + A^2*a*c^2)*d*f))/x) - 1/4*\text{sqrt}((B^2*a - A^2*c - 2*(a*c^2*d - a^2*c*f)*\text{sqrt}(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^2*d - a^2*c*f))*\log(((A*B^3*a + A^3*B*c)*f*x + (A^2*B*c^2*d - A^2*B*a*c*f - (B*a*c^3*d^2 - 2*B*a^2*c^2*d*f + B*a^3*c*f^2)*\text{sqrt}(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))*\text{sqrt}(f*x^2 + d)*\text{sqrt}((B^2*a - A^2*c - 2*(a*c^2*d - a^2*c*f)*\text{sqrt}(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^2*d - a^2*c*f)) - \text{sqrt}(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2))*((B^2*a*c^2 + A^2*c^3)*d^2 - (B^2*a^2*c + A^2*a*c^2)*d*f))/x) + 1/4*\text{sqrt}((B^2*a - A^2*c - 2*(a*c^2*d - a^2*c*f)*\text{sqrt}(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^2*d - a^2*c*f))*\log(((A*B^3*a + A^3*B*c)*f*x - (A^2*B*c^2*d - A^2*B*a*c*f - (B*a*c^3*d^2 - 2*B*a^2*c^2*d*f + B*a^3*c*f^2)*\text{sqrt}(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))*\text{sqrt}(f*x^2 + d)*\text{sqrt}((B^2*a - A^2*c - 2*(a*c^2*d - a^2*c*f)*\text{sqrt}(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^2*d - a^2*c*f)) - \text{sqrt}(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2))*((B^2*a*c^2 + A^2*c^3)*d^2 - (B^2*a^2*c + A^2*a*c^2)*d*f))/x)$

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+a)/(f*x^2+d)^(1/2),x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.02, size = 608, normalized size = 6.02

$$\frac{A \ln \left(\frac{\frac{2\sqrt{-ac} \left(x - \frac{\sqrt{-ac}}{c} \right)^f}{c} - \frac{2(af-cd)}{c} + 2\sqrt{-\frac{af-cd}{c}} \sqrt{\left(x - \frac{\sqrt{-ac}}{c} \right)^2 + \frac{2\sqrt{-ac} \left(x - \frac{\sqrt{-ac}}{c} \right)^f - \frac{af-cd}{c}}}{x - \frac{\sqrt{-ac}}{c}} \right)}{2\sqrt{-ac} \sqrt{-\frac{af-cd}{c}}} + \frac{A \ln \left(\frac{\frac{2\sqrt{-ac} \left(x + \frac{\sqrt{-ac}}{c} \right)^f}{c} - \frac{2(af-cd)}{c} + 2\sqrt{-\frac{af-cd}{c}} \sqrt{\left(x + \frac{\sqrt{-ac}}{c} \right)^2 + \frac{2\sqrt{-ac} \left(x + \frac{\sqrt{-ac}}{c} \right)^f - \frac{af-cd}{c}}}{x + \frac{\sqrt{-ac}}{c}} \right)}{2\sqrt{-ac} \sqrt{-\frac{af-cd}{c}}} \right)}{2\sqrt{-ac} \sqrt{-\frac{af-cd}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x+A)/(c*x^2+a)/(f*x^2+d)^(1/2),x)

[Out]
$$\begin{aligned} & -1/2/(-a*c)^{(1/2)}/(-a*f-c*d)/c)^{(1/2)}*\ln((-2*(a*f-c*d)/c+2*f*(-a*c)^{(1/2)}/ \\ & c*(x-(-a*c)^{(1/2)}/c)+2*(-a*f-c*d)/c)^{(1/2)}*((x-(-a*c)^{(1/2)}/c)^{2*f+2*f*(-a \\ & *c)^{(1/2)}/c*(x-(-a*c)^{(1/2)}/c)-(a*f-c*d)/c)^{(1/2)})/(x-(-a*c)^{(1/2)}/c))*A-1/ \\ & 2/c/(-a*f-c*d)/c)^{(1/2)}*\ln((-2*(a*f-c*d)/c+2*f*(-a*c)^{(1/2)}/c*(x-(-a*c)^{(1 \\ & /2)}/c)+2*(-a*f-c*d)/c)^{(1/2)}*((x-(-a*c)^{(1/2)}/c)^{2*f+2*f*(-a*c)^{(1/2)}/c*(x \\ & -(-a*c)^{(1/2)}/c)-(a*f-c*d)/c)^{(1/2)})/(x-(-a*c)^{(1/2)}/c))*B+1/2/(-a*c)^{(1/2) \\ & /(-a*f-c*d)/c)^{(1/2)}*\ln((-2*(a*f-c*d)/c-2*f*(-a*c)^{(1/2)}/c*(x+(-a*c)^{(1/2) \\ & /c)+2*(-a*f-c*d)/c)^{(1/2)}*((x+(-a*c)^{(1/2)}/c)^{2*f-2*f*(-a*c)^{(1/2)}/c*(x+(- \\ & a*c)^{(1/2)}/c)-(a*f-c*d)/c)^{(1/2)})/(x+(-a*c)^{(1/2)}/c))*A-1/2/c/(-a*f-c*d)/c \\ &)^{(1/2)}*\ln((-2*(a*f-c*d)/c-2*f*(-a*c)^{(1/2)}/c*(x+(-a*c)^{(1/2)}/c)+2*(-a*f-c \\ & *d)/c)^{(1/2)}*((x+(-a*c)^{(1/2)}/c)^{2*f-2*f*(-a*c)^{(1/2)}/c*(x+(-a*c)^{(1/2)}/c)- \\ & (a*f-c*d)/c)^{(1/2)})/(x+(-a*c)^{(1/2)}/c))*B \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx + A}{(cx^2 + a)\sqrt{fx^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+a)/(f*x^2+d)^(1/2),x, algorithm="maxima")

[Out] integrate((B*x + A)/((c*x^2 + a)*sqrt(f*x^2 + d)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\left\{ \begin{array}{ll} \frac{B \operatorname{atan}\left(\frac{c \sqrt{f x^2+d}}{\sqrt{a c f-c^2 d}}\right)}{\sqrt{a c f-c^2 d}} + \frac{A \operatorname{atan}\left(\frac{x \sqrt{c d-a f}}{\sqrt{a} \sqrt{f x^2+d}}\right)}{\sqrt{-a(a f-c d)}} & \text{if } 0 < c d - a f \\ \frac{A \ln\left(\frac{\sqrt{a(f x^2+d)}+x \sqrt{a f-c d}}{\sqrt{a(f x^2+d)}-x \sqrt{a f-c d}}\right)}{2 \sqrt{a(a f-c d)}} + \frac{B \operatorname{atan}\left(\frac{c \sqrt{f x^2+d}}{\sqrt{a c f-c^2 d}}\right)}{\sqrt{a c f-c^2 d}} & \text{if } c d - a f < 0 \\ \int \frac{A+B x}{(c x^2+a) \sqrt{f x^2+d}} dx & \text{if } c d - a f \notin \mathbb{R} \vee a f = c d \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x)/((a + c*x^2)*(d + f*x^2)^(1/2)),x)`

[Out] `piecewise(0 < - a*f + c*d, (B*atan((c*(d + f*x^2)^(1/2))/(- c^2*d + a*c*f)^(1/2)))/(- c^2*d + a*c*f)^(1/2) + (A*atan((x*(- a*f + c*d)^(1/2))/(a^(1/2)*(d + f*x^2)^(1/2))))/(-a*(a*f - c*d))^(1/2), - a*f + c*d < 0, (A*log(((a*(d + f*x^2)^(1/2) + x*(a*f - c*d)^(1/2)))/((a*(d + f*x^2)^(1/2) - x*(a*f - c*d)^(1/2)))))/(2*(a*(a*f - c*d))^(1/2)) + (B*atan((c*(d + f*x^2)^(1/2))/(- c^2*d + a*c*f)^(1/2)))/(- c^2*d + a*c*f)^(1/2), ~in(- a*f + c*d, 'real') | a*f == c*d, int((A + B*x)/((a + c*x^2)*(d + f*x^2)^(1/2)), x))`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx}{(a + cx^2) \sqrt{d + fx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x+A)/(c*x**2+a)/(f*x**2+d)**(1/2),x)`

[Out] `Integral((A + B*x)/((a + c*x**2)*sqrt(d + f*x**2)), x)`

$$3.25 \quad \int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x-2x^2}} dx$$

Optimal. Leaf size=139

$$\frac{1}{2}\sqrt{\sqrt{10}-\frac{13}{5}} \tan^{-1}\left(\frac{(1+4\sqrt{10})x+3(4-\sqrt{10})}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right) + \frac{1}{2}\sqrt{\frac{13}{5}+\sqrt{10}} \tanh^{-1}\left(\frac{(1-4\sqrt{10})x+3(4+\sqrt{10})}{2\sqrt{\sqrt{10}-1}\sqrt{-2x^2+3x+1}}\right)$$

[Out] 1/10*arctan(1/2*(12-3*10^(1/2)+x*(1+4*10^(1/2)))/(-2*x^2+3*x+1)^(1/2)/(1+10^(1/2))^(1/2))*(-65+25*10^(1/2))^(1/2)+1/10*arctanh(1/2*(x*(1-4*10^(1/2))+12+3*10^(1/2))/(-2*x^2+3*x+1)^(1/2)/(-1+10^(1/2))^(1/2))*(65+25*10^(1/2))^(1/2)

Rubi [A] time = 0.22, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1032, 724, 204, 206}

$$\frac{1}{2}\sqrt{\sqrt{10}-\frac{13}{5}} \tan^{-1}\left(\frac{(1+4\sqrt{10})x+3(4-\sqrt{10})}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right) + \frac{1}{2}\sqrt{\frac{13}{5}+\sqrt{10}} \tanh^{-1}\left(\frac{(1-4\sqrt{10})x+3(4+\sqrt{10})}{2\sqrt{\sqrt{10}-1}\sqrt{-2x^2+3x+1}}\right)$$

Antiderivative was successfully verified.

[In] Int[(2 + x)/((2 + 4*x - 3*x^2)*Sqrt[1 + 3*x - 2*x^2]),x]

[Out] (Sqrt[-13/5 + Sqrt[10]]*ArcTan[(3*(4 - Sqrt[10]) + (1 + 4*Sqrt[10]))*x]/(2*Sqrt[1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2]))/2 + (Sqrt[13/5 + Sqrt[10]]*ArcTanh[(3*(4 + Sqrt[10]) + (1 - 4*Sqrt[10]))*x]/(2*Sqrt[-1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2]))/2

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol]
:> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol]
:> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned} \int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x-2x^2}} dx &= \frac{1}{5}(5-4\sqrt{10}) \int \frac{1}{(4-2\sqrt{10}-6x)\sqrt{1+3x-2x^2}} dx + \frac{1}{5}(5+4\sqrt{10}) \int \frac{1}{(4+2\sqrt{10}-6x)\sqrt{1+3x-2x^2}} dx \\ &= -\left(\frac{1}{5}(2(5-4\sqrt{10}))\right) \text{Subst}\left(\int \frac{1}{144+72(4-2\sqrt{10})-8(4-2\sqrt{10})^2-x^2} dx, \frac{4-2\sqrt{10}-6x}{2}\right) \\ &= \frac{1}{10}\sqrt{-65+25\sqrt{10}} \tan^{-1}\left(\frac{3(4-\sqrt{10})+(1+4\sqrt{10})x}{2\sqrt{1+\sqrt{10}}\sqrt{1+3x-2x^2}}\right) + \frac{1}{10}\sqrt{65+25\sqrt{10}} \tan^{-1}\left(\frac{3(4+\sqrt{10})+(1+4\sqrt{10})x}{2\sqrt{1+\sqrt{10}}\sqrt{1+3x-2x^2}}\right) \end{aligned}$$

Mathematica [A] time = 0.30, size = 140, normalized size = 1.01

$$\frac{(4\sqrt{10}-5) \tan^{-1}\left(\frac{4\sqrt{10}x+x-3\sqrt{10}+12}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right) + 3\sqrt{5(7+2\sqrt{10})} \tanh^{-1}\left(\frac{-4\sqrt{10}x+x+3(4+\sqrt{10})}{2\sqrt{\sqrt{10}-1}\sqrt{-2x^2+3x+1}}\right)}{10\sqrt{1+\sqrt{10}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + x)/((2 + 4*x - 3*x^2)*Sqrt[1 + 3*x - 2*x^2]), x]
```

```
[Out] ((-5 + 4*Sqrt[10])*ArcTan[(12 - 3*Sqrt[10] + x + 4*Sqrt[10]*x)/(2*Sqrt[1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])] + 3*Sqrt[5*(7 + 2*Sqrt[10])]*ArcTanh[(3*(4 + Sqrt[10]) + x - 4*Sqrt[10]*x)/(2*Sqrt[-1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/(10*Sqrt[1 + Sqrt[10]])
```

fricas [B] time = 0.93, size = 322, normalized size = 2.32

$$\frac{2}{5} \sqrt{5} \sqrt{5 \sqrt{5} \sqrt{2} - 13} \arctan \left(\frac{\sqrt{2} (2 \sqrt{5} x - \sqrt{2} x) \sqrt{5 \sqrt{5} \sqrt{2} - 13} \sqrt{\frac{\sqrt{5} \sqrt{2} (3x^2 + 2x) + 6x^2 - 2(\sqrt{5} \sqrt{2} x + 2x + 2) \sqrt{-2x^2 + 3x + 1}}{x^2}}}{\sqrt{2} (2 \sqrt{5} x - \sqrt{2} x) \sqrt{5 \sqrt{5} \sqrt{2} - 13} \sqrt{\frac{\sqrt{5} \sqrt{2} (3x^2 + 2x) + 6x^2 - 2(\sqrt{5} \sqrt{2} x + 2x + 2) \sqrt{-2x^2 + 3x + 1}}{x^2}}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(1/2),x, algorithm="fricas")

[Out] 2/5*sqrt(5)*sqrt(5*sqrt(5)*sqrt(2) - 13)*arctan(1/18*(sqrt(2)*(2*sqrt(5)*x - sqrt(2)*x)*sqrt(5*sqrt(5)*sqrt(2) - 13)*sqrt((sqrt(5)*sqrt(2)*(3*x^2 + 2*x) + 6*x^2 - 2*(sqrt(5)*sqrt(2)*x + 2*x + 2)*sqrt(-2*x^2 + 3*x + 1) + 10*x + 4)/x^2) + 2*(sqrt(2)*(4*x - 1) + sqrt(5)*(x + 2) - sqrt(-2*x^2 + 3*x + 1) * (2*sqrt(5) - sqrt(2))) * sqrt(5*sqrt(5)*sqrt(2) - 13))/x) - 1/10*sqrt(5)*sqrt(5*sqrt(5)*sqrt(2) + 13)*log((9*sqrt(5)*sqrt(2)*x + (4*sqrt(5)*x - 7*sqrt(2)*x)*sqrt(5*sqrt(5)*sqrt(2) + 13) - 18*x + 18*sqrt(-2*x^2 + 3*x + 1) - 18)/x) + 1/10*sqrt(5)*sqrt(5*sqrt(5)*sqrt(2) + 13)*log((9*sqrt(5)*sqrt(2)*x - (4*sqrt(5)*x - 7*sqrt(2)*x)*sqrt(5*sqrt(5)*sqrt(2) + 13) - 18*x + 18*sqrt(-2*x^2 + 3*x + 1) - 18)/x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(1/2),x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.07, size = 324, normalized size = 2.33

$$\frac{2\sqrt{10} \operatorname{arctanh} \left(\frac{-1 + \sqrt{10} + \frac{9 \left(\frac{1}{3} - \frac{4\sqrt{10}}{3} \right) \left(x - \frac{2}{3} - \frac{\sqrt{10}}{3} \right)}{2}}{\sqrt{-1 + \sqrt{10}} \sqrt{-18 \left(x - \frac{2}{3} - \frac{\sqrt{10}}{3} \right)^2 + 9 \left(\frac{1}{3} - \frac{4\sqrt{10}}{3} \right) \left(x - \frac{2}{3} - \frac{\sqrt{10}}{3} \right) - 1 + \sqrt{10}}} \right)}{5\sqrt{-1 + \sqrt{10}}} + \frac{\operatorname{arctanh} \left(\frac{-1 + \sqrt{10} + \frac{9 \left(\frac{1}{3} - \frac{4\sqrt{10}}{3} \right)}{2}}{\sqrt{-1 + \sqrt{10}} \sqrt{-18 \left(x - \frac{2}{3} - \frac{\sqrt{10}}{3} \right)^2 + 9 \left(\frac{1}{3} - \frac{4\sqrt{10}}{3} \right) \left(x - \frac{2}{3} - \frac{\sqrt{10}}{3} \right) - 1 + \sqrt{10}}} \right)}{2\sqrt{-1 + \sqrt{10}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(1/2),x)

[Out] 2/5*10^(1/2)/(1+10^(1/2))^(1/2)*arctan(9/2*(-2/9-2/9*10^(1/2)+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2)))/(1+10^(1/2))^(1/2)/(-18*(x-2/3+1/3*10^(1/2))^2+

$9*(1/3+4/3*10^{(1/2)})*(x-2/3+1/3*10^{(1/2)})-1-10^{(1/2)}^{(1/2)}-1/2/(1+10^{(1/2)})^{(1/2)}*\arctan(9/2*(-2/9-2/9*10^{(1/2)}+(1/3+4/3*10^{(1/2)})*(x-2/3+1/3*10^{(1/2)})))/(1+10^{(1/2)})^{(1/2)}/(-18*(x-2/3+1/3*10^{(1/2)})^2+9*(1/3+4/3*10^{(1/2)})*(x-2/3+1/3*10^{(1/2)})-1-10^{(1/2)})^{(1/2)}+2/5*10^{(1/2)}/(-1+10^{(1/2)})^{(1/2)}*\arctanh(9/2*(-2/9+2/9*10^{(1/2)}+(1/3-4/3*10^{(1/2)})*(x-2/3-1/3*10^{(1/2)})))/(-1+10^{(1/2)})^{(1/2)}/(-18*(x-2/3-1/3*10^{(1/2)})^2+9*(1/3-4/3*10^{(1/2)})*(x-2/3-1/3*10^{(1/2)})-1+10^{(1/2)})^{(1/2)}+1/2/(-1+10^{(1/2)})^{(1/2)}*\arctanh(9/2*(-2/9+2/9*10^{(1/2)}+(1/3-4/3*10^{(1/2)})*(x-2/3-1/3*10^{(1/2)})))/(-1+10^{(1/2)})^{(1/2)}/(-18*(x-2/3-1/3*10^{(1/2)})^2+9*(1/3-4/3*10^{(1/2)})*(x-2/3-1/3*10^{(1/2)})-1+10^{(1/2)})^{(1/2)}$

maxima [B] time = 1.07, size = 361, normalized size = 2.60

$$-\frac{1}{20}\sqrt{10}\left(\frac{\sqrt{10}\arcsin\left(\frac{8\sqrt{17}\sqrt{10}x}{17|6x+2\sqrt{10}-4|}+\frac{2\sqrt{17}x}{17|6x+2\sqrt{10}-4|}-\frac{6\sqrt{17}\sqrt{10}}{17|6x+2\sqrt{10}-4|}+\frac{24\sqrt{17}}{17|6x+2\sqrt{10}-4|}\right)}{\sqrt{\sqrt{10}+1}}-\sqrt{10}\log\left(-\frac{2}{9}\sqrt{10}+\dots\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(1/2),x, algorithm="maxima")

[Out] -1/20*sqrt(10)*(sqrt(10)*arcsin(8/17*sqrt(17)*sqrt(10)*x/abs(6*x + 2*sqrt(10) - 4) + 2/17*sqrt(17)*x/abs(6*x + 2*sqrt(10) - 4) - 6/17*sqrt(17)*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 24/17*sqrt(17)/abs(6*x + 2*sqrt(10) - 4))/sqrt(sqrt(10) + 1) - sqrt(10)*log(-2/9*sqrt(10) + 2/3*sqrt(-2*x^2 + 3*x + 1)*sqrt(sqrt(10) - 1)/abs(6*x - 2*sqrt(10) - 4) + 2/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) - 2/9/abs(6*x - 2*sqrt(10) - 4) + 1/18)/sqrt(sqrt(10) - 1) - 8*arcsin(8/17*sqrt(17)*sqrt(10)*x/abs(6*x + 2*sqrt(10) - 4) + 2/17*sqrt(17)*x/abs(6*x + 2*sqrt(10) - 4) - 6/17*sqrt(17)*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 24/17*sqrt(17)/abs(6*x + 2*sqrt(10) - 4))/sqrt(sqrt(10) + 1) - 8*log(-2/9*sqrt(10) + 2/3*sqrt(-2*x^2 + 3*x + 1)*sqrt(sqrt(10) - 1)/abs(6*x - 2*sqrt(10) - 4) + 2/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) - 2/9/abs(6*x - 2*sqrt(10) - 4) + 1/18)/sqrt(sqrt(10) - 1))

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+2}{\sqrt{-2x^2+3x+1}(-3x^2+4x+2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 2)/((3*x - 2*x^2 + 1)^(1/2)*(4*x - 3*x^2 + 2)),x)

[Out] int((x + 2)/((3*x - 2*x^2 + 1)^(1/2)*(4*x - 3*x^2 + 2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{3x^2\sqrt{-2x^2+3x+1}-4x\sqrt{-2x^2+3x+1}-2\sqrt{-2x^2+3x+1}} dx - \int \frac{2}{3x^2\sqrt{-2x^2+3x+1}-4x\sqrt{-2x^2+3x+1}-2\sqrt{-2x^2+3x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(-3*x**2+4*x+2)/(-2*x**2+3*x+1)**(1/2), x)

[Out] -Integral(x/(3*x**2*sqrt(-2*x**2 + 3*x + 1) - 4*x*sqrt(-2*x**2 + 3*x + 1) - 2*sqrt(-2*x**2 + 3*x + 1)), x) - Integral(2/(3*x**2*sqrt(-2*x**2 + 3*x + 1) - 4*x*sqrt(-2*x**2 + 3*x + 1) - 2*sqrt(-2*x**2 + 3*x + 1)), x)

$$3.26 \quad \int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{3/2}} dx$$

Optimal. Leaf size=166

$$-\frac{2(14x+15)}{17\sqrt{-2x^2+3x+1}} - \frac{9}{2}\sqrt{\frac{1}{5}}(\sqrt{10}-3) \tan^{-1}\left(\frac{(1+4\sqrt{10})x+3(4-\sqrt{10})}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right) + \frac{9}{2}\sqrt{\frac{1}{5}}(3+\sqrt{10}) \tanh^{-1}\left(\frac{(1-4\sqrt{10})x+3(4+\sqrt{10})}{2\sqrt{-1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)$$

[Out] $-2/17*(15+14*x)/(-2*x^2+3*x+1)^{(1/2)}-9/10*\arctan(1/2*(12-3*10^{(1/2)}+x*(1+4*10^{(1/2)}))/(-2*x^2+3*x+1)^{(1/2)/(1+10^{(1/2)})^{(1/2)}}*(-15+5*10^{(1/2)})^{(1/2)}+9/10*\operatorname{arctanh}(1/2*(x*(1-4*10^{(1/2)})+12+3*10^{(1/2)}))/(-2*x^2+3*x+1)^{(1/2)/(-1+10^{(1/2)})^{(1/2)}}*(15+5*10^{(1/2)})^{(1/2)}$

Rubi [A] time = 0.22, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1016, 12, 1032, 724, 204, 206}

$$-\frac{2(14x+15)}{17\sqrt{-2x^2+3x+1}} - \frac{9}{2}\sqrt{\frac{1}{5}}(\sqrt{10}-3) \tan^{-1}\left(\frac{(1+4\sqrt{10})x+3(4-\sqrt{10})}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right) + \frac{9}{2}\sqrt{\frac{1}{5}}(3+\sqrt{10}) \tanh^{-1}\left(\frac{(1-4\sqrt{10})x+3(4+\sqrt{10})}{2\sqrt{-1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)$$

Antiderivative was successfully verified.

[In] Int[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x - 2*x^2)^(3/2)), x]

[Out] $(-2*(15+14*x))/(17*\operatorname{Sqrt}[1+3*x-2*x^2]) - (9*\operatorname{Sqrt}[(-3+\operatorname{Sqrt}[10])/5]*\operatorname{ArcTan}[(3*(4-\operatorname{Sqrt}[10])+(1+4*\operatorname{Sqrt}[10])*x)/(2*\operatorname{Sqrt}[1+\operatorname{Sqrt}[10]]*\operatorname{Sqrt}[1+3*x-2*x^2])])/2 + (9*\operatorname{Sqrt}[(3+\operatorname{Sqrt}[10])/5]*\operatorname{ArcTanh}[(3*(4+\operatorname{Sqrt}[10])+(1-4*\operatorname{Sqrt}[10])*x)/(2*\operatorname{Sqrt}[-1+\operatorname{Sqrt}[10]]*\operatorname{Sqrt}[1+3*x-2*x^2])])/2$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1016

Int[((g_.) + (h_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1) * (d + e*x + f*x^2)^(q + 1) * (g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*c*d - 2*a*c*e + a*b*f))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !(IntegerQ[p] && ILtQ[q, -1])

Rule 1032

Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{3/2}} dx &= -\frac{2(15+14x)}{17\sqrt{1+3x-2x^2}} + \frac{2}{17} \int \frac{153x}{2(2+4x-3x^2)\sqrt{1+3x-2x^2}} dx \\
&= -\frac{2(15+14x)}{17\sqrt{1+3x-2x^2}} + 9 \int \frac{x}{(2+4x-3x^2)\sqrt{1+3x-2x^2}} dx \\
&= -\frac{2(15+14x)}{17\sqrt{1+3x-2x^2}} + \frac{1}{5} (9(5-\sqrt{10})) \int \frac{1}{(4-2\sqrt{10}-6x)\sqrt{1+3x-2x^2}} dx \\
&= -\frac{2(15+14x)}{17\sqrt{1+3x-2x^2}} - \frac{1}{5} (18(5-\sqrt{10})) \text{Subst} \left(\int \frac{1}{144+72(4-2\sqrt{10})x} dx \right) \\
&= -\frac{2(15+14x)}{17\sqrt{1+3x-2x^2}} - \frac{9}{2} \sqrt{\frac{1}{5}} (-3+\sqrt{10}) \tan^{-1} \left(\frac{3(4-\sqrt{10})+(1+4\sqrt{10})x}{2\sqrt{1+\sqrt{10}}\sqrt{1+3x-2x^2}} \right)
\end{aligned}$$

Mathematica [A] time = 0.38, size = 167, normalized size = 1.01

$$\frac{1}{170} \left(153\sqrt{5(3+\sqrt{10})} \tanh^{-1} \left(\frac{-4\sqrt{10}x+x+3(4+\sqrt{10})}{2\sqrt{\sqrt{10}-1}\sqrt{-2x^2+3x+1}} \right) - \frac{153\sqrt{5(\sqrt{10}-3)}\sqrt{-2x^2+3x+1} \tan^{-1} \left(\frac{4\sqrt{10}x-3}{2\sqrt{1+\sqrt{10}}\sqrt{1+3x-2x^2}} \right)}{\sqrt{-2x^2+3x+1}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x - 2*x^2)^(3/2)), x]

[Out] (-((300 + 280*x + 153*Sqrt[5*(-3 + Sqrt[10]])*Sqrt[1 + 3*x - 2*x^2]*ArcTan[(12 - 3*Sqrt[10] + x + 4*Sqrt[10]*x)/(2*Sqrt[1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/Sqrt[1 + 3*x - 2*x^2]) + 153*Sqrt[5*(3 + Sqrt[10])]*ArcTanh[(3*(4 + Sqrt[10]) + x - 4*Sqrt[10]*x)/(2*Sqrt[-1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/170

fricas [B] time = 0.90, size = 344, normalized size = 2.07

$$612\sqrt{5}(2x^2 - 3x - 1)\sqrt{\sqrt{10} - 3} \arctan \left(\frac{\sqrt{10}\sqrt{5}\sqrt{2}x\sqrt{\sqrt{10}-3}\sqrt{\frac{6x^2+\sqrt{10}(3x^2+2x)-2\sqrt{-2x^2+3x+1}(\sqrt{10}x+2x+2)+10x+4}{x^2}}}{10x} \right) + 2(\sqrt{10}\sqrt{5}\sqrt{2}x\sqrt{\sqrt{10}-3}\sqrt{\frac{6x^2+\sqrt{10}(3x^2+2x)-2\sqrt{-2x^2+3x+1}(\sqrt{10}x+2x+2)+10x+4}{x^2}})$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(3/2),x, algorithm="fricas")
[Out] -1/170*(612*sqrt(5)*(2*x^2 - 3*x - 1)*sqrt(sqrt(10) - 3)*arctan(1/10*(sqrt(10)*sqrt(5)*sqrt(2)*x*sqrt(sqrt(10) - 3)*sqrt((6*x^2 + sqrt(10)*(3*x^2 + 2*x) - 2*sqrt(-2*x^2 + 3*x + 1)*(sqrt(10)*x + 2*x + 2) + 10*x + 4)/x^2) + 2*(sqrt(10)*sqrt(5)*(x + 1) - sqrt(10)*sqrt(5)*sqrt(-2*x^2 + 3*x + 1) + 5*sqrt(5)*x)*sqrt(sqrt(10) - 3))/x) + 153*sqrt(5)*(2*x^2 - 3*x - 1)*sqrt(sqrt(10) + 3)*log(9*(5*sqrt(10)*x + (3*sqrt(10)*sqrt(5)*x - 10*sqrt(5)*x)*sqrt(sqrt(10) + 3) - 10*x + 10*sqrt(-2*x^2 + 3*x + 1) - 10)/x) - 153*sqrt(5)*(2*x^2 - 3*x - 1)*sqrt(sqrt(10) + 3)*log(9*(5*sqrt(10)*x - (3*sqrt(10)*sqrt(5)*x - 10*sqrt(5)*x)*sqrt(sqrt(10) + 3) - 10*x + 10*sqrt(-2*x^2 + 3*x + 1) - 10)/x) + 600*x^2 - 20*sqrt(-2*x^2 + 3*x + 1)*(14*x + 15) - 900*x - 300)/(2*x^2 - 3*x - 1)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.03, size = 760, normalized size = 4.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(3/2),x)
```

```
[Out] 26/255*10^(1/2)/(-1/9-1/9*10^(1/2))/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(1/2)+32/765/(-1/9-1/9*10^(1/2))/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(1/2)*10^(1/2)*x-62/153/(-1/9-1/9*10^(1/2))/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(1/2)*x+7/51/(-1/9-1/9*10^(1/2))/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(1/2)+2/5*10^(1/2)/(-1/9-1/9*10^(1/2))/((1+10^(1/2))^(1/2)*arctan(9/2*(-2/9-2/9*10^(1/2)+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))))/(1+10^(1/2))^(1/2)/(-18*(x-2/3+1/3*10^(1/2))^2+9*(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1-10^(1/2))^(1/2))-1/2/(-1/9-1/9*10^(1/2))/((1+10^(1/2))^(1/2)*arctan(9/2*(-2/9-2/9*10^(1/2)+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))))/(1+10^(1/2))^(1/2)/(-18*(x-2/3+1/3*10^(1/2))^2+9*(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1-10^(1/2))^(1/2))-26/255*10^(1/2)/(-1/9+1/9*10^(1/2))/(-2*(x-2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/2))^(1/2)-32/765/(-1/9+1/9*10^(1/2))/(-2*(x-2/3-1/3*10^(1/2))^2
```

$$\begin{aligned}
& + (1/3 - 4/3 * 10^{(1/2)}) * (x - 2/3 - 1/3 * 10^{(1/2)} - 1/9 + 1/9 * 10^{(1/2)})^{(1/2)} * 10^{(1/2)} * x \\
& - 62/153 / (-1/9 + 1/9 * 10^{(1/2)}) / (-2 * (x - 2/3 - 1/3 * 10^{(1/2)})^2 + (1/3 - 4/3 * 10^{(1/2)}) * (\\
& x - 2/3 - 1/3 * 10^{(1/2)} - 1/9 + 1/9 * 10^{(1/2)})^{(1/2)} * x + 7/51 / (-1/9 + 1/9 * 10^{(1/2)}) / (-2 * \\
& (x - 2/3 - 1/3 * 10^{(1/2)})^2 + (1/3 - 4/3 * 10^{(1/2)}) * (x - 2/3 - 1/3 * 10^{(1/2)} - 1/9 + 1/9 * 10^{(1/2)})^{(1/2)} \\
& + 2/5 * 10^{(1/2)} / (-1/9 + 1/9 * 10^{(1/2)}) / (-1 + 10^{(1/2)})^{(1/2)} * \operatorname{arctanh}(9/ \\
& 2 * (-2/9 + 2/9 * 10^{(1/2)} + (1/3 - 4/3 * 10^{(1/2)}) * (x - 2/3 - 1/3 * 10^{(1/2)})) / (-1 + 10^{(1/2)}) \\
& ^{(1/2)} / (-18 * (x - 2/3 - 1/3 * 10^{(1/2)})^2 + 9 * (1/3 - 4/3 * 10^{(1/2)}) * (x - 2/3 - 1/3 * 10^{(1/2)} \\
&) - 1 + 10^{(1/2)})^{(1/2)} + 1/2 / (-1/9 + 1/9 * 10^{(1/2)}) / (-1 + 10^{(1/2)})^{(1/2)} * \operatorname{arctanh}(9/ \\
& 2 * (-2/9 + 2/9 * 10^{(1/2)} + (1/3 - 4/3 * 10^{(1/2)}) * (x - 2/3 - 1/3 * 10^{(1/2)})) / (-1 + 10^{(1/2)}) \\
& ^{(1/2)} / (-18 * (x - 2/3 - 1/3 * 10^{(1/2)})^2 + 9 * (1/3 - 4/3 * 10^{(1/2)}) * (x - 2/3 - 1/3 * 10^{(1/2)} \\
&) - 1 + 10^{(1/2)})^{(1/2)}
\end{aligned}$$

maxima [B] time = 1.11, size = 678, normalized size = 4.08

$$\frac{1}{340} \sqrt{10} \left(\frac{124 \sqrt{10} x}{\sqrt{10} \sqrt{-2x^2 + 3x + 1} + \sqrt{-2x^2 + 3x + 1}} - \frac{124 \sqrt{10} x}{\sqrt{10} \sqrt{-2x^2 + 3x + 1} - \sqrt{-2x^2 + 3x + 1}} + \frac{153 \sqrt{10} \operatorname{arcsin}\left(\frac{8x}{17\sqrt{10}}\right)}{\sqrt{10} \sqrt{-2x^2 + 3x + 1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(3/2),x, algorithm="maxima")

[Out] 1/340*sqrt(10)*(124*sqrt(10)*x/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1)) - 124*sqrt(10)*x/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1)) + 153*sqrt(10)*arcsin(8/17*sqrt(17)*sqrt(10)*x/abs(6*x + 2*sqrt(10) - 4) + 2/17*sqrt(17)*x/abs(6*x + 2*sqrt(10) - 4) - 6/17*sqrt(17)*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 24/17*sqrt(17)/abs(6*x + 2*sqrt(10) - 4))/(sqrt(10)*sqrt(sqrt(10) + 1) + sqrt(sqrt(10) + 1)) - 128*x/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1)) - 128*x/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1)) - 1224*arcsin(8/17*sqrt(17)*sqrt(10)*x/abs(6*x + 2*sqrt(10) - 4) + 2/17*sqrt(17)*x/abs(6*x + 2*sqrt(10) - 4) - 6/17*sqrt(17)*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 24/17*sqrt(17)/abs(6*x + 2*sqrt(10) - 4))/(sqrt(10)*sqrt(sqrt(10) + 1) + sqrt(sqrt(10) + 1)) + 153*sqrt(10)*log(-2/9*sqrt(10) + 2/3*sqrt(-2*x^2 + 3*x + 1)*sqrt(sqrt(10) - 1)/abs(6*x - 2*sqrt(10) - 4) + 2/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) - 2/9/abs(6*x - 2*sqrt(10) - 4) + 1/18)/(sqrt(10) - 1)^(3/2) - 42*sqrt(10)/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1)) + 42*sqrt(10)/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1)) + 1224*log(-2/9*sqrt(10) + 2/3*sqrt(-2*x^2 + 3*x + 1)*sqrt(sqrt(10) - 1)/abs(6*x - 2*sqrt(10) - 4) + 2/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) - 2/9/abs(6*x - 2*sqrt(10) - 4) + 1/18)/(sqrt(10) - 1)^(3/2) - 312/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1)) - 312/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1))

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+2}{(-2x^2+3x+1)^{3/2}(-3x^2+4x+2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 2)/((3*x - 2*x^2 + 1)^(3/2)*(4*x - 3*x^2 + 2)), x)

[Out] int((x + 2)/((3*x - 2*x^2 + 1)^(3/2)*(4*x - 3*x^2 + 2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-6x^4\sqrt{-2x^2+3x+1} + 17x^3\sqrt{-2x^2+3x+1} - 5x^2\sqrt{-2x^2+3x+1} - 10x\sqrt{-2x^2+3x+1} - 2\sqrt{-2x^2+3x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(-3*x**2+4*x+2)/(-2*x**2+3*x+1)**(3/2), x)

[Out] -Integral(x/(-6*x**4*sqrt(-2*x**2 + 3*x + 1) + 17*x**3*sqrt(-2*x**2 + 3*x + 1) - 5*x**2*sqrt(-2*x**2 + 3*x + 1) - 10*x*sqrt(-2*x**2 + 3*x + 1) - 2*sqrt(-2*x**2 + 3*x + 1)), x) - Integral(2/(-6*x**4*sqrt(-2*x**2 + 3*x + 1) + 17*x**3*sqrt(-2*x**2 + 3*x + 1) - 5*x**2*sqrt(-2*x**2 + 3*x + 1) - 10*x*sqrt(-2*x**2 + 3*x + 1) - 2*sqrt(-2*x**2 + 3*x + 1)), x)

$$3.27 \quad \int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{5/2}} dx$$

Optimal. Leaf size=193

$$-\frac{2(14x+15)}{51(-2x^2+3x+1)^{3/2}} - \frac{2(4814x+291)}{867\sqrt{-2x^2+3x+1}} + \frac{9}{2}\sqrt{\frac{1}{5}(17\sqrt{10}-53)} \tan^{-1}\left(\frac{(1+4\sqrt{10})x+3(4-\sqrt{10})}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right) + \frac{9}{2}\sqrt{\frac{1}{5}}$$

[Out] $-2/51*(15+14*x)/(-2*x^2+3*x+1)^(3/2)-2/867*(291+4814*x)/(-2*x^2+3*x+1)^(1/2)+9/10*\arctan(1/2*(12-3*10^(1/2)+x*(1+4*10^(1/2)))/(-2*x^2+3*x+1)^(1/2)/(1+10^(1/2))^(1/2))*(-265+85*10^(1/2))^(1/2)+9/10*\operatorname{arctanh}(1/2*(x*(1-4*10^(1/2))+12+3*10^(1/2))/(-2*x^2+3*x+1)^(1/2)/(-1+10^(1/2))^(1/2))*(265+85*10^(1/2))^(1/2)$

Rubi [A] time = 0.27, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1016, 1060, 1032, 724, 204, 206}

$$-\frac{2(14x+15)}{51(-2x^2+3x+1)^{3/2}} - \frac{2(4814x+291)}{867\sqrt{-2x^2+3x+1}} + \frac{9}{2}\sqrt{\frac{1}{5}(17\sqrt{10}-53)} \tan^{-1}\left(\frac{(1+4\sqrt{10})x+3(4-\sqrt{10})}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right) + \frac{9}{2}\sqrt{\frac{1}{5}}$$

Antiderivative was successfully verified.

[In] Int[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x - 2*x^2)^(5/2)), x]

[Out] $(-2*(15+14*x))/(51*(1+3*x-2*x^2)^(3/2)) - (2*(291+4814*x))/(867*\operatorname{Sqrt}[1+3*x-2*x^2]) + (9*\operatorname{Sqrt}[(-53+17*\operatorname{Sqrt}[10])/5]*\operatorname{ArcTan}[(3*(4-\operatorname{Sqrt}[10]))+(1+4*\operatorname{Sqrt}[10])*x]/(2*\operatorname{Sqrt}[1+\operatorname{Sqrt}[10]]*\operatorname{Sqrt}[1+3*x-2*x^2]))/2 + (9*\operatorname{Sqrt}[(53+17*\operatorname{Sqrt}[10])/5]*\operatorname{ArcTanh}[(3*(4+\operatorname{Sqrt}[10]))+(1-4*\operatorname{Sqrt}[10])*x]/(2*\operatorname{Sqrt}[-1+\operatorname{Sqrt}[10]]*\operatorname{Sqrt}[1+3*x-2*x^2]))/2$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_.))*Sqrt[(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2]), x_Symbol]
:> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1016

```
Int[((g_.) + (h_.)*(x_.))*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^(p_.)*((d_.) + (e_.)*(x_.) + (f_.)*(x_.)^2)^(q_.), x_Symbol]
:> Simp[((a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*c*d - 2*a*c*e + a*b*f))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !(IntegerQ[p] && ILtQ[q, -1])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_.))/(((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)*Sqrt[(d_.) + (e_.)*(x_.) + (f_.)*(x_.)^2]), x_Symbol]
:> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1060

```
Int[((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^(p_.)*((A_.) + (B_.)*(x_.) + (C_.)*(x_.)^2)*((d_.) + (e_.)*(x_.) + (f_.)*(x_.)^2)^(q_.), x_Symbol]
:> Simp[((a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))
```

```
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
 \int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{5/2}} dx &= -\frac{2(15+14x)}{51(1+3x-2x^2)^{3/2}} + \frac{2}{51} \int \frac{-56 + \frac{235x}{2} + 84x^2}{(2+4x-3x^2)(1+3x-2x^2)^{3/2}} dx \\
 &= -\frac{2(15+14x)}{51(1+3x-2x^2)^{3/2}} - \frac{2(291+4814x)}{867\sqrt{1+3x-2x^2}} + \frac{4}{867} \int \frac{\frac{7803}{2} + \frac{23}{2}x}{(2+4x-3x^2)\sqrt{1+3x-2x^2}} dx \\
 &= -\frac{2(15+14x)}{51(1+3x-2x^2)^{3/2}} - \frac{2(291+4814x)}{867\sqrt{1+3x-2x^2}} + \frac{1}{5} (27(5-2\sqrt{10})) \int \frac{1}{(4-3x+2x^2)\sqrt{1+3x-2x^2}} dx \\
 &= -\frac{2(15+14x)}{51(1+3x-2x^2)^{3/2}} - \frac{2(291+4814x)}{867\sqrt{1+3x-2x^2}} - \frac{1}{5} (54(5-2\sqrt{10})) \text{Subst} \int \frac{1}{(4-3u+2u^2)\sqrt{1+3u-2u^2}} du \\
 &= -\frac{2(15+14x)}{51(1+3x-2x^2)^{3/2}} - \frac{2(291+4814x)}{867\sqrt{1+3x-2x^2}} + \frac{9}{2} \sqrt{\frac{1}{5}(-53+17\sqrt{10})} \tan^{-1} \left(\frac{3(\sqrt{10}-4) - (1+4\sqrt{10})x}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}} \right)
 \end{aligned}$$

Mathematica [A] time = 0.60, size = 185, normalized size = 0.96

$$\frac{3}{10} \sqrt{1+\sqrt{10}} (7\sqrt{10}-25) \tan^{-1} \left(\frac{3(\sqrt{10}-4) - (1+4\sqrt{10})x}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}} \right) - \frac{3}{10} \sqrt{\sqrt{10}-1} (25+7\sqrt{10}) \tanh^{-1} \left(\frac{4\sqrt{10}-2x}{2\sqrt{\sqrt{10}-1}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x - 2*x^2)^(5/2)),x]

[Out] $(-2*(546 + 5925*x + 13860*x^2 - 9628*x^3))/(867*(1 + 3*x - 2*x^2)^(3/2)) + (3*\sqrt{1 + \sqrt{10}}*(-25 + 7*\sqrt{10})*\text{ArcTan}[(3*(-4 + \sqrt{10}) - (1 + 4*\sqrt{10})*x)/(2*\sqrt{1 + \sqrt{10}}*\sqrt{1 + 3*x - 2*x^2})])/10 - (3*\sqrt{-1 + \sqrt{10}}*(25 + 7*\sqrt{10})*\text{ArcTanh}[(-3*(4 + \sqrt{10}) + (-1 + 4*\sqrt{10})*x)/(2*\sqrt{-1 + \sqrt{10}}*\sqrt{1 + 3*x - 2*x^2})])/10$

fricas [B] time = 0.99, size = 439, normalized size = 2.27

$$\frac{43680x^4 - 131040x^3 - 31212\sqrt{5}(4x^4 - 12x^3 + 5x^2 + 6x + 1)\sqrt{17\sqrt{10} - 53} \arctan\left(\frac{\sqrt{2}(\sqrt{10}\sqrt{5x+10}\sqrt{5x})\sqrt{17}}{\dots}\right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(5/2),x, algorithm="fricas")

[Out] $-1/8670*(43680*x^4 - 131040*x^3 - 31212*\sqrt{5}*(4*x^4 - 12*x^3 + 5*x^2 + 6*x + 1)*\sqrt{17*\sqrt{10} - 53}*\arctan(1/90*(\sqrt{2}*(\sqrt{10}*\sqrt{5}*x + 10*\sqrt{5})*x)*\sqrt{17*\sqrt{10} - 53}*\sqrt{(6*x^2 + \sqrt{10}*(3*x^2 + 2*x) - 2*\sqrt{-2*x^2 + 3*x + 1}*(\sqrt{10}*x + 2*x + 2) + 10*x + 4)/x^2}) + 2*(\sqrt{10}*\sqrt{5}*(6*x + 1) - \sqrt{-2*x^2 + 3*x + 1}*(\sqrt{10}*\sqrt{5} + 10*\sqrt{5})) + 5*\sqrt{5}*(3*x + 2))*\sqrt{17*\sqrt{10} - 53})/x - 7803*\sqrt{5}*(4*x^4 - 12*x^3 + 5*x^2 + 6*x + 1)*\sqrt{17*\sqrt{10} + 53}*\log(9*(45*\sqrt{10}*x + (13*\sqrt{10}*\sqrt{5})*x - 40*\sqrt{5})*x)*\sqrt{17*\sqrt{10} + 53} - 90*x + 90*\sqrt{-2*x^2 + 3*x + 1} - 90)/x + 7803*\sqrt{5}*(4*x^4 - 12*x^3 + 5*x^2 + 6*x + 1)*\sqrt{17*\sqrt{10} + 53}*\log(9*(45*\sqrt{10}*x - (13*\sqrt{10}*\sqrt{5})*x - 40*\sqrt{5})*x)*\sqrt{17*\sqrt{10} + 53} - 90*x + 90*\sqrt{-2*x^2 + 3*x + 1} - 90)/x + 54600*x^2 - 20*(9628*x^3 - 13860*x^2 - 5925*x - 546)*\sqrt{-2*x^2 + 3*x + 1} + 65520*x + 10920)/(4*x^4 - 12*x^3 + 5*x^2 + 6*x + 1)$

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(5/2),x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.02, size = 1560, normalized size = 8.08

Expression too large to display

$$\begin{aligned} & /2)/(-2*(x-2/3-1/3*10^{(1/2)})^2+(1/3-4/3*10^{(1/2)})*(x-2/3-1/3*10^{(1/2)})-1/9+ \\ & 1/9*10^{(1/2)})^{(3/2)}-62/153/(-1/9+1/9*10^{(1/2)})^2/(-2*(x-2/3-1/3*10^{(1/2)})^2 \\ & +(1/3-4/3*10^{(1/2)})*(x-2/3-1/3*10^{(1/2)})-1/9+1/9*10^{(1/2)})^{(1/2)}*x-62/459/(\\ & -1/9+1/9*10^{(1/2)})/(-2*(x-2/3-1/3*10^{(1/2)})^2+(1/3-4/3*10^{(1/2)})*(x-2/3-1/3 \\ & *10^{(1/2)})-1/9+1/9*10^{(1/2)})^{(3/2)}*x+128/13005*10^{(1/2)}/(-1/9+1/9*10^{(1/2)}) \\ & /(-2*(x-2/3-1/3*10^{(1/2)})^2+(1/3-4/3*10^{(1/2)})*(x-2/3-1/3*10^{(1/2)})-1/9+1/9 \\ & *10^{(1/2)})^{(1/2)}-992/7803/(-1/9+1/9*10^{(1/2)})/(-2*(x-2/3-1/3*10^{(1/2)})^2+(1 \\ & /3-4/3*10^{(1/2)})*(x-2/3-1/3*10^{(1/2)})-1/9+1/9*10^{(1/2)})^{(1/2)}*x-128/13005*1 \\ & 0^{(1/2)}/(-1/9-1/9*10^{(1/2)})/(-2*(x-2/3+1/3*10^{(1/2)})^2+(1/3+4/3*10^{(1/2)})*(\\ & x-2/3+1/3*10^{(1/2)})-1/9-1/9*10^{(1/2)})^{(1/2)}-992/7803/(-1/9-1/9*10^{(1/2)})/(- \\ & 2*(x-2/3+1/3*10^{(1/2)})^2+(1/3+4/3*10^{(1/2)})*(x-2/3+1/3*10^{(1/2)})-1/9-1/9*10 \\ & ^{(1/2)})^{(1/2)}*x \end{aligned}$$

maxima [B] time = 1.26, size = 1276, normalized size = 6.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(5/2),x, algorithm="maxima")
[Out] 1/17340*sqrt(10)*(2108*sqrt(10)*x/(sqrt(10)*(-2*x^2 + 3*x + 1)^(3/2) + (-2*x^2 + 3*x + 1)^(3/2)) - 2108*sqrt(10)*x/(sqrt(10)*(-2*x^2 + 3*x + 1)^(3/2) - (-2*x^2 + 3*x + 1)^(3/2)) - 56916*sqrt(10)*x/(2*sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + 11*sqrt(-2*x^2 + 3*x + 1)) + 56916*sqrt(10)*x/(2*sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - 11*sqrt(-2*x^2 + 3*x + 1)) + 1984*sqrt(10)*x/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1)) - 1984*sqrt(10)*x/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1)) - 70227*sqrt(10)*arcsin(8/17*sqrt(17)*sqrt(10)*x/abs(6*x + 2*sqrt(10) - 4) + 2/17*sqrt(17)*x/abs(6*x + 2*sqrt(10) - 4) - 6/17*sqrt(17)*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 24/17*sqrt(17)/abs(6*x + 2*sqrt(10) - 4))/(2*sqrt(10)*sqrt(sqrt(10) + 1) + 11*sqrt(sqrt(10) + 1)) - 2176*x/(sqrt(10)*(-2*x^2 + 3*x + 1)^(3/2) + (-2*x^2 + 3*x + 1)^(3/2)) - 2176*x/(sqrt(10)*(-2*x^2 + 3*x + 1)^(3/2) - (-2*x^2 + 3*x + 1)^(3/2)) + 58752*x/(2*sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + 11*sqrt(-2*x^2 + 3*x + 1)) + 58752*x/(2*sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - 11*sqrt(-2*x^2 + 3*x + 1)) - 2048*x/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1)) - 2048*x/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1)) + 561816*arcsin(8/17*sqrt(17)*sqrt(10)*x/abs(6*x + 2*sqrt(10) - 4) + 2/17*sqrt(17)*x/abs(6*x + 2*sqrt(10) - 4) - 6/17*sqrt(17)*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 24/17*sqrt(17)/abs(6*x + 2*sqrt(10) - 4))/(2*sqrt(10)*sqrt(sqrt(10) + 1) + 11*sqrt(sqrt(10) + 1)) - 714*sqrt(10)/(sqrt(10)*(-2*x^2 + 3*x + 1)^(3/2) + (-2*x^2 + 3*x + 1)^(3/2)) + 714*sqrt(10)/(sqrt(10)*(-2*x^2 + 3*x + 1)^(3/2) - (-2*x^2 + 3*x + 1)^(3/2)) + 19278*sqrt(10)/(2*sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + 11*sqrt(-2*x^2 + 3*x + 1)) - 19278*sqrt(10)/(2*sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - 11*sqrt(-2*x^2 + 3*x + 1)) - 1488*sqrt(10)/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1)) + 1488*sqrt(10)/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1))
```

) $\sqrt{-2x^2 + 3x + 1} - \sqrt{-2x^2 + 3x + 1}) - 5304/(\sqrt{10})*(-2x^2 + 3x + 1)^{(3/2)} + (-2x^2 + 3x + 1)^{(3/2)}) - 5304/(\sqrt{10})*(-2x^2 + 3x + 1)^{(3/2)} - (-2x^2 + 3x + 1)^{(3/2)}) + 143208/(2*\sqrt{10}*\sqrt{-2x^2 + 3x + 1}) + 11*\sqrt{-2x^2 + 3x + 1}) + 143208/(2*\sqrt{10}*\sqrt{-2x^2 + 3x + 1}) - 11*\sqrt{-2x^2 + 3x + 1}) + 1536/(\sqrt{10}*\sqrt{-2x^2 + 3x + 1}) + \sqrt{-2x^2 + 3x + 1}) + 1536/(\sqrt{10}*\sqrt{-2x^2 + 3x + 1}) - \sqrt{-2x^2 + 3x + 1}) + 70227*\sqrt{10}*\log(-2/9*\sqrt{10} + 2/3*\sqrt{-2x^2 + 3x + 1})*\sqrt{\sqrt{10} - 1}/\text{abs}(6x - 2*\sqrt{10} - 4) + 2/9*\sqrt{10}/\text{abs}(6x - 2*\sqrt{10} - 4) - 2/9/\text{abs}(6x - 2*\sqrt{10} - 4) + 1/18)/(\sqrt{10} - 1)^{(5/2)} + 561816*\log(-2/9*\sqrt{10} + 2/3*\sqrt{-2x^2 + 3x + 1})*\sqrt{\sqrt{10} - 1}/\text{abs}(6x - 2*\sqrt{10} - 4) + 2/9*\sqrt{10}/\text{abs}(6x - 2*\sqrt{10} - 4) - 2/9/\text{abs}(6x - 2*\sqrt{10} - 4) + 1/18)/(\sqrt{10} - 1)^{(5/2)})$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+2}{(-2x^2+3x+1)^{5/2}(-3x^2+4x+2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x + 2)/((3*x - 2*x^2 + 1)^(5/2)*(4*x - 3*x^2 + 2)), x)`

[Out] `int((x + 2)/((3*x - 2*x^2 + 1)^(5/2)*(4*x - 3*x^2 + 2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{12x^6\sqrt{-2x^2+3x+1} - 52x^5\sqrt{-2x^2+3x+1} + 55x^4\sqrt{-2x^2+3x+1} + 22x^3\sqrt{-2x^2+3x+1} - 31x^2\sqrt{-2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2+x)/(-3*x**2+4*x+2)/(-2*x**2+3*x+1)**(5/2), x)`

[Out] `-Integral(x/(12*x**6*sqrt(-2*x**2 + 3*x + 1) - 52*x**5*sqrt(-2*x**2 + 3*x + 1) + 55*x**4*sqrt(-2*x**2 + 3*x + 1) + 22*x**3*sqrt(-2*x**2 + 3*x + 1) - 31*x**2*sqrt(-2*x**2 + 3*x + 1) - 16*x*sqrt(-2*x**2 + 3*x + 1) - 2*sqrt(-2*x**2 + 3*x + 1)), x) - Integral(2/(12*x**6*sqrt(-2*x**2 + 3*x + 1) - 52*x**5*sqrt(-2*x**2 + 3*x + 1) + 55*x**4*sqrt(-2*x**2 + 3*x + 1) + 22*x**3*sqrt(-2*x**2 + 3*x + 1) - 31*x**2*sqrt(-2*x**2 + 3*x + 1) - 16*x*sqrt(-2*x**2 + 3*x + 1) - 2*sqrt(-2*x**2 + 3*x + 1)), x)`

$$3.28 \quad \int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x+2x^2}} dx$$

Optimal. Leaf size=151

$$\frac{1}{2}\sqrt{1-\frac{7\sqrt{\frac{2}{5}}}{5}} \tanh^{-1}\left(\frac{(17+4\sqrt{10})x+3(4+\sqrt{10})}{2\sqrt{55+17\sqrt{10}}\sqrt{2x^2+3x+1}}\right) - \frac{1}{2}\sqrt{1+\frac{7\sqrt{\frac{2}{5}}}{5}} \tanh^{-1}\left(\frac{(17-4\sqrt{10})x+3(4-\sqrt{10})}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)$$

[Out] 1/10*arctanh(1/2*(12+3*10^(1/2)+x*(17+4*10^(1/2)))/(2*x^2+3*x+1)^(1/2)/(55+17*10^(1/2))^(1/2))*(25-7*10^(1/2))^(1/2)-1/10*arctanh(1/2*(x*(17-4*10^(1/2))+12-3*10^(1/2))/(2*x^2+3*x+1)^(1/2)/(55-17*10^(1/2))^(1/2))*(25+7*10^(1/2))^(1/2)

Rubi [A] time = 0.23, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1032, 724, 206}

$$\frac{1}{2}\sqrt{1-\frac{7\sqrt{\frac{2}{5}}}{5}} \tanh^{-1}\left(\frac{(17+4\sqrt{10})x+3(4+\sqrt{10})}{2\sqrt{55+17\sqrt{10}}\sqrt{2x^2+3x+1}}\right) - \frac{1}{2}\sqrt{1+\frac{7\sqrt{\frac{2}{5}}}{5}} \tanh^{-1}\left(\frac{(17-4\sqrt{10})x+3(4-\sqrt{10})}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)$$

Antiderivative was successfully verified.

[In] Int[(2 + x)/((2 + 4*x - 3*x^2)*Sqrt[1 + 3*x + 2*x^2]), x]

[Out] -(Sqrt[1 + (7*Sqrt[2/5])/5]*ArcTanh[(3*(4 - Sqrt[10]) + (17 - 4*Sqrt[10]))*x]/(2*Sqrt[55 - 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2]))/2 + (Sqrt[1 - (7*Sqrt[2/5])/5]*ArcTanh[(3*(4 + Sqrt[10]) + (17 + 4*Sqrt[10]))*x]/(2*Sqrt[55 + 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2]))/2

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned} \int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x+2x^2}} dx &= \frac{1}{5}(5-4\sqrt{10}) \int \frac{1}{(4-2\sqrt{10}-6x)\sqrt{1+3x+2x^2}} dx + \frac{1}{5}(5+4\sqrt{10}) \int \frac{1}{(4+2\sqrt{10}-6x)\sqrt{1+3x+2x^2}} dx \\ &= -\left(\frac{1}{5}(2(5-4\sqrt{10}))\right) \text{Subst}\left(\int \frac{1}{144+72(4-2\sqrt{10})+8(4-2\sqrt{10})^2-x}\right) \\ &= -\frac{1}{10}\sqrt{25+7\sqrt{10}} \tanh^{-1}\left(\frac{3(4-\sqrt{10})+(17-4\sqrt{10})x}{2\sqrt{55-17\sqrt{10}}\sqrt{1+3x+2x^2}}\right) + \frac{1}{10}\sqrt{25-7\sqrt{10}} \tanh^{-1}\left(\frac{3(4+\sqrt{10})+(17+4\sqrt{10})x}{2\sqrt{55+17\sqrt{10}}\sqrt{1+3x+2x^2}}\right) \end{aligned}$$

Mathematica [A] time = 0.36, size = 148, normalized size = 0.98

$$\frac{(5-4\sqrt{10}) \tanh^{-1}\left(\frac{-4\sqrt{10}x+17x-3\sqrt{10}+12}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right) + 3\sqrt{285-90\sqrt{10}} \tanh^{-1}\left(\frac{(17+4\sqrt{10})x+3(4+\sqrt{10})}{2\sqrt{55+17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)}{10\sqrt{55-17\sqrt{10}}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(2 + x)/((2 + 4*x - 3*x^2)*Sqrt[1 + 3*x + 2*x^2]), x]

[Out] ((5 - 4*Sqrt[10])*ArcTanh[(12 - 3*Sqrt[10] + 17*x - 4*Sqrt[10]*x)/(2*Sqrt[55 - 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])]) + 3*Sqrt[285 - 90*Sqrt[10]]*ArcTanh[(3*(4 + Sqrt[10]) + (17 + 4*Sqrt[10])*x)/(2*Sqrt[55 + 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/(10*Sqrt[55 - 17*Sqrt[10]])

fricas [B] time = 1.23, size = 245, normalized size = 1.62

$$\frac{1}{10}\sqrt{7\sqrt{10}+25} \log\left(-\frac{3\sqrt{10}x+(\sqrt{10}x-4x)\sqrt{7\sqrt{10}+25}+6x-6\sqrt{2x^2+3x+1}+6}{x}\right) - \frac{1}{10}\sqrt{7\sqrt{10}+25}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(1/2),x, algorithm="fricas")

[Out] $\frac{1}{10}\sqrt{7\sqrt{10} + 25}\log(-3\sqrt{10}x + (\sqrt{10}x - 4x)\sqrt{7\sqrt{10} + 25} + 6x - 6\sqrt{2x^2 + 3x + 1} + 6)/x - \frac{1}{10}\sqrt{7\sqrt{10} + 25}\log(-3\sqrt{10}x - (\sqrt{10}x - 4x)\sqrt{7\sqrt{10} + 25} + 6x - 6\sqrt{2x^2 + 3x + 1} + 6)/x + \frac{1}{10}\sqrt{-7\sqrt{10} + 25}\log((3\sqrt{10}x + (\sqrt{10}x + 4x)\sqrt{-7\sqrt{10} + 25} - 6x + 6\sqrt{2x^2 + 3x + 1} - 6)/x - \frac{1}{10}\sqrt{-7\sqrt{10} + 25}\log((3\sqrt{10}x - (\sqrt{10}x + 4x)\sqrt{-7\sqrt{10} + 25} - 6x + 6\sqrt{2x^2 + 3x + 1} - 6)/x)$

giac [A] time = 0.48, size = 93, normalized size = 0.62

$0.169235232112667 \log\left(-\sqrt{2}x + \sqrt{2x^2 + 3x + 1} + 5.90976932712000\right) - 0.686556214893333 \log\left(-\sqrt{2}x + \dots\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(1/2),x, algorithm="giac")

[Out] $0.169235232112667*\log(-\sqrt{2}*x + \sqrt{2*x^2 + 3*x + 1} + 5.90976932712000) - 0.686556214893333*\log(-\sqrt{2}*x + \sqrt{2*x^2 + 3*x + 1} - 0.176527156327000) + 0.686556214893333*\log(-\sqrt{2}*x + \sqrt{2*x^2 + 3*x + 1} - 0.919278730509000) - 0.169235232112667*\log(-\sqrt{2}*x + \sqrt{2*x^2 + 3*x + 1} - 1.04272727395000)$

maple [A] time = 0.05, size = 186, normalized size = 1.23

$$\frac{(-8 + \sqrt{10})\sqrt{10} \operatorname{arctanh}\left(\frac{55 - 17\sqrt{10} + \frac{9\left(\frac{17}{3} - \frac{4\sqrt{10}}{3}\right)\left(x - \frac{2}{3} + \frac{\sqrt{10}}{3}\right)}{2}}{\sqrt{55 - 17\sqrt{10}} \sqrt{18\left(x - \frac{2}{3} + \frac{\sqrt{10}}{3}\right)^2 + 9\left(\frac{17}{3} - \frac{4\sqrt{10}}{3}\right)\left(x - \frac{2}{3} + \frac{\sqrt{10}}{3}\right) + 55 - 17\sqrt{10}}}\right)}{20\sqrt{55 - 17\sqrt{10}}} + \frac{(8 + \sqrt{10})\sqrt{10} \operatorname{arctanh}\left(\dots\right)}{20\sqrt{55 - 17\sqrt{10}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(1/2),x)

[Out] $\frac{1}{20}*(-8+10^{(1/2)})*10^{(1/2)}/(55-17*10^{(1/2)})^{(1/2)}*\operatorname{arctanh}\left(\frac{9/2*(110/9-34/9*10^{(1/2)}+(17/3-4/3*10^{(1/2)})*(x-2/3+1/3*10^{(1/2)}))}{(55-17*10^{(1/2)})^{(1/2)}}\right)/((18*(x-2/3+1/3*10^{(1/2)})^2+9*(17/3-4/3*10^{(1/2)})*(x-2/3+1/3*10^{(1/2)})+55-17*10^{(1/2)})^{(1/2)}) + \frac{1}{20}*(8+10^{(1/2)})*10^{(1/2)}/(55+17*10^{(1/2)})^{(1/2)}*\operatorname{arctanh}\left(\frac{9/2*(110/9+34/9*10^{(1/2)}+(17/3+4/3*10^{(1/2)})*(x-2/3-1/3*10^{(1/2)}))}{(55+17*10^{(1/2)})^{(1/2)}}\right)/((18*(x-2/3-1/3*10^{(1/2)})^2+9*(17/3+4/3*10^{(1/2)})*(x-2/3-1/3*10^{(1/2)})+55+17*10^{(1/2)})^{(1/2)})$

maxima [B] time = 1.07, size = 363, normalized size = 2.40

$$\frac{1}{60} \sqrt{10} \left(\frac{3 \sqrt{10} \log \left(\frac{2}{9} \sqrt{10} + \frac{2 \sqrt{2x^2+3x+1} \sqrt{17 \sqrt{10}+55}}{3|6x-2\sqrt{10}-4|} + \frac{34 \sqrt{10}}{9|6x-2\sqrt{10}-4|} + \frac{110}{9|6x-2\sqrt{10}-4|} + \frac{17}{18} \right)}{\sqrt{17 \sqrt{10} + 55}} + \frac{\sqrt{10} \log \left(-\frac{2}{9} \sqrt{10} \right)}{\sqrt{17 \sqrt{10} + 55}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(1/2),x, algorithm="maxima")

[Out] 1/60*sqrt(10)*(3*sqrt(10)*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1)*sqrt(17*sqrt(10) + 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) + 110/9/abs(6*x - 2*sqrt(10) - 4) + 17/18)/sqrt(17*sqrt(10) + 55) + sqrt(10)*log(-2/9*sqrt(10) + 2*sqrt(2*x^2 + 3*x + 1)*sqrt(-17/9*sqrt(10) + 55/9)/abs(6*x + 2*sqrt(10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 110/9/abs(6*x + 2*sqrt(10) - 4) + 17/18)/sqrt(-17/9*sqrt(10) + 55/9) + 24*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1)*sqrt(17*sqrt(10) + 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) + 110/9/abs(6*x - 2*sqrt(10) - 4) + 17/18)/sqrt(17*sqrt(10) + 55) - 8*log(-2/9*sqrt(10) + 2*sqrt(2*x^2 + 3*x + 1)*sqrt(-17/9*sqrt(10) + 55/9)/abs(6*x + 2*sqrt(10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 110/9/abs(6*x + 2*sqrt(10) - 4) + 17/18)/sqrt(-17/9*sqrt(10) + 55/9))

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+2}{\sqrt{2x^2+3x+1}(-3x^2+4x+2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 2)/((3*x + 2*x^2 + 1)^(1/2)*(4*x - 3*x^2 + 2)),x)

[Out] int((x + 2)/((3*x + 2*x^2 + 1)^(1/2)*(4*x - 3*x^2 + 2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{3x^2\sqrt{2x^2+3x+1}-4x\sqrt{2x^2+3x+1}-2\sqrt{2x^2+3x+1}} dx - \int \frac{2}{3x^2\sqrt{2x^2+3x+1}-4x\sqrt{2x^2+3x+1}-2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(-3*x**2+4*x+2)/(2*x**2+3*x+1)**(1/2),x)

[Out] -Integral(x/(3*x**2*sqrt(2*x**2 + 3*x + 1) - 4*x*sqrt(2*x**2 + 3*x + 1) - 2*sqrt(2*x**2 + 3*x + 1)), x) - Integral(2/(3*x**2*sqrt(2*x**2 + 3*x + 1) - 4*x*sqrt(2*x**2 + 3*x + 1) - 2*sqrt(2*x**2 + 3*x + 1)), x)

$$3.29 \quad \int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{3/2}} dx$$

Optimal. Leaf size=174

$$\frac{2(22x+21)}{5\sqrt{2x^2+3x+1}} - \frac{1}{10} \sqrt{\frac{3}{5}} (2065+653\sqrt{10}) \tanh^{-1} \left(\frac{(17-4\sqrt{10})x+3(4-\sqrt{10})}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}} \right) + \frac{1}{10} \sqrt{\frac{3}{5}} (2065-653\sqrt{10})$$

[Out] 2/5*(21+22*x)/(2*x^2+3*x+1)^(1/2)+1/50*arctanh(1/2*(12+3*10^(1/2)+x*(17+4*10^(1/2)))/(2*x^2+3*x+1)^(1/2)/(55+17*10^(1/2))^(1/2))*(30975-9795*10^(1/2))^(1/2)-1/50*arctanh(1/2*(x*(17-4*10^(1/2))+12-3*10^(1/2))/(2*x^2+3*x+1)^(1/2)/(55-17*10^(1/2))^(1/2))*(30975+9795*10^(1/2))^(1/2)

Rubi [A] time = 0.25, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1016, 1032, 724, 206}

$$\frac{2(22x+21)}{5\sqrt{2x^2+3x+1}} - \frac{1}{10} \sqrt{\frac{3}{5}} (2065+653\sqrt{10}) \tanh^{-1} \left(\frac{(17-4\sqrt{10})x+3(4-\sqrt{10})}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}} \right) + \frac{1}{10} \sqrt{\frac{3}{5}} (2065-653\sqrt{10})$$

Antiderivative was successfully verified.

[In] Int[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x + 2*x^2)^(3/2)), x]

[Out] (2*(21 + 22*x))/(5*Sqrt[1 + 3*x + 2*x^2]) - (Sqrt[(3*(2065 + 653*Sqrt[10]))/5]*ArcTanh[(3*(4 - Sqrt[10]) + (17 - 4*Sqrt[10])*x)/(2*Sqrt[55 - 17*Sqrt[10]])*Sqrt[1 + 3*x + 2*x^2]])/10 + (Sqrt[(3*(2065 - 653*Sqrt[10]))/5]*ArcTanh[(3*(4 + Sqrt[10]) + (17 + 4*Sqrt[10])*x)/(2*Sqrt[55 + 17*Sqrt[10]])*Sqrt[1 + 3*x + 2*x^2]])/10

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_) + (e_)*(x_)*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1016

```

Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1) * (d + e*x + f*x^2)^(q + 1) * (g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*c*d - 2*a*c*e + a*b*f))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1])

```

Rule 1032

```

Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]

```

Rubi steps

$$\begin{aligned}
\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{3/2}} dx &= \frac{2(21+22x)}{5\sqrt{1+3x+2x^2}} - \frac{2}{15} \int \frac{-72 + \frac{81x}{2}}{(2+4x-3x^2)\sqrt{1+3x+2x^2}} dx \\
&= \frac{2(21+22x)}{5\sqrt{1+3x+2x^2}} - \frac{1}{5} (9(3-\sqrt{10})) \int \frac{1}{(4+2\sqrt{10}-6x)\sqrt{1+3x+2x^2}} dx \\
&= \frac{2(21+22x)}{5\sqrt{1+3x+2x^2}} + \frac{1}{5} (18(3-\sqrt{10})) \text{Subst} \left(\int \frac{1}{144+72(4+2\sqrt{10})} \right) \\
&= \frac{2(21+22x)}{5\sqrt{1+3x+2x^2}} - \frac{1}{10} \sqrt{\frac{3}{5}} (2065+653\sqrt{10}) \tanh^{-1} \left(\frac{3(4-\sqrt{10}) + \sqrt{2x^2+3x+1}}{2\sqrt{55-17\sqrt{10}}} \right)
\end{aligned}$$

Mathematica [A] time = 0.57, size = 172, normalized size = 0.99

$$\frac{1}{50} \left(\frac{\sqrt{30975-9795\sqrt{10}} \sqrt{2x^2+3x+1} \tanh^{-1} \left(\frac{4\sqrt{10}x+17x+3\sqrt{10}+12}{2\sqrt{55+17\sqrt{10}} \sqrt{2x^2+3x+1}} \right) + 440x + 420}{\sqrt{2x^2+3x+1}} - \sqrt{30975+9795\sqrt{10}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x + 2*x^2)^(3/2)),x]

[Out] (-(Sqrt[30975 + 9795*Sqrt[10]]*ArcTanh[(12 - 3*Sqrt[10] + 17*x - 4*Sqrt[10]*x)/(2*Sqrt[55 - 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])]) + (420 + 440*x + Sqrt[30975 - 9795*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2]*ArcTanh[(12 + 3*Sqrt[10] + 17*x + 4*Sqrt[10]*x)/(2*Sqrt[55 + 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/Sqrt[1 + 3*x + 2*x^2])/50

fricas [B] time = 0.97, size = 365, normalized size = 2.10

$$\frac{\sqrt{5}(2x^2+3x+1)\sqrt{1959\sqrt{10}+6195} \log\left(-\frac{45\sqrt{10}x+(41\sqrt{10}\sqrt{5}x-130\sqrt{5}x)\sqrt{1959\sqrt{10}+6195+90x-90\sqrt{2x^2+3x+1}+90}}{x}\right)}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(3/2),x, algorithm="fricas")

```
[Out] 1/50*(sqrt(5)*(2*x^2 + 3*x + 1)*sqrt(1959*sqrt(10) + 6195)*log(-(45*sqrt(10)
)*x + (41*sqrt(10)*sqrt(5)*x - 130*sqrt(5)*x)*sqrt(1959*sqrt(10) + 6195) +
90*x - 90*sqrt(2*x^2 + 3*x + 1) + 90)/x) - sqrt(5)*(2*x^2 + 3*x + 1)*sqrt(1
959*sqrt(10) + 6195)*log(-(45*sqrt(10)*x - (41*sqrt(10)*sqrt(5)*x - 130*sqrt
(5)*x)*sqrt(1959*sqrt(10) + 6195) + 90*x - 90*sqrt(2*x^2 + 3*x + 1) + 90)/
x) + sqrt(5)*(2*x^2 + 3*x + 1)*sqrt(-1959*sqrt(10) + 6195)*log((45*sqrt(10)
*x + (41*sqrt(10)*sqrt(5)*x + 130*sqrt(5)*x)*sqrt(-1959*sqrt(10) + 6195) -
90*x + 90*sqrt(2*x^2 + 3*x + 1) - 90)/x) - sqrt(5)*(2*x^2 + 3*x + 1)*sqrt(-
1959*sqrt(10) + 6195)*log((45*sqrt(10)*x - (41*sqrt(10)*sqrt(5)*x + 130*sqrt
(5)*x)*sqrt(-1959*sqrt(10) + 6195) - 90*x + 90*sqrt(2*x^2 + 3*x + 1) - 90)
/x) + 840*x^2 + 20*sqrt(2*x^2 + 3*x + 1)*(22*x + 21) + 1260*x + 420)/(2*x^2
+ 3*x + 1)
```

giac [A] time = 0.50, size = 112, normalized size = 0.64

$$\frac{2(22x + 21)}{5\sqrt{2x^2 + 3x + 1}} + 0.0140045514133333 \log\left(-\sqrt{2}x + \sqrt{2x^2 + 3x + 1} + 5.90976932712000\right) - 4.9779316862000$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(3/2),x, algorithm="giac")
```

```
[Out] 2/5*(22*x + 21)/sqrt(2*x^2 + 3*x + 1) + 0.0140045514133333*log(-sqrt(2)*x +
sqrt(2*x^2 + 3*x + 1) + 5.90976932712000) - 4.97793168620000*log(-sqrt(2)*
x + sqrt(2*x^2 + 3*x + 1) - 0.176527156327000) + 4.97793168620000*log(-sqrt
(2)*x + sqrt(2*x^2 + 3*x + 1) - 0.919278730509000) - 0.0140045514125333*log
(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 1.04272727395000)
```

maple [B] time = 0.02, size = 466, normalized size = 2.68

$$(-8 + \sqrt{10})\sqrt{10} \left[-\frac{\operatorname{arctanh}\left(\frac{9\left(\frac{17}{3} - \frac{4\sqrt{10}}{3}\right)\left(x - \frac{2}{3} + \frac{\sqrt{10}}{3}\right)}{55 - 17\sqrt{10} + 2}\right)}{\sqrt{55 - 17\sqrt{10}} \sqrt{18\left(x - \frac{2}{3} + \frac{\sqrt{10}}{3}\right)^2 + 9\left(\frac{17}{3} - \frac{4\sqrt{10}}{3}\right)\left(x - \frac{2}{3} + \frac{\sqrt{10}}{3}\right) + 55 - 17\sqrt{10}}}\right] + \frac{1}{3\left(\frac{55}{9} - \frac{17\sqrt{10}}{9}\right)\sqrt{2\left(x - \frac{2}{3} + \frac{\sqrt{10}}{3}\right)^2 + \left(\frac{17}{3} - \frac{4\sqrt{10}}{3}\right)\left(x - \frac{2}{3} + \frac{\sqrt{10}}{3}\right) + 55 - 17\sqrt{10}}}$$

20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(3/2),x)
```

```
[Out] -1/20*(-8+10^(1/2))*10^(1/2)*(1/3/(55/9-17/9*10^(1/2)))/(2*(x-2/3+1/3*10^(1/
2))^2+(17/3-4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))+55/9-17/9*10^(1/2))^(1/2)-1/
3*(17/3-4/3*10^(1/2))/(55/9-17/9*10^(1/2))*(3+4*x)/(440/9-136/9*10^(1/2)-(1
```

$$\frac{7/3-4/3*10^{(1/2)})^2/(2*(x-2/3+1/3*10^{(1/2)})^2+(17/3-4/3*10^{(1/2)})*(x-2/3+1/3*10^{(1/2)})+55/9-17/9*10^{(1/2)})^{(1/2)}-1/(55/9-17/9*10^{(1/2)})/(55-17*10^{(1/2)})^{(1/2)}*\operatorname{arctanh}(9/2*(110/9-34/9*10^{(1/2)}+(17/3-4/3*10^{(1/2)})*(x-2/3+1/3*10^{(1/2)})))/(55-17*10^{(1/2)})^{(1/2)}/(18*(x-2/3+1/3*10^{(1/2)})^2+9*(17/3-4/3*10^{(1/2)})*(x-2/3+1/3*10^{(1/2)})+55-17*10^{(1/2)})^{(1/2)})-1/20*(8+10^{(1/2)})*10^{(1/2)}*(1/3/(55/9+17/9*10^{(1/2)})/(2*(x-2/3-1/3*10^{(1/2)})^2+(17/3+4/3*10^{(1/2)})*(x-2/3-1/3*10^{(1/2)})+55/9+17/9*10^{(1/2)})^{(1/2)}-1/3*(17/3+4/3*10^{(1/2)})/(55/9+17/9*10^{(1/2)})*(3+4*x)/(440/9+136/9*10^{(1/2)}-(17/3+4/3*10^{(1/2)})^2)/(2*(x-2/3-1/3*10^{(1/2)})^2+(17/3+4/3*10^{(1/2)})*(x-2/3-1/3*10^{(1/2)})+55/9+17/9*10^{(1/2)})^{(1/2)}-1/(55/9+17/9*10^{(1/2)})/(55+17*10^{(1/2)})^{(1/2)}*\operatorname{arctanh}(9/2*(110/9+34/9*10^{(1/2)}+(17/3+4/3*10^{(1/2)})*(x-2/3-1/3*10^{(1/2)})))/(55+17*10^{(1/2)})^{(1/2)}/(18*(x-2/3-1/3*10^{(1/2)})^2+9*(17/3+4/3*10^{(1/2)})*(x-2/3-1/3*10^{(1/2)})+55+17*10^{(1/2)})^{(1/2)})$$

maxima [B] time = 1.10, size = 668, normalized size = 3.84

$$-\frac{1}{60}\sqrt{10}\left(\frac{588\sqrt{10}x}{17\sqrt{10}\sqrt{2x^2+3x+1}+55\sqrt{2x^2+3x+1}}-\frac{588\sqrt{10}x}{17\sqrt{10}\sqrt{2x^2+3x+1}-55\sqrt{2x^2+3x+1}}\right)+\frac{1}{17\sqrt{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(3/2),x, algorithm="maxima")

[Out]
$$-1/60*\sqrt{10}*(588*\sqrt{10}*x/(17*\sqrt{10}*\sqrt{2*x^2+3*x+1})+55*\sqrt{10}*\sqrt{2*x^2+3*x+1})-588*\sqrt{10}*x/(17*\sqrt{10}*\sqrt{2*x^2+3*x+1}-55*\sqrt{10}*\sqrt{2*x^2+3*x+1})+2112*x/(17*\sqrt{10}*\sqrt{2*x^2+3*x+1})+55*\sqrt{10}*\sqrt{2*x^2+3*x+1})+2112*x/(17*\sqrt{10}*\sqrt{2*x^2+3*x+1}-55*\sqrt{10}*\sqrt{2*x^2+3*x+1})-27*\sqrt{10}*\log(2/9*\sqrt{10}+2/3*\sqrt{2*x^2+3*x+1})*\sqrt{17*\sqrt{10}+55}/\operatorname{abs}(6*x-2*\sqrt{10}-4)+34/9*\sqrt{10}/\operatorname{abs}(6*x-2*\sqrt{10}-4)+110/9/\operatorname{abs}(6*x-2*\sqrt{10}-4)+17/18)/(17*\sqrt{10}+55)^{(3/2)}-\sqrt{10}*\log(-2/9*\sqrt{10}+2*\sqrt{2*x^2+3*x+1})*\sqrt{(-17/9*\sqrt{10}+55/9)/\operatorname{abs}(6*x+2*\sqrt{10}-4)}-34/9*\sqrt{10}/\operatorname{abs}(6*x+2*\sqrt{10}-4)+110/9/\operatorname{abs}(6*x+2*\sqrt{10}-4)+17/18)/(-17/9*\sqrt{10}+55/9)^{(3/2)}+450*\sqrt{10}/(17*\sqrt{10}*\sqrt{2*x^2+3*x+1})+55*\sqrt{10}*\sqrt{2*x^2+3*x+1})-450*\sqrt{10}/(17*\sqrt{10}*\sqrt{2*x^2+3*x+1}-55*\sqrt{10}*\sqrt{2*x^2+3*x+1})-216*\log(2/9*\sqrt{10}+2/3*\sqrt{2*x^2+3*x+1})*\sqrt{17*\sqrt{10}+55}/\operatorname{abs}(6*x-2*\sqrt{10}-4)+34/9*\sqrt{10}/\operatorname{abs}(6*x-2*\sqrt{10}-4)+110/9/\operatorname{abs}(6*x-2*\sqrt{10}-4)+17/18)/(17*\sqrt{10}+55)^{(3/2)}+8*\log(-2/9*\sqrt{10}+2*\sqrt{2*x^2+3*x+1})*\sqrt{(-17/9*\sqrt{10}+55/9)/\operatorname{abs}(6*x+2*\sqrt{10}-4)}-34/9*\sqrt{10}/\operatorname{abs}(6*x+2*\sqrt{10}-4)+110/9/\operatorname{abs}(6*x+2*\sqrt{10}-4)+17/18)/(-17/9*\sqrt{10}+55/9)^{(3/2)}+1656/(17*\sqrt{10}*\sqrt{2*x^2+3*x+1})+55*\sqrt{10}*\sqrt{2*x^2+3*x+1})+1656/(17*\sqrt{10}*\sqrt{2*x^2+3*x+1}-55*\sqrt{10}*\sqrt{2*x^2+3*x+1}))$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+2}{(2x^2+3x+1)^{3/2}(-3x^2+4x+2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 2)/((3*x + 2*x^2 + 1)^(3/2)*(4*x - 3*x^2 + 2)), x)

[Out] int((x + 2)/((3*x + 2*x^2 + 1)^(3/2)*(4*x - 3*x^2 + 2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{6x^4\sqrt{2x^2+3x+1} + x^3\sqrt{2x^2+3x+1} - 13x^2\sqrt{2x^2+3x+1} - 10x\sqrt{2x^2+3x+1} - 2\sqrt{2x^2+3x+1}} dx - \int$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(-3*x**2+4*x+2)/(2*x**2+3*x+1)**(3/2), x)

[Out] -Integral(x/(6*x**4*sqrt(2*x**2 + 3*x + 1) + x**3*sqrt(2*x**2 + 3*x + 1) - 13*x**2*sqrt(2*x**2 + 3*x + 1) - 10*x*sqrt(2*x**2 + 3*x + 1) - 2*sqrt(2*x**2 + 3*x + 1)), x) - Integral(2/(6*x**4*sqrt(2*x**2 + 3*x + 1) + x**3*sqrt(2*x**2 + 3*x + 1) - 13*x**2*sqrt(2*x**2 + 3*x + 1) - 10*x*sqrt(2*x**2 + 3*x + 1) - 2*sqrt(2*x**2 + 3*x + 1)), x)

$$3.30 \quad \int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{5/2}} dx$$

Optimal. Leaf size=197

$$\frac{2(22x+21)}{15(2x^2+3x+1)^{3/2}} + \frac{2(230x+273)}{15\sqrt{2x^2+3x+1}} - \frac{1}{50} \sqrt{\frac{1}{3}(4885115+1544809\sqrt{10})} \tanh^{-1} \left(\frac{(17-4\sqrt{10})x+3(4-\sqrt{10})}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}} \right)$$

[Out] 2/15*(21+22*x)/(2*x^2+3*x+1)^(3/2)+2/15*(273+230*x)/(2*x^2+3*x+1)^(1/2)+1/150*arctanh(1/2*(12+3*10^(1/2)+x*(17+4*10^(1/2)))/(2*x^2+3*x+1)^(1/2)/(55+17*10^(1/2))^(1/2))*(14655345-4634427*10^(1/2))^(1/2)-1/150*arctanh(1/2*(x*(17-4*10^(1/2))+12-3*10^(1/2))/(2*x^2+3*x+1)^(1/2)/(55-17*10^(1/2))^(1/2))*(14655345+4634427*10^(1/2))^(1/2)

Rubi [A] time = 0.30, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1016, 1060, 1032, 724, 206}

$$\frac{2(22x+21)}{15(2x^2+3x+1)^{3/2}} + \frac{2(230x+273)}{15\sqrt{2x^2+3x+1}} - \frac{1}{50} \sqrt{\frac{1}{3}(4885115+1544809\sqrt{10})} \tanh^{-1} \left(\frac{(17-4\sqrt{10})x+3(4-\sqrt{10})}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}} \right)$$

Antiderivative was successfully verified.

[In] Int[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x + 2*x^2)^(5/2)), x]

[Out] (2*(21 + 22*x))/(15*(1 + 3*x + 2*x^2)^(3/2)) + (2*(273 + 230*x))/(15*Sqrt[1 + 3*x + 2*x^2]) - (Sqrt[(4885115 + 1544809*Sqrt[10])/3]*ArcTanh[(3*(4 - Sqrt[10]) + (17 - 4*Sqrt[10])*x)/(2*Sqrt[55 - 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/50 + (Sqrt[(4885115 - 1544809*Sqrt[10])/3]*ArcTanh[(3*(4 + Sqrt[10]) + (17 + 4*Sqrt[10])*x)/(2*Sqrt[55 + 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/50

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,

d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1016

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1) * (d + e*x + f*x^2)^(q + 1) * (g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*c*d - 2*a*c*e + a*b*f))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
```

```

*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
+ q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{5/2}} dx &= \frac{2(21+22x)}{15(1+3x+2x^2)^{3/2}} - \frac{2}{45} \int \frac{-480 - \frac{813x}{2} + 396x^2}{(2+4x-3x^2)(1+3x+2x^2)^{3/2}} dx \\
&= \frac{2(21+22x)}{15(1+3x+2x^2)^{3/2}} + \frac{2(273+230x)}{15\sqrt{1+3x+2x^2}} + \frac{4}{675} \int \frac{\frac{23355}{2} - \frac{273}{2}}{(2+4x-3x^2)\sqrt{1+3x+2x^2}} dx \\
&= \frac{2(21+22x)}{15(1+3x+2x^2)^{3/2}} + \frac{2(273+230x)}{15\sqrt{1+3x+2x^2}} - \frac{1}{25} (3(335-106\sqrt{10})) \int \frac{1}{2+4x-3x^2} dx \\
&= \frac{2(21+22x)}{15(1+3x+2x^2)^{3/2}} + \frac{2(273+230x)}{15\sqrt{1+3x+2x^2}} + \frac{1}{25} (6(335-106\sqrt{10})) \operatorname{Subst}\left(\int \frac{1}{-3u^2+4u+2} du, u, 2+4x-3x^2\right) \\
&= \frac{2(21+22x)}{15(1+3x+2x^2)^{3/2}} + \frac{2(273+230x)}{15\sqrt{1+3x+2x^2}} - \frac{1}{50} \sqrt{\frac{1}{3}(4885115+154480x)} \operatorname{ArcTanh}\left[\frac{(4\sqrt{10}-17)x+3(\sqrt{10}-4)}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right]
\end{aligned}$$

Mathematica [A] time = 0.71, size = 190, normalized size = 0.96

$$\frac{1}{450} \left(\sqrt{55-17\sqrt{10}} (7289+2305\sqrt{10}) \operatorname{tanh}^{-1} \left(\frac{(4\sqrt{10}-17)x+3(\sqrt{10}-4)}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}} \right) - \sqrt{55+17\sqrt{10}} (2305\sqrt{10} - \dots) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(2+x)/((2+4*x-3*x^2)*(1+3*x+2*x^2)^(5/2)),x]

[Out] ((60*(294+1071*x+1236*x^2+460*x^3))/(1+3*x+2*x^2)^(3/2)+Sqrt[55-17*Sqrt[10]]*(7289+2305*Sqrt[10]))*ArcTanh[(3*(-4+Sqrt[10])+(-17+...

$$\frac{4\sqrt{10}x}{(2\sqrt{55-17\sqrt{10}})\sqrt{1+3x+2x^2}} - \sqrt{55+17\sqrt{10}} \cdot (-7289+2305\sqrt{10}) \operatorname{ArcTanh}\left(\frac{-3(4+\sqrt{10})-(17+4\sqrt{10})x}{(2\sqrt{55+17\sqrt{10}})\sqrt{1+3x+2x^2}}\right) / 450$$

fricas [B] time = 1.02, size = 435, normalized size = 2.21

$$23520x^4 + 70560x^3 + \sqrt{3}(4x^4 + 12x^3 + 13x^2 + 6x + 1)\sqrt{1544809\sqrt{10} + 4885115} \log\left(-\frac{243\sqrt{10}x + (893\sqrt{10}\sqrt{3}x}{\dots}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(5/2),x, algorithm="fricas")

[Out] 1/150*(23520*x^4 + 70560*x^3 + sqrt(3)*(4*x^4 + 12*x^3 + 13*x^2 + 6*x + 1)*sqrt(1544809*sqrt(10) + 4885115)*log(-(243*sqrt(10)*x + (893*sqrt(10)*sqrt(3)*x - 2824*sqrt(3)*x)*sqrt(1544809*sqrt(10) + 4885115) + 486*x - 486*sqrt(2*x^2 + 3*x + 1) + 486)/x) - sqrt(3)*(4*x^4 + 12*x^3 + 13*x^2 + 6*x + 1)*sqrt(1544809*sqrt(10) + 4885115)*log(-(243*sqrt(10)*x - (893*sqrt(10)*sqrt(3)*x - 2824*sqrt(3)*x)*sqrt(1544809*sqrt(10) + 4885115) + 486*x - 486*sqrt(2*x^2 + 3*x + 1) + 486)/x) + sqrt(3)*(4*x^4 + 12*x^3 + 13*x^2 + 6*x + 1)*sqrt(-1544809*sqrt(10) + 4885115)*log((243*sqrt(10)*x + (893*sqrt(10)*sqrt(3)*x + 2824*sqrt(3)*x)*sqrt(-1544809*sqrt(10) + 4885115) - 486*x + 486*sqrt(2*x^2 + 3*x + 1) - 486)/x) - sqrt(3)*(4*x^4 + 12*x^3 + 13*x^2 + 6*x + 1)*sqrt(-1544809*sqrt(10) + 4885115)*log((243*sqrt(10)*x - (893*sqrt(10)*sqrt(3)*x + 2824*sqrt(3)*x)*sqrt(-1544809*sqrt(10) + 4885115) - 486*x + 486*sqrt(2*x^2 + 3*x + 1) - 486)/x) + 76440*x^2 + 20*(460*x^3 + 1236*x^2 + 1071*x + 294)*sqrt(2*x^2 + 3*x + 1) + 35280*x + 5880)/(4*x^4 + 12*x^3 + 13*x^2 + 6*x + 1)

giac [A] time = 0.54, size = 121, normalized size = 0.61

$$\frac{2((4(115x+309)x+1071)x+294)}{15(2x^2+3x+1)^{\frac{3}{2}}} + 0.00115890443050800 \log\left(-\sqrt{2}x + \sqrt{2x^2+3x+1} + 5.9097693271200\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(5/2),x, algorithm="giac")

[Out] 2/15*((4*(115*x + 309)*x + 1071)*x + 294)/(2*x^2 + 3*x + 1)^(3/2) + 0.00115890443050800*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) + 5.90976932712000) - 36.0928986365333*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 0.176527156327000) + 36.0928986365333*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 0.919278730509000) - 0.00115890442528267*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 1.04272727395000)

maple [B] time = 0.02, size = 878, normalized size = 4.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^{(5/2)}, x)$

[Out]
$$\begin{aligned} & -1/20*(-8+10^{(1/2)})*10^{(1/2)}*(1/9/(55/9-17/9*10^{(1/2)})/(2*(x-2/3+1/3*10^{(1/2)}))^{(2)} \\ & + (17/3-4/3*10^{(1/2)})*(x-2/3+1/3*10^{(1/2)})+55/9-17/9*10^{(1/2)})^{(3/2)}-1/ \\ & 6*(17/3-4/3*10^{(1/2)})/(55/9-17/9*10^{(1/2)})*(2/3*(4*x+3)/(440/9-136/9*10^{(1/2)} \\ & - (17/3-4/3*10^{(1/2)})^2)/(2*(x-2/3+1/3*10^{(1/2)})^2+(17/3-4/3*10^{(1/2)})*(x- \\ & 2/3+1/3*10^{(1/2)})+55/9-17/9*10^{(1/2)})^{(3/2)}+32/3/(440/9-136/9*10^{(1/2)}- \\ & (17/3-4/3*10^{(1/2)})^2)^2*(4*x+3)/(2*(x-2/3+1/3*10^{(1/2)})^2+(17/3-4/3*10^{(1/2)}) \\ & *(x-2/3+1/3*10^{(1/2)})+55/9-17/9*10^{(1/2)})^{(1/2)}+1/3/(55/9-17/9*10^{(1/2)})*(1 \\ & / (55/9-17/9*10^{(1/2)})/(2*(x-2/3+1/3*10^{(1/2)})^2+(17/3-4/3*10^{(1/2)})*(x-2/3+ \\ & 1/3*10^{(1/2)})+55/9-17/9*10^{(1/2)})^{(1/2)}- (17/3-4/3*10^{(1/2)})/(55/9-17/9*10^{(1/2)} \\ &)*(4*x+3)/(440/9-136/9*10^{(1/2)}- (17/3-4/3*10^{(1/2)})^2)/(2*(x-2/3+1/3*10^{(1/2)} \\ & ^{(1/2)})^2+(17/3-4/3*10^{(1/2)})*(x-2/3+1/3*10^{(1/2)})+55/9-17/9*10^{(1/2)})^{(1/2)} \\ &)-3/(55/9-17/9*10^{(1/2)})/(55-17*10^{(1/2)})^{(1/2)}*\text{arctanh}(9/2*(110/9-34/9*10^{(1/2)} \\ & + (17/3-4/3*10^{(1/2)})*(x-2/3+1/3*10^{(1/2)}))/(55-17*10^{(1/2)})^{(1/2)})/(18*(x-2/3+1/3*10^{(1/2)})^2 \\ & +9*(17/3-4/3*10^{(1/2)})*(x-2/3+1/3*10^{(1/2)})+55-17*10^{(1/2)})^{(1/2)})) \\ & -1/20*(8+10^{(1/2)})*10^{(1/2)}*(1/9/(55/9+17/9*10^{(1/2)})/(2*(x-2/3-1/3*10^{(1/2)}))^{(2)} \\ & + (17/3+4/3*10^{(1/2)})*(x-2/3-1/3*10^{(1/2)})+55/9+17/9*10^{(1/2)})^{(3/2)}-1/6*(17/3+4/3*10^{(1/2)}) \\ & / (55/9+17/9*10^{(1/2)})*(2/3*(4*x+3)/(440/9+136/9*10^{(1/2)}- (17/3+4/3*10^{(1/2)})^2) \\ & / (2*(x-2/3-1/3*10^{(1/2)})^2+(17/3+4/3*10^{(1/2)})*(x-2/3-1/3*10^{(1/2)})+55/9+17/9*10^{(1/2)})^{(3/2)} \\ & +32/3/(440/9+136/9*10^{(1/2)}- (17/3+4/3*10^{(1/2)})^2)^2*(4*x+3)/(2*(x-2/3-1/3*10^{(1/2)})^2 \\ & + (17/3+4/3*10^{(1/2)})*(x-2/3-1/3*10^{(1/2)})+55/9+17/9*10^{(1/2)})^{(1/2)}+1/3/(55/9+17/9*10^{(1/2)}) \\ & *(1/(55/9+17/9*10^{(1/2)})/(2*(x-2/3-1/3*10^{(1/2)})^2+(17/3+4/3*10^{(1/2)})*(x-2/3-1/3*10^{(1/2)}) \\ & +55/9+17/9*10^{(1/2)})^{(1/2)}- (17/3+4/3*10^{(1/2)})/(55/9+17/9*10^{(1/2)})*(4*x+3)/(440/9+136/9*10^{(1/2)} \\ & - (17/3+4/3*10^{(1/2)})^2)/(2*(x-2/3-1/3*10^{(1/2)})^2+(17/3+4/3*10^{(1/2)})*(x-2/3-1/3*10^{(1/2)}) \\ & +55/9+17/9*10^{(1/2)})^{(1/2)}-3/(55/9+17/9*10^{(1/2)})/(55+17*10^{(1/2)})^{(1/2)}*\text{arctanh}(9/2*(110/9+34/9*10^{(1/2)} \\ & + (17/3+4/3*10^{(1/2)})*(x-2/3-1/3*10^{(1/2)}))/(55+17*10^{(1/2)})^{(1/2)})/(18*(x-2/3-1/3*10^{(1/2)})^2 \\ & +9*(17/3+4/3*10^{(1/2)})*(x-2/3-1/3*10^{(1/2)})+55+17*10^{(1/2)})^{(1/2)})) \end{aligned}$$

maxima [B] time = 1.24, size = 1276, normalized size = 6.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^{(5/2)}, x, \text{algorithm}="maxima")$

```
[Out] -1/300*sqrt(10)*(980*sqrt(10)*x/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) + 55*(
2*x^2 + 3*x + 1)^(3/2)) - 980*sqrt(10)*x/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/
2) - 55*(2*x^2 + 3*x + 1)^(3/2)) + 5292*sqrt(10)*x/(374*sqrt(10)*sqrt(2*x^2
+ 3*x + 1) + 1183*sqrt(2*x^2 + 3*x + 1)) - 5292*sqrt(10)*x/(374*sqrt(10)*s
qrt(2*x^2 + 3*x + 1) - 1183*sqrt(2*x^2 + 3*x + 1)) - 15680*sqrt(10)*x/(17*s
qrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqrt(2*x^2 + 3*x + 1)) + 15680*sqrt(10)*
x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(2*x^2 + 3*x + 1)) + 3520*x/(
17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) + 55*(2*x^2 + 3*x + 1)^(3/2)) + 3520*x/(
17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) - 55*(2*x^2 + 3*x + 1)^(3/2)) + 19008*
x/(374*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 1183*sqrt(2*x^2 + 3*x + 1)) + 19008
*x/(374*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 1183*sqrt(2*x^2 + 3*x + 1)) - 5632
0*x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqrt(2*x^2 + 3*x + 1)) - 56320*
x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(2*x^2 + 3*x + 1)) + 750*sqrt
(10)/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) + 55*(2*x^2 + 3*x + 1)^(3/2)) - 7
50*sqrt(10)/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) - 55*(2*x^2 + 3*x + 1)^(3/
2)) + 4050*sqrt(10)/(374*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 1183*sqrt(2*x^2 +
3*x + 1)) - 4050*sqrt(10)/(374*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 1183*sqrt(
2*x^2 + 3*x + 1)) - 11760*sqrt(10)/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*
sqrt(2*x^2 + 3*x + 1)) + 11760*sqrt(10)/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1)
- 55*sqrt(2*x^2 + 3*x + 1)) + 2760/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) + 5
5*(2*x^2 + 3*x + 1)^(3/2)) + 2760/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) - 55
*(2*x^2 + 3*x + 1)^(3/2)) + 14904/(374*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 118
3*sqrt(2*x^2 + 3*x + 1)) + 14904/(374*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 1183
*sqrt(2*x^2 + 3*x + 1)) - 42240/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sq
rt(2*x^2 + 3*x + 1)) - 42240/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(2*
x^2 + 3*x + 1)) - 1215*sqrt(10)*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1
))*sqrt(17*sqrt(10) + 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x
- 2*sqrt(10) - 4) + 110/9/abs(6*x - 2*sqrt(10) - 4) + 17/18)/(17*sqrt(10) +
55)^(5/2) - 5*sqrt(10)*log(-2/9*sqrt(10) + 2*sqrt(2*x^2 + 3*x + 1))*sqrt(-1
7/9*sqrt(10) + 55/9)/abs(6*x + 2*sqrt(10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*
sqrt(10) - 4) + 110/9/abs(6*x + 2*sqrt(10) - 4) + 17/18)/(-17/9*sqrt(10) +
55/9)^(5/2) - 9720*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1))*sqrt(17*sq
rt(10) + 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x - 2*sqrt(10)
- 4) + 110/9/abs(6*x - 2*sqrt(10) - 4) + 17/18)/(17*sqrt(10) + 55)^(5/2) +
40*log(-2/9*sqrt(10) + 2*sqrt(2*x^2 + 3*x + 1))*sqrt(-17/9*sqrt(10) + 55/9)/
abs(6*x + 2*sqrt(10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 110/9
/abs(6*x + 2*sqrt(10) - 4) + 17/18)/(-17/9*sqrt(10) + 55/9)^(5/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+2}{(2x^2+3x+1)^{5/2}(-3x^2+4x+2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x + 2)/((3*x + 2*x^2 + 1)^(5/2)*(4*x - 3*x^2 + 2)), x)`

[Out] `int((x + 2)/((3*x + 2*x^2 + 1)^(5/2)*(4*x - 3*x^2 + 2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{12x^6\sqrt{2x^2+3x+1} + 20x^5\sqrt{2x^2+3x+1} - 17x^4\sqrt{2x^2+3x+1} - 58x^3\sqrt{2x^2+3x+1} - 47x^2\sqrt{2x^2+3x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2+x)/(-3*x**2+4*x+2)/(2*x**2+3*x+1)**(5/2), x)`

[Out] `-Integral(x/(12*x**6*sqrt(2*x**2 + 3*x + 1) + 20*x**5*sqrt(2*x**2 + 3*x + 1) - 17*x**4*sqrt(2*x**2 + 3*x + 1) - 58*x**3*sqrt(2*x**2 + 3*x + 1) - 47*x**2*sqrt(2*x**2 + 3*x + 1) - 16*x*sqrt(2*x**2 + 3*x + 1) - 2*sqrt(2*x**2 + 3*x + 1)), x) - Integral(2/(12*x**6*sqrt(2*x**2 + 3*x + 1) + 20*x**5*sqrt(2*x**2 + 3*x + 1) - 17*x**4*sqrt(2*x**2 + 3*x + 1) - 58*x**3*sqrt(2*x**2 + 3*x + 1) - 47*x**2*sqrt(2*x**2 + 3*x + 1) - 16*x*sqrt(2*x**2 + 3*x + 1) - 2*sqrt(2*x**2 + 3*x + 1)), x)`

$$3.31 \quad \int \frac{1+x}{(4+2x+x^2)\sqrt{5+2x+x^2}} dx$$

Optimal. Leaf size=15

$$-\tanh^{-1}\left(\sqrt{x^2+2x+5}\right)$$

[Out] -arctanh((x^2+2*x+5)^(1/2))

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1024, 206}

$$-\tanh^{-1}\left(\sqrt{x^2+2x+5}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + x)/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]), x]

[Out] -ArcTanh[Sqrt[5 + 2*x + x^2]]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1024

Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*g, Subst[Int[1/(b*d - a*e - b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] && EqQ[h*e - 2*g*f, 0]

Rubi steps

$$\begin{aligned} \int \frac{1+x}{(4+2x+x^2)\sqrt{5+2x+x^2}} dx &= -\left(2 \operatorname{Subst}\left(\int \frac{1}{2-2x^2} dx, x, \sqrt{5+2x+x^2}\right)\right) \\ &= -\tanh^{-1}\left(\sqrt{5+2x+x^2}\right) \end{aligned}$$

Mathematica [C] time = 0.04, size = 79, normalized size = 5.27

$$\frac{1}{2} \left(-\tanh^{-1} \left(\frac{-i\sqrt{3}x - i\sqrt{3} + 4}{\sqrt{x^2 + 2x + 5}} \right) - \tanh^{-1} \left(\frac{i\sqrt{3}x + i\sqrt{3} + 4}{\sqrt{x^2 + 2x + 5}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x)/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]

[Out] (-ArcTanh[(4 - I*Sqrt[3] - I*Sqrt[3]*x)/Sqrt[5 + 2*x + x^2]] - ArcTanh[(4 + I*Sqrt[3] + I*Sqrt[3]*x)/Sqrt[5 + 2*x + x^2]])/2

fricas [B] time = 0.87, size = 49, normalized size = 3.27

$$\frac{1}{2} \log \left(x^2 - \sqrt{x^2 + 2x + 5}(x + 2) + 3x + 6 \right) - \frac{1}{2} \log \left(x^2 - \sqrt{x^2 + 2x + 5}x + x + 4 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="fricas")

[Out] 1/2*log(x^2 - sqrt(x^2 + 2*x + 5)*(x + 2) + 3*x + 6) - 1/2*log(x^2 - sqrt(x^2 + 2*x + 5)*x + x + 4)

giac [B] time = 0.35, size = 31, normalized size = 2.07

$$-\frac{1}{2} \log \left(\sqrt{x^2 + 2x + 5} + 1 \right) + \frac{1}{2} \log \left(\sqrt{x^2 + 2x + 5} - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="giac")

[Out] -1/2*log(sqrt(x^2 + 2*x + 5) + 1) + 1/2*log(sqrt(x^2 + 2*x + 5) - 1)

maple [A] time = 0.02, size = 14, normalized size = 0.93

$$-\operatorname{arctanh} \left(\sqrt{x^2 + 2x + 5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+1)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x)

[Out] -arctanh((x^2+2*x+5)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x^2+2x+5}(x^2+2x+4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="maxima")

[Out] integrate((x + 1)/(sqrt(x^2 + 2*x + 5)*(x^2 + 2*x + 4)), x)

mupad [B] time = 3.76, size = 13, normalized size = 0.87

$$-\operatorname{atanh}\left(\sqrt{x^2 + 2x + 5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 1)/((2*x + x^2 + 4)*(2*x + x^2 + 5)^(1/2)),x)

[Out] -atanh((2*x + x^2 + 5)^(1/2))

sympy [B] time = 7.00, size = 36, normalized size = 2.40

$$\frac{\log\left(-1 + \frac{1}{\sqrt{x^2+2x+5}}\right)}{2} - \frac{\log\left(1 + \frac{1}{\sqrt{x^2+2x+5}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(x**2+2*x+4)/(x**2+2*x+5)**(1/2),x)

[Out] log(-1 + 1/sqrt(x**2 + 2*x + 5))/2 - log(1 + 1/sqrt(x**2 + 2*x + 5))/2

$$3.32 \quad \int \frac{4+x}{(4+2x+x^2)\sqrt{5+2x+x^2}} dx$$

Optimal. Leaf size=44

$$\sqrt{3} \tan^{-1}\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right) - \tanh^{-1}\left(\sqrt{x^2+2x+5}\right)$$

[Out] $-\operatorname{arctanh}((x^2+2x+5)^{(1/2)})+\operatorname{arctan}(1/3*(1+x)*3^{(1/2)}/(x^2+2x+5)^{(1/2}))*3^{(1/2)}$

Rubi [A] time = 0.05, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1025, 982, 204, 1024, 206}

$$\sqrt{3} \tan^{-1}\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right) - \tanh^{-1}\left(\sqrt{x^2+2x+5}\right)$$

Antiderivative was successfully verified.

[In] Int[(4 + x)/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]), x]

[Out] Sqrt[3]*ArcTan[(1 + x)/(Sqrt[3]*Sqrt[5 + 2*x + x^2])] - ArcTanh[Sqrt[5 + 2*x + x^2]]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 982

Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0]

Rule 1024

```
Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[-2*g, Subst[Int[1/(b*d - a*e - b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] && EqQ[h*e - 2*g*f, 0]
```

Rule 1025

```
Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := -Dist[(h*e - 2*g*f)/(2*f), Int[1/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{4+x}{(4+2x+x^2)\sqrt{5+2x+x^2}} dx &= \frac{1}{2} \int \frac{2+2x}{(4+2x+x^2)\sqrt{5+2x+x^2}} dx + 3 \int \frac{1}{(4+2x+x^2)\sqrt{5+2x+x^2}} dx \\ &= -\left(2 \operatorname{Subst}\left(\int \frac{1}{2-2x^2} dx, x, \sqrt{5+2x+x^2}\right)\right) - 12 \operatorname{Subst}\left(\int \frac{1}{-24-2x^2} dx, x, \sqrt{5+2x+x^2}\right) \\ &= \sqrt{3} \tan^{-1}\left(\frac{1+x}{\sqrt{3}\sqrt{5+2x+x^2}}\right) - \tanh^{-1}\left(\sqrt{5+2x+x^2}\right) \end{aligned}$$

Mathematica [C] time = 0.05, size = 101, normalized size = 2.30

$$-\frac{1}{2}(1+i\sqrt{3}) \tanh^{-1}\left(\frac{-i\sqrt{3}x-i\sqrt{3}+4}{\sqrt{x^2+2x+5}}\right) - \frac{1}{2}(1-i\sqrt{3}) \tanh^{-1}\left(\frac{i\sqrt{3}x+i\sqrt{3}+4}{\sqrt{x^2+2x+5}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + x)/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]), x]

[Out] -1/2*((1 + I*Sqrt[3])*ArcTanh[(4 - I*Sqrt[3] - I*Sqrt[3]*x)/Sqrt[5 + 2*x + x^2]]) - ((1 - I*Sqrt[3])*ArcTanh[(4 + I*Sqrt[3] + I*Sqrt[3]*x)/Sqrt[5 + 2*x + x^2]])/2

fricas [B] time = 0.99, size = 106, normalized size = 2.41

$$-\sqrt{3} \arctan\left(-\frac{1}{3}\sqrt{3}(x+2) + \frac{1}{3}\sqrt{3}\sqrt{x^2+2x+5}\right) + \sqrt{3} \arctan\left(-\frac{1}{3}\sqrt{3}x + \frac{1}{3}\sqrt{3}\sqrt{x^2+2x+5}\right) + \frac{1}{2} \log(x^2 - \dots)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="fricas")

[Out] -sqrt(3)*arctan(-1/3*sqrt(3)*(x + 2) + 1/3*sqrt(3)*sqrt(x^2 + 2*x + 5)) + sqrt(3)*arctan(-1/3*sqrt(3)*x + 1/3*sqrt(3)*sqrt(x^2 + 2*x + 5)) + 1/2*log(x^2 - sqrt(x^2 + 2*x + 5)*(x + 2) + 3*x + 6) - 1/2*log(x^2 - sqrt(x^2 + 2*x + 5)*x + x + 4)

giac [B] time = 0.37, size = 108, normalized size = 2.45

$$-\sqrt{3} \arctan\left(-\frac{1}{3}\sqrt{3}\left(x - \sqrt{x^2 + 2x + 5} + 2\right)\right) + \sqrt{3} \arctan\left(-\frac{1}{3}\sqrt{3}\left(x - \sqrt{x^2 + 2x + 5}\right)\right) + \frac{1}{2} \log\left(\left(x - \sqrt{x^2 + 2x + 5}\right)^2 + 4x - 4\sqrt{x^2 + 2x + 5} + 7\right) - \frac{1}{2} \log\left(\left(x - \sqrt{x^2 + 2x + 5}\right)^2 + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="giac")

[Out] -sqrt(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5) + 2)) + sqrt(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5))) + 1/2*log((x - sqrt(x^2 + 2*x + 5))^2 + 4*x - 4*sqrt(x^2 + 2*x + 5) + 7) - 1/2*log((x - sqrt(x^2 + 2*x + 5))^2 + 3)

maple [A] time = 0.01, size = 40, normalized size = 0.91

$$-\operatorname{arctanh}\left(\sqrt{x^2 + 2x + 5}\right) + \sqrt{3} \arctan\left(\frac{\sqrt{3}(2x + 2)}{6\sqrt{x^2 + 2x + 5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x)

[Out] -arctanh((x^2+2*x+5)^(1/2))+3^(1/2)*arctan(1/6*3^(1/2)/(x^2+2*x+5)^(1/2)*(2*x+2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x + 4}{\sqrt{x^2 + 2x + 5}(x^2 + 2x + 4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="maxima")

[Out] integrate((x + 4)/(sqrt(x^2 + 2*x + 5)*(x^2 + 2*x + 4)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x + 4}{(x^2 + 2x + 4) \sqrt{x^2 + 2x + 5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x + 4)/((2*x + x^2 + 4)*(2*x + x^2 + 5)^(1/2)), x)`

[Out] `int((x + 4)/((2*x + x^2 + 4)*(2*x + x^2 + 5)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x + 4}{(x^2 + 2x + 4) \sqrt{x^2 + 2x + 5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4+x)/(x**2+2*x+4)/(x**2+2*x+5)**(1/2), x)`

[Out] `Integral((x + 4)/((x**2 + 2*x + 4)*sqrt(x**2 + 2*x + 5)), x)`

$$3.33 \quad \int \frac{1+2x}{(3+x+x^2)\sqrt{5+x+x^2}} dx$$

Optimal. Leaf size=24

$$-\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{x^2+x+5}}{\sqrt{2}}\right)$$

[Out] $-\operatorname{arctanh}(1/2*(x^2+x+5)^{(1/2)*2^{(1/2)}}*2^{(1/2)})$

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1024, 206}

$$-\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{x^2+x+5}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(1 + 2*x)/((3 + x + x^2)*\operatorname{Sqrt}[5 + x + x^2]), x]$

[Out] $-(\operatorname{Sqrt}[2]*\operatorname{ArcTanh}[\operatorname{Sqrt}[5 + x + x^2]/\operatorname{Sqrt}[2]])$

Rule 206

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /;$ $\operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{Gt} Q[a, 0] \ || \ \operatorname{Lt} Q[b, 0])$

Rule 1024

$\operatorname{Int}[(g_ + (h_)*(x_))/((a_ + (b_)*(x_) + (c_)*(x_)^2)*\operatorname{Sqrt}[(d_ + (e_)*(x_) + (f_)*(x_)^2)]), x_Symbol] \rightarrow \operatorname{Dist}[-2*g, \operatorname{Subst}[\operatorname{Int}[1/(b*d - a*e - b*x^2), x], x, \operatorname{Sqrt}[d + e*x + f*x^2]], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, e, f, g, h\}, x \ \&\& \operatorname{NeQ}[b^2 - 4*a*c, 0] \ \&\& \operatorname{NeQ}[e^2 - 4*d*f, 0] \ \&\& \operatorname{EqQ}[c*e - b*f, 0] \ \&\& \operatorname{EqQ}[h*e - 2*g*f, 0]$

Rubi steps

$$\begin{aligned} \int \frac{1+2x}{(3+x+x^2)\sqrt{5+x+x^2}} dx &= -\left(2 \operatorname{Subst}\left(\int \frac{1}{2-x^2} dx, x, \sqrt{5+x+x^2}\right)\right) \\ &= -\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{5+x+x^2}}{\sqrt{2}}\right) \end{aligned}$$

Mathematica [C] time = 0.06, size = 90, normalized size = 3.75

$$\frac{\tanh^{-1}\left(\frac{-2i\sqrt{11}x-i\sqrt{11}+19}{4\sqrt{2}\sqrt{x^2+x+5}}\right) + \tanh^{-1}\left(\frac{2i\sqrt{11}x+i\sqrt{11}+19}{4\sqrt{2}\sqrt{x^2+x+5}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x)/((3 + x + x^2)*Sqrt[5 + x + x^2]), x]

[Out] -((ArcTanh[(19 - I*Sqrt[11] - (2*I)*Sqrt[11]*x)/(4*Sqrt[2]*Sqrt[5 + x + x^2]]) + ArcTanh[(19 + I*Sqrt[11] + (2*I)*Sqrt[11]*x)/(4*Sqrt[2]*Sqrt[5 + x + x^2])])/Sqrt[2])

fricas [A] time = 1.10, size = 34, normalized size = 1.42

$$\frac{1}{2}\sqrt{2}\log\left(\frac{x^2 - 2\sqrt{2}\sqrt{x^2 + x + 5} + x + 7}{x^2 + x + 3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x^2+x+3)/(x^2+x+5)^(1/2), x, algorithm="fricas")

[Out] 1/2*sqrt(2)*log((x^2 - 2*sqrt(2)*sqrt(x^2 + x + 5) + x + 7)/(x^2 + x + 3))

giac [B] time = 0.36, size = 39, normalized size = 1.62

$$-\frac{1}{2}\sqrt{2}\log\left(\sqrt{2} + \sqrt{x^2 + x + 5}\right) + \frac{1}{2}\sqrt{2}\log\left(-\sqrt{2} + \sqrt{x^2 + x + 5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x^2+x+3)/(x^2+x+5)^(1/2), x, algorithm="giac")

[Out] -1/2*sqrt(2)*log(sqrt(2) + sqrt(x^2 + x + 5)) + 1/2*sqrt(2)*log(-sqrt(2) + sqrt(x^2 + x + 5))

maple [A] time = 0.02, size = 20, normalized size = 0.83

$$-\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{x^2 + x + 5}\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+2*x)/(x^2+x+3)/(x^2+x+5)^(1/2), x)

[Out] -arctanh(1/2*(x^2+x+5)^(1/2)*2^(1/2))*2^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+1}{\sqrt{x^2+x+5}(x^2+x+3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="maxima")

[Out] integrate((2*x + 1)/(sqrt(x^2 + x + 5)*(x^2 + x + 3)), x)

mupad [B] time = 3.78, size = 19, normalized size = 0.79

$$-\sqrt{2} \operatorname{atanh}\left(\frac{\sqrt{2} \sqrt{x^2+x+5}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x + 1)/((x + x^2 + 3)*(x + x^2 + 5)^(1/2)),x)

[Out] -2^(1/2)*atanh((2^(1/2)*(x + x^2 + 5)^(1/2))/2)

sympy [A] time = 6.81, size = 68, normalized size = 2.83

$$2 \left(\begin{array}{l} \left(\begin{array}{l} \frac{\sqrt{2} \operatorname{acoth}\left(\frac{\sqrt{2}}{\sqrt{x^2+x+5}}\right)}{2} \\ -\frac{\sqrt{2} \operatorname{atanh}\left(\frac{\sqrt{2}}{\sqrt{x^2+x+5}}\right)}{2} \end{array} \right) \text{ for } \frac{1}{x^2+x+5} > \frac{1}{2} \\ \left(\begin{array}{l} \frac{\sqrt{2} \operatorname{acoth}\left(\frac{\sqrt{2}}{\sqrt{x^2+x+5}}\right)}{2} \\ -\frac{\sqrt{2} \operatorname{atanh}\left(\frac{\sqrt{2}}{\sqrt{x^2+x+5}}\right)}{2} \end{array} \right) \text{ for } \frac{1}{x^2+x+5} < \frac{1}{2} \end{array} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x**2+x+3)/(x**2+x+5)**(1/2),x)

[Out] 2*Piecewise((-sqrt(2)*acoth(sqrt(2)/sqrt(x**2 + x + 5))/2, 1/(x**2 + x + 5) > 1/2), (-sqrt(2)*atanh(sqrt(2)/sqrt(x**2 + x + 5))/2, 1/(x**2 + x + 5) < 1/2))

$$3.34 \quad \int \frac{x}{(3+x+x^2)\sqrt{5+x+x^2}} dx$$

Optimal. Leaf size=56

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{2}{11}}(2x+1)}{\sqrt{x^2+x+5}}\right)}{\sqrt{22}} - \frac{\tanh^{-1}\left(\frac{\sqrt{x^2+x+5}}{\sqrt{2}}\right)}{\sqrt{2}}$$

[Out] $-1/2*\operatorname{arctanh}(1/2*(x^2+x+5)^{(1/2)}*2^{(1/2)})*2^{(1/2)}-1/22*\operatorname{arctan}(1/11*(1+2*x))*22^{(1/2)}/(x^2+x+5)^{(1/2)}*22^{(1/2)}$

Rubi [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1025, 982, 204, 1024, 206}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{2}{11}}(2x+1)}{\sqrt{x^2+x+5}}\right)}{\sqrt{22}} - \frac{\tanh^{-1}\left(\frac{\sqrt{x^2+x+5}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[x/((3 + x + x^2)*Sqrt[5 + x + x^2]),x]

[Out] $-(\operatorname{ArcTan}[(\operatorname{Sqrt}[2/11]*(1 + 2*x))/\operatorname{Sqrt}[5 + x + x^2]]/\operatorname{Sqrt}[22]) - \operatorname{ArcTanh}[\operatorname{Sqrt}[5 + x + x^2]/\operatorname{Sqrt}[2]]/\operatorname{Sqrt}[2]$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 982

Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0]

]

Rule 1024

```
Int[((g_) + (h_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[-2*g, Subst[Int[1/(b*d - a*e - b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] && EqQ[h*e - 2*g*f, 0]
```

Rule 1025

```
Int[((g_) + (h_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := -Dist[(h*e - 2*g*f)/(2*f), Int[1/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x}{(3+x+x^2)\sqrt{5+x+x^2}} dx &= -\left(\frac{1}{2} \int \frac{1}{(3+x+x^2)\sqrt{5+x+x^2}} dx\right) + \frac{1}{2} \int \frac{1+2x}{(3+x+x^2)\sqrt{5+x+x^2}} dx \\ &= \text{Subst}\left(\int \frac{1}{-11-2x^2} dx, x, \frac{1+2x}{\sqrt{5+x+x^2}}\right) - \text{Subst}\left(\int \frac{1}{2-x^2} dx, x, \sqrt{5+x+x^2}\right) \\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{\frac{2}{11}}(1+2x)}{\sqrt{5+x+x^2}}\right)}{\sqrt{22}} - \frac{\tanh^{-1}\left(\frac{\sqrt{5+x+x^2}}{\sqrt{2}}\right)}{\sqrt{2}} \end{aligned}$$

Mathematica [C] time = 0.06, size = 114, normalized size = 2.04

$$\frac{-\left((\sqrt{11}-i)\tanh^{-1}\left(\frac{-2i\sqrt{11}x-i\sqrt{11}+19}{4\sqrt{2}\sqrt{x^2+x+5}}\right)\right)-\left(\sqrt{11}+i\right)\tanh^{-1}\left(\frac{2i\sqrt{11}x+i\sqrt{11}+19}{4\sqrt{2}\sqrt{x^2+x+5}}\right)}{2\sqrt{22}}$$

Antiderivative was successfully verified.

[In] Integrate[x/((3 + x + x^2)*Sqrt[5 + x + x^2]),x]

[Out] $(-((-I + \text{Sqrt}[11]) * \text{ArcTanh}[(19 - I * \text{Sqrt}[11] - (2 * I) * \text{Sqrt}[11] * x) / (4 * \text{Sqrt}[2] * \text{Sqrt}[5 + x + x^2])]) - (I + \text{Sqrt}[11]) * \text{ArcTanh}[(19 + I * \text{Sqrt}[11] + (2 * I) * \text{Sqrt}[11] * x) / (4 * \text{Sqrt}[2] * \text{Sqrt}[5 + x + x^2])])) / (2 * \text{Sqrt}[22])$

fricas [B] time = 1.10, size = 307, normalized size = 5.48

$$-\frac{1}{33} \sqrt{11} \sqrt{6} \sqrt{3} \arctan\left(\frac{2}{33} \sqrt{11} \sqrt{3} \sqrt{\sqrt{6} \sqrt{3} (2x+1) + 6x^2 - \sqrt{x^2+x+5} (2\sqrt{6} \sqrt{3} + 6x+3) + 6x+30} + \frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="fricas")`

[Out] $-1/33 * \text{sqrt}(11) * \text{sqrt}(6) * \text{sqrt}(3) * \arctan(2/33 * \text{sqrt}(11) * \text{sqrt}(3) * \text{sqrt}(\text{sqrt}(6) * \text{sqrt}(3) * (2 * x + 1) + 6 * x^2 - \text{sqrt}(x^2 + x + 5) * (2 * \text{sqrt}(6) * \text{sqrt}(3) + 6 * x + 3) + 6 * x + 30) + 1/33 * \text{sqrt}(11) * (2 * \text{sqrt}(6) * \text{sqrt}(3) + 6 * x + 3) - 2/11 * \text{sqrt}(11) * \text{sqrt}(x^2 + x + 5)) + 1/33 * \text{sqrt}(11) * \text{sqrt}(6) * \text{sqrt}(3) * \arctan(-1/33 * \text{sqrt}(11) * (2 * \text{sqrt}(6) * \text{sqrt}(3) - 6 * x - 3) + 1/33 * \text{sqrt}(11) * \text{sqrt}(-12 * \text{sqrt}(6) * \text{sqrt}(3) * (2 * x + 1) + 72 * x^2 + 12 * \text{sqrt}(x^2 + x + 5) * (2 * \text{sqrt}(6) * \text{sqrt}(3) - 6 * x - 3) + 72 * x + 360) - 2/11 * \text{sqrt}(11) * \text{sqrt}(x^2 + x + 5)) + 1/12 * \text{sqrt}(6) * \text{sqrt}(3) * \log(12 * \text{sqrt}(6) * \text{sqrt}(3) * (2 * x + 1) + 72 * x^2 - 12 * \text{sqrt}(x^2 + x + 5) * (2 * \text{sqrt}(6) * \text{sqrt}(3) + 6 * x + 3) + 72 * x + 360) - 1/12 * \text{sqrt}(6) * \text{sqrt}(3) * \log(-12 * \text{sqrt}(6) * \text{sqrt}(3) * (2 * x + 1) + 72 * x^2 + 12 * \text{sqrt}(x^2 + x + 5) * (2 * \text{sqrt}(6) * \text{sqrt}(3) - 6 * x - 3) + 72 * x + 360))$

giac [B] time = 0.32, size = 133, normalized size = 2.38

$$\frac{1}{22} \sqrt{11} \sqrt{2} \arctan\left(-\frac{1}{11} \sqrt{11} (2x + 2\sqrt{2} - 2\sqrt{x^2+x+5} + 1)\right) - \frac{1}{22} \sqrt{11} \sqrt{2} \arctan\left(-\frac{1}{11} \sqrt{11} (2x - 2\sqrt{2} - 2\sqrt{x^2+x+5} + 1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="giac")`

[Out] $1/22 * \text{sqrt}(11) * \text{sqrt}(2) * \arctan(-1/11 * \text{sqrt}(11) * (2 * x + 2 * \text{sqrt}(2) - 2 * \text{sqrt}(x^2 + x + 5) + 1)) - 1/22 * \text{sqrt}(11) * \text{sqrt}(2) * \arctan(-1/11 * \text{sqrt}(11) * (2 * x - 2 * \text{sqrt}(2) - 2 * \text{sqrt}(x^2 + x + 5) + 1)) + 1/4 * \text{sqrt}(2) * \log(324 * (2 * x + 2 * \text{sqrt}(2) - 2 * \text{sqrt}(x^2 + x + 5) + 1)^2 + 3564) - 1/4 * \text{sqrt}(2) * \log(324 * (2 * x - 2 * \text{sqrt}(2) - 2 * \text{sqrt}(x^2 + x + 5) + 1)^2 + 3564)$

maple [A] time = 0.01, size = 45, normalized size = 0.80

$$-\frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{x^2+x+5} \sqrt{2}}{2}\right)}{2} - \frac{\sqrt{22} \operatorname{arctan}\left(\frac{(2x+1)\sqrt{22}}{11\sqrt{x^2+x+5}}\right)}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(x^2+x+3)/(x^2+x+5)^(1/2),x)`

[Out] `-1/2*arctanh(1/2*(x^2+x+5)^(1/2)*2^(1/2))*2^(1/2)-1/22*arctan(1/11*(1+2*x)*22^(1/2)/(x^2+x+5)^(1/2))*22^(1/2)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^2 + x + 5}(x^2 + x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="maxima")`

[Out] `integrate(x/(sqrt(x^2 + x + 5)*(x^2 + x + 3)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{(x^2 + x + 3)\sqrt{x^2 + x + 5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/((x + x^2 + 3)*(x + x^2 + 5)^(1/2)),x)`

[Out] `int(x/((x + x^2 + 3)*(x + x^2 + 5)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(x^2 + x + 3)\sqrt{x^2 + x + 5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x**2+x+3)/(x**2+x+5)**(1/2),x)`

[Out] `Integral(x/((x**2 + x + 3)*sqrt(x**2 + x + 5)), x)`

$$3.35 \quad \int \frac{A+Bx}{\sqrt{d+ex+fx^2} (ae+bex+bf x^2)^2} dx$$

Optimal. Leaf size=249

$$\frac{(Be - 2Af) (8aef - b(4df + e^2)) \tanh^{-1} \left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}} \right)}{2e^{3/2}f(bd - ae)^{3/2}(be - 4af)^{3/2}} - \frac{\sqrt{d+ex+fx^2} (e(Ab - 2aB) - bx(Be - 2Af))}{e(bd - ae)(be - 4af)(ae + bex + bf x^2)} +$$

[Out] 1/2*(-2*A*f+B*e)*(8*a*e*f-b*(4*d*f+e^2))*arctanh((2*f*x+e)*(-a*e+b*d)^(1/2)/e^(1/2)/(-4*a*f+b*e)^(1/2)/(f*x^2+e*x+d)^(1/2))/e^(3/2)/(-a*e+b*d)^(3/2)/f/(-4*a*f+b*e)^(3/2)+1/2*B*arctanh(b^(1/2)*(f*x^2+e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/(-a*e+b*d)^(3/2)/f/b^(1/2)-((A*b-2*B*a)*e-b*(-2*A*f+B*e)*x)*(f*x^2+e*x+d)^(1/2)/e/(-a*e+b*d)/(-4*a*f+b*e)/(b*f*x^2+b*e*x+a*e)

Rubi [A] time = 0.91, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {1016, 1025, 982, 208, 1024}

$$\frac{(Be - 2Af) (8aef - b(4df + e^2)) \tanh^{-1} \left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}} \right)}{2e^{3/2}f(bd - ae)^{3/2}(be - 4af)^{3/2}} - \frac{\sqrt{d+ex+fx^2} (e(Ab - 2aB) - bx(Be - 2Af))}{e(bd - ae)(be - 4af)(ae + bex + bf x^2)} +$$

Antiderivative was successfully verified.

[In] Int[(A + B*x)/(Sqrt[d + e*x + f*x^2]*(a*e + b*e*x + b*f*x^2)^2), x]

[Out] -((((A*b - 2*a*B)*e - b*(B*e - 2*A*f)*x)*Sqrt[d + e*x + f*x^2])/(e*(b*d - a*e)*(b*e - 4*a*f)*(a*e + b*e*x + b*f*x^2))) + ((B*e - 2*A*f)*(8*a*e*f - b*(e^2 + 4*d*f))*ArcTanh[(Sqrt[b*d - a*e]*(e + 2*f*x))/(Sqrt[e]*Sqrt[b*e - 4*a*f]*Sqrt[d + e*x + f*x^2]])/(2*e^(3/2)*(b*d - a*e)^(3/2)*f*(b*e - 4*a*f)^(3/2)) + (B*ArcTanh[(Sqrt[b]*Sqrt[d + e*x + f*x^2])/Sqrt[b*d - a*e]])/(2*Sqrt[b]*(b*d - a*e)^(3/2)*f)

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 982

Int[1/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,

e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0]

Rule 1016

Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1) * (d + e*x + f*x^2)^(q + 1) * (g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*c*d - 2*a*c*e + a*b*f))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !(IntegerQ[p] && ILtQ[q, -1])

Rule 1024

Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*g, Subst[Int[1/(b*d - a*e - b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] && EqQ[h*e - 2*g*f, 0]

Rule 1025

Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := -Dist[(h*e - 2*g*f)/(2*f), Int[1/(a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2], x], x] + Dist[h/(2*f), Int[(e + 2*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] && NeQ[h*e - 2*g*f, 0]

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx}{\sqrt{d + ex + fx^2} (ae + bex + bfx^2)^2} dx &= -\frac{((Ab - 2aB)e - b(Be - 2Af)x)\sqrt{d + ex + fx^2}}{e(bd - ae)(be - 4af)(ae + bex + bfx^2)} - \int \frac{-\frac{1}{2}b(bd - ae)f^2(2bBde - \dots)}{\dots} \\
&= -\frac{((Ab - 2aB)e - b(Be - 2Af)x)\sqrt{d + ex + fx^2}}{e(bd - ae)(be - 4af)(ae + bex + bfx^2)} - \frac{B \int \frac{e+2fx}{\sqrt{d+ex+fx^2}(ae+be \dots)}}{4(bd - ae)} \\
&= -\frac{((Ab - 2aB)e - b(Be - 2Af)x)\sqrt{d + ex + fx^2}}{e(bd - ae)(be - 4af)(ae + bex + bfx^2)} + \frac{(Be) \text{Subst} \left(\int \frac{\dots}{bde-a \dots} \right)}{2} \\
&= -\frac{((Ab - 2aB)e - b(Be - 2Af)x)\sqrt{d + ex + fx^2}}{e(bd - ae)(be - 4af)(ae + bex + bfx^2)} + \frac{(Be - 2Af)(8aef - \dots)}{2e}
\end{aligned}$$

Mathematica [B] time = 1.49, size = 767, normalized size = 3.08

$$-\frac{(ae + bx(e + fx)) \log(b(e + 2fx) - \sqrt{b} \sqrt{e} \sqrt{be - 4af}) (-8abef(Be - 2Af) - b^{3/2}Be^{5/2}\sqrt{be - 4af} + 4a\sqrt{b}Be^{3/2})}{\dots}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x)/(Sqrt[d + e*x + f*x^2]*(a*e + b*e*x + b*f*x^2)^2), x]

[Out]
$$\begin{aligned}
& -1/4*(4*b*Sqrt[e]*Sqrt[b*d - a*e]*f*Sqrt[b*e - 4*a*f]*Sqrt[d + x*(e + f*x)] \\
& *(-(B*e*(2*a + b*x)) + A*b*(e + 2*f*x)) - (-(b^(3/2)*B*e^(5/2)*Sqrt[b*e - 4 \\
& *a*f]) + 4*a*Sqrt[b]*B*e^(3/2)*f*Sqrt[b*e - 4*a*f] - 8*a*b*e*f*(B*e - 2*A*f \\
&) + b^2*(B*e - 2*A*f)*(e^2 + 4*d*f))*(a*e + b*x*(e + f*x))*Log[-(Sqrt[b]*Sqr \\
& rt[e]*Sqrt[b*e - 4*a*f]) + b*(e + 2*f*x)] + (b^(3/2)*B*e^(5/2)*Sqrt[b*e - 4 \\
& *a*f] - 4*a*Sqrt[b]*B*e^(3/2)*f*Sqrt[b*e - 4*a*f] - 8*a*b*e*f*(B*e - 2*A*f) \\
& + b^2*(B*e - 2*A*f)*(e^2 + 4*d*f))*(a*e + b*x*(e + f*x))*Log[Sqrt[b]*Sqrt[\\
& e]*Sqrt[b*e - 4*a*f] + b*(e + 2*f*x)] - (b^(3/2)*B*e^(5/2)*Sqrt[b*e - 4*a*f \\
&] - 4*a*Sqrt[b]*B*e^(3/2)*f*Sqrt[b*e - 4*a*f] - 8*a*b*e*f*(B*e - 2*A*f) + b \\
& ^2*(B*e - 2*A*f)*(e^2 + 4*d*f))*(a*e + b*x*(e + f*x))*Log[Sqrt[b]*(e^(3/2)* \\
& Sqrt[b*e - 4*a*f] + Sqrt[b]*(e^2 - 4*d*f) + 2*Sqrt[e]*f*Sqrt[b*e - 4*a*f]*x \\
& - 4*Sqrt[b*d - a*e]*f*Sqrt[d + x*(e + f*x)])] + (-(b^(3/2)*B*e^(5/2)*Sqrt[\\
& b*e - 4*a*f]) + 4*a*Sqrt[b]*B*e^(3/2)*f*Sqrt[b*e - 4*a*f] - 8*a*b*e*f*(B*e \\
& - 2*A*f) + b^2*(B*e - 2*A*f)*(e^2 + 4*d*f))*(a*e + b*x*(e + f*x))*Log[Sqrt[\\
& b]*(e^(3/2)*Sqrt[b*e - 4*a*f] - Sqrt[b]*(e^2 - 4*d*f) + 2*Sqrt[e]*f*Sqrt[b* \\
& e - 4*a*f]*x + 4*Sqrt[b*d - a*e]*f*Sqrt[d + x*(e + f*x)])]/(b*e^(3/2)*(b*d \\
& - a*e)^(3/2)*f*(b*e - 4*a*f)^(3/2)*(a*e + b*x*(e + f*x)))
\end{aligned}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas")
```

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Unable to divide, perhaps due to rounding er
ror%%{%%{1, [2]%%}, [8,2,0,0,0]%%}+%%{%%{[-4, [1]%%}, 0]: [1,0,%%{-1,
[1]%%}]}%%, [7,2,1,0,0]%%}+%%{%%{6, [1]%%}, [6,2,2,0,0]%%}+%%{%%{-4, [2]
%%}, [6,2,0,0,1]%%}+%%{%%{8, [2]%%}, [6,1,1,1,0]%%}+%%{%%{-4,0]: [1,0,
%%{-1, [1]%%}]}%%, [5,2,3,0,0]%%}+%%{%%{12, [1]%%}, 0]: [1,0,%%{-1, [1]
%%}]}%%, [5,2,1,0,1]%%}+%%{%%{-24, [1]%%}, 0]: [1,0,%%{-1, [1]%%}]}%%
}, [5,1,2,1,0]%%}+%%{1, [4,2,4,0,0]%%}+%%{%%{-14, [1]%%}, [4,2,2,0,1]%%}
+%%{%%{6, [2]%%}, [4,2,0,0,2]%%}+%%{%%{26, [1]%%}, [4,1,3,1,0]%%}+%%{%%
%%{-16, [2]%%}, [4,1,1,1,1]%%}+%%{%%{16, [2]%%}, [4,0,2,2,0]%%}+%%{%%{8
,0]: [1,0,%%{-1, [1]%%}]}%%, [3,2,3,0,1]%%}+%%{%%{-12, [1]%%}, 0]: [1,0
,%%{-1, [1]%%}]}%%, [3,2,1,0,2]%%}+%%{%%{-12,0]: [1,0,%%{-1, [1]%%}]}%%,
[3,1,4,1,0]%%}+%%{%%{32, [1]%%}, 0]: [1,0,%%{-1, [1]%%}]}%%, [3,1,2,1,
1]%%}+%%{%%{-32, [1]%%}, 0]: [1,0,%%{-1, [1]%%}]}%%, [3,0,3,2,0]%%}+
%%{-2, [2,2,4,0,1]%%}+%%{%%{10, [1]%%}, [2,2,2,0,2]%%}+%%{%%{-4, [2]%%}
, [2,2,0,0,3]%%}+%%{2, [2,1,5,1,0]%%}+%%{%%{-28, [1]%%}, [2,1,3,1,1]%%}+
%%{%%{8, [2]%%}, [2,1,1,1,2]%%}+%%{%%{24, [1]%%}, [2,0,4,2,0]%%}+%%{%%
{-4,0]: [1,0,%%{-1, [1]%%}]}%%, [1,2,3,0,2]%%}+%%{%%{4, [1]%%}, 0]: [1
,0,%%{-1, [1]%%}]}%%, [1,2,1,0,3]%%}+%%{%%{12,0]: [1,0,%%{-1, [1]%%}]}%%
, [1,1,4,1,1]%%}+%%{%%{-8, [1]%%}, 0]: [1,0,%%{-1, [1]%%}]}%%, [1,1,2,1
,2]%%}+%%{%%{-8,0]: [1,0,%%{-1, [1]%%}]}%%, [1,0,5,2,0]%%}+%%{1, [0,2,4,
0,2]%%}+%%{%%{-2, [1]%%}, [0,2,2,0,3]%%}+%%{%%{1, [2]%%}, [0,2,0,0,4]%%
}+%%{-2, [0,1,5,1,1]%%}+%%{%%{2, [1]%%}, [0,1,3,1,2]%%}+%%{1, [0,0,6,2,
0]%%} / %%{%%{1, [3]%%}, [8,2,0,0,0]%%}+%%{%%{poly1[%%{-4, [2]%%}, 0]: [
1,0,%%{-1, [1]%%}]}%%, [7,2,1,0,0]%%}+%%{%%{6, [2]%%}, [6,2,2,0,0]%%}+%%
%%{-4, [3]%%}, [6,2,0,0,1]%%}+%%{%%{8, [3]%%}, [6,1,1,1,0]%%}+%%{%%{p
```

```

oly1[%%{-4, [1]%%}, 0] : [1, 0, %%{-1, [1]%%}]%%, [5, 2, 3, 0, 0]%%}+%%{%%{poly1
[%%{12, [2]%%}, 0] : [1, 0, %%{-1, [1]%%}]%%, [5, 2, 1, 0, 1]%%}+%%{%%{poly1[%%
{-24, [2]%%}, 0] : [1, 0, %%{-1, [1]%%}]%%, [5, 1, 2, 1, 0]%%}+%%{%%{1, [1]%%}, [
4, 2, 4, 0, 0]%%}+%%{%%{-14, [2]%%}, [4, 2, 2, 0, 1]%%}+%%{%%{6, [3]%%}, [4, 2, 0
, 0, 2]%%}+%%{%%{26, [2]%%}, [4, 1, 3, 1, 0]%%}+%%{%%{-16, [3]%%}, [4, 1, 1, 1, 1
]%%}+%%{%%{16, [3]%%}, [4, 0, 2, 2, 0]%%}+%%{%%{poly1[%%{8, [1]%%}, 0] : [1, 0
, %%{-1, [1]%%}]%%, [3, 2, 3, 0, 1]%%}+%%{%%{poly1[%%{-12, [2]%%}, 0] : [1, 0, %%
{-1, [1]%%}]%%, [3, 2, 1, 0, 2]%%}+%%{%%{poly1[%%{-12, [1]%%}, 0] : [1, 0, %%{-
1, [1]%%}]%%, [3, 1, 4, 1, 0]%%}+%%{%%{poly1[%%{32, [2]%%}, 0] : [1, 0, %%{-1, [1
]%%}]%%, [3, 1, 2, 1, 1]%%}+%%{%%{poly1[%%{-32, [2]%%}, 0] : [1, 0, %%{-1, [1]%%
}]%%, [3, 0, 3, 2, 0]%%}+%%{%%{-2, [1]%%}, [2, 2, 4, 0, 1]%%}+%%{%%{10, [2]%%
}, [2, 2, 2, 0, 2]%%}+%%{%%{-4, [3]%%}, [2, 2, 0, 0, 3]%%}+%%{%%{2, [1]%%}, [2, 1
, 5, 1, 0]%%}+%%{%%{-28, [2]%%}, [2, 1, 3, 1, 1]%%}+%%{%%{8, [3]%%}, [2, 1, 1, 1,
2]%%}+%%{%%{24, [2]%%}, [2, 0, 4, 2, 0]%%}+%%{%%{poly1[%%{-4, [1]%%}, 0] : [1
, 0, %%{-1, [1]%%}]%%, [1, 2, 3, 0, 2]%%}+%%{%%{poly1[%%{4, [2]%%}, 0] : [1, 0, %%
{-1, [1]%%}]%%, [1, 2, 1, 0, 3]%%}+%%{%%{poly1[%%{12, [1]%%}, 0] : [1, 0, %%{-1
, [1]%%}]%%, [1, 1, 4, 1, 1]%%}+%%{%%{poly1[%%{-8, [2]%%}, 0] : [1, 0, %%{-1, [1
]%%}]%%, [1, 1, 2, 1, 2]%%}+%%{%%{poly1[%%{-8, [1]%%}, 0] : [1, 0, %%{-1, [1]%%
}]%%, [1, 0, 5, 2, 0]%%}+%%{%%{1, [1]%%}, [0, 2, 4, 0, 2]%%}+%%{%%{-2, [2]%%}, [
0, 2, 2, 0, 3]%%}+%%{%%{1, [3]%%}, [0, 2, 0, 0, 4]%%}+%%{%%{-2, [1]%%}, [0, 1, 5,
1, 1]%%}+%%{%%{2, [2]%%}, [0, 1, 3, 1, 2]%%}+%%{%%{1, [1]%%}, [0, 0, 6, 2, 0]%%
} Error: Bad Argument Value

```

maple [B] time = 0.05, size = 3606, normalized size = 14.48

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((B*x+A)/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^{(1/2)}, x)$

[Out]
$$-1/e/(4*a*f-b*e)/(a*e-b*d)/(x+1/2/f*e-1/2/b/f*(-b*e*(4*a*f-b*e))^{(1/2)})*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)^2*f+(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)-1/b*(a*e-b*d))^{(1/2)}*A+1/2/f/(4*a*f-b*e)/(a*e-b*d)/(x+1/2/f*e-1/2/b/f*(-b*e*(4*a*f-b*e))^{(1/2)})*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)^2*f+(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)-1/b*(a*e-b*d))^{(1/2)}*B-1/2/f/e/(4*a*f-b*e)/b/(a*e-b*d)/(x+1/2/f*e-1/2/b/f*(-b*e*(4*a*f-b*e))^{(1/2)})*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)^2*f+(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)-1/b*(a*e-b*d))^{(1/2)}*B*(-b*e*(4*a*f-b*e))^{(1/2)}+1/2/e/(4*a*f-b*e)/b*(-b*e*(4*a*f-b*e))^{(1/2)}/(a*e-b*d)/(-1/b*(a*e-b*d))^{(1/2)}*\ln((-2/b*(a*e-b*d)+(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)+2*(-1/b*(a*e-b*d))^{(1/2)}*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)^2*f+(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)-1/b*(a*e-b*d))^{(1/2)})/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)*A-1/4/f/(4*a*f-b*e)/b*(-b*e*(4*a*f-b*e))^{(1/2)}/(a*e-b*d)/(-1/b*(a*e-b$$

$$\begin{aligned}
& d))^{(1/2)} * \ln((-2/b*(a*e-b*d)+(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x-1/2*(-b*e+(-b*e \\
& *(4*a*f-b*e))^{(1/2)})/b/f)+2*(-1/b*(a*e-b*d))^{(1/2)}*((x-1/2*(-b*e+(-b*e*(4*a \\
& *f-b*e))^{(1/2)})/b/f)^2*f+(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x-1/2*(-b*e+(-b*e*(4*a \\
& *f-b*e))^{(1/2)})/b/f)-1/b*(a*e-b*d))^{(1/2)})/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f) \\
& *B-1/4/f/b/(a*e-b*d)/(-1/b*(a*e-b*d))^{(1/2)} * \ln((-2/b*(a*e-b*d)+ \\
& (-b*e*(4*a*f-b*e))^{(1/2)}/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)+2*(- \\
& 1/b*(a*e-b*d))^{(1/2)}*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)^2*f+(-b*e \\
& *(4*a*f-b*e))^{(1/2)}/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)-1/b*(a*e- \\
& b*d))^{(1/2)})/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f) *B-2/e/(4*a*f-b*e) \\
& /(-b*e*(4*a*f-b*e))^{(1/2)}/(-1/b*(a*e-b*d))^{(1/2)} * \ln((-2/b*(a*e-b*d)+(-b*e*(\\
& 4*a*f-b*e))^{(1/2)}/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)+2*(-1/b*(a* \\
& e-b*d))^{(1/2)}*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)^2*f+(-b*e*(4*a*f \\
& -b*e))^{(1/2)}/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)-1/b*(a*e-b*d))^{(\\
& 1/2)})/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f) *A*f+1/(4*a*f-b*e)/(-b*e* \\
& (4*a*f-b*e))^{(1/2)}/(-1/b*(a*e-b*d))^{(1/2)} * \ln((-2/b*(a*e-b*d)+(-b*e*(4*a*f-b \\
& *e))^{(1/2)}/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)+2*(-1/b*(a*e-b*d)) \\
& ^{(1/2)}*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)^2*f+(-b*e*(4*a*f-b*e))^{(\\
& 1/2)}/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)-1/b*(a*e-b*d))^{(1/2)})/(\\
& x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f) *B+2/e/(4*a*f-b*e)/(-b*e*(4*a*f- \\
& b*e))^{(1/2)}/(-1/b*(a*e-b*d))^{(1/2)} * \ln((-2/b*(a*e-b*d)-(-b*e*(4*a*f-b*e))^{(1 \\
& /2)}/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)+2*(-1/b*(a*e-b*d))^{(1/2)}*((\\
& x+1/2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)^2*f-(-b*e*(4*a*f-b*e))^{(1/2)}/b*(\\
& x+1/2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)-1/b*(a*e-b*d))^{(1/2)})/(x+1/2*(b*e \\
& +(-b*e*(4*a*f-b*e))^{(1/2)})/b/f) *A*f-1/(4*a*f-b*e)/(-b*e*(4*a*f-b*e))^{(1/2) \\
& }/(-1/b*(a*e-b*d))^{(1/2)} * \ln((-2/b*(a*e-b*d)-(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x+1/ \\
& 2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)+2*(-1/b*(a*e-b*d))^{(1/2)}*((x+1/2*(b*e \\
& +(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)^2*f-(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x+1/2*(b*e+ \\
& (-b*e*(4*a*f-b*e))^{(1/2)})/b/f)-1/b*(a*e-b*d))^{(1/2)})/(x+1/2*(b*e+(-b*e*(4*a \\
& *f-b*e))^{(1/2)})/b/f) *B-1/e/(4*a*f-b*e)/(a*e-b*d)/(x+1/2*f*e+1/2/b/f*(-b*e* \\
& (4*a*f-b*e))^{(1/2)}*((x+1/2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)^2*f-(-b*e*(\\
& 4*a*f-b*e))^{(1/2)}/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)-1/b*(a*e-b*d \\
&))^{(1/2)} *A+1/2/f/(4*a*f-b*e)/(a*e-b*d)/(x+1/2*f*e+1/2/b/f*(-b*e*(4*a*f-b*e) \\
&))^{(1/2)}*((x+1/2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)^2*f-(-b*e*(4*a*f-b*e)) \\
& ^{(1/2)}/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)-1/b*(a*e-b*d))^{(1/2)} *B+ \\
& 1/2/f/e/(4*a*f-b*e)/b/(a*e-b*d)/(x+1/2*f*e+1/2/b/f*(-b*e*(4*a*f-b*e))^{(1/2) \\
& }*((x+1/2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)^2*f-(-b*e*(4*a*f-b*e))^{(1/2)}/ \\
& b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)-1/b*(a*e-b*d))^{(1/2)} *B*(-b*e*(\\
& 4*a*f-b*e))^{(1/2)}-1/2/e/(4*a*f-b*e)/b*(-b*e*(4*a*f-b*e))^{(1/2)}/(a*e-b*d)/(- \\
& 1/b*(a*e-b*d))^{(1/2)} * \ln((-2/b*(a*e-b*d)-(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x+1/2*(\\
& b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)+2*(-1/b*(a*e-b*d))^{(1/2)}*((x+1/2*(b*e+(- \\
& b*e*(4*a*f-b*e))^{(1/2)})/b/f)^2*f-(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x+1/2*(b*e+(- \\
& *e*(4*a*f-b*e))^{(1/2)})/b/f)-1/b*(a*e-b*d))^{(1/2)})/(x+1/2*(b*e+(-b*e*(4*a*f- \\
& b*e))^{(1/2)})/b/f) *A+1/4/f/(4*a*f-b*e)/b*(-b*e*(4*a*f-b*e))^{(1/2)}/(a*e-b*d) \\
& /(-1/b*(a*e-b*d))^{(1/2)} * \ln((-2/b*(a*e-b*d)-(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x+1/ \\
& 2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)+2*(-1/b*(a*e-b*d))^{(1/2)}*((x+1/2*(b*e
\end{aligned}$$

$$+(-b*e*(4*a*f-b*e))^{(1/2)}/b/f)^2*f-(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)}/b/f)-1/b*(a*e-b*d))^{(1/2)})/(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)}/b/f))*B-1/4/f/b/(a*e-b*d)/(-1/b*(a*e-b*d))^{(1/2)}*\ln((-2/b*(a*e-b*d)-(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)}/b/f)+2*(-1/b*(a*e-b*d))^{(1/2)}*((x+1/2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)}/b/f)^2*f-(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)}/b/f)-1/b*(a*e-b*d))^{(1/2)})/(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)}/b/f))*B$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx + A}{(bf x^2 + bex + ae)^2 \sqrt{fx^2 + ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")

[Out] integrate((B*x + A)/((b*f*x^2 + b*e*x + a*e)^2*sqrt(f*x^2 + e*x + d)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + Bx}{(bf x^2 + bex + ae)^2 \sqrt{fx^2 + ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x)/((a*e + b*e*x + b*f*x^2)^2*(d + e*x + f*x^2)^(1/2)),x)

[Out] int((A + B*x)/((a*e + b*e*x + b*f*x^2)^2*(d + e*x + f*x^2)^(1/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(b*f*x**2+b*e*x+a*e)**2/(f*x**2+e*x+d)**(1/2),x)

[Out] Timed out

$$3.36 \quad \int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^2} dx$$

Optimal. Leaf size=48

$$\frac{2(-2ah + x(2cg - bh) + bg)}{d^2(b^2 - 4ac)\sqrt{a + bx + cx^2}}$$

[Out] $-2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(-4*a*c+b^2)/d^2/(c*x^2+b*x+a)^(1/2)$

Rubi [A] time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {998, 636}

$$\frac{2(-2ah + x(2cg - bh) + bg)}{d^2(b^2 - 4ac)\sqrt{a + bx + cx^2}}$$

Antiderivative was successfully verified.

[In] Int[((g + h*x)*Sqrt[a + b*x + c*x^2])/(a*d + b*d*x + c*d*x^2)^2, x]

[Out] $(-2*(b*g - 2*a*h + (2*c*g - b*h)*x))/((b^2 - 4*a*c)*d^2*Sqrt[a + b*x + c*x^2])$

Rule 636

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] := Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0]

Rule 998

Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_.), x_Symbol] := Dist[(c/f)^p, Int[(g + h*x)^m*(d + e*x + f*x^2)^(p + q), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q}, x] && EqQ[c*d - a*f, 0] && EqQ[b*d - a*e, 0] && (IntegerQ[p] || GtQ[c/f, 0]) && (!IntegerQ[q] || LeafCount[d + e*x + f*x^2] <= LeafCount[a + b*x + c*x^2])

Rubi steps

$$\int \frac{(g + hx)\sqrt{a + bx + cx^2}}{(ad + bdx + cd^2x^2)^2} dx = \frac{\int \frac{g+hx}{(a+bx+cx^2)^{3/2}} dx}{d^2}$$

$$= -\frac{2(bg - 2ah + (2cg - bh)x)}{(b^2 - 4ac)d^2\sqrt{a + bx + cx^2}}$$

Mathematica [A] time = 0.19, size = 46, normalized size = 0.96

$$\frac{4ah - 2bg + 2bhx - 4cgx}{d^2(b^2 - 4ac)\sqrt{a + x(b + cx)}}$$

Antiderivative was successfully verified.

[In] Integrate[((g + h*x)*Sqrt[a + b*x + c*x^2])/(a*d + b*d*x + c*d*x^2)^2,x]

[Out] (-2*b*g + 4*a*h - 4*c*g*x + 2*b*h*x)/((b^2 - 4*a*c)*d^2*Sqrt[a + x*(b + c*x)])

fricas [A] time = 1.97, size = 85, normalized size = 1.77

$$\frac{2\sqrt{cx^2 + bx + a}(bg - 2ah + (2cg - bh)x)}{(b^2c - 4ac^2)d^2x^2 + (b^3 - 4abc)d^2x + (ab^2 - 4a^2c)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="fricas")

[Out] -2*sqrt(c*x^2 + b*x + a)*(b*g - 2*a*h + (2*c*g - b*h)*x)/((b^2*c - 4*a*c^2)*d^2*x^2 + (b^3 - 4*a*b*c)*d^2*x + (a*b^2 - 4*a^2*c)*d^2)

giac [A] time = 0.27, size = 81, normalized size = 1.69

$$\frac{2\left(\frac{(2cd^2g-bd^2h)x}{b^2d^4-4acd^4} + \frac{bd^2g-2ad^2h}{b^2d^4-4acd^4}\right)}{\sqrt{cx^2 + bx + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="giac")

[Out] $-2*((2*c*d^2*g - b*d^2*h)*x/(b^2*d^4 - 4*a*c*d^4) + (b*d^2*g - 2*a*d^2*h)/(b^2*d^4 - 4*a*c*d^4))/\sqrt{c*x^2 + b*x + a}$

maple [A] time = 0.00, size = 48, normalized size = 1.00

$$\frac{2(bhx - 2cgx + 2ah - bg)}{\sqrt{cx^2 + bx + a} (4ac - b^2) d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((h*x+g)*(c*x^2+b*x+a)^{(1/2)}/(c*d*x^2+b*d*x+a*d)^2, x)$

[Out] $-2/(c*x^2+b*x+a)^{(1/2)}*(b*h*x-2*c*g*x+2*a*h-b*g)/d^2/(4*a*c-b^2)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + bx + a} (hx + g)}{(cdx^2 + bdx + ad)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((h*x+g)*(c*x^2+b*x+a)^{(1/2)}/(c*d*x^2+b*d*x+a*d)^2, x, \text{algorithm}="maxima")$

[Out] $\text{integrate}(\sqrt{c*x^2 + b*x + a}*(h*x + g)/(c*d*x^2 + b*d*x + a*d)^2, x)$

mupad [B] time = 3.75, size = 49, normalized size = 1.02

$$\frac{4ah - 2bg + 2bhx - 4cgx}{(b^2d^2 - 4acd^2) \sqrt{cx^2 + bx + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(((g + h*x)*(a + b*x + c*x^2)^{(1/2)})/(a*d + b*d*x + c*d*x^2)^2, x)$

[Out] $(4*a*h - 2*b*g + 2*b*h*x - 4*c*g*x)/((b^2*d^2 - 4*a*c*d^2)*(a + b*x + c*x^2)^{(1/2)})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{g}{a\sqrt{a+bx+cx^2} + bx\sqrt{a+bx+cx^2} + cx^2\sqrt{a+bx+cx^2}} dx + \int \frac{hx}{a\sqrt{a+bx+cx^2} + bx\sqrt{a+bx+cx^2} + cx^2\sqrt{a+bx+cx^2}} dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x**2+b*x+a)**(1/2)/(c*d*x**2+b*d*x+a*d)**2,x)
```

```
[Out] (Integral(g/(a*sqrt(a + b*x + c*x**2) + b*x*sqrt(a + b*x + c*x**2) + c*x**2*sqrt(a + b*x + c*x**2)), x) + Integral(h*x/(a*sqrt(a + b*x + c*x**2) + b*x*sqrt(a + b*x + c*x**2) + c*x**2*sqrt(a + b*x + c*x**2)), x))/d**2
```

$$3.37 \quad \int \frac{3+2x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx$$

Optimal. Leaf size=17

$$\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

[Out] arctanh(x/(-x^2-4*x-3)^(1/2))

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1027, 206}

$$\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

[In] Int[(3 + 2*x)/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]

[Out] ArcTanh[x/Sqrt[-3 - 4*x - x^2]]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1027

Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[g, Subst[Int[1/(a + (c*d - a*f)*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] && EqQ[2*h*d - g*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{3+2x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx &= 3 \text{Subst} \left(\int \frac{1}{3-3x^2} dx, x, \frac{x}{\sqrt{-3-4x-x^2}} \right) \\ &= \tanh^{-1} \left(\frac{x}{\sqrt{-3-4x-x^2}} \right) \end{aligned}$$

Mathematica [C] time = 0.29, size = 165, normalized size = 9.71

$$\frac{1}{6} \left(\sqrt{1-2i\sqrt{2}} (\sqrt{2}+i) \tanh^{-1} \left(\frac{(2-i\sqrt{2})x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}} \sqrt{-x^2-4x-3}} \right) + \sqrt{1+2i\sqrt{2}} (\sqrt{2}-i) \tanh^{-1} \left(\frac{(2+i\sqrt{2})x+2}{\sqrt{2-4i\sqrt{2}} \sqrt{-x^2-4x-3}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(3 + 2*x)/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)), x]

[Out] (Sqrt[1 - (2*I)*Sqrt[2]]*(I + Sqrt[2])*ArcTanh[(2 - (2*I)*Sqrt[2] + (2 - I*Sqrt[2])*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])] + Sqrt[1 + (2*I)*Sqrt[2]]*(-I + Sqrt[2])*ArcTanh[(2 + (2*I)*Sqrt[2] + (2 + I*Sqrt[2])*x)/(Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])])/6

fricas [B] time = 1.34, size = 56, normalized size = 3.29

$$-\frac{1}{4} \log \left(-\frac{2\sqrt{-x^2-4x-3}x+4x+3}{x^2} \right) + \frac{1}{4} \log \left(\frac{2\sqrt{-x^2-4x-3}x-4x-3}{x^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+2*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2), x, algorithm="fricas")

[Out] -1/4*log(-(2*sqrt(-x^2 - 4*x - 3))*x + 4*x + 3)/x^2) + 1/4*log((2*sqrt(-x^2 - 4*x - 3))*x - 4*x - 3)/x^2)

giac [B] time = 0.23, size = 98, normalized size = 5.76

$$\frac{1}{2} \log \left(\frac{2(\sqrt{-x^2-4x-3}-1)}{x+2} + \frac{3(\sqrt{-x^2-4x-3}-1)^2}{(x+2)^2} + 1 \right) - \frac{1}{2} \log \left(\frac{2(\sqrt{-x^2-4x-3}-1)}{x+2} + \frac{(\sqrt{-x^2-4x-3}-1)^2}{(x+2)^2} + 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+2*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2), x, algorithm="giac")

[Out] 1/2*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) - 1/2*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)

maple [B] time = 0.02, size = 94, normalized size = 5.53

$$\frac{\sqrt{3} \sqrt{4} \sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2} - 12} \operatorname{arctanh}\left(\frac{3x}{\left(-x-\frac{3}{2}\right) \sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2} - 12}}\right)}{6 \sqrt{\frac{\frac{x^2}{\left(-x-\frac{3}{2}\right)^2} - 4}{\left(\frac{x}{-x-\frac{3}{2}} + 1\right)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3+2*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2), x)`

[Out] `-1/6*3^(1/2)*4^(1/2)/((x^2/(-3/2-x)^2-4)/(x/(-3/2-x)+1)^2)^(1/2)/(x/(-3/2-x)+1)*(3*x^2/(-3/2-x)^2-12)^(1/2)*arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2))`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x + 3}{(2x^2 + 4x + 3)\sqrt{-x^2 - 4x - 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3+2*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2), x, algorithm="maxima")`

[Out] `integrate((2*x + 3)/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{2x + 3}{\sqrt{-x^2 - 4x - 3} (2x^2 + 4x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x + 3)/((-4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)`

[Out] `int((2*x + 3)/((-4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x + 3}{\sqrt{-(x+1)(x+3)} (2x^2 + 4x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral((2*x + 3)/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```


$$3.38 \quad \int \frac{3+4x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx$$

Optimal. Leaf size=86

$$\sqrt{2} \tan^{-1} \left(\frac{1 - \frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}} \right) - \sqrt{2} \tan^{-1} \left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}} + 1}{\sqrt{2}} \right) + \tanh^{-1} \left(\frac{x}{\sqrt{-x^2-4x-3}} \right)$$

[Out] arctanh(x/(-x^2-4*x-3)^(1/2))+arctan(1/2*(1+(-3-x)/(-x^2-4*x-3)^(1/2))*2^(1/2))*2^(1/2)-arctan(1/2*(1+(3+x)/(-x^2-4*x-3)^(1/2))*2^(1/2))*2^(1/2)

Rubi [A] time = 0.18, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {1028, 986, 12, 1026, 1161, 618, 204, 1027, 206}

$$\sqrt{2} \tan^{-1} \left(\frac{1 - \frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}} \right) - \sqrt{2} \tan^{-1} \left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}} + 1}{\sqrt{2}} \right) + \tanh^{-1} \left(\frac{x}{\sqrt{-x^2-4x-3}} \right)$$

Antiderivative was successfully verified.

[In] Int[(3 + 4*x)/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)), x]

[Out] Sqrt[2]*ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]] - Sqrt[2]*ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]] + ArcTanh[x/Sqrt[-3 - 4*x - x^2]]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e - b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e - b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1026

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 - 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1027

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[g, Subst[Int[1/(a + (c*d - a*f)*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] && EqQ[2*h*d - g*e, 0]
```

Rule 1028

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := -Dist[(2*h*d - g*e)/e, Int[1/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] && NeQ[2*h*d - g*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
```

, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))

Rubi steps

$$\begin{aligned}
 \int \frac{3+4x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx &= - \left(3 \int \frac{1}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx \right) - \int \frac{-6-4x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx \\
 &= \frac{1}{2} \int \frac{-6-4x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx - \frac{1}{2} \int \frac{4x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx \\
 &= 2 \tanh^{-1} \left(\frac{x}{\sqrt{-3-4x-x^2}} \right) + 2 \int \frac{x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx - 3 \int \frac{1}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx \\
 &= \tanh^{-1} \left(\frac{x}{\sqrt{-3-4x-x^2}} \right) + 16 \operatorname{Subst} \left(\int \frac{1+3x^2}{-4-8x^2-36x^4} dx, x, \frac{1-x}{\sqrt{-3-4x-x^2}} \right) \\
 &= \tanh^{-1} \left(\frac{x}{\sqrt{-3-4x-x^2}} \right) - \frac{2}{3} \operatorname{Subst} \left(\int \frac{1}{\frac{1}{3} - \frac{2x}{3} + x^2} dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}} \right) \\
 &= \tanh^{-1} \left(\frac{x}{\sqrt{-3-4x-x^2}} \right) + \frac{4}{3} \operatorname{Subst} \left(\int \frac{1}{-\frac{8}{9} - x^2} dx, x, \frac{2}{3} \left(-1 + \frac{3+x}{\sqrt{-3-4x-x^2}} \right) \right) \\
 &= \sqrt{2} \tan^{-1} \left(\frac{1 - \frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}} \right) - \sqrt{2} \tan^{-1} \left(\frac{1 + \frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}} \right) + \tanh^{-1} \left(\frac{x}{\sqrt{-3-4x-x^2}} \right)
 \end{aligned}$$

Mathematica [C] time = 0.11, size = 150, normalized size = 1.74

$$-\frac{1}{2}i \left(\sqrt{1+2i\sqrt{2}} \tanh^{-1} \left(\frac{(2-i\sqrt{2})x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}} \sqrt{-x^2-4x-3}} \right) - \sqrt{1-2i\sqrt{2}} \tanh^{-1} \left(\frac{(2+i\sqrt{2})x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}} \sqrt{-x^2-4x-3}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(3 + 4*x)/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)), x]

[Out] (-1/2*I)*(Sqrt[1 + (2*I)*Sqrt[2]]*ArcTanh[(2 - (2*I)*Sqrt[2] + (2 - I*Sqrt[2]))*x]/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])) - Sqrt[1 - (2*I)*Sqr

t[2]]*ArcTanh[(2 + (2*I)*Sqrt[2] + (2 + I*Sqrt[2])*x)/(Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]])

fricas [A] time = 1.02, size = 132, normalized size = 1.53

$$\frac{1}{2} \sqrt{2} \arctan\left(\frac{\sqrt{2}x + 3\sqrt{2}\sqrt{-x^2 - 4x - 3}}{2(2x + 3)}\right) + \frac{1}{2} \sqrt{2} \arctan\left(-\frac{\sqrt{2}x - 3\sqrt{2}\sqrt{-x^2 - 4x - 3}}{2(2x + 3)}\right) - \frac{1}{4} \log\left(-\frac{2\sqrt{-x^2 - 4x - 3}}{2x + 3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+4*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")

[Out] 1/2*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) + 1/2*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) - 1/4*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) + 1/4*log((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)

giac [B] time = 0.27, size = 163, normalized size = 1.90

$$\sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} \left(\frac{3(\sqrt{-x^2 - 4x - 3} - 1)}{x + 2} + 1\right)\right) + \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} \left(\frac{\sqrt{-x^2 - 4x - 3} - 1}{x + 2} + 1\right)\right) + \frac{1}{2} \log\left(\frac{2(\sqrt{-x^2 - 4x - 3} - 1)}{x + 2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+4*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")

[Out] sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/2*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) - 1/2*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)

maple [A] time = 0.01, size = 123, normalized size = 1.43

$$\frac{\sqrt{3} \sqrt{4} \sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2} - 12} \left(-\operatorname{arctanh}\left(\frac{3x}{\left(-x-\frac{3}{2}\right) \sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2} - 12}}\right) + \sqrt{2} \arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2} - 12} \sqrt{2}}{6}\right) \right)}{6 \sqrt{\frac{\frac{x^2}{\left(-x-\frac{3}{2}\right)^2} - 4}{\left(\frac{x}{-x-\frac{3}{2}} + 1\right)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*x+3)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)`

[Out] $\frac{1}{6} \cdot 3^{1/2} \cdot 4^{1/2} \cdot (3/(-x-3/2)^2 \cdot x^2 - 12)^{1/2} \cdot (2^{1/2} \cdot \arctan(1/6 \cdot (3/(-x-3/2)^2 \cdot x^2 - 12)^{1/2} \cdot 2^{1/2}) - \operatorname{arctanh}(3/(-x-3/2)/(3/(-x-3/2)^2 \cdot x^2 - 12)^{1/2} \cdot x)) / ((1/(-x-3/2)^2 \cdot x^2 - 4)/(1/(-x-3/2) \cdot x + 1)^2)^{1/2} / (1/(-x-3/2) \cdot x + 1)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x + 3}{(2x^2 + 4x + 3)\sqrt{-x^2 - 4x - 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3+4*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")`

[Out] `integrate((4*x + 3)/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4x + 3}{\sqrt{-x^2 - 4x - 3} (2x^2 + 4x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*x + 3)/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)`

[Out] `int((4*x + 3)/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x + 3}{\sqrt{-(x + 1)(x + 3)} (2x^2 + 4x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3+4*x)/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)`

[Out] `Integral((4*x + 3)/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)`

$$3.39 \quad \int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^{3/2}} dx$$

Optimal. Leaf size=136

$$\frac{h\sqrt{a+bx+cx^2} \log(a+bx+cx^2)}{2cd\sqrt{ad+bdx+cdx^2}} - \frac{\sqrt{a+bx+cx^2} (2cg-bh) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{cd\sqrt{b^2-4ac}\sqrt{ad+bdx+cdx^2}}$$

[Out] $1/2*h*\ln(c*x^2+b*x+a)*(c*x^2+b*x+a)^{(1/2)}/c/d/(c*d*x^2+b*d*x+a*d)^{(1/2)}-(-b*h+2*c*g)*\operatorname{arctanh}((2*c*x+b)/(-4*a*c+b^2)^{(1/2)})*(c*x^2+b*x+a)^{(1/2)}/c/d/(-4*a*c+b^2)^{(1/2)}/(c*d*x^2+b*d*x+a*d)^{(1/2)}$

Rubi [A] time = 0.11, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {999, 634, 618, 206, 628}

$$\frac{h\sqrt{a+bx+cx^2} \log(a+bx+cx^2)}{2cd\sqrt{ad+bdx+cdx^2}} - \frac{\sqrt{a+bx+cx^2} (2cg-bh) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{cd\sqrt{b^2-4ac}\sqrt{ad+bdx+cdx^2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(g+h*x)*\operatorname{Sqrt}[a+b*x+c*x^2]/(a*d+b*d*x+c*d*x^2)^{(3/2)},x]$

[Out] $-(((2*c*g-b*h)*\operatorname{Sqrt}[a+b*x+c*x^2]*\operatorname{ArcTanh}[(b+2*c*x)/\operatorname{Sqrt}[b^2-4*a*c]])/(c*\operatorname{Sqrt}[b^2-4*a*c]*d*\operatorname{Sqrt}[a*d+b*d*x+c*d*x^2]))+(h*\operatorname{Sqrt}[a+b*x+c*x^2]*\operatorname{Log}[a+b*x+c*x^2])/(2*c*d*\operatorname{Sqrt}[a*d+b*d*x+c*d*x^2])$

Rule 206

$\operatorname{Int}[(a_+ + (b_+)(x_+)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 618

$\operatorname{Int}[(a_+ + (b_+)(x_+) + (c_+)(x_+)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Dist}[-2, \operatorname{Subst}[\operatorname{Int}[1/\operatorname{Simp}[b^2-4*a*c-x^2, x], x], x, b+2*c*x], x] /; \operatorname{FreeQ}\{a, b, c\}, x \ \&\& \operatorname{NeQ}[b^2-4*a*c, 0]$

Rule 628

$\operatorname{Int}[(d_+ + (e_+)(x_+))/((a_+ + (b_+)(x_+) + (c_+)(x_+)^2), x_Symbol] \rightarrow \operatorname{Simp}[(d*\operatorname{Log}[\operatorname{RemoveContent}[a+b*x+c*x^2, x]])/b, x] /; \operatorname{FreeQ}\{a, b, c, d,$

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 999

Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x + c*x^2)^FracPart[p])/(d^IntPart[p]*(d + e*x + f*x^2)^FracPart[p]), Int[(g + h*x)^m*(d + e*x + f*x^2)^(p + q), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q}, x] && EqQ[c*d - a*f, 0] && EqQ[b*d - a*e, 0] && !IntegerQ[p] && !IntegerQ[q] && !GtQ[c/f, 0]

Rubi steps

$$\begin{aligned} \int \frac{(g + hx)\sqrt{a + bx + cx^2}}{(ad + bdx + cdx^2)^{3/2}} dx &= \frac{\sqrt{a + bx + cx^2} \int \frac{g+hx}{ad+bdx+cdx^2} dx}{\sqrt{ad + bdx + cdx^2}} \\ &= \frac{\left(h\sqrt{a + bx + cx^2}\right) \int \frac{bd+2cdx}{ad+bdx+cdx^2} dx}{2cd\sqrt{ad + bdx + cdx^2}} + \frac{\left((2cdg - bdh)\sqrt{a + bx + cx^2}\right) \int \frac{1}{ad+bdx+cdx^2} dx}{2cd\sqrt{ad + bdx + cdx^2}} \\ &= \frac{h\sqrt{a + bx + cx^2} \log(a + bx + cx^2)}{2cd\sqrt{ad + bdx + cdx^2}} - \frac{\left((2cdg - bdh)\sqrt{a + bx + cx^2}\right) \text{Subst}\left(\int \frac{1}{u} du\right)}{cd\sqrt{ad + bdx + cdx^2}} \\ &= -\frac{(2cg - bh)\sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2 - 4ac}d\sqrt{ad + bdx + cdx^2}} + \frac{h\sqrt{a + bx + cx^2} \log(a + bx + cx^2)}{2cd\sqrt{ad + bdx + cdx^2}} \end{aligned}$$

Mathematica [A] time = 0.09, size = 108, normalized size = 0.79

$$\frac{(a + x(b + cx))^{3/2} \left((4cg - 2bh) \tan^{-1} \left(\frac{b+2cx}{\sqrt{4ac-b^2}} \right) + h\sqrt{4ac-b^2} \log(a + x(b + cx)) \right)}{2c\sqrt{4ac-b^2} (d(a + x(b + cx)))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((g + h*x)*Sqrt[a + b*x + c*x^2])/(a*d + b*d*x + c*d*x^2)^(3/2), x]

[Out] ((a + x*(b + c*x))^(3/2)*((4*c*g - 2*b*h)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a*c]*h*Log[a + x*(b + c*x)]))/(2*c*Sqrt[-b^2 + 4*a*c]*(d*(a + x*(b + c*x)))^(3/2))

fricas [F] time = 119.13, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\sqrt{cdx^2 + bdx + ad} \sqrt{cx^2 + bx + a} (hx + g)}{c^2 d^2 x^4 + 2bcd^2 x^3 + 2abd^2 x + (b^2 + 2ac)d^2 x^2 + a^2 d^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^(3/2), x, algorithm="fricas")

[Out] integral(sqrt(c*d*x^2 + b*d*x + a*d)*sqrt(c*x^2 + b*x + a)*(h*x + g)/(c^2*d^2*x^4 + 2*b*c*d^2*x^3 + 2*a*b*d^2*x + (b^2 + 2*a*c)*d^2*x^2 + a^2*d^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + bx + a} (hx + g)}{(cdx^2 + bdx + ad)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^(3/2), x, algorithm="giac")

[Out] integrate(sqrt(c*x^2 + b*x + a)*(h*x + g)/(c*d*x^2 + b*d*x + a*d)^(3/2), x)

maple [A] time = 0.03, size = 121, normalized size = 0.89

$$\frac{\sqrt{(cx^2 + bx + a)d} \left(-2bh \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right) + 4cg \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right) + \sqrt{4ac-b^2} h \ln(cx^2 + bx + a) \right)}{2\sqrt{cx^2 + bx + a} \sqrt{4ac-b^2} cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^(3/2), x)

[Out] 1/2/(c*x^2+b*x+a)^(1/2)*(d*(c*x^2+b*x+a))^(1/2)*(h*ln(c*x^2+b*x+a)*(4*a*c-b^2)^(1/2)-2*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*h+4*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c*g)/d^2/c/(4*a*c-b^2)^(1/2)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^(3/2),x, algorithm m="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g + hx) \sqrt{cx^2 + bx + a}}{(cdx^2 + bdx + ad)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((g + h*x)*(a + b*x + c*x^2)^(1/2))/(a*d + b*d*x + c*d*x^2)^(3/2), x)

[Out] int(((g + h*x)*(a + b*x + c*x^2)^(1/2))/(a*d + b*d*x + c*d*x^2)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g + hx) \sqrt{a + bx + cx^2}}{(d(a + bx + cx^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)*(c*x**2+b*x+a)**(1/2)/(c*d*x**2+b*d*x+a*d)**(3/2), x)

[Out] Integral((g + h*x)*sqrt(a + b*x + c*x**2)/(d*(a + b*x + c*x**2))**(3/2), x)

3.40 $\int x^2 \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2} dx$

Optimal. Leaf size=212

$$\frac{ac^2 \sqrt{a^2 + 2abx + b^2x^2} \tanh^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a+bx)} - \frac{\sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2} (8bc - 15adx)}{60d^2(a+bx)} - \frac{acx \sqrt{a^2 + 2abx + b^2x^2}}{8d(a+bx)}$$

[Out] $\frac{1}{5} b^2 x^2 (d x^2 + c)^{3/2} ((b x + a)^2)^{1/2} / d (b x + a) - 1/60 (-15 a d x + 8 b^2 c) (d x^2 + c)^{3/2} ((b x + a)^2)^{1/2} / d^2 (b x + a) - 1/8 a c^2 \operatorname{arctanh}(x d^{1/2}) / (d x^2 + c)^{1/2} ((b x + a)^2)^{1/2} / d^{3/2} (b x + a) - 1/8 a c x ((b x + a)^2)^{1/2} (d x^2 + c)^{1/2} / d (b x + a)$

Rubi [A] time = 0.12, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {1001, 833, 780, 195, 217, 206}

$$\frac{ac^2 \sqrt{a^2 + 2abx + b^2x^2} \tanh^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a+bx)} - \frac{\sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2} (8bc - 15adx)}{60d^2(a+bx)} - \frac{acx \sqrt{a^2 + 2abx + b^2x^2}}{8d(a+bx)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^2 \sqrt{a^2 + 2 a b x + b^2 x^2} \sqrt{c + d x^2}, x]$

[Out] $-(a c x \sqrt{a^2 + 2 a b x + b^2 x^2} \sqrt{c + d x^2}) / (8 d (a + b x)) + (b x^2 \sqrt{a^2 + 2 a b x + b^2 x^2} (c + d x^2)^{3/2}) / (5 d (a + b x)) - ((8 b^2 c - 15 a d x) \sqrt{a^2 + 2 a b x + b^2 x^2} (c + d x^2)^{3/2}) / (60 d^2 (a + b x)) - (a c^2 \sqrt{a^2 + 2 a b x + b^2 x^2} \operatorname{ArcTanh}[(\sqrt{d} x) / \sqrt{c + d x^2}]) / (8 d^{3/2} (a + b x))$

Rule 195

$\text{Int}[(a + b x^n)^p, x_Symbol] \rightarrow \text{Simp}[(x (a + b x^n)^p) / (n p + 1), x] + \text{Dist}[(a^n x^p) / (n p + 1), \text{Int}[(a + b x^n)^{p-1}, x], x] /;$ Free Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 206

$\text{Int}[(a + b x^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1 * \operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2] x) / \operatorname{Rt}[a, 2]]) / (\operatorname{Rt}[a, 2] * \operatorname{Rt}[-b, 2]), x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 780

```
Int[((d_) + (e_)*(x_))*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x
_Symbol] := Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 833

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p
_), x_Symbol] := Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x]
/; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(GtQ[m, 0] && EqQ[f, 0])
```

Rule 1001

```
Int[((g_) + (h_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_
) + (f_)*(x_)^2)^(q_), x_Symbol] := Dist[(a + b*x + c*x^2)^FracPart[p]/((4
*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(b + 2*c*x)^(2
*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, g, h, m, p, q}, x] && Eq
Q[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2 \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2} dx &= \frac{\sqrt{a^2 + 2abx + b^2x^2} \int x^2 (2ab + 2b^2x) \sqrt{c + dx^2} dx}{2ab + 2b^2x} \\
&= \frac{bx^2 \sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2}}{5d(a + bx)} + \frac{\sqrt{a^2 + 2abx + b^2x^2} \int x (-4b^2c + 1}{5d(2ab + 2b^2x} \\
&= \frac{bx^2 \sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2}}{5d(a + bx)} - \frac{(8bc - 15adx) \sqrt{a^2 + 2abx + b^2x^2}}{60d^2(a + bx)} \\
&= -\frac{acx \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{8d(a + bx)} + \frac{bx^2 \sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2}}{5d(a + bx)} \\
&= -\frac{acx \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{8d(a + bx)} + \frac{bx^2 \sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2}}{5d(a + bx)} \\
&= -\frac{acx \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{8d(a + bx)} + \frac{bx^2 \sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2}}{5d(a + bx)}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 129, normalized size = 0.61

$$\frac{\sqrt{(a + bx)^2} \sqrt{c + dx^2} \left(\sqrt{\frac{dx^2}{c} + 1} (15adx (c + 2dx^2) + 8b (-2c^2 + cd^2x^2 + 3d^2x^4)) - 15ac^{3/2} \sqrt{d} \sinh^{-1} \left(\frac{\sqrt{d}x}{\sqrt{c}} \right) \right)}{120d^2(a + bx) \sqrt{\frac{dx^2}{c} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]

[Out] (Sqrt[(a + b*x)^2]*Sqrt[c + d*x^2]*(Sqrt[1 + (d*x^2)/c]*(15*a*d*x*(c + 2*d*x^2) + 8*b*(-2*c^2 + c*d*x^2 + 3*d^2*x^4)) - 15*a*c^(3/2)*Sqrt[d]*ArcSinh[(Sqrt[d]*x)/Sqrt[c]]))/(120*d^2*(a + b*x)*Sqrt[1 + (d*x^2)/c])

fricas [A] time = 0.94, size = 175, normalized size = 0.83

$$\left[\frac{15ac^2\sqrt{d} \log(-2dx^2 + 2\sqrt{dx^2 + c}\sqrt{d}x - c) + 2(24bd^2x^4 + 30ad^2x^3 + 8bcdx^2 + 15acdx - 16bc^2)\sqrt{dx^2 + c}}{240d^2}, \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="fricas")

[Out] [1/240*(15*a*c^2*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(24*b*d^2*x^4 + 30*a*d^2*x^3 + 8*b*c*d*x^2 + 15*a*c*d*x - 16*b*c^2)*sqrt(d*x^2 + c))/d^2, 1/120*(15*a*c^2*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (24*b*d^2*x^4 + 30*a*d^2*x^3 + 8*b*c*d*x^2 + 15*a*c*d*x - 16*b*c^2)*sqrt(d*x^2 + c))/d^2]

giac [A] time = 0.25, size = 117, normalized size = 0.55

$$\frac{ac^2 \log\left(\left|-\sqrt{d}x + \sqrt{dx^2 + c}\right|\right) \operatorname{sgn}(bx + a)}{8d^{\frac{3}{2}}} + \frac{1}{120} \sqrt{dx^2 + c} \left(\left(2 \left(3 \left(4bx \operatorname{sgn}(bx + a) + 5a \operatorname{sgn}(bx + a) \right) x + \frac{4bc}{d} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="giac")

[Out] 1/8*a*c^2*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sgn(b*x + a)/d^(3/2) + 1/120*sqrt(d*x^2 + c)*((2*(3*(4*b*x*sgn(b*x + a) + 5*a*sgn(b*x + a))*x + 4*b*c*sgn(b*x + a)/d)*x + 15*a*c*sgn(b*x + a)/d)*x - 16*b*c^2*sgn(b*x + a)/d^2)

maple [C] time = 0.05, size = 103, normalized size = 0.49

$$\frac{\left(15ac^2d \ln\left(\sqrt{d}x + \sqrt{dx^2 + c}\right) + 15\sqrt{dx^2 + c} acd^{\frac{3}{2}}x - 24(dx^2 + c)^{\frac{3}{2}}bd^{\frac{3}{2}}x^2 - 30(dx^2 + c)^{\frac{3}{2}}ad^{\frac{3}{2}}x + 16(dx^2 + c)^{\frac{3}{2}}cd^{\frac{3}{2}} \right)}{120d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x)

[Out] -1/120*csgn(b*x+a)*(-24*d^(3/2)*(d*x^2+c)^(3/2)*x^2*b-30*d^(3/2)*(d*x^2+c)^(3/2)*x*a+16*d^(1/2)*(d*x^2+c)^(3/2)*b*c+15*d^(3/2)*(d*x^2+c)^(1/2)*x*a*c+15*ln(x*d^(1/2)+(d*x^2+c)^(1/2))*a*c^2*d)/d^(5/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx^2 + c} \sqrt{(bx + a)^2 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)*x^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \sqrt{(a + bx)^2} \sqrt{dx^2 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2), x)`

[Out] `int(x^2*((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{c + dx^2} \sqrt{(a + bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2), x)`

[Out] `Integral(x**2*sqrt(c + d*x**2)*sqrt((a + b*x)**2), x)`

3.41 $\int x\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2} dx$

Optimal. Leaf size=161

$$\frac{bc^2\sqrt{a^2 + 2abx + b^2x^2} \tanh^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a + bx)} - \frac{bcx\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{8d(a + bx)} + \frac{(4a + 3bx)\sqrt{a^2 + 2abx + b^2x^2} (c - dx)}{12d(a + bx)}$$

[Out] 1/12*(3*b*x+4*a)*(d*x^2+c)^(3/2)*((b*x+a)^2)^(1/2)/d/(b*x+a)-1/8*b*c^2*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))*((b*x+a)^2)^(1/2)/d^(3/2)/(b*x+a)-1/8*b*c*x*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/d/(b*x+a)

Rubi [A] time = 0.07, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1001, 780, 195, 217, 206}

$$\frac{bc^2\sqrt{a^2 + 2abx + b^2x^2} \tanh^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a + bx)} - \frac{bcx\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{8d(a + bx)} + \frac{(4a + 3bx)\sqrt{a^2 + 2abx + b^2x^2} (c - dx)}{12d(a + bx)}$$

Antiderivative was successfully verified.

[In] Int[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]

[Out] -(b*c*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(8*d*(a + b*x)) + ((4*a + 3*b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + d*x^2)^(3/2))/(12*d*(a + b*x)) - (b*c^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*d^(3/2)*(a + b*x))

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 780

Int[((d_) + (e_)*(x_))*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x
_Symbol] := Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]

Rule 1001

Int[((g_) + (h_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_
) + (f_)*(x_)^2)^(q_), x_Symbol] := Dist[(a + b*x + c*x^2)^FracPart[p]/((4
c)^IntPart[p](b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(b + 2*c*x)^(2
p)(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, g, h, m, p, q}, x] && Eq
Q[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
 \int x\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2} dx &= \frac{\sqrt{a^2 + 2abx + b^2x^2} \int x(2ab + 2b^2x) \sqrt{c + dx^2} dx}{2ab + 2b^2x} \\
 &= \frac{(4a + 3bx)\sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2}}{12d(a + bx)} - \frac{(b^2c\sqrt{a^2 + 2abx + b^2x^2}) \int \sqrt{c + dx^2} dx}{2d(2ab + 2b^2x)} \\
 &= -\frac{bcx\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{8d(a + bx)} + \frac{(4a + 3bx)\sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2}}{12d(a + bx)} \\
 &= -\frac{bcx\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{8d(a + bx)} + \frac{(4a + 3bx)\sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2}}{12d(a + bx)} \\
 &= -\frac{bcx\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{8d(a + bx)} + \frac{(4a + 3bx)\sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2}}{12d(a + bx)}
 \end{aligned}$$

Mathematica [A] time = 0.12, size = 117, normalized size = 0.73

$$\frac{\sqrt{(a+bx)^2} \sqrt{c+dx^2} \left(\sqrt{d} \sqrt{\frac{dx^2}{c} + 1} (8a(c+dx^2) + 3bx(c+2dx^2)) - 3bc^{3/2} \sinh^{-1} \left(\frac{\sqrt{d}x}{\sqrt{c}} \right) \right)}{24d^{3/2}(a+bx)\sqrt{\frac{dx^2}{c} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2], x]

[Out] (Sqrt[(a + b*x)^2]*Sqrt[c + d*x^2]*(Sqrt[d]*Sqrt[1 + (d*x^2)/c]*(8*a*(c + d*x^2) + 3*b*x*(c + 2*d*x^2)) - 3*b*c^(3/2)*ArcSinh[(Sqrt[d]*x)/Sqrt[c]]))/(24*d^(3/2)*(a + b*x)*Sqrt[1 + (d*x^2)/c])

fricas [A] time = 1.00, size = 157, normalized size = 0.98

$$\left[\frac{3bc^2\sqrt{d} \log\left(-2dx^2 + 2\sqrt{dx^2 + c}\sqrt{d}x - c\right) + 2\left(6bd^2x^3 + 8ad^2x^2 + 3bcdx + 8acd\right)\sqrt{dx^2 + c}}{48d^2}, \frac{3bc^2\sqrt{-d} \arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2 + c}}\right)}{48d^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2), x, algorithm="fricas")

[Out] [1/48*(3*b*c^2*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(6*b*d^2*x^3 + 8*a*d^2*x^2 + 3*b*c*d*x + 8*a*c*d)*sqrt(d*x^2 + c))/d^2, 1/24*(3*b*c^2*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (6*b*d^2*x^3 + 8*a*d^2*x^2 + 3*b*c*d*x + 8*a*c*d)*sqrt(d*x^2 + c))/d^2]

giac [A] time = 0.21, size = 98, normalized size = 0.61

$$\frac{bc^2 \log\left(\left|-\sqrt{d}x + \sqrt{dx^2 + c}\right|\right) \operatorname{sgn}(bx + a)}{8d^{3/2}} + \frac{1}{24} \sqrt{dx^2 + c} \left(\left(2(3bx \operatorname{sgn}(bx + a) + 4a \operatorname{sgn}(bx + a))x + \frac{3bc \operatorname{sgn}(bx + a)}{d} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2), x, algorithm="giac")

[Out] 1/8*b*c^2*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sgn(b*x + a)/d^(3/2) + 1/24*sqrt(d*x^2 + c)*((2*(3*b*x*sgn(b*x + a) + 4*a*sgn(b*x + a))*x + 3*b*c*sgn(b*x + a)/d)*x + 8*a*c*sgn(b*x + a)/d)

maple [C] time = 0.01, size = 83, normalized size = 0.52

$$\frac{\left(-3bc^2 \ln\left(\sqrt{d}x + \sqrt{dx^2 + c}\right) - 3\sqrt{dx^2 + c}bc\sqrt{d}x + 6(dx^2 + c)^{3/2}b\sqrt{d}x + 8(dx^2 + c)^{3/2}a\sqrt{d}\right) \operatorname{csgn}(bx + a)}{24d^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x)`

[Out] $1/24 * \text{csgn}(b*x+a) * (6*d^{1/2} * (d*x^2+c)^{3/2} * x*b + 8*a * (d*x^2+c)^{3/2} * d^{1/2} - 3*d^{1/2} * (d*x^2+c)^{1/2} * x*b*c - 3*\ln(d^{1/2}*x+(d*x^2+c)^{1/2}))*b*c^2 / d^{3/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx^2 + c} \sqrt{(bx + a)^2} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)*x, x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \sqrt{(a + bx)^2} \sqrt{dx^2 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2),x)`

[Out] `int(x*((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \sqrt{c + dx^2} \sqrt{(a + bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2),x)`

[Out] `Integral(x*sqrt(c + d*x**2)*sqrt((a + b*x)**2), x)`

3.42 $\int \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2} dx$

Optimal. Leaf size=148

$$\frac{b\sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2}}{3d(a + bx)} + \frac{ax\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{2(a + bx)} + \frac{ac\sqrt{a^2 + 2abx + b^2x^2} \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}(a + bx)}$$

[Out] $1/3*b*(d*x^2+c)^{(3/2)*((b*x+a)^2)^{(1/2)}/d/(b*x+a)+1/2*a*c*\arctanh(x*d^{(1/2)}/(d*x^2+c)^{(1/2)))*((b*x+a)^2)^{(1/2)/(b*x+a)/d^{(1/2)+1/2*a*x*((b*x+a)^2)^{(1/2)*(d*x^2+c)^{(1/2)/(b*x+a)}$

Rubi [A] time = 0.06, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {970, 641, 195, 217, 206}

$$\frac{b\sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2}}{3d(a + bx)} + \frac{ax\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{2(a + bx)} + \frac{ac\sqrt{a^2 + 2abx + b^2x^2} \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}(a + bx)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]

[Out] $(a*x*\text{Sqrt}[a^2 + 2*a*b*x + b^2*x^2]*\text{Sqrt}[c + d*x^2])/(2*(a + b*x)) + (b*\text{Sqrt}[a^2 + 2*a*b*x + b^2*x^2]*(c + d*x^2)^{(3/2)})/(3*d*(a + b*x)) + (a*c*\text{Sqrt}[a^2 + 2*a*b*x + b^2*x^2]*\text{ArcTanh}[(\text{Sqrt}[d]*x)/\text{Sqrt}[c + d*x^2]])/(2*\text{Sqrt}[d]*(a + b*x))$

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 970

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] := Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned} \int \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2} dx &= \frac{\sqrt{a^2 + 2abx + b^2x^2} \int (2ab + 2b^2x) \sqrt{c + dx^2} dx}{2ab + 2b^2x} \\ &= \frac{b\sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2}}{3d(a + bx)} + \frac{(2ab\sqrt{a^2 + 2abx + b^2x^2}) \int \sqrt{c + dx^2} dx}{2ab + 2b^2x} \\ &= \frac{ax\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{2(a + bx)} + \frac{b\sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2}}{3d(a + bx)} + \frac{(abc)}{3d} \int \sqrt{c + dx^2} dx \\ &= \frac{ax\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{2(a + bx)} + \frac{b\sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2}}{3d(a + bx)} + \frac{(abc)}{3d} \int \sqrt{c + dx^2} dx \\ &= \frac{ax\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{2(a + bx)} + \frac{b\sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2}}{3d(a + bx)} + \frac{ac\sqrt{c + dx^2}}{3d} \end{aligned}$$

Mathematica [A] time = 0.06, size = 85, normalized size = 0.57

$$\frac{\sqrt{(a + bx)^2} \left(\sqrt{c + dx^2} (3adx + 2b(c + dx^2)) + 3ac\sqrt{d} \log \left(\sqrt{d} \sqrt{c + dx^2} + dx \right) \right)}{6d(a + bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2], x]
```

[Out] $(\sqrt{(a + b*x)^2} * (\sqrt{c + d*x^2} * (3*a*d*x + 2*b*(c + d*x^2)) + 3*a*c*\sqrt{d} * \log[d*x + \sqrt{d}*\sqrt{c + d*x^2}]]) / (6*d*(a + b*x))$

fricas [A] time = 0.79, size = 128, normalized size = 0.86

$$\left[\frac{3ac\sqrt{d} \log\left(-2dx^2 - 2\sqrt{dx^2 + c}\sqrt{d}x - c\right) + 2(2bdx^2 + 3adx + 2bc)\sqrt{dx^2 + c}}{12d}, -\frac{3ac\sqrt{-d} \arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2 + c}}\right)}{d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="fricas")`

[Out] $[1/12*(3*a*c*\sqrt{d}*\log(-2*d*x^2 - 2*\sqrt{d*x^2 + c}*\sqrt{d}*x - c) + 2*(2*b*d*x^2 + 3*a*d*x + 2*b*c)*\sqrt{d*x^2 + c})/d, -1/6*(3*a*c*\sqrt{-d}*\arctan(\sqrt{-d}*x/\sqrt{d*x^2 + c}) - (2*b*d*x^2 + 3*a*d*x + 2*b*c)*\sqrt{d*x^2 + c})/d]$

giac [A] time = 0.27, size = 79, normalized size = 0.53

$$-\frac{ac \log\left(\left|-\sqrt{d}x + \sqrt{dx^2 + c}\right|\right) \operatorname{sgn}(bx + a)}{2\sqrt{d}} + \frac{1}{6}\sqrt{dx^2 + c}\left((2bx\operatorname{sgn}(bx + a) + 3a\operatorname{sgn}(bx + a))x + \frac{2bc\operatorname{sgn}(bx + a)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="giac")`

[Out] $-1/2*a*c*\log(\operatorname{abs}(-\sqrt{d}*x + \sqrt{d*x^2 + c}))*\operatorname{sgn}(b*x + a)/\sqrt{d} + 1/6*\sqrt{d*x^2 + c}*((2*b*x*\operatorname{sgn}(b*x + a) + 3*a*\operatorname{sgn}(b*x + a))*x + 2*b*c*\operatorname{sgn}(b*x + a))/d$

maple [C] time = 0.01, size = 65, normalized size = 0.44

$$\frac{\left(3acd \ln\left(\sqrt{d}x + \sqrt{dx^2 + c}\right) + 3\sqrt{dx^2 + c} a d^{\frac{3}{2}}x + 2(dx^2 + c)^{\frac{3}{2}} b\sqrt{d}\right) \operatorname{csgn}(bx + a)}{6d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x)`

[Out] $1/6*\operatorname{csgn}(b*x+a)*(2*b*(d*x^2+c)^{(3/2)}*d^{(1/2)}+3*d^{(3/2)}*(d*x^2+c)^{(1/2)}*x+a+3*\ln(d^{(1/2)}*x+(d*x^2+c)^{(1/2)})*a*c*d)/d^{(3/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx^2 + c} \sqrt{(bx + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{(a + bx)^2} \sqrt{dx^2 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2),x)

[Out] int(((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c + dx^2} \sqrt{(a + bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2),x)

[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x)**2), x)

$$3.43 \quad \int \frac{\sqrt{a^2+2abx+b^2x^2} \sqrt{c+dx^2}}{x} dx$$

Optimal. Leaf size=160

$$\frac{\sqrt{a^2+2abx+b^2x^2}(2a+bx)\sqrt{c+dx^2}}{2(a+bx)} + \frac{bc\sqrt{a^2+2abx+b^2x^2} \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}(a+bx)} - \frac{a\sqrt{c}\sqrt{a^2+2abx+b^2x^2} \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{a+bx}$$

[Out] $-a*\arctanh((d*x^2+c)^{(1/2)}/c^{(1/2)})*c^{(1/2)}*((b*x+a)^2)^{(1/2)}/(b*x+a)+1/2*b*c*\arctanh(x*d^{(1/2)}/(d*x^2+c)^{(1/2)})*((b*x+a)^2)^{(1/2)}/(b*x+a)/d^{(1/2)}+1/2*(b*x+2*a)*((b*x+a)^2)^{(1/2)}*(d*x^2+c)^{(1/2)}/(b*x+a)$

Rubi [A] time = 0.12, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {1001, 815, 844, 217, 206, 266, 63, 208}

$$\frac{\sqrt{a^2+2abx+b^2x^2}(2a+bx)\sqrt{c+dx^2}}{2(a+bx)} + \frac{bc\sqrt{a^2+2abx+b^2x^2} \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}(a+bx)} - \frac{a\sqrt{c}\sqrt{a^2+2abx+b^2x^2} \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{a+bx}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x,x]

[Out] $((2*a + b*x)*\text{Sqrt}[a^2 + 2*a*b*x + b^2*x^2]*\text{Sqrt}[c + d*x^2])/(2*(a + b*x)) + (b*c*\text{Sqrt}[a^2 + 2*a*b*x + b^2*x^2]*\text{ArcTanh}[(\text{Sqrt}[d]*x)/\text{Sqrt}[c + d*x^2]])/(2*\text{Sqrt}[d]*(a + b*x)) - (a*\text{Sqrt}[c]*\text{Sqrt}[a^2 + 2*a*b*x + b^2*x^2]*\text{ArcTanh}[\text{Sqrt}[c + d*x^2]/\text{Sqrt}[c]])/(a + b*x)$

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 206

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

`Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

Rule 217

`Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]`

Rule 266

`Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]`

Rule 815

`Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])`

Rule 844

`Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]`

Rule 1001

`Int[((g_) + (h_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_) + (f_)*(x_)^2)^(q_), x_Symbol] := Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, g, h, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0]`

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{x} dx &= \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(2ab+2b^2x)\sqrt{c+dx^2}}{x} dx}{2ab + 2b^2x} \\
&= \frac{(2a + bx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{2(a + bx)} + \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{4abcd+2b^2c}{x\sqrt{c+dx^2}}}{2d(2ab + 2b^2x)} \\
&= \frac{(2a + bx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{2(a + bx)} + \frac{(2abc\sqrt{a^2 + 2abx + b^2x^2}) \int \frac{1}{x\sqrt{c+dx^2}}}{2ab + 2b^2x} \\
&= \frac{(2a + bx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{2(a + bx)} + \frac{(abc\sqrt{a^2 + 2abx + b^2x^2}) \operatorname{Subst}}{2ab + 2b^2x} \\
&= \frac{(2a + bx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{2(a + bx)} + \frac{bc\sqrt{a^2 + 2abx + b^2x^2} \tanh^{-1}\left(\frac{\sqrt{dx^2+c}}{\sqrt{c}}\right)}{2\sqrt{d}(a + bx)} \\
&= \frac{(2a + bx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{2(a + bx)} + \frac{bc\sqrt{a^2 + 2abx + b^2x^2} \tanh^{-1}\left(\frac{\sqrt{dx^2+c}}{\sqrt{c}}\right)}{2\sqrt{d}(a + bx)}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 139, normalized size = 0.87

$$\frac{\sqrt{(a + bx)^2} \left(\sqrt{d} \sqrt{\frac{dx^2}{c} + 1} \left((2a + bx)\sqrt{c + dx^2} - 2a\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right) \right) + b\sqrt{c} \sqrt{c + dx^2} \sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right) \right)}{2\sqrt{d}(a + bx)\sqrt{\frac{dx^2}{c} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x,x]

[Out] (Sqrt[(a + b*x)^2]*(b*Sqrt[c]*Sqrt[c + d*x^2]*ArcSinh[(Sqrt[d]*x)/Sqrt[c]] + Sqrt[d]*Sqrt[1 + (d*x^2)/c]*((2*a + b*x)*Sqrt[c + d*x^2] - 2*a*Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]))/(2*Sqrt[d]*(a + b*x)*Sqrt[1 + (d*x^2)/c])

fricas [A] time = 0.99, size = 341, normalized size = 2.13

$$\left[\frac{bc\sqrt{d} \log\left(-2dx^2 - 2\sqrt{dx^2+c}\sqrt{d}x - c\right) + 2a\sqrt{c}d \log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c+2c}}{x^2}\right) + 2(bdx + 2ad)\sqrt{dx^2+c}}{4d}, \dots \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x,x, algorithm="fricas")

[Out] [1/4*(b*c*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*a*sqrt(c)*d*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(b*d*x + 2*a*d)*sqrt(d*x^2 + c))/d, -1/2*(b*c*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - a*sqrt(c)*d*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - (b*d*x + 2*a*d)*sqrt(d*x^2 + c))/d, 1/4*(4*a*sqrt(-c)*d*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + b*c*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(b*d*x + 2*a*d)*sqrt(d*x^2 + c))/d, -1/2*(b*c*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - 2*a*sqrt(-c)*d*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - (b*d*x + 2*a*d)*sqrt(d*x^2 + c))/d]

giac [A] time = 0.25, size = 102, normalized size = 0.64

$$\frac{2ac \arctan\left(-\frac{\sqrt{d}x - \sqrt{dx^2+c}}{\sqrt{-c}}\right) \operatorname{sgn}(bx+a) - bc \log\left(\left|-\sqrt{d}x + \sqrt{dx^2+c}\right|\right) \operatorname{sgn}(bx+a)}{\sqrt{-c}} - \frac{bc \log\left(\left|-\sqrt{d}x + \sqrt{dx^2+c}\right|\right) \operatorname{sgn}(bx+a)}{2\sqrt{d}} + \frac{1}{2} \sqrt{dx^2+c} (bx \operatorname{sgn}(bx+a))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x,x, algorithm="giac")

[Out] 2*a*c*arctan(-(sqrt(d)*x - sqrt(d*x^2 + c))/sqrt(-c))*sgn(b*x + a)/sqrt(-c) - 1/2*b*c*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sgn(b*x + a)/sqrt(d) + 1/2*sqrt(d*x^2 + c)*(b*x*sgn(b*x + a) + 2*a*sgn(b*x + a))

maple [C] time = 0.01, size = 94, normalized size = 0.59

$$\frac{\left(2a\sqrt{c}\sqrt{d}\ln\left(\frac{2c+2\sqrt{dx^2+c}\sqrt{c}}{x}\right) - bc\ln\left(\sqrt{d}x + \sqrt{dx^2+c}\right) - \sqrt{dx^2+c}b\sqrt{d}x - 2\sqrt{dx^2+c}a\sqrt{d}\right) \operatorname{csgn}(bx+a)}{2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x,x)

[Out] -1/2*c*sgn(b*x+a)*(2*c^(1/2)*d^(1/2)*ln(2*(c^(1/2)*(d*x^2+c)^(1/2)+c)/x)*a-d^(1/2)*(d*x^2+c)^(1/2)*x*b-2*d^(1/2)*(d*x^2+c)^(1/2)*a-ln(d^(1/2)*x+(d*x^2+c)^(1/2))*b*c)/d^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2+c}\sqrt{(bx+a)^2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x,x, algorithm="maxima")

[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)/x, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{(a+bx)^2} \sqrt{dx^2+c}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2))/x,x)

[Out] int((((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2))/x, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2} \sqrt{(a+bx)^2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2)/x,x)

[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x)**2)/x, x)

$$3.44 \quad \int \frac{\sqrt{a^2+2abx+b^2x^2} \sqrt{c+dx^2}}{x^2} dx$$

Optimal. Leaf size=156

$$\frac{\sqrt{a^2+2abx+b^2x^2}(a-bx)\sqrt{c+dx^2}}{x(a+bx)} + \frac{a\sqrt{d}\sqrt{a^2+2abx+b^2x^2} \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{a+bx} - \frac{b\sqrt{c}\sqrt{a^2+2abx+b^2x^2} \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{a+bx}$$

[Out] $-b \cdot \text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right) \cdot \sqrt{c+dx^2} \cdot \sqrt{a^2+2abx+b^2x^2} \cdot \frac{1}{x(a+bx)} + a \cdot \sqrt{d} \cdot \sqrt{a^2+2abx+b^2x^2} \cdot \text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right) \cdot \frac{1}{a+bx} - b \cdot \sqrt{c} \cdot \sqrt{a^2+2abx+b^2x^2} \cdot \text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right) \cdot \frac{1}{a+bx}$

Rubi [A] time = 0.11, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {1001, 813, 844, 217, 206, 266, 63, 208}

$$\frac{\sqrt{a^2+2abx+b^2x^2}(a-bx)\sqrt{c+dx^2}}{x(a+bx)} + \frac{a\sqrt{d}\sqrt{a^2+2abx+b^2x^2} \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{a+bx} - \frac{b\sqrt{c}\sqrt{a^2+2abx+b^2x^2} \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{a+bx}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x^2,x]

[Out] $-\left(\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x(a+bx)}\right) + \frac{a\sqrt{d}\sqrt{a^2+2abx+b^2x^2}\text{ArcTanh}\left[\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right]}{a+bx} - \frac{b\sqrt{c}\sqrt{a^2+2abx+b^2x^2}\text{ArcTanh}\left[\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right]}{a+bx}$

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m+1)-1)*(c-(a*d)/b+(d*x^p)/b)^n, x], x, (a+b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c-a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 206

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]]/a, x] \text{ /; FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

Rule 217

$\text{Int}[1/\text{Sqrt}[(a_ + (b_)*(x_)^2)], x_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(1 - b*x^2)], x], x, x/\text{Sqrt}[a + b*x^2]] \text{ /; FreeQ}\{a, b\}, x] \ \&\& \ !\text{GtQ}[a, 0]$

Rule 266

$\text{Int}[(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}), x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p}], x, x^n], x] \text{ /; FreeQ}\{a, b, m, n, p\}, x] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 813

$\text{Int}[(d_ + (e_)*(x_))^{(m_)}*((f_ + (g_)*(x_))*(a_ + (c_)*(x_)^2)^{(p_)}), x_Symbol] \rightarrow \text{Simp}[(d + e*x)^{(m + 1)}*(e*f*(m + 2*p + 2) - d*g*(2*p + 1) + e*g*(m + 1)*x*(a + c*x^2)^p]/(e^2*(m + 1)*(m + 2*p + 2)), x] + \text{Dist}[p/(e^2*(m + 1)*(m + 2*p + 2)), \text{Int}[(d + e*x)^{(m + 1)}*(a + c*x^2)^{(p - 1)}*\text{Simp}[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x], x], x] \text{ /; FreeQ}\{a, c, d, e, f, g, m\}, x] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{RationalQ}[p] \ \&\& \ p > 0 \ \&\& \ (\text{LtQ}[m, -1] \ || \ \text{EqQ}[p, 1] \ || \ (\text{IntegerQ}[p] \ \&\& \ !\text{RationalQ}[m])) \ \&\& \ \text{NeQ}[m, -1] \ \&\& \ !\text{ILtQ}[m + 2*p + 1, 0] \ \&\& \ (\text{IntegerQ}[m] \ || \ \text{IntegerQ}[p] \ || \ \text{IntegersQ}[2*m, 2*p])$

Rule 844

$\text{Int}[(d_ + (e_)*(x_))^{(m_)}*((f_ + (g_)*(x_))*(a_ + (c_)*(x_)^2)^{(p_)}), x_Symbol] \rightarrow \text{Dist}[g/e, \text{Int}[(d + e*x)^{(m + 1)}*(a + c*x^2)^p], x] + \text{Dist}[(e*f - d*g)/e, \text{Int}[(d + e*x)^m*(a + c*x^2)^p], x] \text{ /; FreeQ}\{a, c, d, e, f, g, m, p\}, x] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ !\text{GtQ}[m, 0]$

Rule 1001

$\text{Int}[(g_ + (h_)*(x_))^{(m_)}*((a_ + (b_)*(x_) + (c_)*(x_)^2)^{(p_)}*((d_ + (f_)*(x_)^2)^{(q_)}), x_Symbol] \rightarrow \text{Dist}[(a + b*x + c*x^2)^{\text{FracPart}[p]}/((4*c)^{\text{IntPart}[p]}*(b + 2*c*x)^{(2*\text{FracPart}[p])}), \text{Int}[(g + h*x)^m*(b + 2*c*x)^{(2*p)}*(d + f*x^2)^q], x] \text{ /; FreeQ}\{a, b, c, d, f, g, h, m, p, q\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{x^2} dx &= \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(2ab + 2b^2x)\sqrt{c + dx^2}}{x^2} dx}{2ab + 2b^2x} \\
&= -\frac{(a - bx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{x(a + bx)} - \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{-4b^2c - 4abd x}{x\sqrt{c + dx^2}}}{2(2ab + 2b^2x)} \\
&= -\frac{(a - bx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{x(a + bx)} + \frac{(2b^2c\sqrt{a^2 + 2abx + b^2x^2}) \int \frac{1}{x\sqrt{c + dx^2}}}{2ab + 2b^2x} \\
&= -\frac{(a - bx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{x(a + bx)} + \frac{(b^2c\sqrt{a^2 + 2abx + b^2x^2}) \text{Subst} \left(\frac{1}{x\sqrt{c + dx^2}} \right)}{2ab + 2b^2x} \\
&= -\frac{(a - bx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{x(a + bx)} + \frac{a\sqrt{d} \sqrt{a^2 + 2abx + b^2x^2} \tanh^{-1} \left(\frac{\sqrt{c + dx^2}}{\sqrt{c}} \right)}{a + bx} \\
&= -\frac{(a - bx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{x(a + bx)} + \frac{a\sqrt{d} \sqrt{a^2 + 2abx + b^2x^2} \tanh^{-1} \left(\frac{\sqrt{c + dx^2}}{\sqrt{c}} \right)}{a + bx}
\end{aligned}$$

Mathematica [A] time = 0.28, size = 118, normalized size = 0.76

$$\frac{\sqrt{(a + bx)^2} \left(\frac{(bx - a)\sqrt{c + dx^2}}{x} + \frac{a\sqrt{c}\sqrt{d} \sqrt{\frac{dx^2}{c} + 1} \sinh^{-1} \left(\frac{\sqrt{d}x}{\sqrt{c}} \right)}{\sqrt{c + dx^2}} - b\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c + dx^2}}{\sqrt{c}} \right) \right)}{a + bx}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x^2,x]

[Out] (Sqrt[(a + b*x)^2]*(((-a + b*x)*Sqrt[c + d*x^2])/x + (a*Sqrt[c]*Sqrt[d]*Sqrt[1 + (d*x^2)/c]*ArcSinh[(Sqrt[d]*x)/Sqrt[c]]/Sqrt[c + d*x^2] - b*Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]))/(a + b*x)

fricas [A] time = 1.05, size = 333, normalized size = 2.13

$$\left[\frac{a\sqrt{d}x \log \left(-2dx^2 - 2\sqrt{dx^2 + c}\sqrt{d}x - c \right) + b\sqrt{c}x \log \left(-\frac{dx^2 - 2\sqrt{dx^2 + c}\sqrt{c} + 2c}{x^2} \right) + 2\sqrt{dx^2 + c}(bx - a) - 2a\sqrt{-d}x a}{2x}, \dots \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^2,x, algorithm="fricas")

[Out] [1/2*(a*sqrt(d)*x*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + b*sqrt(c)*x*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*sqrt(d*x^2 + c)*(b*x - a))/x, -1/2*(2*a*sqrt(-d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - b*sqrt(c)*x*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*sqrt(d*x^2 + c)*(b*x - a))/x, 1/2*(2*b*sqrt(-c)*x*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + a*sqrt(d)*x*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*sqrt(d*x^2 + c)*(b*x - a))/x, -(a*sqrt(-d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - b*sqrt(-c)*x*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - sqrt(d*x^2 + c)*(b*x - a))/x]

giac [A] time = 0.26, size = 126, normalized size = 0.81

$$\frac{2bc \arctan\left(-\frac{\sqrt{d}x - \sqrt{dx^2+c}}{\sqrt{-c}}\right) \operatorname{sgn}(bx+a)}{\sqrt{-c}} - a\sqrt{d} \log\left(\left|-\sqrt{d}x + \sqrt{dx^2+c}\right|\right) \operatorname{sgn}(bx+a) + \sqrt{dx^2+c} b \operatorname{sgn}(bx+a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^2,x, algorithm="giac")

[Out] 2*b*c*arctan(-(sqrt(d)*x - sqrt(d*x^2 + c))/sqrt(-c))*sgn(b*x + a)/sqrt(-c) - a*sqrt(d)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sgn(b*x + a) + sqrt(d*x^2 + c)*b*sgn(b*x + a) + 2*a*c*sqrt(d)*sgn(b*x + a)/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)

maple [C] time = 0.01, size = 118, normalized size = 0.76

$$\frac{\left(acdx \ln\left(\sqrt{d}x + \sqrt{dx^2+c}\right) - bc^{\frac{3}{2}}\sqrt{d}x \ln\left(\frac{2c+2\sqrt{dx^2+c}\sqrt{c}}{x}\right) + \sqrt{dx^2+c} a d^{\frac{3}{2}}x^2 + \sqrt{dx^2+c} bc\sqrt{d}x - (dx^2+c)\right)}{c\sqrt{d}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^2,x)

[Out] csgn(b*x+a)*(d^(3/2)*(d*x^2+c)^(1/2)*x^2*a-d^(1/2)*c^(3/2)*ln(2*(c+(d*x^2+c)^(1/2)*c^(1/2)))/x)*x*b-(d*x^2+c)^(3/2)*a*d^(1/2)+(d*x^2+c)^(1/2)*b*c*d^(1/2)*x+ln(d^(1/2)*x+(d*x^2+c)^(1/2))*x*a*c*d/c/x/d^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2+c} \sqrt{(bx+a)^2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^2,x, algorithm="maxima")

[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)/x^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{(a+bx)^2} \sqrt{dx^2+c}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2))/x^2,x)

[Out] int((((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2))/x^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2} \sqrt{(a+bx)^2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2)/x**2,x)

[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x)**2)/x**2, x)

$$3.45 \quad \int \frac{\sqrt{a^2+2abx+b^2x^2} \sqrt{c+dx^2}}{x^3} dx$$

Optimal. Leaf size=161

$$\frac{\sqrt{a^2+2abx+b^2x^2}(a+2bx)\sqrt{c+dx^2}}{2x^2(a+bx)} + \frac{b\sqrt{d}\sqrt{a^2+2abx+b^2x^2} \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{a+bx} - \frac{ad\sqrt{a^2+2abx+b^2x^2} \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{c}(a+bx)}$$

[Out] $-1/2*a*d*\operatorname{arctanh}((d*x^2+c)^{(1/2)}/c^{(1/2)})*((b*x+a)^2)^{(1/2)}/(b*x+a)/c^{(1/2)}$
 $+b*\operatorname{arctanh}(x*d^{(1/2)}/(d*x^2+c)^{(1/2)})*d^{(1/2)}*((b*x+a)^2)^{(1/2)}/(b*x+a)-1/2$
 $*(2*b*x+a)*((b*x+a)^2)^{(1/2)}*(d*x^2+c)^{(1/2)}/x^2/(b*x+a)$

Rubi [A] time = 0.12, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {1001, 811, 844, 217, 206, 266, 63, 208}

$$\frac{\sqrt{a^2+2abx+b^2x^2}(a+2bx)\sqrt{c+dx^2}}{2x^2(a+bx)} + \frac{b\sqrt{d}\sqrt{a^2+2abx+b^2x^2} \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{a+bx} - \frac{ad\sqrt{a^2+2abx+b^2x^2} \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{c}(a+bx)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Sqrt}[a^2+2*a*b*x+b^2*x^2]*\operatorname{Sqrt}[c+d*x^2])/x^3,x]$

[Out] $-((a+2*b*x)*\operatorname{Sqrt}[a^2+2*a*b*x+b^2*x^2]*\operatorname{Sqrt}[c+d*x^2])/(2*x^2*(a+b*x)) + (b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[a^2+2*a*b*x+b^2*x^2]*\operatorname{ArcTanh}[(\operatorname{Sqrt}[d]*x)/\operatorname{Sqrt}[c+d*x^2]])/(a+b*x) - (a*d*\operatorname{Sqrt}[a^2+2*a*b*x+b^2*x^2]*\operatorname{ArcTanh}[\operatorname{Sqrt}[c+d*x^2]/\operatorname{Sqrt}[c]])/(2*\operatorname{Sqrt}[c]*(a+b*x))$

Rule 63

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] := \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a+b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 206

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x_Symbol] := \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \mid\mid \operatorname{LtQ}[b, 0])$

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 811

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g)*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] && !ILtQ[m + 2*p + 3, 0]

Rule 844

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]

Rule 1001

Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)*((d_.) + (f_.)*(x_)^2)^(q_.), x_Symbol] := Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, g, h, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{x^3} dx &= \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(2ab+2b^2x)\sqrt{c+dx^2}}{x^3} dx}{2ab + 2b^2x} \\
&= -\frac{(a + 2bx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{2x^2(a + bx)} - \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{-4abcd-8b^2c}{x\sqrt{c+dx^2}} dx}{4c(2ab + 2b^2x)} \\
&= -\frac{(a + 2bx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{2x^2(a + bx)} + \frac{(abd\sqrt{a^2 + 2abx + b^2x^2}) \int \frac{1}{x\sqrt{c+dx^2}} dx}{2ab + 2b^2x} \\
&= -\frac{(a + 2bx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{2x^2(a + bx)} + \frac{(abd\sqrt{a^2 + 2abx + b^2x^2}) \operatorname{Subst}\left(\int \frac{1}{u\sqrt{c+du^2}} du, x, a+bx\right)}{2(2ab + 2b^2x)} \\
&= -\frac{(a + 2bx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{2x^2(a + bx)} + \frac{b\sqrt{d} \sqrt{a^2 + 2abx + b^2x^2} \operatorname{tanh}^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{a + bx} \\
&= -\frac{(a + 2bx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{2x^2(a + bx)} + \frac{b\sqrt{d} \sqrt{a^2 + 2abx + b^2x^2} \operatorname{tanh}^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{a + bx}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 126, normalized size = 0.78

$$\frac{\sqrt{(a + bx)^2} \sqrt{c + dx^2} \left(c(a + 2bx)\sqrt{\frac{dx^2}{c} + 1} + adx^2 \operatorname{tanh}^{-1}\left(\sqrt{\frac{dx^2}{c} + 1}\right) - 2b\sqrt{c} \sqrt{d} x^2 \sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right) \right)}{2cx^2(a + bx)\sqrt{\frac{dx^2}{c} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x^3,x]

[Out] -1/2*(Sqrt[(a + b*x)^2]*Sqrt[c + d*x^2]*(c*(a + 2*b*x)*Sqrt[1 + (d*x^2)/c] - 2*b*Sqrt[c]*Sqrt[d]*x^2*ArcSinh[(Sqrt[d]*x)/Sqrt[c]] + a*d*x^2*ArcTanh[Sqrt[1 + (d*x^2)/c]]))/(c*x^2*(a + b*x)*Sqrt[1 + (d*x^2)/c])

fricas [A] time = 0.76, size = 377, normalized size = 2.34

$$\left[\frac{2bc\sqrt{d}x^2 \log\left(-2dx^2 - 2\sqrt{dx^2 + c}\sqrt{d}x - c\right) + a\sqrt{c}dx^2 \log\left(-\frac{dx^2 - 2\sqrt{dx^2 + c}\sqrt{c} + 2c}{x^2}\right) - 2(2bcx + ac)\sqrt{dx^2 + c}}{4cx^2}, - \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^3,x, algorithm="fricas")

[Out] [1/4*(2*b*c*sqrt(d)*x^2*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + a*sqrt(c)*d*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(2*b*c*x + a*c)*sqrt(d*x^2 + c))/(c*x^2), -1/4*(4*b*c*sqrt(-d)*x^2*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - a*sqrt(c)*d*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(2*b*c*x + a*c)*sqrt(d*x^2 + c))/(c*x^2), 1/2*(a*sqrt(-c)*d*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + b*c*sqrt(d)*x^2*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - (2*b*c*x + a*c)*sqrt(d*x^2 + c))/(c*x^2), -1/2*(2*b*c*sqrt(-d)*x^2*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - a*sqrt(-c)*d*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (2*b*c*x + a*c)*sqrt(d*x^2 + c))/(c*x^2)]

giac [A] time = 0.28, size = 199, normalized size = 1.24

$$\frac{ad \arctan\left(-\frac{\sqrt{d}x - \sqrt{dx^2+c}}{\sqrt{-c}}\right) \operatorname{sgn}(bx+a)}{\sqrt{-c}} - b\sqrt{d} \log\left(\left|-\sqrt{d}x + \sqrt{dx^2+c}\right|\right) \operatorname{sgn}(bx+a) + \frac{\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^3 ad \operatorname{sgn}(bx+a)}{\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^3,x, algorithm="giac")

[Out] a*d*arctan(-(sqrt(d)*x - sqrt(d*x^2 + c))/sqrt(-c))*sgn(b*x + a)/sqrt(-c) - b*sqrt(d)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sgn(b*x + a) + ((sqrt(d)*x - sqrt(d*x^2 + c))^3*a*d*sgn(b*x + a) + 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c*sqrt(d)*sgn(b*x + a) + (sqrt(d)*x - sqrt(d*x^2 + c))*a*c*d*sgn(b*x + a) - 2*b*c^2*sqrt(d)*sgn(b*x + a))/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^2

maple [C] time = 0.02, size = 141, normalized size = 0.88

$$\frac{\left(a\sqrt{c} d^{\frac{3}{2}} x^2 \ln\left(\frac{2c+2\sqrt{dx^2+c} \sqrt{c}}{x}\right) - 2bcd x^2 \ln\left(\sqrt{d} x + \sqrt{dx^2+c}\right) - 2\sqrt{dx^2+c} b d^{\frac{3}{2}} x^3 - \sqrt{dx^2+c} a d^{\frac{3}{2}} x^2 + 2(d x^2 + c)^{\frac{3}{2}}\right) \operatorname{sgn}(bx+a)}{2c\sqrt{d} x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^3,x)

[Out] -1/2*c*sgn(b*x+a)*(c^(1/2)*ln(2*(c+(d*x^2+c)^(1/2)*c^(1/2))/x)*d^(3/2)*x^2*a - 2*d^(3/2)*(d*x^2+c)^(1/2)*x^3*b+2*(d*x^2+c)^(3/2)*b*d^(1/2)*x-(d*x^2+c)^(1/2)*a*d^(3/2)*x^2-2*ln(d^(1/2)*x+(d*x^2+c)^(1/2))*x^2*b*c*d+(d*x^2+c)^(3/2)*a*d^(1/2))/x^2/c/d^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2 + c} \sqrt{(bx + a)^2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^3,x, algorithm="maxima")

[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)/x^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{(a + bx)^2} \sqrt{dx^2 + c}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2))/x^3,x)

[Out] int((((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2))/x^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^2} \sqrt{(a + bx)^2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2)/x**3,x)

[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x)**2)/x**3, x)

3.46 $\int x^2 \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2} dx$

Optimal. Leaf size=317

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} (2dx + e) \sqrt{c + dx^2 + ex} (2ad(4cd - 5e^2) - b(12cde - 7e^3)) \sqrt{a^2 + 2abx + b^2x^2} (c + dx^2 + ex)}{128d^4(a + bx)}$$

[Out] $\frac{1}{5} b x^2 (d x^2 + e x + c)^{3/2} ((b x + a)^2)^{1/2} / d (b x + a) - 1/240 (32 b^3 c d + 50 a^2 d e - 35 b^2 e^2 - 6 d (10 a d - 7 b e) x) (d x^2 + e x + c)^{3/2} ((b x + a)^2)^{1/2} / d^3 (b x + a) - 1/256 (4 c d - e^2) (8 a^2 c d^2 - 10 a d e^2 - 12 b^2 c d e + 7 b^2 e^3) \operatorname{arctanh}(1/2 (2 d x + e) / d)^{1/2} / (d x^2 + e x + c)^{1/2} ((b x + a)^2)^{1/2} / d^{9/2} (b x + a) - 1/128 (2 a d (4 c d - 5 e^2) - b (12 c d e - 7 e^3)) (2 d x + e) ((b x + a)^2)^{1/2} (d x^2 + e x + c)^{1/2} / d^4 (b x + a)$

Rubi [A] time = 0.33, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1000, 832, 779, 612, 621, 206}

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} (c + dx^2 + ex)^{3/2} (-6dx(10ad - 7be) + 50ade + 32bcd - 35be^2) \sqrt{a^2 + 2abx + b^2x^2} (2dx + e)}{240d^3(a + bx)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^2 \operatorname{Sqrt}[a^2 + 2 a b x + b^2 x^2] \operatorname{Sqrt}[c + e x + d x^2], x]$

[Out] $-\frac{((2 a d (4 c d - 5 e^2) - b (12 c d e - 7 e^3)) (e + 2 d x) \operatorname{Sqrt}[a^2 + 2 a b x + b^2 x^2] \operatorname{Sqrt}[c + e x + d x^2]) / (128 d^4 (a + b x)) + (b x^2 \operatorname{Sqrt}[a^2 + 2 a b x + b^2 x^2] (c + e x + d x^2)^{3/2}) / (5 d (a + b x)) - ((32 b^3 c d + 50 a^2 d e - 35 b^2 e^2 - 6 d (10 a d - 7 b e) x) \operatorname{Sqrt}[a^2 + 2 a b x + b^2 x^2] (c + e x + d x^2)^{3/2}) / (240 d^3 (a + b x)) - ((4 c d - e^2) (8 a^2 c d^2 - 12 b^2 c d e - 10 a d e^2 + 7 b^2 e^3) \operatorname{Sqrt}[a^2 + 2 a b x + b^2 x^2] \operatorname{ArcTanh}[(e + 2 d x) / (2 \operatorname{Sqrt}[d] \operatorname{Sqrt}[c + e x + d x^2])]) / (256 d^{9/2} (a + b x))}{240 d^3 (a + b x)}$

Rule 206

$\operatorname{Int}[(a_) + (b_.)(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1 \operatorname{ArcTanh}[\operatorname{Rt}[-b, 2] x] / \operatorname{Rt}[a, 2]) / (\operatorname{Rt}[a, 2] \operatorname{Rt}[-b, 2]), x] /;$ $\operatorname{FreeQ}\{a, b\}, x \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

Rule 612

$\operatorname{Int}[(a_.) + (b_.)(x_) + (c_.)(x_)^2)^{(p_)}, x_Symbol] \rightarrow \operatorname{Simp}[(b + 2 c x) (a + b x + c x^2)^p / (2 c (2 p + 1)), x] - \operatorname{Dist}[(p (b^2 - 4 a c)) / (2 c (2 p + 1)), \operatorname{Int}[(a + b x + c x^2)^{(p - 1)}, x], x] /;$ $\operatorname{FreeQ}\{a, b, c\}, x \&\& \mathbb{N}$

$eQ[b^2 - 4ac, 0] \ \&\& \ GtQ[p, 0] \ \&\& \ IntegerQ[4p]$

Rule 621

$Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] \ :> \ Dist[2, Subst[In$
 $t[1/(4c - x^2), x], x, (b + 2cx)/Sqrt[a + bx + cx^2]], x] \ ; \ FreeQ[\{a,$
 $b, c\}, x] \ \&\& \ NeQ[b^2 - 4ac, 0]$

Rule 779

$Int[((d_) + (e_)*(x_))*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c_)*($
 $x_)^2)^{(p_)}, x_Symbol] \ :> \ -Simp[((b*eg*(p + 2) - c*(ef + dg)*(2p + 3) -$
 $2c*eg*(p + 1)*x)*(a + bx + cx^2)^{(p + 1)})/(2c^2*(p + 1)*(2p + 3)), x$
 $] + Dist[(b^2*eg*(p + 2) - 2a*c*eg + c*(2c*d*f - b*(ef + dg))*(2p +$
 $3))/(2c^2*(2p + 3)), Int[(a + bx + cx^2)^p, x], x] \ ; \ FreeQ[\{a, b, c, d,$
 $e, f, g, p\}, x] \ \&\& \ NeQ[b^2 - 4ac, 0] \ \&\& \ !LeQ[p, -1]$

Rule 832

$Int[((d_) + (e_)*(x_))^{(m_)}*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c$
 $_)*(x_)^2)^{(p_)}, x_Symbol] \ :> \ Simp[(g*(d + ex)^m*(a + bx + cx^2)^{(p +$
 $1)})/(c*(m + 2p + 2)), x] + Dist[1/(c*(m + 2p + 2)), Int[(d + ex)^{(m - 1)}$
 $*(a + bx + cx^2)^p*Simp[m*(c*d*f - a*eg) + d*(2c*f - b*g)*(p + 1) + (m*$
 $(c*ef + c*d*g - b*eg) + e*(p + 1)*(2c*f - b*g))*x, x], x] \ ; \ FreeQ[\{$
 $a, b, c, d, e, f, g, p\}, x] \ \&\& \ NeQ[b^2 - 4ac, 0] \ \&\& \ NeQ[c*d^2 - b*d*e + a$
 $*e^2, 0] \ \&\& \ GtQ[m, 0] \ \&\& \ NeQ[m + 2p + 2, 0] \ \&\& \ (IntegerQ[m] \ || \ IntegerQ[p]$
 $\ || \ IntegersQ[2m, 2p]) \ \&\& \ !(IGtQ[m, 0] \ \&\& \ EqQ[f, 0])$

Rule 1000

$Int[((g_) + (h_)*(x_))^{(m_)}*((a_) + (b_)*(x_) + (c_)*(x_)^2)^{(p_)}*((d_$
 $) + (e_)*(x_) + (f_)*(x_)^2)^{(q_)}, x_Symbol] \ :> \ Dist[(a + bx + cx^2)^{Fr$
 $acPart[p]/((4c)^{IntPart[p]}*(b + 2cx)^{(2*FracPart[p])}), Int[(g + hx)^m*($
 $b + 2cx)^{(2p)}*(d + ex + fx^2)^q, x], x] \ ; \ FreeQ[\{a, b, c, d, e, f, g,$
 $h, m, p, q\}, x] \ \&\& \ EqQ[b^2 - 4ac, 0]$

Rubi steps

$$\begin{aligned}
\int x^2 \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2} dx &= \frac{\sqrt{a^2 + 2abx + b^2x^2} \int x^2 (2ab + 2b^2x) \sqrt{c + ex + dx^2} dx}{2ab + 2b^2x} \\
&= \frac{bx^2 \sqrt{a^2 + 2abx + b^2x^2} (c + ex + dx^2)^{3/2}}{5d(a + bx)} + \frac{\sqrt{a^2 + 2abx + b^2x^2} \int x (c + ex + dx^2)^{3/2} dx}{5d(a + bx)} \\
&= \frac{bx^2 \sqrt{a^2 + 2abx + b^2x^2} (c + ex + dx^2)^{3/2}}{5d(a + bx)} - \frac{(32bcd + 50ade - 35be^2) \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{128d^4(a + bx)} \\
&= -\frac{(8acd^2 - 12bcde - 10ade^2 + 7be^3) (e + 2dx) \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{128d^4(a + bx)} \\
&= -\frac{(8acd^2 - 12bcde - 10ade^2 + 7be^3) (e + 2dx) \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{128d^4(a + bx)} \\
&= -\frac{(8acd^2 - 12bcde - 10ade^2 + 7be^3) (e + 2dx) \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{128d^4(a + bx)}
\end{aligned}$$

Mathematica [A] time = 0.27, size = 198, normalized size = 0.62

$$\frac{\sqrt{(a + bx)^2} \left(-\frac{5(2ad(4cd - 5e^2) + b(7e^3 - 12cde)) \left((4cd - e^2) \tanh^{-1} \left(\frac{2dx + e}{2\sqrt{d} \sqrt{c + x(dx + e)}} \right) + 2\sqrt{d} (2dx + e) \sqrt{c + x(dx + e)} \right)}{256d^{7/2}} + \frac{(c + x(dx + e))^{3/2} (10ad(6dx - 5e) - 35be^2)}{48d^2} \right)}{5d(a + bx)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2], x]

[Out] (Sqrt[(a + b*x)^2]*(b*x^2*(c + x*(e + d*x))^(3/2) + ((c + x*(e + d*x))^(3/2)*(-32*b*c*d + 7*b*e*(5*e - 6*d*x) + 10*a*d*(-5*e + 6*d*x)))/(48*d^2) - (5*(2*a*d*(4*c*d - 5*e^2) + b*(-12*c*d*e + 7*e^3))*(2*Sqrt[d]*(e + 2*d*x)*Sqrt[c + x*(e + d*x)] + (4*c*d - e^2)*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + x*(e + d*x)])]))/(256*d^(7/2)))/(5*d*(a + b*x))

fricas [A] time = 1.03, size = 517, normalized size = 1.63

$$\left[-\frac{15(32ac^2d^3 - 48bc^2d^2e - 48acd^2e^2 + 40bcde^3 + 10ade^4 - 7be^5)\sqrt{d} \log\left(8d^2x^2 + 8dex + 4\sqrt{dx^2 + ex + c}\right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/7680*(15*(32*a*c^2*d^3 - 48*b*c^2*d^2*e - 48*a*c*d^2*e^2 + 40*b*c*d*e^3 \\ & + 10*a*d*e^4 - 7*b*e^5)*\sqrt{d}*\log(8*d^2*x^2 + 8*d*e*x + 4*\sqrt{d*x^2 + e} \\ & *x + c)*(2*d*x + e)*\sqrt{d} + 4*c*d + e^2) - 4*(384*b*d^5*x^4 - 256*b*c^2*d \\ & ^3 - 520*a*c*d^3*e + 460*b*c*d^2*e^2 + 150*a*d^2*e^3 - 105*b*d*e^4 + 48*(10 \\ & *a*d^5 + b*d^4*e)*x^3 + 8*(16*b*c*d^4 + 10*a*d^4*e - 7*b*d^3*e^2)*x^2 + 2*(\\ & 120*a*c*d^4 - 116*b*c*d^3*e - 50*a*d^3*e^2 + 35*b*d^2*e^3)*x)*\sqrt{d*x^2 + \\ & e*x + c))/d^5, 1/3840*(15*(32*a*c^2*d^3 - 48*b*c^2*d^2*e - 48*a*c*d^2*e^2 + \\ & 40*b*c*d*e^3 + 10*a*d*e^4 - 7*b*e^5)*\sqrt{-d}*\arctan(1/2*\sqrt{d*x^2 + e*x \\ & + c)*(2*d*x + e)*\sqrt{-d}/(d^2*x^2 + d*e*x + c*d) + 2*(384*b*d^5*x^4 - 256 \\ & *b*c^2*d^3 - 520*a*c*d^3*e + 460*b*c*d^2*e^2 + 150*a*d^2*e^3 - 105*b*d*e^4 \\ & + 48*(10*a*d^5 + b*d^4*e)*x^3 + 8*(16*b*c*d^4 + 10*a*d^4*e - 7*b*d^3*e^2)*x \\ & ^2 + 2*(120*a*c*d^4 - 116*b*c*d^3*e - 50*a*d^3*e^2 + 35*b*d^2*e^3)*x)*\sqrt{d*x^2 + \\ & e*x + c))/d^5] \end{aligned}$$

giac [A] time = 0.31, size = 368, normalized size = 1.16

$$\frac{1}{1920} \sqrt{dx^2 + xe + c} \left(2 \left(4 \left(6 \left(8bx \operatorname{sgn}(bx + a) + \frac{10ad^4 \operatorname{sgn}(bx + a) + bd^3 e \operatorname{sgn}(bx + a)}{d^4} \right) x + \frac{16bcd^3 \operatorname{sgn}(bx + a)}{d^4} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="giac")

[Out]
$$\begin{aligned} & 1/1920*\sqrt{d*x^2 + x*e + c}*(2*(4*(6*(8*b*x*\operatorname{sgn}(b*x + a) + (10*a*d^4*\operatorname{sgn}(b \\ & *x + a) + b*d^3*e*\operatorname{sgn}(b*x + a))/d^4)*x + (16*b*c*d^3*\operatorname{sgn}(b*x + a) + 10*a*d^ \\ & 3*e*\operatorname{sgn}(b*x + a) - 7*b*d^2*e^2*\operatorname{sgn}(b*x + a))/d^4)*x + (120*a*c*d^3*\operatorname{sgn}(b*x \\ & + a) - 116*b*c*d^2*e*\operatorname{sgn}(b*x + a) - 50*a*d^2*e^2*\operatorname{sgn}(b*x + a) + 35*b*d*e^3* \\ & \operatorname{sgn}(b*x + a))/d^4)*x - (256*b*c^2*d^2*\operatorname{sgn}(b*x + a) + 520*a*c*d^2*e*\operatorname{sgn}(b*x \\ & + a) - 460*b*c*d*e^2*\operatorname{sgn}(b*x + a) - 150*a*d*e^3*\operatorname{sgn}(b*x + a) + 105*b*e^4*\operatorname{sg} \\ & n(b*x + a))/d^4) + 1/256*(32*a*c^2*d^3*\operatorname{sgn}(b*x + a) - 48*b*c^2*d^2*e*\operatorname{sgn}(b* \\ & x + a) - 48*a*c*d^2*e^2*\operatorname{sgn}(b*x + a) + 40*b*c*d*e^3*\operatorname{sgn}(b*x + a) + 10*a*d*e \\ & ^4*\operatorname{sgn}(b*x + a) - 7*b*e^5*\operatorname{sgn}(b*x + a))*\log(\operatorname{abs}(-2*(\sqrt{d})*x - \sqrt{d*x^2 \\ & + x*e + c}))*\sqrt{d} - e))/d^{(9/2)} \end{aligned}$$

maple [C] time = 0.02, size = 530, normalized size = 1.67

$$\left(-480a^2c^2d^4 \ln \left(\frac{2dx+e+2\sqrt{dx^2+ex+c}\sqrt{d}}{2\sqrt{d}} \right) + 720acd^3e^2 \ln \left(\frac{2dx+e+2\sqrt{dx^2+ex+c}\sqrt{d}}{2\sqrt{d}} \right) - 150ad^2e^4 \ln \left(\frac{2dx+e+2\sqrt{dx^2+ex+c}\sqrt{d}}{2\sqrt{d}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x)`

[Out] $\frac{1}{3840} \text{csgn}(b*x+a) * (768*(d*x^2+e*x+c)^{(3/2)} * d^{(9/2)} * x^2 * b + 960*(d*x^2+e*x+c)^{(3/2)} * d^{(9/2)} * x * a - 672*(d*x^2+e*x+c)^{(3/2)} * d^{(7/2)} * x * b * e - 800*(d*x^2+e*x+c)^{(3/2)} * d^{(7/2)} * a * e - 512*(d*x^2+e*x+c)^{(3/2)} * d^{(7/2)} * b * c + 560*(d*x^2+e*x+c)^{(3/2)} * d^{(5/2)} * b * e^2 - 480*(d*x^2+e*x+c)^{(1/2)} * d^{(9/2)} * x * a * c + 600*(d*x^2+e*x+c)^{(1/2)} * d^{(7/2)} * x * a * e^2 + 720*(d*x^2+e*x+c)^{(1/2)} * d^{(7/2)} * x * b * c * e - 420*(d*x^2+e*x+c)^{(1/2)} * d^{(5/2)} * x * b * e^3 - 240*(d*x^2+e*x+c)^{(1/2)} * d^{(7/2)} * a * c * e + 300*(d*x^2+e*x+c)^{(1/2)} * d^{(5/2)} * a * e^3 + 360*(d*x^2+e*x+c)^{(1/2)} * d^{(5/2)} * b * c * e^2 - 210*(d*x^2+e*x+c)^{(1/2)} * d^{(3/2)} * b * e^4 - 480 * \ln(1/2 * (2 * (d*x^2+e*x+c)^{(1/2)} * d^{(1/2)} + 2 * d * x + e) / d^{(1/2)}) * a * c^2 * d^4 + 720 * \ln(1/2 * (2 * (d*x^2+e*x+c)^{(1/2)} * d^{(1/2)} + 2 * d * x + e) / d^{(1/2)}) * a * c * d^3 * e^2 - 150 * \ln(1/2 * (2 * (d*x^2+e*x+c)^{(1/2)} * d^{(1/2)} + 2 * d * x + e) / d^{(1/2)}) * a * d^2 * e^4 + 720 * \ln(1/2 * (2 * (d*x^2+e*x+c)^{(1/2)} * d^{(1/2)} + 2 * d * x + e) / d^{(1/2)}) * b * c^2 * d^3 * e - 600 * \ln(1/2 * (2 * (d*x^2+e*x+c)^{(1/2)} * d^{(1/2)} + 2 * d * x + e) / d^{(1/2)}) * b * c * d^2 * e^3 + 105 * \ln(1/2 * (2 * (d*x^2+e*x+c)^{(1/2)} * d^{(1/2)} + 2 * d * x + e) / d^{(1/2)}) * b * d * e^5) / d^{(11/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx^2 + ex + c} \sqrt{(bx + a)^2} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)*x^2, x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \sqrt{(a + bx)^2} \sqrt{dx^2 + ex + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2),x)`

[Out] `int(x^2*((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{c + dx^2 + ex} \sqrt{(a + bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2),x)`

[Out] `Integral(x**2*sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2), x)`

3.47 $\int x\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2} dx$

Optimal. Leaf size=227

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} (4cd - e^2) (8ade + 4bcd - 5be^2) \tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{128d^{7/2}(a+bx)} - \frac{\sqrt{a^2 + 2abx + b^2x^2} (2dx + e)\sqrt{c + ex + dx^2}}{64d^3(a+bx)}$$

[Out] $1/24*(6*b*d*x+8*a*d-5*b*e)*(d*x^2+e*x+c)^{(3/2)*((b*x+a)^2)^{(1/2)}/d^2/(b*x+a)}$
 $-1/128*(4*c*d-e^2)*(8*a*d*e+4*b*c*d-5*b*e^2)*\operatorname{arctanh}(1/2*(2*d*x+e)/d^{(1/2)})$
 $/(d*x^2+e*x+c)^{(1/2)*((b*x+a)^2)^{(1/2)}/d^{(7/2)/(b*x+a)}-1/64*(8*a*d*e+4*b*c$
 $*d-5*b*e^2)*(2*d*x+e)*((b*x+a)^2)^{(1/2)*(d*x^2+e*x+c)^{(1/2)}/d^3/(b*x+a)}$

Rubi [A] time = 0.13, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {1000, 779, 612, 621, 206}

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} (2dx + e)\sqrt{c + dx^2 + ex} (8ade + 4bcd - 5be^2)}{64d^3(a+bx)} - \frac{\sqrt{a^2 + 2abx + b^2x^2} (4cd - e^2) (8ade + 4bcd - 5be^2)}{128d^{7/2}(a+bx)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x\sqrt{a^2 + 2*a*b*x + b^2*x^2}*\sqrt{c + e*x + d*x^2}, x]$

[Out] $-((4*b*c*d + 8*a*d*e - 5*b*e^2)*(e + 2*d*x)*\sqrt{a^2 + 2*a*b*x + b^2*x^2}*\sqrt{c + e*x + d*x^2})/(64*d^3*(a + b*x)) + ((8*a*d - 5*b*e + 6*b*d*x)*\sqrt{a^2 + 2*a*b*x + b^2*x^2}*(c + e*x + d*x^2)^{(3/2)})/(24*d^2*(a + b*x)) - ((4*c*d - e^2)*(4*b*c*d + 8*a*d*e - 5*b*e^2)*\sqrt{a^2 + 2*a*b*x + b^2*x^2}*\operatorname{ArcTanh}[(e + 2*d*x)/(2*\sqrt{d}*\sqrt{c + e*x + d*x^2})])/(128*d^{(7/2)}*(a + b*x))$

Rule 206

$\operatorname{Int}[(a + (b*x)^2)^{-1}, x_Symbol] := \operatorname{Simp}[(1*\operatorname{ArcTanh}[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

$\operatorname{Int}[(a + (b*x + c*x^2)^p), x_Symbol] := \operatorname{Simp}[(b + 2*c*x)*(a + b*x + c*x^2)^p/(2*c*(2*p + 1)), x] - \operatorname{Dist}[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), \operatorname{Int}[(a + b*x + c*x^2)^{p-1}, x], x] /;$ FreeQ[{a, b, c}, x] && NegQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

`Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]`

Rule 779

`Int[((d_) + (e_)*(x_))*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := -Simp[((b*e*g*(p + 2) - c*(e*f + d*g))*(2*p + 3) - 2*c*e*g*(p + 1)*x*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]`

Rule 1000

`Int[((g_) + (h_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0]`

Rubi steps

$$\begin{aligned}
 \int x \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2} dx &= \frac{\sqrt{a^2 + 2abx + b^2x^2} \int x(2ab + 2b^2x) \sqrt{c + ex + dx^2} dx}{2ab + 2b^2x} \\
 &= \frac{(8ad - 5be + 6bdx) \sqrt{a^2 + 2abx + b^2x^2} (c + ex + dx^2)^{3/2}}{24d^2(a + bx)} - \frac{(b(4bcd + 8ade - 5be^2)(e + 2dx) \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2})}{64d^3(a + bx)} + \\
 &= -\frac{(4bcd + 8ade - 5be^2)(e + 2dx) \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{64d^3(a + bx)} + \\
 &= -\frac{(4bcd + 8ade - 5be^2)(e + 2dx) \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{64d^3(a + bx)} + \\
 &= -\frac{(4bcd + 8ade - 5be^2)(e + 2dx) \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{64d^3(a + bx)} +
 \end{aligned}$$

Mathematica [A] time = 0.13, size = 147, normalized size = 0.65

$$\frac{\sqrt{(a+bx)^2} \left((c+x(dx+e))^{3/2} (8ad+6bdx-5be) - \frac{3(8ade+4bcd-5be^2) \left((4cd-e^2) \tanh^{-1} \left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+x(dx+e)}} \right) + 2\sqrt{d}(2dx+e)\sqrt{c+x(dx+e)} \right)}{16d^{3/2}} \right)}{24d^2(a+bx)}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]

[Out] (Sqrt[(a + b*x)^2]*((8*a*d - 5*b*e + 6*b*d*x)*(c + x*(e + d*x))^(3/2) - (3*(4*b*c*d + 8*a*d*e - 5*b*e^2)*(2*Sqrt[d]*(e + 2*d*x)*Sqrt[c + x*(e + d*x)] + (4*c*d - e^2)*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + x*(e + d*x)])))/(16*d^(3/2))))/(24*d^2*(a + b*x))

fricas [A] time = 0.90, size = 391, normalized size = 1.72

$$\frac{3(16bc^2d^2 + 32acd^2e - 24bcde^2 - 8ade^3 + 5be^4)\sqrt{d} \log(8d^2x^2 + 8dex - 4\sqrt{dx^2 + ex + c}(2dx + e)\sqrt{d} + 4c)}{24d^2(a+bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="fricas")

[Out] [1/768*(3*(16*b*c^2*d^2 + 32*a*c*d^2*e - 24*b*c*d*e^2 - 8*a*d*e^3 + 5*b*e^4)*sqrt(d)*log(8*d^2*x^2 + 8*d*e*x - 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + 4*(48*b*d^4*x^3 + 64*a*c*d^3 - 52*b*c*d^2*e - 24*a*d^2*e^2 + 15*b*d*e^3 + 8*(8*a*d^4 + b*d^3*e)*x^2 + 2*(12*b*c*d^3 + 8*a*d^3*e - 5*b*d^2*e^2)*x)*sqrt(d*x^2 + e*x + c))/d^4, 1/384*(3*(16*b*c^2*d^2 + 32*a*c*d^2*e - 24*b*c*d*e^2 - 8*a*d*e^3 + 5*b*e^4)*sqrt(-d)*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(48*b*d^4*x^3 + 64*a*c*d^3 - 52*b*c*d^2*e - 24*a*d^2*e^2 + 15*b*d*e^3 + 8*(8*a*d^4 + b*d^3*e)*x^2 + 2*(12*b*c*d^3 + 8*a*d^3*e - 5*b*d^2*e^2)*x)*sqrt(d*x^2 + e*x + c))/d^4]

giac [A] time = 0.28, size = 268, normalized size = 1.18

$$\frac{1}{192} \sqrt{dx^2 + xe + c} \left(2 \left(4 \left(6bx \operatorname{sgn}(bx + a) + \frac{8ad^3 \operatorname{sgn}(bx + a) + bd^2 e \operatorname{sgn}(bx + a)}{d^3} \right) x + \frac{12bcd^2 \operatorname{sgn}(bx + a) + 8ad^3 \operatorname{sgn}(bx + a)}{d^3} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="giac")

[Out] $\frac{1}{192}\sqrt{d^2x^2 + x^2e + c} \left(2 \left(4 \left(6bx \operatorname{sgn}(bx+a) + (8ad^3 \operatorname{sgn}(bx+a) + b^2d^2e \operatorname{sgn}(bx+a)) \right) / d^3 \right) x + (12b^2cd^2 \operatorname{sgn}(bx+a) + 8ad^2e \operatorname{sgn}(bx+a) - 5b^2d^2e^2 \operatorname{sgn}(bx+a)) / d^3 \right) x + (64ac^2d^2 \operatorname{sgn}(bx+a) - 52b^2cd^2e \operatorname{sgn}(bx+a) - 24ad^2e^2 \operatorname{sgn}(bx+a) + 15b^2e^3 \operatorname{sgn}(bx+a)) / d^3 + 1/128 \left(16b^2c^2d^2 \operatorname{sgn}(bx+a) + 32a^2cd^2e \operatorname{sgn}(bx+a) - 24b^2cd^2e^2 \operatorname{sgn}(bx+a) - 8ad^2e^3 \operatorname{sgn}(bx+a) + 5b^2e^4 \operatorname{sgn}(bx+a) \right) \log(\operatorname{abs}(-2(\sqrt{d}x - \sqrt{d^2x^2 + x^2e + c})\sqrt{d} - e)) / d^{7/2}$

maple [C] time = 0.01, size = 381, normalized size = 1.68

$$\left(-96ac d^3 e \ln \left(\frac{2dx+e+2\sqrt{d^2x^2+ex+c} \sqrt{d}}{2\sqrt{d}} \right) + 24a d^2 e^3 \ln \left(\frac{2dx+e+2\sqrt{d^2x^2+ex+c} \sqrt{d}}{2\sqrt{d}} \right) - 48b c^2 d^3 \ln \left(\frac{2dx+e+2\sqrt{d^2x^2+ex+c} \sqrt{d}}{2\sqrt{d}} \right) + 7 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x)

[Out] $\frac{1}{384}c \operatorname{sgn}(bx+a) \left(96d^{7/2} (d^2x^2+ex+c)^{3/2} x^2 b + 128d^{7/2} (d^2x^2+ex+c)^{3/2} a - 80d^{5/2} (d^2x^2+ex+c)^{3/2} b^2 e - 96d^{7/2} (d^2x^2+ex+c)^{1/2} x^2 a e - 48d^{7/2} (d^2x^2+ex+c)^{1/2} x^2 b^2 c + 60d^{5/2} (d^2x^2+ex+c)^{1/2} x^2 b^2 e^2 - 48d^{5/2} (d^2x^2+ex+c)^{1/2} a^2 e^2 - 24d^{5/2} (d^2x^2+ex+c)^{1/2} b^2 c e + 30d^{3/2} (d^2x^2+ex+c)^{1/2} b^2 e^3 - 96 \ln \left(\frac{1}{2} (2d^2x+e+2(d^2x^2+ex+c)^{1/2}) d^{1/2} \right) / d^{1/2} \right) a^2 c d^3 e + 24 \ln \left(\frac{1}{2} (2d^2x+e+2(d^2x^2+ex+c)^{1/2}) d^{1/2} \right) / d^{1/2} \right) a^2 d^2 e^3 - 48 \ln \left(\frac{1}{2} (2d^2x+e+2(d^2x^2+ex+c)^{1/2}) d^{1/2} \right) / d^{1/2} \right) b^2 c^2 d^3 + 72 \ln \left(\frac{1}{2} (2d^2x+e+2(d^2x^2+ex+c)^{1/2}) d^{1/2} \right) / d^{1/2} \right) b^2 c d^2 e^2 - 15 \ln \left(\frac{1}{2} (2d^2x+e+2(d^2x^2+ex+c)^{1/2}) d^{1/2} \right) / d^{1/2} \right) b^2 d e^4 / d^{9/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx^2 + ex + c} \sqrt{(bx + a)^2} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)*x, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x \sqrt{(a + bx)^2} \sqrt{dx^2 + ex + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2), x)`

[Out] `int(x*((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{c + dx^2 + ex}\sqrt{(a + bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2), x)`

[Out] `Integral(x*sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2), x)`

3.48 $\int \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2} dx$

Optimal. Leaf size=198

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} (4cd - e^2) (2ad - be) \tanh^{-1} \left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}} \right)}{16d^{5/2}(a+bx)} + \frac{\sqrt{a^2 + 2abx + b^2x^2} (2dx + e)(2ad - be)\sqrt{c + dx^2}}{8d^2(a+bx)}$$

[Out] 1/3*b*(d*x^2+e*x+c)^(3/2)*((b*x+a)^2)^(1/2)/d/(b*x+a)+1/16*(2*a*d-b*e)*(4*c*d-e^2)*arctanh(1/2*(2*d*x+e)/d^(1/2)/(d*x^2+e*x+c)^(1/2))*((b*x+a)^2)^(1/2)/d^(5/2)/(b*x+a)+1/8*(2*a*d-b*e)*(2*d*x+e)*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/d^2/(b*x+a)

Rubi [A] time = 0.10, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {969, 640, 612, 621, 206}

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} (4cd - e^2) (2ad - be) \tanh^{-1} \left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}} \right)}{16d^{5/2}(a+bx)} + \frac{\sqrt{a^2 + 2abx + b^2x^2} (2dx + e)(2ad - be)\sqrt{c + dx^2}}{8d^2(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]

[Out] ((2*a*d - b*e)*(e + 2*d*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/(8*d^2*(a + b*x)) + (b*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + e*x + d*x^2)^(3/2))/(3*d*(a + b*x)) + ((2*a*d - b*e)*(4*c*d - e^2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(16*d^(5/2)*(a + b*x))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621


```
Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol
] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 969

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_) + (e_)*(x_) + (f_)*(x_)
^2)^(q_), x_Symbol] := Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p
]*(b + 2*c*x)^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] && !In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}
 \int \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2} dx &= \frac{\sqrt{a^2 + 2abx + b^2x^2} \int (2ab + 2b^2x) \sqrt{c + ex + dx^2} dx}{2ab + 2b^2x} \\
 &= \frac{b\sqrt{a^2 + 2abx + b^2x^2} (c + ex + dx^2)^{3/2}}{3d(a + bx)} + \frac{(b(2ad - be)\sqrt{a^2 + 2abx + b^2x^2})}{d(2ab + 2b^2x)} \\
 &= \frac{(2ad - be)(e + 2dx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{8d^2(a + bx)} + \frac{b\sqrt{a^2 + 2abx + b^2x^2}}{d(2ab + 2b^2x)} \\
 &= \frac{(2ad - be)(e + 2dx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{8d^2(a + bx)} + \frac{b\sqrt{a^2 + 2abx + b^2x^2}}{d(2ab + 2b^2x)} \\
 &= \frac{(2ad - be)(e + 2dx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{8d^2(a + bx)} + \frac{b\sqrt{a^2 + 2abx + b^2x^2}}{d(2ab + 2b^2x)}
 \end{aligned}$$

Mathematica [A] time = 0.13, size = 134, normalized size = 0.68

$$\frac{\sqrt{(a + bx)^2} \left(2\sqrt{d} \sqrt{c + x(dx + e)} (6ad(2dx + e) + b(8cd + 8d^2x^2 + 2dex - 3e^2)) + 3(4cd - e^2)(2ad - be) \tanh^{-1} \left(\frac{2dx + e}{\sqrt{c + x(dx + e)}} \right) \right)}{48d^{5/2}(a + bx)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2], x]

[Out] (Sqrt[(a + b*x)^2]*(2*Sqrt[d]*Sqrt[c + x*(e + d*x)]*(6*a*d*(e + 2*d*x) + b*(8*c*d - 3*e^2 + 2*d*e*x + 8*d^2*x^2)) + 3*(2*a*d - b*e)*(4*c*d - e^2)*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + x*(e + d*x)])])/(48*d^(5/2)*(a + b*x))

fricas [A] time = 0.93, size = 287, normalized size = 1.45

$$\frac{3(8acd^2 - 4bcde - 2ade^2 + be^3)\sqrt{d} \log\left(8d^2x^2 + 8dex + 4\sqrt{dx^2 + ex + c}(2dx + e)\sqrt{d} + 4cd + e^2\right) + 4(8bd^3 - 3e^2d + 2dex + 8d^2x^2)\sqrt{d}}{96d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2), x, algorithm="fricas")

[Out] [1/96*(3*(8*a*c*d^2 - 4*b*c*d*e - 2*a*d*e^2 + b*e^3)*sqrt(d)*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + 4*(8*b*d^3*x^2 + 8*b*c*d^2 + 6*a*d^2*e - 3*b*d*e^2 + 2*(6*a*d^3 + b*d^2*e)*x)*sqrt(d*x^2 + e*x + c))/d^3, -1/48*(3*(8*a*c*d^2 - 4*b*c*d*e - 2*a*d*e^2 + b*e^3)*sqrt(-d)*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) - 2*(8*b*d^3*x^2 + 8*b*c*d^2 + 6*a*d^2*e - 3*b*d*e^2 + 2*(6*a*d^3 + b*d^2*e)*x)*sqrt(d*x^2 + e*x + c))/d^3]

giac [A] time = 0.30, size = 185, normalized size = 0.93

$$\frac{1}{24} \sqrt{dx^2 + xe + c} \left(2 \left(4bx \operatorname{sgn}(bx + a) + \frac{6ad^2 \operatorname{sgn}(bx + a) + b \operatorname{desgn}(bx + a)}{d^2} \right) x + \frac{8bcd \operatorname{sgn}(bx + a) + 6adesgn(bx + a)}{d^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2), x, algorithm="giac")

[Out] 1/24*sqrt(d*x^2 + x*e + c)*(2*(4*b*x*sgn(b*x + a) + (6*a*d^2*sgn(b*x + a) + b*d*e*sgn(b*x + a))/d^2)*x + (8*b*c*d*sgn(b*x + a) + 6*a*d*e*sgn(b*x + a) - 3*b*e^2*sgn(b*x + a))/d^2) - 1/16*(8*a*c*d^2*sgn(b*x + a) - 4*b*c*d*e*sgn(b*x + a) - 2*a*d*e^2*sgn(b*x + a) + b*e^3*sgn(b*x + a))*log(abs(-2*(sqrt(d)*x - sqrt(d*x^2 + x*e + c))*sqrt(d) - e))/d^(5/2)

maple [C] time = 0.01, size = 257, normalized size = 1.30

$$\frac{\left(24ac d^3 \ln\left(\frac{2dx+e+2\sqrt{dx^2+ex+c}\sqrt{d}}{2\sqrt{d}}\right) - 6a d^2 e^2 \ln\left(\frac{2dx+e+2\sqrt{dx^2+ex+c}\sqrt{d}}{2\sqrt{d}}\right) - 12bc d^2 e \ln\left(\frac{2dx+e+2\sqrt{dx^2+ex+c}\sqrt{d}}{2\sqrt{d}}\right) + 3bd e^3 \right)}{96d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x)`

[Out] $\frac{1}{48} \text{csgn}(b*x+a) * (16*d^{(5/2)} * (d*x^2+e*x+c)^{(3/2)} * b + 24*d^{(7/2)} * (d*x^2+e*x+c)^{(1/2)} * x * a - 12*d^{(5/2)} * (d*x^2+e*x+c)^{(1/2)} * x * b * e + 12*d^{(5/2)} * (d*x^2+e*x+c)^{(1/2)} * a * e - 6*d^{(3/2)} * (d*x^2+e*x+c)^{(1/2)} * b * e^2 + 24 * \ln(1/2 * (2*d*x+e+2*(d*x^2+e*x+c)^{(1/2)} * d^{(1/2)})) / d^{(1/2)}) * a * c * d^3 - 6 * \ln(1/2 * (2*d*x+e+2*(d*x^2+e*x+c)^{(1/2)} * d^{(1/2)})) / d^{(1/2)}) * a * d^2 * e^2 - 12 * \ln(1/2 * (2*d*x+e+2*(d*x^2+e*x+c)^{(1/2)} * d^{(1/2)})) / d^{(1/2)}) * b * c * d^2 * e + 3 * \ln(1/2 * (2*d*x+e+2*(d*x^2+e*x+c)^{(1/2)} * d^{(1/2)})) / d^{(1/2)}) * b * d * e^3) / d^{(7/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx^2 + ex + c} \sqrt{(bx + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{(a + bx)^2} \sqrt{dx^2 + ex + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2),x)`

[Out] `int(((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c + dx^2 + ex} \sqrt{(a + bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2),x)`

[Out] `Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2), x)`

$$3.49 \quad \int \frac{\sqrt{a^2+2abx+b^2x^2} \sqrt{c+ex+dx^2}}{x} dx$$

Optimal. Leaf size=211

$$\frac{\sqrt{a^2+2abx+b^2x^2} (4ade+4bcd-be^2) \tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{8d^{3/2}(a+bx)} + \frac{\sqrt{a^2+2abx+b^2x^2} \sqrt{c+dx^2+ex} (4ad+2bdx)}{4d(a+bx)}$$

[Out] 1/8*(4*a*d*e+4*b*c*d-b*e^2)*arctanh(1/2*(2*d*x+e)/d^(1/2)/(d*x^2+e*x+c)^(1/2))*((b*x+a)^2)^(1/2)/d^(3/2)/(b*x+a)-a*arctanh(1/2*(e*x+2*c)/c^(1/2)/(d*x^2+e*x+c)^(1/2))*c^(1/2)*((b*x+a)^2)^(1/2)/(b*x+a)+1/4*(2*b*d*x+4*a*d+b*e)*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/d/(b*x+a)

Rubi [A] time = 0.22, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1000, 814, 843, 621, 206, 724}

$$\frac{\sqrt{a^2+2abx+b^2x^2} (4ade+4bcd-be^2) \tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{8d^{3/2}(a+bx)} + \frac{\sqrt{a^2+2abx+b^2x^2} \sqrt{c+dx^2+ex} (4ad+2bdx)}{4d(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x,x]

[Out] ((4*a*d + b*e + 2*b*d*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/((4*d*(a + b*x)) + ((4*b*c*d + 4*a*d*e - b*e^2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(8*d^(3/2)*(a + b*x)) - (a*Sqrt[c]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + e*x + d*x^2])])/(a + b*x)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol]
:> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1)))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1000

```
Int[((g_.) + (h_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol]
:> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{x} dx &= \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(2ab+2b^2x)\sqrt{c+ex+dx^2}}{x} dx}{2ab + 2b^2x} \\
&= \frac{(4ad + be + 2bdx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{4d(a + bx)} - \frac{\sqrt{a^2 + 2abx + b^2x^2}}{4d(a + bx)} \\
&= \frac{(4ad + be + 2bdx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{4d(a + bx)} + \frac{(2abc\sqrt{a^2 + 2abx + b^2x^2})}{4d(a + bx)} \\
&= \frac{(4ad + be + 2bdx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{4d(a + bx)} - \frac{(4abc\sqrt{a^2 + 2abx + b^2x^2})}{4d(a + bx)} \\
&= \frac{(4ad + be + 2bdx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{4d(a + bx)} + \frac{(4bcd + 4ade - be^2)}{4d(a + bx)}
\end{aligned}$$

Mathematica [A] time = 0.20, size = 149, normalized size = 0.71

$$\frac{\sqrt{(a + bx)^2} \left((4ade + 4bcd - be^2) \tanh^{-1} \left(\frac{2dx + e}{2\sqrt{d} \sqrt{c + x(dx + e)}} \right) + 2\sqrt{d} \left(\sqrt{c + x(dx + e)} (4ad + b(2dx + e)) - 4a\sqrt{c} d \tan^{-1} \left(\frac{2dx + e}{2\sqrt{d} \sqrt{c + x(dx + e)}} \right) \right) \right)}{8d^{3/2}(a + bx)}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x,x]

[Out] (Sqrt[(a + b*x)^2]*((4*b*c*d + 4*a*d*e - b*e^2)*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + x*(e + d*x)])] + 2*Sqrt[d]*(Sqrt[c + x*(e + d*x)]*(4*a*d + b*(e + 2*d*x)) - 4*a*Sqrt[c]*d*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + x*(e + d*x)])))]/(8*d^(3/2)*(a + b*x))

fricas [A] time = 2.54, size = 651, normalized size = 3.09

$$\left[\frac{8a\sqrt{c}d^2 \log\left(\frac{8cex + (4cd + e^2)x^2 - 4\sqrt{dx^2 + ex + c}(ex + 2c)\sqrt{c + 8c^2}}{x^2}\right) - (4bcd + 4ade - be^2)\sqrt{d} \log(8d^2x^2 + 8dex - 4\sqrt{dx^2 + ex + c})}{16d^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x,x, algorithm="fricas")

```
[Out] [1/16*(8*a*sqrt(c)*d^2*log((8*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) - (4*b*c*d + 4*a*d*e - b*e^2)*sqrt(d)*log(8*d^2*x^2 + 8*d*e*x - 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + 4*(2*b*d^2*x + 4*a*d^2 + b*d*e)*sqrt(d*x^2 + e*x + c))/d^2, 1/8*(4*a*sqrt(c)*d^2*log((8*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) - (4*b*c*d + 4*a*d*e - b*e^2)*sqrt(-d)*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(2*b*d^2*x + 4*a*d^2 + b*d*e)*sqrt(d*x^2 + e*x + c))/d^2, 1/16*(16*a*sqrt(-c)*d^2*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c*d*x^2 + c*e*x + c^2)) - (4*b*c*d + 4*a*d*e - b*e^2)*sqrt(d)*log(8*d^2*x^2 + 8*d*e*x - 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + 4*(2*b*d^2*x + 4*a*d^2 + b*d*e)*sqrt(d*x^2 + e*x + c))/d^2, 1/8*(8*a*sqrt(-c)*d^2*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c*d*x^2 + c*e*x + c^2)) - (4*b*c*d + 4*a*d*e - b*e^2)*sqrt(-d)*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(2*b*d^2*x + 4*a*d^2 + b*d*e)*sqrt(d*x^2 + e*x + c))/d^2]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(b*x
+a)]Warning, replacing 0 by `u`, a substitution variable should perhaps be
purged.Warning, replacing 0 by `u`, a substitution variable should perhaps
be purged.Warning, replacing 0 by `u`, a substitution variable should perhaps
be purged.index.cc index_m operator + Error: Bad Argument Value
```

maple [C] time = 0.01, size = 214, normalized size = 1.01

$$\frac{\left(8a\sqrt{c}d^{\frac{5}{2}}\ln\left(\frac{ex+2c+2\sqrt{dx^2+ex+c}\sqrt{c}}{x}\right) - 4ad^2e\ln\left(\frac{2dx+e+2\sqrt{dx^2+ex+c}\sqrt{d}}{2\sqrt{d}}\right) - 4bc d^2\ln\left(\frac{2dx+e+2\sqrt{dx^2+ex+c}\sqrt{d}}{2\sqrt{d}}\right) + bde^2\right)}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x,x)
```

```
[Out] -1/8*csgn(b*x+a)*(8*c^(1/2)*d^(5/2)*ln((2*c+e*x+2*c^(1/2)*(d*x^2+e*x+c)^(1/2))/x)*a-4*d^(5/2)*(d*x^2+e*x+c)^(1/2)*x*b-8*d^(5/2)*(d*x^2+e*x+c)^(1/2)*a-2*d^(3/2)*(d*x^2+e*x+c)^(1/2)*b*e-4*d^2*ln(1/2*(2*d*x+e+2*(d*x^2+e*x+c)^(1/2))
```

$2) * d^{(1/2)} / d^{(1/2)} * a * e^{-4 * \ln(1/2 * (2 * d * x + e + 2 * (d * x^2 + e * x + c)^{(1/2)} * d^{(1/2)}) / d^{(1/2)})} * b * c * d^2 + \ln(1/2 * (2 * d * x + e + 2 * (d * x^2 + e * x + c)^{(1/2)} * d^{(1/2)}) / d^{(1/2)}) * b * d * e^2 / d^{(5/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2 + ex + c} \sqrt{(bx + a)^2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x,x, algorithm="maxima")

[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)/x, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{(a + bx)^2} \sqrt{dx^2 + ex + c}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x,x)

[Out] int((((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^2 + ex} \sqrt{(a + bx)^2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2)/x,x)

[Out] Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2)/x, x)

$$3.50 \quad \int \frac{\sqrt{a^2+2abx+b^2x^2} \sqrt{c+ex+dx^2}}{x^2} dx$$

Optimal. Leaf size=202

$$\frac{\sqrt{a^2+2abx+b^2x^2}(a-bx)\sqrt{c+dx^2+ex}}{x(a+bx)} + \frac{\sqrt{a^2+2abx+b^2x^2}(2ad+be) \tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{2\sqrt{d}(a+bx)} - \frac{\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)}$$

[Out] $-1/2*(a*e+2*b*c)*\operatorname{arctanh}(1/2*(e*x+2*c)/c^{(1/2)}/(d*x^2+e*x+c)^{(1/2)})*((b*x+a)^2)^{(1/2)}/(b*x+a)/c^{(1/2)}+1/2*(2*a*d+b*e)*\operatorname{arctanh}(1/2*(2*d*x+e)/d^{(1/2)}/(d*x^2+e*x+c)^{(1/2)})*((b*x+a)^2)^{(1/2)}/(b*x+a)/d^{(1/2)}-(-b*x+a)*((b*x+a)^2)^{(1/2)}*(d*x^2+e*x+c)^{(1/2)}/x/(b*x+a)$

Rubi [A] time = 0.19, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1000, 812, 843, 621, 206, 724}

$$\frac{\sqrt{a^2+2abx+b^2x^2}(a-bx)\sqrt{c+dx^2+ex}}{x(a+bx)} + \frac{\sqrt{a^2+2abx+b^2x^2}(2ad+be) \tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{2\sqrt{d}(a+bx)} - \frac{\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x^2,x]

[Out] $-(((a-b*x)*\operatorname{Sqrt}[a^2+2*a*b*x+b^2*x^2]*\operatorname{Sqrt}[c+e*x+d*x^2])/(x*(a+b*x))) + ((2*a*d+b*e)*\operatorname{Sqrt}[a^2+2*a*b*x+b^2*x^2]*\operatorname{ArcTanh}[(e+2*d*x)/(2*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[c+e*x+d*x^2])])/(2*\operatorname{Sqrt}[d]*(a+b*x)) - ((2*b*c+a*e)*\operatorname{Sqrt}[a^2+2*a*b*x+b^2*x^2]*\operatorname{ArcTanh}[(2*c+e*x)/(2*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[c+e*x+d*x^2])])/(2*\operatorname{Sqrt}[c]*(a+b*x))$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol]
:= Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:= Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x]
+ Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m + 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !RationalQ[m])) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:= Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1000

```
Int[((g_.) + (h_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol]
:= Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, p, q}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{x^2} dx &= \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(2ab+2b^2x)\sqrt{c+ex+dx^2}}{x^2} dx}{2ab + 2b^2x} \\
&= -\frac{(a - bx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{x(a + bx)} - \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{2ab}{x^2} dx}{2(2ab)} \\
&= -\frac{(a - bx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{x(a + bx)} + \frac{\left(b(2bc + ae)\sqrt{a^2 + 2abx + b^2x^2}\right)}{2a} \\
&= -\frac{(a - bx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{x(a + bx)} - \frac{\left(2b(2bc + ae)\sqrt{a^2 + 2abx + b^2x^2}\right)}{2a} \\
&= -\frac{(a - bx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{x(a + bx)} + \frac{(2ad + be)\sqrt{a^2 + 2abx + b^2x^2}}{2a}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 155, normalized size = 0.77

$$\frac{\sqrt{(a + bx)^2} \left(\sqrt{c} x(2ad + be) \tanh^{-1} \left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+x(dx+e)}} \right) + \sqrt{d} \left(2\sqrt{c} (bx - a)\sqrt{c + x(dx + e)} - x(ae + 2bc) \tanh^{-1} \left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+x(dx+e)}} \right) \right) \right)}{2\sqrt{c}\sqrt{d}x(a + bx)}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x^2,x]

[Out] (Sqrt[(a + b*x)^2]*(Sqrt[c]*(2*a*d + b*e)*x*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + x*(e + d*x)])]) + Sqrt[d]*(2*Sqrt[c]*(-a + b*x)*Sqrt[c + x*(e + d*x)] - (2*b*c + a*e)*x*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + x*(e + d*x)])]))/(2*Sqrt[c]*Sqrt[d]*x*(a + b*x))

fricas [A] time = 1.40, size = 647, normalized size = 3.20

$$\left[\frac{(2acd + bce)\sqrt{d}x \log\left(8d^2x^2 + 8dex + 4\sqrt{dx^2 + ex + c}(2dx + e)\sqrt{d} + 4cd + e^2\right) + (2bcd + ade)\sqrt{c}x \log\left(\frac{8cdx + \dots}{4cdx}\right)}{4cdx} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^2,x, algorithm="fricas")

```
[Out] [1/4*((2*a*c*d + b*c*e)*sqrt(d)*x*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + (2*b*c*d + a*d*e)*sqrt(c)*x*log((8*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) + 4*(b*c*d*x - a*c*d)*sqrt(d*x^2 + e*x + c))/(c*d*x), -1/4*(2*(2*a*c*d + b*c*e)*sqrt(-d)*x*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) - (2*b*c*d + a*d*e)*sqrt(c)*x*log((8*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) - 4*(b*c*d*x - a*c*d)*sqrt(d*x^2 + e*x + c))/(c*d*x), 1/4*(2*(2*b*c*d + a*d*e)*sqrt(-c)*x*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c*d*x^2 + c*e*x + c^2)) + (2*a*c*d + b*c*e)*sqrt(d)*x*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + 4*(b*c*d*x - a*c*d)*sqrt(d*x^2 + e*x + c))/(c*d*x), 1/2*((2*b*c*d + a*d*e)*sqrt(-c)*x*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c*d*x^2 + c*e*x + c^2)) - (2*a*c*d + b*c*e)*sqrt(-d)*x*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(b*c*d*x - a*c*d)*sqrt(d*x^2 + e*x + c))/(c*d*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(b*x
+a)]Unable to divide, perhaps due to rounding error%%{2,[4,4,0]%%}+%%{%%
%{-16,[1]%%},[4,2,1]%%}+%%{%%{32,[2]%%},[4,0,2]%%}+%%{-4,[2,4,1]%%}
+%%{%%{32,[1]%%},[2,2,2]%%}+%%{%%{-64,[2]%%},[2,0,3]%%}+%%{2,[0,4,
2]%%}+%%{%%{-16,[1]%%},[0,2,3]%%}+%%{%%{32,[2]%%},[0,0,4]%%} / %%
{4,[4,0,0]%%}+%%{-8,[2,0,1]%%}+%%{4,[0,0,2]%%} Error: Bad Argument Val
ue
```

maple [C] time = 0.01, size = 249, normalized size = 1.23

$$\frac{\left(2ac d^2 x \ln\left(\frac{2dx+e+2\sqrt{dx^2+ex+c} \sqrt{d}}{2\sqrt{d}}\right) - a\sqrt{c} d^{\frac{3}{2}} ex \ln\left(\frac{ex+2c+2\sqrt{dx^2+ex+c} \sqrt{c}}{x}\right) - 2b c^{\frac{3}{2}} d^{\frac{3}{2}} x \ln\left(\frac{ex+2c+2\sqrt{dx^2+ex+c} \sqrt{c}}{x}\right) + bcd\right)}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^2,x)
```

```
[Out] 1/2*csgn(b*x+a)*(2*d^(5/2)*(d*x^2+e*x+c)^(1/2)*x^2*a-2*d^(3/2)*c^(3/2)*ln((
e*x+2*c+2*(d*x^2+e*x+c)^(1/2)*c^(1/2))/x)*x*b-d^(3/2)*c^(1/2)*ln((e*x+2*c+2
```

$(d*x^2+e*x+c)^{(1/2)}*c^{(1/2)}/x)*x*a*e-2*d^{(3/2)}*(d*x^2+e*x+c)^{(3/2)}*a+2*d^{(3/2)}*(d*x^2+e*x+c)^{(1/2)}*x*a*e+2*d^{(3/2)}*(d*x^2+e*x+c)^{(1/2)}*x*b*c+2*\ln(1/2*(2*d*x+e+2*(d*x^2+e*x+c)^{(1/2)}*d^{(1/2)})/d^{(1/2)})*x*a*c*d^2+\ln(1/2*(2*d*x+e+2*(d*x^2+e*x+c)^{(1/2)}*d^{(1/2)})/d^{(1/2)})*d*x*b*c*e)/c/x/d^{(3/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2 + ex + c} \sqrt{(bx + a)^2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^2,x, algorithm="maxima")

[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)/x^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{(a + bx)^2} \sqrt{dx^2 + ex + c}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^2,x)

[Out] int((((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^2 + ex} \sqrt{(a + bx)^2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2)/x**2,x)

[Out] Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2)/x**2, x)

$$3.51 \quad \int \frac{\sqrt{a^2+2abx+b^2x^2} \sqrt{c+ex+dx^2}}{x^3} dx$$

Optimal. Leaf size=215

$$\frac{\sqrt{a^2+2abx+b^2x^2} (4acd - ae^2 + 4bce) \tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+dx^2+ex}}\right)}{8c^{3/2}(a+bx)} - \frac{\sqrt{a^2+2abx+b^2x^2} \sqrt{c+dx^2+ex} (xae+4bc)}{4cx^2(a+bx)}$$

[Out] $-1/8*(4*a*c*d-a*e^2+4*b*c*e)*\operatorname{arctanh}(1/2*(e*x+2*c)/c^{(1/2)}/(d*x^2+e*x+c)^{(1/2)})*((b*x+a)^2)^{(1/2)}/c^{(3/2)}/(b*x+a)+b*\operatorname{arctanh}(1/2*(2*d*x+e)/d^{(1/2)}/(d*x^2+e*x+c)^{(1/2)})*d^{(1/2)}*((b*x+a)^2)^{(1/2)}/(b*x+a)-1/4*(2*a*c+(a*e+4*b*c)*x)*((b*x+a)^2)^{(1/2)}*(d*x^2+e*x+c)^{(1/2)}/c/x^2/(b*x+a)$

Rubi [A] time = 0.18, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1000, 810, 843, 621, 206, 724}

$$\frac{\sqrt{a^2+2abx+b^2x^2} (4acd - ae^2 + 4bce) \tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+dx^2+ex}}\right)}{8c^{3/2}(a+bx)} - \frac{\sqrt{a^2+2abx+b^2x^2} \sqrt{c+dx^2+ex} (xae+4bc)}{4cx^2(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x^3,x]

[Out] $-((2*a*c + (4*b*c + a*e)*x)*\operatorname{Sqrt}[a^2 + 2*a*b*x + b^2*x^2]*\operatorname{Sqrt}[c + e*x + d*x^2])/(4*c*x^2*(a + b*x)) + (b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[a^2 + 2*a*b*x + b^2*x^2]*\operatorname{ArcTan}[\operatorname{h}[(e + 2*d*x)/(2*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[c + e*x + d*x^2])]])/(a + b*x) - ((4*a*c*d + 4*b*c*e - a*e^2)*\operatorname{Sqrt}[a^2 + 2*a*b*x + b^2*x^2]*\operatorname{ArcTanh}[(2*c + e*x)/(2*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[c + e*x + d*x^2])])/(8*c^{(3/2)}*(a + b*x))$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol]
:> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x)/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m - 2*p) + e*f*(m + 2*p + 2)))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] && !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1000

```
Int[((g_.) + (h_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol]
:> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, p, q}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{x^3} dx &= \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(2ab+2b^2x)\sqrt{c+ex+dx^2}}{x^3} dx}{2ab + 2b^2x} \\
&= -\frac{(2ac + (4bc + ae)x)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{4cx^2(a + bx)} - \frac{\sqrt{a^2 + 2abx}}{4cx^2(a + bx)} \\
&= -\frac{(2ac + (4bc + ae)x)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{4cx^2(a + bx)} + \frac{(2b^2d\sqrt{a^2 + 2abx})}{4cx^2(a + bx)} \\
&= -\frac{(2ac + (4bc + ae)x)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{4cx^2(a + bx)} + \frac{(4b^2d\sqrt{a^2 + 2abx})}{4cx^2(a + bx)} \\
&= -\frac{(2ac + (4bc + ae)x)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{4cx^2(a + bx)} + \frac{b\sqrt{d}\sqrt{a^2 + 2abx}}{4cx^2(a + bx)}
\end{aligned}$$

Mathematica [A] time = 0.22, size = 161, normalized size = 0.75

$$\frac{\sqrt{(a + bx)^2} \left(x^2 (4acd - ae^2 + 4bce) \tanh^{-1} \left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+x(dx+e)}} \right) + 2\sqrt{c}\sqrt{c+x(dx+e)} (2ac + aex + 4bcx) - 8bc^{3/2}\sqrt{a^2 + 2abx} \right)}{8c^{3/2}x^2(a + bx)}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x^3,x]

[Out] -1/8*(Sqrt[(a + b*x)^2]*(2*Sqrt[c]*(2*a*c + 4*b*c*x + a*e*x)*Sqrt[c + x*(e + d*x)] - 8*b*c^(3/2)*Sqrt[d]*x^2*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + x*(e + d*x)])] + (4*a*c*d + 4*b*c*e - a*e^2)*x^2*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + x*(e + d*x)])]))/(c^(3/2)*x^2*(a + b*x))

fricas [A] time = 1.81, size = 693, normalized size = 3.22

$$\left[\frac{8bc^2\sqrt{d}x^2 \log\left(8d^2x^2 + 8dex + 4\sqrt{dx^2 + ex + c}(2dx + e)\sqrt{d} + 4cd + e^2\right) - (4acd + 4bce - ae^2)\sqrt{c}x^2 \log\left(\frac{8c}{2\sqrt{c}\sqrt{c+x(dx+e)}}\right)}{16c^2x^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^3,x, algorithm="fricas")


```
[Out] [1/16*(8*b*c^2*sqrt(d)*x^2*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e*x + c)
)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) - (4*a*c*d + 4*b*c*e - a*e^2)*sqrt(c)*
x^2*log((8*c*e*x + (4*c*d + e^2)*x^2 + 4*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*
sqrt(c) + 8*c^2)/x^2) - 4*(2*a*c^2 + (4*b*c^2 + a*c*e)*x)*sqrt(d*x^2 + e*x
+ c))/(c^2*x^2), -1/16*(16*b*c^2*sqrt(-d)*x^2*arctan(1/2*sqrt(d*x^2 + e*x +
c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + (4*a*c*d + 4*b*c*e - a*
e^2)*sqrt(c)*x^2*log((8*c*e*x + (4*c*d + e^2)*x^2 + 4*sqrt(d*x^2 + e*x + c)
*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) + 4*(2*a*c^2 + (4*b*c^2 + a*c*e)*x)*sqrt
(d*x^2 + e*x + c))/(c^2*x^2), 1/8*(4*b*c^2*sqrt(d)*x^2*log(8*d^2*x^2 + 8*d*
e*x + 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + (4*a*c*d
+ 4*b*c*e - a*e^2)*sqrt(-c)*x^2*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*
c)*sqrt(-c)/(c*d*x^2 + c*e*x + c^2)) - 2*(2*a*c^2 + (4*b*c^2 + a*c*e)*x)*sq
rt(d*x^2 + e*x + c))/(c^2*x^2), -1/8*(8*b*c^2*sqrt(-d)*x^2*arctan(1/2*sqrt(
d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) - (4*a*c*d +
4*b*c*e - a*e^2)*sqrt(-c)*x^2*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)
)*sqrt(-c)/(c*d*x^2 + c*e*x + c^2)) + 2*(2*a*c^2 + (4*b*c^2 + a*c*e)*x)*sqrt
(d*x^2 + e*x + c))/(c^2*x^2)]
```

giac [B] time = 0.41, size = 450, normalized size = 2.09

$$-b\sqrt{d} \log \left(\left| -2 \left(\sqrt{d}x - \sqrt{dx^2 + xe + c} \right) d - \sqrt{d}e \right| \right) \operatorname{sgn}(bx + a) + \frac{(4acds\operatorname{gn}(bx + a) + 4bces\operatorname{gn}(bx + a) - ae^2\operatorname{sgn}(bx + a))}{4\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -b*sqrt(d)*log(abs(-2*(sqrt(d)*x - sqrt(d*x^2 + x*e + c))*d - sqrt(d)*e))*s
gn(b*x + a) + 1/4*(4*a*c*d*sgn(b*x + a) + 4*b*c*e*sgn(b*x + a) - a*e^2*sgn(
b*x + a))*arctan(-(sqrt(d)*x - sqrt(d*x^2 + x*e + c))/sqrt(-c))/(sqrt(-c)*c
) + 1/4*(4*(sqrt(d)*x - sqrt(d*x^2 + x*e + c))^3*a*c*d*sgn(b*x + a) + 4*(sq
rt(d)*x - sqrt(d*x^2 + x*e + c))^3*b*c*e*sgn(b*x + a) + 8*(sqrt(d)*x - sqrt
(d*x^2 + x*e + c))^2*b*c^2*sqrt(d)*sgn(b*x + a) + 8*(sqrt(d)*x - sqrt(d*x^2
+ x*e + c))^2*a*c*sqrt(d)*e*sgn(b*x + a) + 4*(sqrt(d)*x - sqrt(d*x^2 + x*e
+ c))*a*c^2*d*sgn(b*x + a) + (sqrt(d)*x - sqrt(d*x^2 + x*e + c))^3*a*e^2*s
gn(b*x + a) - 4*(sqrt(d)*x - sqrt(d*x^2 + x*e + c))*b*c^2*e*sgn(b*x + a) -
8*b*c^3*sqrt(d)*sgn(b*x + a) + (sqrt(d)*x - sqrt(d*x^2 + x*e + c))*a*c*e^2*
sgn(b*x + a))/(((sqrt(d)*x - sqrt(d*x^2 + x*e + c))^2 - c)^2*c)
```

maple [C] time = 0.02, size = 358, normalized size = 1.67

$$\frac{\left(-4a c^{\frac{3}{2}} d^{\frac{5}{2}} x^2 \ln \left(\frac{ex+2c+2\sqrt{d}x^2+ex+c}{x} \sqrt{c} \right) + a\sqrt{c} d^{\frac{3}{2}} e^2 x^2 \ln \left(\frac{ex+2c+2\sqrt{d}x^2+ex+c}{x} \sqrt{c} \right) + 8b c^2 d^2 x^2 \ln \left(\frac{2dx+e+2\sqrt{d}x^2+ex+c}{2\sqrt{d}} \right) \right)}{4\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^3,x)`

[Out] $\frac{1}{8} \operatorname{csgn}(b*x+a) * (-4*d^{5/2} * c^{3/2} * \ln((e*x+2*c+2*(d*x^2+e*x+c)^{1/2}) * c^{1/2})) / x * x^2 * a - 2*d^{5/2} * (d*x^2+e*x+c)^{1/2} * x^3 * a * e + 8*d^{5/2} * (d*x^2+e*x+c)^{1/2} * x^3 * b * c - 4*d^{3/2} * c^{3/2} * \ln((e*x+2*c+2*(d*x^2+e*x+c)^{1/2}) * c^{1/2}) / x * x^2 * b * e + 4*d^{5/2} * (d*x^2+e*x+c)^{1/2} * x^2 * a * c + d^{3/2} * c^{1/2} * \ln((e*x+2*c+2*(d*x^2+e*x+c)^{1/2}) * c^{1/2}) / x * x^2 * a * e^2 + 2*d^{3/2} * (d*x^2+e*x+c)^{3/2} * x * a * e - 8*d^{3/2} * (d*x^2+e*x+c)^{3/2} * x * b * c - 2*d^{3/2} * (d*x^2+e*x+c)^{1/2} * x^2 * a * e^2 + 8*d^{3/2} * (d*x^2+e*x+c)^{1/2} * x^2 * b * c * e + 8 * \ln(1/2 * (2*d*x+e+2*(d*x^2+e*x+c)^{1/2}) * d^{1/2}) / d^{1/2} * x^2 * b * c^2 * d^2 - 4*d^{3/2} * (d*x^2+e*x+c)^{3/2} * a * c) / x^2 / c^2 / d^{3/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2 + ex + c} \sqrt{(bx + a)^2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^3,x, algorithm="maxima")`

[Out] `integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)/x^3, x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{(a + bx)^2} \sqrt{dx^2 + ex + c}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^3,x)`

[Out] `int((((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^3, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^2 + ex} \sqrt{(a + bx)^2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2)/x**3,x)`

[Out] `Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2)/x**3, x)`

$$3.52 \quad \int \frac{x^2 \sqrt{a+cx^2}}{d+ex+fx^2} dx$$

Optimal. Leaf size=452

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(af^2 + 2c(e^2 - df)) \left(e\left(e - \sqrt{e^2 - 4df}\right)(af^2 + c(e^2 - 2df)) - 2df(af^2 + c(e^2 - df))\right) \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}f^3} \frac{\sqrt{2}f^3\sqrt{e^2 - 4df}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - \dots)}}{\dots}$$

[Out] 1/2*(a*f^2+2*c*(-d*f+e^2))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/f^3/c^(1/2)-1/2*(-f*x+2*e)*(c*x^2+a)^(1/2)/f^2-1/2*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))^(1/2))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(-2*d*f*(a*f^2+c*(-d*f+e^2))+e*(a*f^2+c*(-2*d*f+e^2))*(e-(-4*d*f+e^2)^(1/2)))/f^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)+1/2*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)))^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(-2*d*f*(a*f^2+c*(-d*f+e^2))+e*(a*f^2+c*(-2*d*f+e^2))*(e+(-4*d*f+e^2)^(1/2)))/f^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)

Rubi [A] time = 1.97, antiderivative size = 452, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1069, 1080, 217, 206, 1034, 725}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(af^2 + 2c(e^2 - df)) \left(e\left(e - \sqrt{e^2 - 4df}\right)(af^2 + c(e^2 - 2df)) - 2df(af^2 + c(e^2 - df))\right) \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}f^3} \frac{\sqrt{2}f^3\sqrt{e^2 - 4df}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - \dots)}}{\dots}$$

Antiderivative was successfully verified.

[In] Int[(x^2*Sqrt[a + c*x^2])/(d + e*x + f*x^2), x]

[Out] -((2*e - f*x)*Sqrt[a + c*x^2])/(2*f^2) + ((a*f^2 + 2*c*(e^2 - d*f))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c]*f^3) - ((e*(e - Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - 2*d*f)) - 2*d*f*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f) - e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f) - e*Sqrt[e^2 - 4*d*f]]) + ((e*(e + Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - 2*d*f)) - 2*d*f*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f) + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f) + e*Sqrt[e^2 - 4*d*f]])

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 725

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (c_)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 1034

Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (f_)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rule 1069

Int[((a_) + (c_)*(x_)^2)^(p_)*((A_) + (C_)*(x_)^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((C*(-(c*e*(2*p + q + 2))) + 2*c*C*f*(p + q + 1)*x)*(a + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(-(a*e))*(C*(c*e)*(q + 1) - c*(C*e)*(2*p + 2*q + 3)) + (p + q + 1)*(a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(-2*A*f)*(2*p + 2*q + 3)))] + (2*p*(c*d - a*f)*(C*(c*e)*(q + 1) - c*(C*e)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(-4*a*c)))*x + (p*(c*e)*(C*(c*e)*(q + 1) - c*(C*e)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(-4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))]*x^2, x], x] /; FreeQ[{a, c, d, e, f, A, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]

Rule 1080

Int[((A_) + (B_)*(x_) + (C_)*(x_)^2)/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (f_)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + f*x^2],

$x], x] + \text{Dist}[1/c, \text{Int}[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*\text{Sqrt}[d + f*x^2]), x], x] /; \text{FreeQ}[\{a, b, c, d, f, A, B, C\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rubi steps

$$\begin{aligned} \int \frac{x^2 \sqrt{a+cx^2}}{d+ex+fx^2} dx &= -\frac{(2e-fx)\sqrt{a+cx^2}}{2f^2} - \frac{\int \frac{acdf-ce(2cd-af)x-c(af^2+2c(e^2-df))x^2}{\sqrt{a+cx^2}(d+ex+fx^2)} dx}{2cf^2} \\ &= -\frac{(2e-fx)\sqrt{a+cx^2}}{2f^2} - \frac{\int \frac{acdf^2+cd(af^2+2c(e^2-df))+(-cef(2cd-af)+ce(af^2+2c(e^2-df)))x}{\sqrt{a+cx^2}(d+ex+fx^2)} dx}{2cf^3} + \frac{(af^2+2c(e^2-df)) \text{Subst}\left(\int \frac{1}{1-cx^2} dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{2f^3} + \frac{(e(e-\sqrt{e^2-4df}))}{2f^3} \\ &= -\frac{(2e-fx)\sqrt{a+cx^2}}{2f^2} + \frac{(af^2+2c(e^2-df)) \text{Subst}\left(\int \frac{1}{1-cx^2} dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{2f^3} + \frac{(e(e-\sqrt{e^2-4df}))}{2f^3} \\ &= -\frac{(2e-fx)\sqrt{a+cx^2}}{2f^2} + \frac{(af^2+2c(e^2-df)) \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}f^3} - \frac{(e(e-\sqrt{e^2-4df}))(af^2+2c(e^2-df))}{2\sqrt{c}f^3} \\ &= -\frac{(2e-fx)\sqrt{a+cx^2}}{2f^2} + \frac{(af^2+2c(e^2-df)) \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}f^3} - \frac{(e(e-\sqrt{e^2-4df}))(af^2+2c(e^2-df))}{2\sqrt{c}f^3} \end{aligned}$$

Mathematica [A] time = 2.50, size = 516, normalized size = 1.14

$$\frac{2f \left(\frac{a^{3/2} \sqrt{\frac{cx^2}{a} + 1} \sinh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a}}\right) + ax + cx^3}{\sqrt{c}} \right)}{\sqrt{a+cx^2}} + \frac{\left(\frac{2df-e^2}{\sqrt{e^2-4df}} + e \right) \left(\sqrt{4af^2-2c(e\sqrt{e^2-4df}+2df-e^2)} \tanh^{-1}\left(\frac{2af+cx(\sqrt{e^2-4df}-e)}{\sqrt{a+cx^2} \sqrt{4af^2-2c(e\sqrt{e^2-4df}+2df-e^2)}} \right) - \sqrt{c}(\sqrt{e^2-4df}) \right)}{f}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*Sqrt[a + c*x^2])/(d + e*x + f*x^2), x]

[Out] $(-2*(e + (e^2 - 2*d*f))/\text{Sqrt}[e^2 - 4*d*f])* \text{Sqrt}[a + c*x^2] - 2*(e + (-e^2 + 2*d*f))/\text{Sqrt}[e^2 - 4*d*f]* \text{Sqrt}[a + c*x^2] + (2*f*(a*x + c*x^3 + (a^{3/2})*\text{Sqrt}[1 + (c*x^2)/a]*\text{ArcSinh}[(\text{Sqrt}[c]*x)/\text{Sqrt}[a]])/\text{Sqrt}[c])/ \text{Sqrt}[a + c*x^2] + ((e + (-e^2 + 2*d*f))/\text{Sqrt}[e^2 - 4*d*f])*(-(\text{Sqrt}[c]*(-e + \text{Sqrt}[e^2 - 4*d*f])$

$$\begin{aligned} &) * \text{ArcTanh}[(\text{Sqrt}[c] * x) / \text{Sqrt}[a + c * x^2]] + \text{Sqrt}[4 * a * f^2 - 2 * c * (-e^2 + 2 * d * f \\ & + e * \text{Sqrt}[e^2 - 4 * d * f])] * \text{ArcTanh}[(2 * a * f + c * (-e + \text{Sqrt}[e^2 - 4 * d * f]) * x) / (\text{Sqr} \\ & \text{t}[4 * a * f^2 - 2 * c * (-e^2 + 2 * d * f + e * \text{Sqrt}[e^2 - 4 * d * f])] * \text{Sqrt}[a + c * x^2])]) / f \\ & + ((e^2 - 2 * d * f + e * \text{Sqrt}[e^2 - 4 * d * f]) * (\text{Sqrt}[c] * (e + \text{Sqrt}[e^2 - 4 * d * f]) * \text{Ar} \\ & \text{cTanh}[(\text{Sqrt}[c] * x) / \text{Sqrt}[a + c * x^2]] + \text{Sqrt}[4 * a * f^2 + 2 * c * (e^2 - 2 * d * f + e * \text{Sqr} \\ & \text{t}[e^2 - 4 * d * f])] * \text{ArcTanh}[(2 * a * f - c * (e + \text{Sqrt}[e^2 - 4 * d * f]) * x) / (\text{Sqrt}[4 * a * f \\ & ^2 + 2 * c * (e^2 - 2 * d * f + e * \text{Sqrt}[e^2 - 4 * d * f])] * \text{Sqrt}[a + c * x^2])])]) / (f * \text{Sqrt}[e \\ & ^2 - 4 * d * f]) / (4 * f^2) \end{aligned}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.04, size = 7739, normalized size = 17.12

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x)

[Out] result too large to display

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h

elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details) Is $4*d*f-e^2$ positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \sqrt{cx^2 + a}}{fx^2 + ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(a + c*x^2)^(1/2))/(d + e*x + f*x^2), x)

[Out] int((x^2*(a + c*x^2)^(1/2))/(d + e*x + f*x^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sqrt{a + cx^2}}{d + ex + fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(c*x**2+a)**(1/2)/(f*x**2+e*x+d), x)

[Out] Integral(x**2*sqrt(a + c*x**2)/(d + e*x + f*x**2), x)

$$3.53 \quad \int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2} dx$$

Optimal. Leaf size=395

$$\frac{(2cdef - (e - \sqrt{e^2 - 4df})(af^2 + c(e^2 - df))) \tanh^{-1} \left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}} \right) (2cdef - (\sqrt{e^2 - 4df}))}{\sqrt{2}f^2\sqrt{e^2 - 4df}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}} +$$

[Out] $-e \operatorname{arctanh}(x \cdot c^{1/2} / (c \cdot x^2 + a)^{1/2}) \cdot c^{1/2} / f^2 + (c \cdot x^2 + a)^{1/2} / f - 1/2 \operatorname{arc} \tanh(1/2 \cdot (2 \cdot a \cdot f - c \cdot x \cdot (e - (-4 \cdot d \cdot f + e^2)^{1/2}))) \cdot 2^{1/2} / (c \cdot x^2 + a)^{1/2} / (2 \cdot a \cdot f^2 + (-(-4 \cdot d \cdot f + e^2)^{1/2} \cdot e - 2 \cdot d \cdot f + e^2) \cdot c)^{1/2}) \cdot (2 \cdot c \cdot d \cdot e \cdot f - (a \cdot f^2 + c \cdot (-d \cdot f + e^2))) \cdot (e - (-4 \cdot d \cdot f + e^2)^{1/2}) / f^2 \cdot 2^{1/2} / (-4 \cdot d \cdot f + e^2)^{1/2} / (2 \cdot a \cdot f^2 + (-(-4 \cdot d \cdot f + e^2)^{1/2} \cdot e - 2 \cdot d \cdot f + e^2) \cdot c)^{1/2} + 1/2 \operatorname{arctanh}(1/2 \cdot (2 \cdot a \cdot f - c \cdot x \cdot (e + (-4 \cdot d \cdot f + e^2)^{1/2}))) \cdot 2^{1/2} / (c \cdot x^2 + a)^{1/2} / (2 \cdot a \cdot f^2 + ((-4 \cdot d \cdot f + e^2)^{1/2} \cdot e - 2 \cdot d \cdot f + e^2) \cdot c)^{1/2}) \cdot (2 \cdot c \cdot d \cdot e \cdot f - (a \cdot f^2 + c \cdot (-d \cdot f + e^2))) \cdot (e + (-4 \cdot d \cdot f + e^2)^{1/2}) / f^2 \cdot 2^{1/2} / (-4 \cdot d \cdot f + e^2)^{1/2} / (2 \cdot a \cdot f^2 + ((-4 \cdot d \cdot f + e^2)^{1/2} \cdot e - 2 \cdot d \cdot f + e^2) \cdot c)^{1/2}$

Rubi [A] time = 0.93, antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1020, 1080, 217, 206, 1034, 725}

$$\frac{(2cdef - (e - \sqrt{e^2 - 4df})(af^2 + c(e^2 - df))) \tanh^{-1} \left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}} \right) (2cdef - (\sqrt{e^2 - 4df}))}{\sqrt{2}f^2\sqrt{e^2 - 4df}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}} +$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x \cdot \text{Sqrt}[a + c \cdot x^2]) / (d + e \cdot x + f \cdot x^2), x]$

[Out] $\text{Sqrt}[a + c \cdot x^2] / f - (\text{Sqrt}[c] \cdot e \cdot \text{ArcTanh}[(\text{Sqrt}[c] \cdot x) / \text{Sqrt}[a + c \cdot x^2]]) / f^2 - ((2 \cdot c \cdot d \cdot e \cdot f - (e - \text{Sqrt}[e^2 - 4 \cdot d \cdot f]) \cdot (a \cdot f^2 + c \cdot (e^2 - d \cdot f))) \cdot \text{ArcTanh}[(2 \cdot a \cdot f - c \cdot (e - \text{Sqrt}[e^2 - 4 \cdot d \cdot f]) \cdot x) / (\text{Sqrt}[2] \cdot \text{Sqrt}[2 \cdot a \cdot f^2 + c \cdot (e^2 - 2 \cdot d \cdot f - e \cdot \text{Sqrt}[e^2 - 4 \cdot d \cdot f])]) \cdot \text{Sqrt}[a + c \cdot x^2]]) / (\text{Sqrt}[2] \cdot f^2 \cdot \text{Sqrt}[e^2 - 4 \cdot d \cdot f] \cdot \text{Sqrt}[2 \cdot a \cdot f^2 + c \cdot (e^2 - 2 \cdot d \cdot f - e \cdot \text{Sqrt}[e^2 - 4 \cdot d \cdot f])]) + ((2 \cdot c \cdot d \cdot e \cdot f - (e + \text{Sqrt}[e^2 - 4 \cdot d \cdot f]) \cdot (a \cdot f^2 + c \cdot (e^2 - d \cdot f))) \cdot \text{ArcTanh}[(2 \cdot a \cdot f - c \cdot (e + \text{Sqrt}[e^2 - 4 \cdot d \cdot f]) \cdot x) / (\text{Sqrt}[2] \cdot \text{Sqrt}[2 \cdot a \cdot f^2 + c \cdot (e^2 - 2 \cdot d \cdot f + e \cdot \text{Sqrt}[e^2 - 4 \cdot d \cdot f])]) \cdot \text{Sqrt}[a + c \cdot x^2]]) / (\text{Sqrt}[2] \cdot f^2 \cdot \text{Sqrt}[e^2 - 4 \cdot d \cdot f] \cdot \text{Sqrt}[2 \cdot a \cdot f^2 + c \cdot (e^2 - 2 \cdot d \cdot f + e \cdot \text{Sqrt}[e^2 - 4 \cdot d \cdot f])])$

Rule 206

$\text{Int}[(a + b \cdot x) \cdot (x^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1 \cdot \text{ArcTanh}[(\text{Rt}[-b, 2] \cdot x) / \text{Rt}[a, 2]]) / (\text{Rt}[a, 2] \cdot \text{Rt}[-b, 2]), x] / ; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{Gt}$

$Q[a, 0] \parallel LtQ[b, 0]$

Rule 217

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{!GtQ}[a, 0]$

Rule 725

$\text{Int}[1/(((d_) + (e_)*(x_))*\text{Sqrt}[(a_) + (c_)*(x_)^2]), x_Symbol] \rightarrow -\text{Subst}[\text{Int}[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/\text{Sqrt}[a + c*x^2]] /; \text{FreeQ}[\{a, c, d, e\}, x]$

Rule 1020

$\text{Int}(((g_) + (h_)*(x_))*((a_) + (c_)*(x_)^2)^{(p_)}*((d_) + (e_)*(x_) + (f_)*(x_)^2)^{(q_)}, x_Symbol] \rightarrow \text{Simp}[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^{(q + 1)})/(2*f*(p + q + 1)), x] + \text{Dist}[1/(2*f*(p + q + 1)), \text{Int}[(a + c*x^2)^{(p - 1)}*(d + e*x + f*x^2)^q*\text{Simp}[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x] /; \text{FreeQ}[\{a, c, d, e, f, g, h, q\}, x] \&\& \text{NeQ}[e^2 - 4*d*f, 0] \&\& \text{GtQ}[p, 0] \&\& \text{NeQ}[p + q + 1, 0]$

Rule 1034

$\text{Int}(((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*\text{Sqrt}[(d_) + (f_)*(x_)^2]), x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(2*c*g - h*(b - q))/q, \text{Int}[1/((b - q + 2*c*x)*\text{Sqrt}[d + f*x^2]), x], x] - \text{Dist}[(2*c*g - h*(b + q))/q, \text{Int}[1/((b + q + 2*c*x)*\text{Sqrt}[d + f*x^2]), x], x]] /; \text{FreeQ}[\{a, b, c, d, f, g, h\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{PosQ}[b^2 - 4*a*c]$

Rule 1080

$\text{Int}(((A_) + (B_)*(x_) + (C_)*(x_)^2)/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*\text{Sqrt}[(d_) + (f_)*(x_)^2]), x_Symbol] \rightarrow \text{Dist}[C/c, \text{Int}[1/\text{Sqrt}[d + f*x^2], x], x] + \text{Dist}[1/c, \text{Int}[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*\text{Sqrt}[d + f*x^2]), x], x] /; \text{FreeQ}[\{a, b, c, d, f, A, B, C\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2} dx &= \frac{\sqrt{a+cx^2}}{f} + \frac{\int \frac{-(cd-af)x-cex^2}{\sqrt{a+cx^2}(d+ex+fx^2)} dx}{f} \\
&= \frac{\sqrt{a+cx^2}}{f} + \frac{\int \frac{cde+(ce^2+f(-cd+af))x}{\sqrt{a+cx^2}(d+ex+fx^2)} dx}{f^2} - \frac{(ce) \int \frac{1}{\sqrt{a+cx^2}} dx}{f^2} \\
&= \frac{\sqrt{a+cx^2}}{f} - \frac{(ce) \operatorname{Subst}\left(\int \frac{1}{1-cx^2} dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{f^2} + \frac{(2cdef - (e - \sqrt{e^2 - 4df})(af^2 + c(e^2 - df)))}{f^2 \sqrt{e^2 - 4df}} \\
&= \frac{\sqrt{a+cx^2}}{f} - \frac{\sqrt{c} e \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f^2} - \frac{(2cdef - (e - \sqrt{e^2 - 4df})(af^2 + c(e^2 - df))) \operatorname{Subst}\left(\int \frac{1}{1-cx^2} dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{f^2 \sqrt{e^2 - 4df}} \\
&= \frac{\sqrt{a+cx^2}}{f} - \frac{\sqrt{c} e \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f^2} - \frac{(2cdef - (e - \sqrt{e^2 - 4df})(af^2 + c(e^2 - df))) \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{2} f^2 \sqrt{e^2 - 4df} \sqrt{2af^2 + c(e^2 - df)}}
\end{aligned}$$

Mathematica [A] time = 1.52, size = 422, normalized size = 1.07

$$\frac{(\sqrt{e^2-4df}-e)\sqrt{4af^2-2c(e\sqrt{e^2-4df}+2df-e^2)}\tanh^{-1}\left(\frac{2af+cx(\sqrt{e^2-4df}-e)}{\sqrt{a+cx^2}\sqrt{4af^2-2c(e\sqrt{e^2-4df}+2df-e^2)}}\right)}{\sqrt{e^2-4df}} + \frac{e\sqrt{4af^2+2c(e\sqrt{e^2-4df}-2df+e^2)}\tanh^{-1}\left(\frac{2af+cx(\sqrt{e^2-4df}+e)}{\sqrt{a+cx^2}\sqrt{4af^2+2c(e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*Sqrt[a + c*x^2])/(d + e*x + f*x^2), x]

[Out]
$$\begin{aligned}
& -1/4*(-4*f*\operatorname{Sqrt}[a + c*x^2] + 4*\operatorname{Sqrt}[c]*e*\operatorname{ArcTanh}[(\operatorname{Sqrt}[c]*x)/\operatorname{Sqrt}[a + c*x^2]]) \\
& + ((-e + \operatorname{Sqrt}[e^2 - 4*d*f])*\operatorname{Sqrt}[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*\operatorname{Sqrt}[e^2 - 4*d*f]])* \\
& \operatorname{ArcTanh}[(2*a*f + c*(-e + \operatorname{Sqrt}[e^2 - 4*d*f])*x)/(\operatorname{Sqrt}[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*\operatorname{Sqrt}[e^2 - 4*d*f]])* \\
& \operatorname{Sqrt}[a + c*x^2])])/\operatorname{Sqrt}[e^2 - 4*d*f] + \operatorname{Sqrt}[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*\operatorname{Sqrt}[e^2 - 4*d*f]])* \\
& \operatorname{ArcTanh}[(2*a*f - c*(e + \operatorname{Sqrt}[e^2 - 4*d*f])*x)/(\operatorname{Sqrt}[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*\operatorname{Sqrt}[e^2 - 4*d*f]])* \\
& \operatorname{Sqrt}[a + c*x^2])]) + (e*\operatorname{Sqrt}[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*\operatorname{Sqrt}[e^2 - 4*d*f]])* \\
& \operatorname{ArcTanh}[(2*a*f - c*(e + \operatorname{Sqrt}[e^2 - 4*d*f])*x)/(\operatorname{Sqrt}[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*\operatorname{Sqrt}[e^2 - 4*d*f]])* \\
& \operatorname{Sqrt}[a + c*x^2])])/\operatorname{Sqrt}[e^2 - 4*d*f]
\end{aligned}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.02, size = 5581, normalized size = 14.13

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x)

[Out] result too large to display

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details)Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\sqrt{cx^2+a}}{fx^2+ex+d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + c*x^2)^(1/2))/(d + e*x + f*x^2),x)
```

```
[Out] int((x*(a + c*x^2)^(1/2))/(d + e*x + f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x*sqrt(a + c*x**2)/(d + e*x + f*x**2), x)
```

$$3.54 \quad \int \frac{\sqrt{a+cx^2}}{d+ex+fx^2} dx$$

Optimal. Leaf size=298

$$\frac{\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)} \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2}f\sqrt{e^2 - 4df}} + \frac{\sqrt{2af^2 + c(e\sqrt{e^2 - 4df})}}{\sqrt{2}f\sqrt{e^2 - 4df}}$$

[Out] arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))*c^(1/2)/f-1/2*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)/f*2^(1/2)/(-4*d*f+e^2)^(1/2)+1/2*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)/f*2^(1/2)/(-4*d*f+e^2)^(1/2)

Rubi [A] time = 0.38, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {991, 217, 206, 1034, 725}

$$\frac{\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)} \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2}f\sqrt{e^2 - 4df}} + \frac{\sqrt{2af^2 + c(e\sqrt{e^2 - 4df})}}{\sqrt{2}f\sqrt{e^2 - 4df}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + c*x^2]/(d + e*x + f*x^2),x]

[Out] (Sqrt[c]*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/f - (Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f]))*x]/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]))/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]) + (Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f]))*x]/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]))/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

`Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]`

Rule 725

`Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[a, c, d, e], x]`

Rule 991

`Int[Sqrt[(a_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^2), x_Symbol]
:= Dist[c/f, Int[1/Sqrt[a + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*f + c*
e*x)/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)), x], x] /; FreeQ[{a, c, d, e, f},
x] && NeQ[e^2 - 4*d*f, 0]`

Rule 1034

`Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
.)*(x)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]`

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a+cx^2}}{d+ex+fx^2} dx &= -\frac{\int \frac{cd-af+cex}{\sqrt{a+cx^2}(d+ex+fx^2)} dx}{f} + \frac{c \int \frac{1}{\sqrt{a+cx^2}} dx}{f} \\ &= \frac{c \operatorname{Subst}\left(\int \frac{1}{1-cx^2} dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{f} + \frac{(2f(cd-af) - ce(e + \sqrt{e^2-4df})) \int \frac{1}{(e + \sqrt{e^2-4df} + 2fx)\sqrt{a+cx^2}} dx}{f\sqrt{e^2-4df}} \\ &= \frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f} - \frac{(2f(cd-af) - ce(e + \sqrt{e^2-4df})) \operatorname{Subst}\left(\int \frac{1}{4af^2+c(e + \sqrt{e^2-4df})^2-x^2} dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{f\sqrt{e^2-4df}} \\ &= \frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f} - \frac{\sqrt{2af^2+c(e^2-2df-e\sqrt{e^2-4df})} \tanh^{-1}\left(\frac{2af-c(e-\sqrt{e^2-4df})}{\sqrt{2}\sqrt{2af^2+c(e^2-2df-e\sqrt{e^2-4df})}}\right)}{\sqrt{2}f\sqrt{e^2-4df}} \end{aligned}$$

Mathematica [A] time = 0.36, size = 282, normalized size = 0.95

$$\frac{-\sqrt{4af^2 - 2c(e\sqrt{e^2 - 4df} + 2df - e^2)} \tanh^{-1}\left(\frac{2af + cx(\sqrt{e^2 - 4df} - e)}{\sqrt{a + cx^2}\sqrt{4af^2 - 2c(e\sqrt{e^2 - 4df} + 2df - e^2)}}\right) + \sqrt{4af^2 + 2c(e\sqrt{e^2 - 4df} - 2f\sqrt{e^2 - 4df}}}{2f\sqrt{e^2 - 4df}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + c*x^2]/(d + e*x + f*x^2), x]

[Out] (2*Sqrt[c]*Sqrt[e^2 - 4*d*f]*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] - Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]) + Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(2*f*Sqrt[e^2 - 4*d*f])

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(1/2)/(f*x^2+e*x+d), x, algorithm="fricas")

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(1/2)/(f*x^2+e*x+d), x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.01, size = 3249, normalized size = 10.90

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+a)^(1/2)/(f*x^2+e*x+d), x)

[Out] -1/2/(-4*d*f+e^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*

$$\begin{aligned}
& 2)) / f) + 1/2 * 2^{(1/2)} * ((-(-4*d*f+e^2)^{(1/2)} * c * e + 2*a*f^2 - 2*c*d*f + c*e^2) / f^2)^{(1/2)} \\
& * (4*(x - 1/2 * (-e + (-4*d*f+e^2)^{(1/2))) / f)^2 * c - 4*c*(e - (-4*d*f+e^2)^{(1/2))) / f * (\\
& x - 1/2 * (-e + (-4*d*f+e^2)^{(1/2))) / f) + 2 * ((-4*d*f+e^2)^{(1/2)} * c * e + 2*a*f^2 - 2*c*d*f \\
& + c*e^2) / f^2)^{(1/2)) / (x - 1/2 * (-e + (-4*d*f+e^2)^{(1/2))) / f) * c * e - 1 / (-4*d*f+e^2)^{(1/2)} \\
& * 2^{(1/2)} / (((-4*d*f+e^2)^{(1/2)} * c * e + 2*a*f^2 - 2*c*d*f + c*e^2) / f^2)^{(1/2)} * \ln \\
& (((-4*d*f+e^2)^{(1/2)} * c * e + 2*a*f^2 - 2*c*d*f + c*e^2) / f^2 - c * (e - (-4*d*f+e^2)^{(1/2))) / f * \\
& (x - 1/2 * (-e + (-4*d*f+e^2)^{(1/2))) / f) + 1/2 * 2^{(1/2)} * ((-(-4*d*f+e^2)^{(1/2)} * c \\
& * e + 2*a*f^2 - 2*c*d*f + c*e^2) / f^2)^{(1/2)} * (4*(x - 1/2 * (-e + (-4*d*f+e^2)^{(1/2))) / f)^2 \\
& * c - 4*c*(e - (-4*d*f+e^2)^{(1/2))) / f * (x - 1/2 * (-e + (-4*d*f+e^2)^{(1/2))) / f) + 2 * ((-4*d \\
& * f+e^2)^{(1/2)} * c * e + 2*a*f^2 - 2*c*d*f + c*e^2) / f^2)^{(1/2)) / (x - 1/2 * (-e + (-4*d*f+e^2)^{(1/2) \\
&)^2)^{(1/2)) / f) * a + 1 / (-4*d*f+e^2)^{(1/2)} / f * 2^{(1/2)} / (((-4*d*f+e^2)^{(1/2)} * c * e + 2*a \\
& * f^2 - 2*c*d*f + c*e^2) / f^2)^{(1/2)} * \ln(((-4*d*f+e^2)^{(1/2)} * c * e + 2*a*f^2 - 2*c*d*f \\
& + c*e^2) / f^2 - c * (e - (-4*d*f+e^2)^{(1/2))) / f * (x - 1/2 * (-e + (-4*d*f+e^2)^{(1/2))) / f) + 1 / \\
& 2 * 2^{(1/2)} * ((-(-4*d*f+e^2)^{(1/2)} * c * e + 2*a*f^2 - 2*c*d*f + c*e^2) / f^2)^{(1/2)} * (4*(x \\
& - 1/2 * (-e + (-4*d*f+e^2)^{(1/2))) / f)^2 * c - 4*c*(e - (-4*d*f+e^2)^{(1/2))) / f * (x - 1/2 * (-e \\
& + (-4*d*f+e^2)^{(1/2))) / f) + 2 * ((-4*d*f+e^2)^{(1/2)} * c * e + 2*a*f^2 - 2*c*d*f + c*e^2) / f \\
& ^2)^{(1/2)) / (x - 1/2 * (-e + (-4*d*f+e^2)^{(1/2))) / f) * c * d - 1/2 / (-4*d*f+e^2)^{(1/2)} / f^ \\
& 2 * 2^{(1/2)} / (((-4*d*f+e^2)^{(1/2)} * c * e + 2*a*f^2 - 2*c*d*f + c*e^2) / f^2)^{(1/2)} * \ln(((-4*d*f+e^2)^{(1/2)} * c * e + 2*a*f^2 - 2*c*d*f + c*e^2) / f^2 - c * (e - (-4*d*f+e^2)^{(1/2))) \\
& / f * (x - 1/2 * (-e + (-4*d*f+e^2)^{(1/2))) / f) + 1/2 * 2^{(1/2)} * ((-(-4*d*f+e^2)^{(1/2)} * c * e + \\
& 2*a*f^2 - 2*c*d*f + c*e^2) / f^2)^{(1/2)} * (4*(x - 1/2 * (-e + (-4*d*f+e^2)^{(1/2))) / f)^2 * c - \\
& 4*c*(e - (-4*d*f+e^2)^{(1/2))) / f * (x - 1/2 * (-e + (-4*d*f+e^2)^{(1/2))) / f) + 2 * ((-4*d*f+ \\
& e^2)^{(1/2)} * c * e + 2*a*f^2 - 2*c*d*f + c*e^2) / f^2)^{(1/2)) / (x - 1/2 * (-e + (-4*d*f+e^2)^{(1/2) \\
&)^2)^{(1/2)) / f) * c * e^2
\end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details)Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^2+a}}{fx^2+ex+d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c*x^2)^(1/2)/(d + e*x + f*x^2),x)

```
[Out] int((a + c*x^2)^(1/2)/(d + e*x + f*x^2), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{\sqrt{a + cx^2}}{d + ex + fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(d + e*x + f*x**2), x)
```

$$3.55 \quad \int \frac{\sqrt{a+cx^2}}{x(d+ex+fx^2)} dx$$

Optimal. Leaf size=358

$$\frac{\left((e - \sqrt{e^2 - 4df}) (cd - af) + 2aef \right) \tanh^{-1} \left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2} \sqrt{a+cx^2} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}} \right) \left((\sqrt{e^2 - 4df} + e) (cd - af) + \sqrt{2} d \sqrt{e^2 - 4df} \right)}{\sqrt{2} d \sqrt{e^2 - 4df} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}} \sqrt{2} d \sqrt{e^2 - 4df}$$

[Out] $-\operatorname{arctanh}\left(\frac{c x^2+a}{a}\right)^{1/2} / a^{1/2} / d+1/2 * \operatorname{arctanh}\left(\frac{1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^{1/2})) * 2^{1/2}}{(c*x^2+a)^{1/2} / (2*a*f^2+(-(-4*d*f+e^2)^{1/2})*e-2*d*f+e^2)*c}\right)^{1/2} * (2*a*e*f+(-a*f+c*d)) * (e-(-4*d*f+e^2)^{1/2}) / d * 2^{1/2} / (-4*d*f+e^2)^{1/2} / (2*a*f^2+(-(-4*d*f+e^2)^{1/2})*e-2*d*f+e^2)*c\right)^{1/2} - 1/2 * \operatorname{arctanh}\left(\frac{1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^{1/2})) * 2^{1/2}}{(c*x^2+a)^{1/2} / (2*a*f^2+((-4*d*f+e^2)^{1/2})*e-2*d*f+e^2)*c}\right)^{1/2} * (2*a*e*f+(-a*f+c*d)) * (e+(-4*d*f+e^2)^{1/2}) / d * 2^{1/2} / (-4*d*f+e^2)^{1/2} / (2*a*f^2+((-4*d*f+e^2)^{1/2})*e-2*d*f+e^2)*c\right)^{1/2}$

Rubi [A] time = 1.31, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6728, 266, 50, 63, 208, 1020, 1034, 725, 206}

$$\frac{\left((e - \sqrt{e^2 - 4df}) (cd - af) + 2aef \right) \tanh^{-1} \left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2} \sqrt{a+cx^2} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}} \right) \left((\sqrt{e^2 - 4df} + e) (cd - af) + \sqrt{2} d \sqrt{e^2 - 4df} \right)}{\sqrt{2} d \sqrt{e^2 - 4df} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}} \sqrt{2} d \sqrt{e^2 - 4df}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sqrt}[a + c*x^2]/(x*(d + e*x + f*x^2)), x]$

[Out] $((2*a*e*f + (c*d - a*f)*(e - \operatorname{Sqrt}[e^2 - 4*d*f])) * \operatorname{ArcTanh}[(2*a*f - c*(e - \operatorname{Sqrt}[e^2 - 4*d*f])*x) / (\operatorname{Sqrt}[2] * \operatorname{Sqrt}[2*a*f^2 + c*(e^2 - 2*d*f - e*\operatorname{Sqrt}[e^2 - 4*d*f])]) * \operatorname{Sqrt}[a + c*x^2]]) / (\operatorname{Sqrt}[2] * d * \operatorname{Sqrt}[e^2 - 4*d*f] * \operatorname{Sqrt}[2*a*f^2 + c*(e^2 - 2*d*f - e*\operatorname{Sqrt}[e^2 - 4*d*f])]) - ((2*a*e*f + (c*d - a*f)*(e + \operatorname{Sqrt}[e^2 - 4*d*f])) * \operatorname{ArcTanh}[(2*a*f - c*(e + \operatorname{Sqrt}[e^2 - 4*d*f])*x) / (\operatorname{Sqrt}[2] * \operatorname{Sqrt}[2*a*f^2 + c*(e^2 - 2*d*f + e*\operatorname{Sqrt}[e^2 - 4*d*f])]) * \operatorname{Sqrt}[a + c*x^2]]) / (\operatorname{Sqrt}[2] * d * \operatorname{Sqrt}[e^2 - 4*d*f] * \operatorname{Sqrt}[2*a*f^2 + c*(e^2 - 2*d*f + e*\operatorname{Sqrt}[e^2 - 4*d*f])]) - (\operatorname{Sqrt}[a] * \operatorname{ArcTanh}[\operatorname{Sqrt}[a + c*x^2] / \operatorname{Sqrt}[a]]) / d$

Rule 50

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)} * ((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(a + b*x)^{(m+1)} * (c + d*x)^n / (b*(m+n+1)), x] + \operatorname{Dist}[(n*(b*c - a*d)) /$

```
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1020

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] := Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q +
1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
```

eQ[p + q + 1, 0]

Rule 1034

Int[((g_.) + (h_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rule 6728

Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{a+cx^2}}{x(d+ex+fx^2)} dx &= \int \left(\frac{\sqrt{a+cx^2}}{dx} + \frac{(-e-fx)\sqrt{a+cx^2}}{d(d+ex+fx^2)} \right) dx \\
 &= \frac{\int \frac{\sqrt{a+cx^2}}{x} dx}{d} + \frac{\int \frac{(-e-fx)\sqrt{a+cx^2}}{d+ex+fx^2} dx}{d} \\
 &= -\frac{\sqrt{a+cx^2}}{d} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x} dx, x, x^2\right)}{2d} + \frac{\int \frac{-aef+f(cd-af)x}{\sqrt{a+cx^2}(d+ex+fx^2)} dx}{df} \\
 &= \frac{a \text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}} dx, x, x^2\right)}{2d} - \frac{(2aef + (cd - af)(e - \sqrt{e^2 - 4df})) \int \frac{1}{(e - \sqrt{e^2 - 4df} + 2fx)\sqrt{a+cx^2}} dx}{d\sqrt{e^2 - 4df}} \\
 &= \frac{a \text{Subst}\left(\int \frac{1}{-\frac{a}{c} + \frac{x^2}{c}} dx, x, \sqrt{a+cx^2}\right)}{cd} + \frac{(2aef + (cd - af)(e - \sqrt{e^2 - 4df})) \text{Subst}\left(\int \frac{1}{4a} dx, x, \sqrt{a+cx^2}\right)}{d\sqrt{e^2 - 4df}} \\
 &= \frac{(2aef + (cd - af)(e - \sqrt{e^2 - 4df})) \tanh^{-1}\left(\frac{2af - c(e - \sqrt{e^2 - 4df})x}{\sqrt{2}\sqrt{2af^2 + c(e^2 - 2df - e\sqrt{e^2 - 4df})}\sqrt{a+cx^2}}\right)}{\sqrt{2}d\sqrt{e^2 - 4df}\sqrt{2af^2 + c(e^2 - 2df - e\sqrt{e^2 - 4df})}} - \frac{(2aef + (cd - af)(e - \sqrt{e^2 - 4df}))}{d\sqrt{e^2 - 4df}}
 \end{aligned}$$

Mathematica [A] time = 0.65, size = 314, normalized size = 0.88

$$\left(\sqrt{e^2 - 4df} + e\right) \sqrt{4af^2 - 2c\left(e\sqrt{e^2 - 4df} + 2df - e^2\right)} \tanh^{-1}\left(\frac{2af + cx\left(\sqrt{e^2 - 4df} - e\right)}{\sqrt{a + cx^2} \sqrt{4af^2 - 2c\left(e\sqrt{e^2 - 4df} + 2df - e^2\right)}}\right) + \left(\sqrt{e^2 - 4df} - \right.$$

$\left. 4df\right)$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + c*x^2]/(x*(d + e*x + f*x^2)),x]

[Out] ((e + Sqrt[e^2 - 4*d*f])*Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]))*Sqrt[a + c*x^2]]) + (-e + Sqrt[e^2 - 4*d*f])*Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])*Sqrt[a + c*x^2]]) - 4*Sqrt[a]*f*Sqrt[e^2 - 4*d*f]*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(4*d*f*Sqrt[e^2 - 4*d*f])

fricas [B] time = 83.15, size = 2266, normalized size = 6.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] [-1/4*(sqrt(2)*d*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))))/(d^2*e^2 - 4*d^3*f))*log((2*a*c*d*e*x - a^2*e^2 + sqrt(2)*(d^3*e^2 - 4*d^4*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))*sqrt(c*x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))))/(d^2*e^2 - 4*d^3*f)) - (a*d^2*e^2 - 4*a*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/x) - sqrt(2)*d*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))))/(d^2*e^2 - 4*d^3*f))*log((2*a*c*d*e*x - a^2*e^2 - sqrt(2)*(d^3*e^2 - 4*d^4*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))*sqrt(c*x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))))/(d^2*e^2 - 4*d^3*f)) - (a*d^2*e^2 - 4*a*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/x) - sqrt(2)*d*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))))/(d^2*e^2 - 4*d^3*f))*log((2*a*c*d*e*x - a^2*e^2 + sqrt(2)*(d^3*e^2 - 4*d^4*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))*sqrt(c*x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))))/(d^2*e^2 - 4*d^3*f)) + (a*d^2*e^2 - 4*a*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/x) + sqrt(2)*d*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))))/(d^2*e^2 - 4*d^3*f))*log((2*a*c*d*e*x - a^2*e^2 - sqrt(2)*(d^3*e^2 - 4*d^4*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))

$$\begin{aligned} &)\sqrt{c*x^2 + a}\sqrt{(2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*f))\sqrt{a^2*e^2/(d^4*e^2 - 4*d^5*f)}}/(d^2*e^2 - 4*d^3*f) + (a*d^2*e^2 - 4*a*d^3*f)\sqrt{a^2*e^2/(d^4*e^2 - 4*d^5*f))/x) - 2*\sqrt{a}\log(-(c*x^2 - 2*\sqrt{c*x^2 + a})\sqrt{a} + 2*a)/x^2)/d, -1/4*(\sqrt{2}*d*\sqrt{(2*c*d^2 + a*e^2 - 2*a*d*f + (d^2*e^2 - 4*d^3*f))\sqrt{a^2*e^2/(d^4*e^2 - 4*d^5*f)}}/(d^2*e^2 - 4*d^3*f))*\log((2*a*c*d*e*x - a^2*e^2 + \sqrt{2}*(d^3*e^2 - 4*d^4*f))\sqrt{a^2*e^2/(d^4*e^2 - 4*d^5*f))\sqrt{c*x^2 + a})\sqrt{(2*c*d^2 + a*e^2 - 2*a*d*f + (d^2*e^2 - 4*d^3*f))\sqrt{a^2*e^2/(d^4*e^2 - 4*d^5*f)}}/(d^2*e^2 - 4*d^3*f)) - (a*d^2*e^2 - 4*a*d^3*f)\sqrt{a^2*e^2/(d^4*e^2 - 4*d^5*f))/x) - \sqrt{2}*d*\sqrt{(2*c*d^2 + a*e^2 - 2*a*d*f + (d^2*e^2 - 4*d^3*f))\sqrt{a^2*e^2/(d^4*e^2 - 4*d^5*f)}}/(d^2*e^2 - 4*d^3*f))*\log((2*a*c*d*e*x - a^2*e^2 - \sqrt{2}*(d^3*e^2 - 4*d^4*f))\sqrt{a^2*e^2/(d^4*e^2 - 4*d^5*f))\sqrt{c*x^2 + a})\sqrt{(2*c*d^2 + a*e^2 - 2*a*d*f + (d^2*e^2 - 4*d^3*f))\sqrt{a^2*e^2/(d^4*e^2 - 4*d^5*f)}}/(d^2*e^2 - 4*d^3*f)) - (a*d^2*e^2 - 4*a*d^3*f)\sqrt{a^2*e^2/(d^4*e^2 - 4*d^5*f))/x) - \sqrt{2}*d*\sqrt{(2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*f))\sqrt{a^2*e^2/(d^4*e^2 - 4*d^5*f)}}/(d^2*e^2 - 4*d^3*f))*\log((2*a*c*d*e*x - a^2*e^2 + \sqrt{2}*(d^3*e^2 - 4*d^4*f))\sqrt{a^2*e^2/(d^4*e^2 - 4*d^5*f))\sqrt{c*x^2 + a})\sqrt{(2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*f))\sqrt{a^2*e^2/(d^4*e^2 - 4*d^5*f)}}/(d^2*e^2 - 4*d^3*f)) + (a*d^2*e^2 - 4*a*d^3*f)\sqrt{a^2*e^2/(d^4*e^2 - 4*d^5*f))/x) + \sqrt{2}*d*\sqrt{(2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*f))\sqrt{a^2*e^2/(d^4*e^2 - 4*d^5*f)}}/(d^2*e^2 - 4*d^3*f))*\log((2*a*c*d*e*x - a^2*e^2 - \sqrt{2}*(d^3*e^2 - 4*d^4*f))\sqrt{a^2*e^2/(d^4*e^2 - 4*d^5*f))\sqrt{c*x^2 + a})\sqrt{(2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*f))\sqrt{a^2*e^2/(d^4*e^2 - 4*d^5*f)}}/(d^2*e^2 - 4*d^3*f)) + (a*d^2*e^2 - 4*a*d^3*f)\sqrt{a^2*e^2/(d^4*e^2 - 4*d^5*f))/x) - 4*\sqrt{-a}\arctan(\sqrt{-a}/\sqrt{c*x^2 + a}))/d] \end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);OUTPUT:Evaluation time: 2Error index.cc index_gcd Error: Bad Argument Value

maple [B] time = 0.02, size = 3544, normalized size = 9.90

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+a)^(1/2)/x/(f*x^2+e*x+d),x)

$$\begin{aligned}
& + \left((x-1/2 * (-e + (-4*d*f + e^2)^{1/2}) / f)^2 * c - c * (e - (-4*d*f + e^2)^{1/2}) / f * (x-1/2 * \right. \\
& (-e + (-4*d*f + e^2)^{1/2}) / f + 1/2 * (-(-4*d*f + e^2)^{1/2} * c * e + 2*a*f^2 - 2*c*d*f + c*e \\
& ^2) / f^2)^{1/2} * e + 1/f / (-e + (-4*d*f + e^2)^{1/2}) * 2^{1/2} / ((-(-4*d*f + e^2)^{1/2} \\
& * c * e + 2*a*f^2 - 2*c*d*f + c*e^2) / f^2)^{1/2} * \ln(((-(-4*d*f + e^2)^{1/2} * c * e + 2*a*f^2 \\
& - 2*c*d*f + c*e^2) / f^2 - c * (e - (-4*d*f + e^2)^{1/2}) / f * (x-1/2 * (-e + (-4*d*f + e^2)^{1/2} \\
&)) / f + 1/2 * 2^{1/2} * ((-(-4*d*f + e^2)^{1/2} * c * e + 2*a*f^2 - 2*c*d*f + c*e^2) / f^2)^{1/2} \\
& * (4 * (x-1/2 * (-e + (-4*d*f + e^2)^{1/2}) / f)^2 * c - 4 * c * (e - (-4*d*f + e^2)^{1/2}) / f * (x \\
& - 1/2 * (-e + (-4*d*f + e^2)^{1/2}) / f) + 2 * (-(-4*d*f + e^2)^{1/2} * c * e + 2*a*f^2 - 2*c*d*f + \\
& c * e^2) / f^2)^{1/2} / (x-1/2 * (-e + (-4*d*f + e^2)^{1/2}) / f) * c * e - 2 * f / (-e + (-4*d*f + e \\
& ^2)^{1/2}) / (-4*d*f + e^2)^{1/2} * 2^{1/2} / ((-(-4*d*f + e^2)^{1/2} * c * e + 2*a*f^2 - 2*c \\
& * d*f + c * e^2) / f^2)^{1/2} * \ln(((-(-4*d*f + e^2)^{1/2} * c * e + 2*a*f^2 - 2*c*d*f + c * e^2) / \\
& f^2 - c * (e - (-4*d*f + e^2)^{1/2}) / f * (x-1/2 * (-e + (-4*d*f + e^2)^{1/2}) / f) + 1/2 * 2^{1/2} \\
&) * ((-(-4*d*f + e^2)^{1/2} * c * e + 2*a*f^2 - 2*c*d*f + c * e^2) / f^2)^{1/2} * (4 * (x-1/2 * (-e \\
& + (-4*d*f + e^2)^{1/2}) / f)^2 * c - 4 * c * (e - (-4*d*f + e^2)^{1/2}) / f * (x-1/2 * (-e + (-4*d*f \\
& + e^2)^{1/2}) / f) + 2 * (-(-4*d*f + e^2)^{1/2} * c * e + 2*a*f^2 - 2*c*d*f + c * e^2) / f^2)^{1/2} \\
&)) / (x-1/2 * (-e + (-4*d*f + e^2)^{1/2}) / f) * a + 2 / (-e + (-4*d*f + e^2)^{1/2}) / (-4*d*f + e \\
& ^2)^{1/2} * 2^{1/2} / ((-(-4*d*f + e^2)^{1/2} * c * e + 2*a*f^2 - 2*c*d*f + c * e^2) / f^2)^{1/2} \\
& * \ln(((-(-4*d*f + e^2)^{1/2} * c * e + 2*a*f^2 - 2*c*d*f + c * e^2) / f^2 - c * (e - (-4*d*f + e^2) \\
& ^{1/2}) / f * (x-1/2 * (-e + (-4*d*f + e^2)^{1/2}) / f) + 1/2 * 2^{1/2} * ((-(-4*d*f + e^2)^{1/2} \\
& * c * e + 2*a*f^2 - 2*c*d*f + c * e^2) / f^2)^{1/2} * (4 * (x-1/2 * (-e + (-4*d*f + e^2)^{1/2}) \\
& / f)^2 * c - 4 * c * (e - (-4*d*f + e^2)^{1/2}) / f * (x-1/2 * (-e + (-4*d*f + e^2)^{1/2}) / f) + 2 * (- \\
& (-4*d*f + e^2)^{1/2} * c * e + 2*a*f^2 - 2*c*d*f + c * e^2) / f^2)^{1/2} / (x-1/2 * (-e + (-4*d * \\
& f + e^2)^{1/2}) / f) * c * d - 1 / f / (-e + (-4*d*f + e^2)^{1/2}) / (-4*d*f + e^2)^{1/2} * 2^{1/2} \\
&)) / ((-(-4*d*f + e^2)^{1/2} * c * e + 2*a*f^2 - 2*c*d*f + c * e^2) / f^2)^{1/2} * \ln(((-(-4*d*f \\
& + e^2)^{1/2} * c * e + 2*a*f^2 - 2*c*d*f + c * e^2) / f^2 - c * (e - (-4*d*f + e^2)^{1/2}) / f * (x-1 / \\
& 2 * (-e + (-4*d*f + e^2)^{1/2}) / f) + 1/2 * 2^{1/2} * ((-(-4*d*f + e^2)^{1/2} * c * e + 2*a*f^2 - \\
& 2*c*d*f + c * e^2) / f^2)^{1/2} * (4 * (x-1/2 * (-e + (-4*d*f + e^2)^{1/2}) / f)^2 * c - 4 * c * (e - \\
& (-4*d*f + e^2)^{1/2}) / f * (x-1/2 * (-e + (-4*d*f + e^2)^{1/2}) / f) + 2 * (-(-4*d*f + e^2)^{1/2} \\
& * c * e + 2*a*f^2 - 2*c*d*f + c * e^2) / f^2)^{1/2} / (x-1/2 * (-e + (-4*d*f + e^2)^{1/2}) / f) \\
&) * c * e^2 + 4 * f / (-e + (-4*d*f + e^2)^{1/2}) / (e + (-4*d*f + e^2)^{1/2}) * a^{1/2} * \ln((2 * a + \\
& 2 * a^{1/2} * (c * x^2 + a)^{1/2}) / x) - 4 * f / (-e + (-4*d*f + e^2)^{1/2}) / (e + (-4*d*f + e^2)^{1/2} \\
&) * (c * x^2 + a)^{1/2}
\end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + a}}{(fx^2 + ex + d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] integrate(sqrt(c*x^2 + a)/((f*x^2 + e*x + d)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^2 + a}}{x(fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c*x^2)^(1/2)/(x*(d + e*x + f*x^2)), x)

[Out] int((a + c*x^2)^(1/2)/(x*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + cx^2}}{x(d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+a)**(1/2)/x/(f*x**2+e*x+d), x)

[Out] Integral(sqrt(a + c*x**2)/(x*(d + e*x + f*x**2)), x)

$$3.56 \quad \int \frac{\sqrt{a+cx^2}}{x^2(d+ex+fx^2)} dx$$

Optimal. Leaf size=382

$$\frac{f(a(e\sqrt{e^2-4df}-2df+e^2)+2cd^2) \tanh^{-1}\left(\frac{2af-cx(e-\sqrt{e^2-4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}} + \frac{f(a(-e\sqrt{e^2-4df}-2df+e^2)+2cd^2) \tanh^{-1}\left(\frac{2af-cx(e+\sqrt{e^2-4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}}$$

[Out] $e \operatorname{arctanh}\left(\frac{(c x^2+a)^{1/2}/a^{1/2}}{d^2-(c x^2+a)^{1/2}/d}\right) \frac{1}{x} - \frac{1}{2} f \operatorname{arctanh}\left(\frac{1/2(2 a f-c x(e-\sqrt{e^2-4 d f}))^{1/2}}{(c x^2+a)^{1/2}/(2 a f^2+(-4 d f+e^2)^{1/2} e-2 d f+e^2)}\right) \frac{1}{d^2} + \frac{1}{2} f \operatorname{arctanh}\left(\frac{1/2(2 a f-c x(e+\sqrt{e^2-4 d f}))^{1/2}}{(c x^2+a)^{1/2}/(2 a f^2+(-4 d f+e^2)^{1/2} e-2 d f+e^2)}\right) \frac{1}{d^2} + \frac{1}{2} f \operatorname{arctanh}\left(\frac{1/2(2 a f-c x(e-\sqrt{e^2-4 d f}))^{1/2}}{(c x^2+a)^{1/2}/(2 a f^2+(-4 d f+e^2)^{1/2} e-2 d f+e^2)}\right) \frac{1}{d^2} + \frac{1}{2} f \operatorname{arctanh}\left(\frac{1/2(2 a f-c x(e+\sqrt{e^2-4 d f}))^{1/2}}{(c x^2+a)^{1/2}/(2 a f^2+(-4 d f+e^2)^{1/2} e-2 d f+e^2)}\right) \frac{1}{d^2}$

Rubi [A] time = 1.42, antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {6728, 277, 217, 206, 266, 50, 63, 208, 1020, 1080, 1034, 725}

$$\frac{f(a(e\sqrt{e^2-4df}-2df+e^2)+2cd^2) \tanh^{-1}\left(\frac{2af-cx(e-\sqrt{e^2-4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}} + \frac{f(a(-e\sqrt{e^2-4df}-2df+e^2)+2cd^2) \tanh^{-1}\left(\frac{2af-cx(e+\sqrt{e^2-4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}\left[\frac{\sqrt{a+cx^2}}{x^2(d+ex+fx^2)}, x\right]$

[Out] $-\frac{\sqrt{a+cx^2}}{d x} - \frac{f(2 c d^2+a(e^2-2 d f+e \sqrt{e^2-4 d f})) \operatorname{ArcTanh}\left[\frac{(2 a f-c(e-\sqrt{e^2-4 d f})) x}{\sqrt{2} \sqrt{2 a f^2+c(e^2-2 d f-e \sqrt{e^2-4 d f})}}\right] \sqrt{a+cx^2}}{\sqrt{2} d^2 \sqrt{e^2-4 d f} \sqrt{2 a f^2+c(e^2-2 d f-e \sqrt{e^2-4 d f})}} + \frac{f(2 c d^2+a(e^2-2 d f-e \sqrt{e^2-4 d f})) \operatorname{ArcTanh}\left[\frac{(2 a f-c(e+\sqrt{e^2-4 d f})) x}{\sqrt{2} \sqrt{2 a f^2+c(e^2-2 d f+e \sqrt{e^2-4 d f})}}\right] \sqrt{a+cx^2}}{\sqrt{2} d^2 \sqrt{e^2-4 d f} \sqrt{2 a f^2+c(e^2-2 d f+e \sqrt{e^2-4 d f})}} + \frac{\sqrt{a} e \operatorname{ArcTanh}\left[\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right]}{d^2}$

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[
m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 725

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (c_)*(x_)^2]), x_Symbol] := -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1020

```
Int[((g_) + (h_)*(x_))*((a_) + (c_)*(x_)^2)^(p_)*((d_) + (e_)*(x_) + (f
_)*(x_)^2)^(q_), x_Symbol] := Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q +
1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]
```

Rule 1034

```
Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (f
_)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1080

```
Int[((A_) + (B_)*(x_) + (C_)*(x_)^2)/(((a_) + (b_)*(x_) + (c_)*(x_)^2)
*Sqrt[(d_) + (f_)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + f*x^2],
x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rule 6728

```
Int[(u_)/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x_Symbol] := With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+cx^2}}{x^2(d+ex+fx^2)} dx &= \int \left(\frac{\sqrt{a+cx^2}}{dx^2} - \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{(e^2-df+efx)\sqrt{a+cx^2}}{d^2(d+ex+fx^2)} \right) dx \\
&= \frac{\int \frac{(e^2-df+efx)\sqrt{a+cx^2}}{d+ex+fx^2} dx}{d^2} + \frac{\int \frac{\sqrt{a+cx^2}}{x^2} dx}{d} - \frac{e \int \frac{\sqrt{a+cx^2}}{x} dx}{d^2} \\
&= \frac{e\sqrt{a+cx^2}}{d^2} - \frac{\sqrt{a+cx^2}}{dx} + \frac{c \int \frac{1}{\sqrt{a+cx^2}} dx}{d} - \frac{e \operatorname{Subst}\left(\int \frac{\sqrt{a+cx}}{x} dx, x, x^2\right)}{2d^2} + \frac{\int \frac{af(e^2-df)-ef}{\sqrt{a+cx^2}} dx}{d^2} \\
&= -\frac{\sqrt{a+cx^2}}{dx} - \frac{c \int \frac{1}{\sqrt{a+cx^2}} dx}{d} + \frac{c \operatorname{Subst}\left(\int \frac{1}{1-cx^2} dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{d} - \frac{(ae) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+cx}} dx, x, x^2\right)}{2d^2} \\
&= -\frac{\sqrt{a+cx^2}}{dx} + \frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{d} - \frac{c \operatorname{Subst}\left(\int \frac{1}{1-cx^2} dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{d} - \frac{(ae) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+cx}} dx, x, x^2\right)}{2d^2} \\
&= -\frac{\sqrt{a+cx^2}}{dx} + \frac{\sqrt{a} e \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2} + \frac{(f(2cd^2 + a(e^2 - 2df - e\sqrt{e^2 - 4df}))) \operatorname{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}} dx, x, x^2\right)}{d^2\sqrt{e^2 - 4df}} \\
&= -\frac{\sqrt{a+cx^2}}{dx} - \frac{f(2cd^2 + a(e^2 - 2df + e\sqrt{e^2 - 4df})) \tanh^{-1}\left(\frac{2af - c(e - \sqrt{e^2 - 4df})x}{\sqrt{2}\sqrt{2af^2 + c(e^2 - 2df - e\sqrt{e^2 - 4df})}}\right)}{\sqrt{2}d^2\sqrt{e^2 - 4df}\sqrt{2af^2 + c(e^2 - 2df - e\sqrt{e^2 - 4df})}}
\end{aligned}$$

Mathematica [A] time = 3.03, size = 569, normalized size = 1.49

$$\frac{(e\sqrt{e^2-4df}-2df+e^2) \left(\sqrt{c}(\sqrt{e^2-4df}-e) \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right) - \sqrt{4af^2-2c(e\sqrt{e^2-4df}+2df-e^2)} \tanh^{-1}\left(\frac{2af+cx(\sqrt{e^2-4df}-e)}{\sqrt{a+cx^2}\sqrt{4af^2-2c(e\sqrt{e^2-4df}+2df-e^2)}}\right) \right)}{f\sqrt{e^2-4df}} + \frac{(-e\sqrt{e^2-4df})}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c(e^2-2df-e\sqrt{e^2-4df})}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + c*x^2]/(x^2*(d + e*x + f*x^2)), x]

[Out] (2*(e + (e^2 - 2*d*f)/Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2] + 2*(e + (-e^2 + 2*d*f)/Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2] - (4*d*(a + c*x^2 - Sqrt[a]*Sqrt[c]*x)*Sqrt[1 + (c*x^2)/a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/(x*Sqrt[a + c*x^2])

$$+ ((e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f})*(\sqrt{c}*(-e + \sqrt{e^2 - 4*d*f}))*\text{ArcTanh}[(\sqrt{c}*x)/\sqrt{a + c*x^2}] - \sqrt{4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*\sqrt{e^2 - 4*d*f})})*\text{ArcTanh}[(2*a*f + c*(-e + \sqrt{e^2 - 4*d*f})*x)/(\sqrt{4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*\sqrt{e^2 - 4*d*f})})*\sqrt{a + c*x^2}]))/(f*\sqrt{e^2 - 4*d*f}) + ((e^2 - 2*d*f - e*\sqrt{e^2 - 4*d*f})*(\sqrt{c}*(e + \sqrt{e^2 - 4*d*f}))*\text{ArcTanh}[(\sqrt{c}*x)/\sqrt{a + c*x^2}] + \sqrt{4*a*f^2 + 2*c*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f})})*\text{ArcTanh}[(2*a*f - c*(e + \sqrt{e^2 - 4*d*f})*x)/(\sqrt{4*a*f^2 + 2*c*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f})})*\sqrt{a + c*x^2}]))/(f*\sqrt{e^2 - 4*d*f}) - 4*e*(\sqrt{a + c*x^2} - \sqrt{a}*\text{ArcTanh}[\sqrt{a + c*x^2}/\sqrt{a}]))/(4*d^2)$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);;OUTPUT:Evaluation time: 5.11Error index.cc index_gc d Error: Bad Argument Value

maple [B] time = 0.02, size = 3703, normalized size = 9.69

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+a)^(1/2)/x^2/(f*x^2+e*x+d),x)

[Out]
$$-2*f^2/(e+(-4*d*f+e^2)^{(1/2)})^2/(-4*d*f+e^2)^{(1/2)}*(4*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c-4*c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)+2*((-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}+2*f/(e+(-4*d*f+e^2)^{(1/2)})^2*c^{(1/2)}*\ln((-1/2*c*(e+(-4*d*f+e^2)^{(1/2)}))/f+c*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f))/c^{(1/2)}+((x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c-c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)+1/2*((-4*d*f+e^2)^{(1/2)}$$

$$\begin{aligned}
&) * c * e + 2 * a * f^2 - 2 * c * d * f + c * e^2) / f^2)^{(1/2)} + 2 * f / (e + (-4 * d * f + e^2)^{(1/2)})^2 / (-4 * d \\
& * f + e^2)^{(1/2)} * c^{(1/2)} * \ln((-1/2 * c * (e + (-4 * d * f + e^2)^{(1/2)}) / f + c * (x + 1/2 * (e + (-4 * d \\
& * f + e^2)^{(1/2)}) / f)) / c^{(1/2)} + ((x + 1/2 * (e + (-4 * d * f + e^2)^{(1/2)}) / f)^2 * c - c * (e + (-4 * d \\
& * f + e^2)^{(1/2)}) / f * (x + 1/2 * (e + (-4 * d * f + e^2)^{(1/2)}) / f) + 1/2 * ((-4 * d * f + e^2)^{(1/2)} * c \\
& * e + 2 * a * f^2 - 2 * c * d * f + c * e^2) / f^2)^{(1/2)}) * e + 2 / (e + (-4 * d * f + e^2)^{(1/2)})^2 * 2^{(1/2)} / \\
& (((-4 * d * f + e^2)^{(1/2)} * c * e + 2 * a * f^2 - 2 * c * d * f + c * e^2) / f^2)^{(1/2)} * \ln((((-4 * d * f + e^2 \\
&)^{(1/2)} * c * e + 2 * a * f^2 - 2 * c * d * f + c * e^2) / f^2 - c * (e + (-4 * d * f + e^2)^{(1/2)}) / f * (x + 1/2 * (e \\
& + (-4 * d * f + e^2)^{(1/2)}) / f) + 1/2 * 2^{(1/2)} * (((-4 * d * f + e^2)^{(1/2)} * c * e + 2 * a * f^2 - 2 * c * d * \\
& f + c * e^2) / f^2)^{(1/2)} * (4 * (x + 1/2 * (e + (-4 * d * f + e^2)^{(1/2)}) / f)^2 * c - 4 * c * (e + (-4 * d * f + \\
& e^2)^{(1/2)}) / f * (x + 1/2 * (e + (-4 * d * f + e^2)^{(1/2)}) / f) + 2 * ((-4 * d * f + e^2)^{(1/2)} * c * e + 2 * \\
& a * f^2 - 2 * c * d * f + c * e^2) / f^2)^{(1/2)} / (x + 1/2 * (e + (-4 * d * f + e^2)^{(1/2)}) / f)) * c * e + 4 * f^2 / \\
& (e + (-4 * d * f + e^2)^{(1/2)})^2 / (-4 * d * f + e^2)^{(1/2)} * 2^{(1/2)} / (((-4 * d * f + e^2)^{(1/2)} * c * e + 2 * a * f^2 - 2 * c * d * f + c * e^2) / f^2)^{(1/2)} * \ln((((-4 * d * f + e^2 \\
& * c * d * f + c * e^2) / f^2 - c * (e + (-4 * d * f + e^2)^{(1/2)}) / f * (x + 1/2 * (e + (-4 * d * f + e^2)^{(1/2)}) / \\
& f) + 1/2 * 2^{(1/2)} * (((-4 * d * f + e^2)^{(1/2)} * c * e + 2 * a * f^2 - 2 * c * d * f + c * e^2) / f^2)^{(1/2)} * (\\
& 4 * (x + 1/2 * (e + (-4 * d * f + e^2)^{(1/2)}) / f)^2 * c - 4 * c * (e + (-4 * d * f + e^2)^{(1/2)}) / f * (x + 1/2 * \\
& (e + (-4 * d * f + e^2)^{(1/2)}) / f) + 2 * ((-4 * d * f + e^2)^{(1/2)} * c * e + 2 * a * f^2 - 2 * c * d * f + c * e^2) / \\
& f^2)^{(1/2)} / (x + 1/2 * (e + (-4 * d * f + e^2)^{(1/2)}) / f)) * a - 4 * f / (e + (-4 * d * f + e^2)^{(1/2)})^2 / \\
& (-4 * d * f + e^2)^{(1/2)} * 2^{(1/2)} / (((-4 * d * f + e^2)^{(1/2)} * c * e + 2 * a * f^2 - 2 * c * d * f + c * e^2) / f^2)^{(1/2)} * \ln((((-4 * d * f + e^2 \\
&)^{(1/2)} * c * e + 2 * a * f^2 - 2 * c * d * f + c * e^2) / f^2 - c * (e + (-4 * d * f + e^2)^{(1/2)}) / f * (x + 1/2 * (e \\
& + (-4 * d * f + e^2)^{(1/2)}) / f) + 1/2 * 2^{(1/2)} * (((-4 * d * f + e^2)^{(1/2)} * c * e + 2 * a * f^2 - 2 * c * d * f + c * e^2) / f^2)^{(1/2)} * (4 * (x + 1/2 * (e + (-4 * d * f + e^2)^{(1/2)}) / f)^2 * c - 4 * c * (e + (-4 * d * f + e^2)^{(1/2)}) / f * (x + 1/2 * (e + (-4 * d * f + e^2)^{(1/2)}) / f) + 2 * ((-4 * d * f + e^2)^{(1/2)} * c * e + 2 * a * f^2 - 2 * c * d * f + c * e^2) / f^2)^{(1/2)} / (x + 1/2 * (e + (-4 * d * f + e^2)^{(1/2)}) / f)) * c * d + 2 / (e + (-4 * d * f + e^2)^{(1/2)})^2 / (-4 * d * f + e^2)^{(1/2)} * 2^{(1/2)} / (((-4 * d * f + e^2)^{(1/2)} * c * e + 2 * a * f^2 - 2 * c * d * f + c * e^2) / f^2)^{(1/2)} * \ln((((-4 * d * f + e^2 \\
& + e^2)^{(1/2)} * c * e + 2 * a * f^2 - 2 * c * d * f + c * e^2) / f^2 - c * (e + (-4 * d * f + e^2)^{(1/2)}) / f * (x + 1/ \\
& 2 * (e + (-4 * d * f + e^2)^{(1/2)}) / f) + 1/2 * 2^{(1/2)} * (((-4 * d * f + e^2)^{(1/2)} * c * e + 2 * a * f^2 - 2 * \\
& c * d * f + c * e^2) / f^2)^{(1/2)} * (4 * (x + 1/2 * (e + (-4 * d * f + e^2)^{(1/2)}) / f)^2 * c - 4 * c * (e + (-4 * \\
& d * f + e^2)^{(1/2)}) / f * (x + 1/2 * (e + (-4 * d * f + e^2)^{(1/2)}) / f) + 2 * ((-4 * d * f + e^2)^{(1/2)} * c * \\
& e + 2 * a * f^2 - 2 * c * d * f + c * e^2) / f^2)^{(1/2)} / (x + 1/2 * (e + (-4 * d * f + e^2)^{(1/2)}) / f)) * c * e \\
& + 2 * 4 * f / (-e + (-4 * d * f + e^2)^{(1/2)}) / (e + (-4 * d * f + e^2)^{(1/2)}) / a * x * (c * x^2 + a)^{(3/2)} - 4 * \\
& f / (-e + (-4 * d * f + e^2)^{(1/2)}) / (e + (-4 * d * f + e^2)^{(1/2)}) * c / a * x * (c * x^2 + a)^{(1/2)} - 4 * f / \\
& (-e + (-4 * d * f + e^2)^{(1/2)}) / (e + (-4 * d * f + e^2)^{(1/2)}) * c^{(1/2)} * \ln(c^{(1/2)} * x + (c * x^2 + \\
& a)^{(1/2)}) + 2 * f^2 / (-e + (-4 * d * f + e^2)^{(1/2)})^2 / (-4 * d * f + e^2)^{(1/2)} * (4 * (x - 1/2 * (-e + \\
& (-4 * d * f + e^2)^{(1/2)}) / f)^2 * c - 4 * c * (e - (-4 * d * f + e^2)^{(1/2)}) / f * (x - 1/2 * (-e + (-4 * d * f + \\
& e^2)^{(1/2)}) / f) + 2 * (-(-4 * d * f + e^2)^{(1/2)} * c * e + 2 * a * f^2 - 2 * c * d * f + c * e^2) / f^2)^{(1/2)} \\
& + 2 * f / (-e + (-4 * d * f + e^2)^{(1/2)})^2 * c^{(1/2)} * \ln((-1/2 * c * (e - (-4 * d * f + e^2)^{(1/2)}) / f + \\
& c * (x - 1/2 * (-e + (-4 * d * f + e^2)^{(1/2)}) / f)) / c^{(1/2)} + ((x - 1/2 * (-e + (-4 * d * f + e^2)^{(1/2)}) / f)^2 * c - c * (e - (-4 * d * f + e^2)^{(1/2)}) / f * (x - 1/2 * (-e + (-4 * d * f + e^2)^{(1/2)}) / f) + 1/2 * (\\
& -(-4 * d * f + e^2)^{(1/2)} * c * e + 2 * a * f^2 - 2 * c * d * f + c * e^2) / f^2)^{(1/2)} - 2 * f / (-e + (-4 * d * f + \\
& e^2)^{(1/2)})^2 / (-4 * d * f + e^2)^{(1/2)} * c^{(1/2)} * \ln((-1/2 * c * (e - (-4 * d * f + e^2)^{(1/2)}) / f + c * (x - 1/2 * (-e + (-4 * d * f + e^2)^{(1/2)}) / f)) / c^{(1/2)} + ((x - 1/2 * (-e + (-4 * d * f + e^2)^{(1/2)}) / f)^2 * c - c * (e - (-4 * d * f + e^2)^{(1/2)}) / f * (x - 1/2 * (-e + (-4 * d * f + e^2)^{(1/2)}) / f) + 1/2 * (\\
& * (-(-4 * d * f + e^2)^{(1/2)} * c * e + 2 * a * f^2 - 2 * c * d * f + c * e^2) / f^2)^{(1/2)}) * e + 2 / (-e + (-4 * d *
\end{aligned}$$

$$f+e^2)^{(1/2)}^2 \cdot 2^{(1/2)} / ((-(-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2-2*c*d*f+c*e^2) / f^{(1/2)})^{(1/2)} * \ln(((-(-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2-2*c*d*f+c*e^2) / f^{(1/2)})^{(1/2)} * \ln(((-(-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2-2*c*d*f+c*e^2) / f^{(1/2)})^{(1/2)} * (4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})) / f)^2 * c-4*c*(e-(-4*d*f+e^2)^{(1/2)}) / f * (x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})) / f + 2*(-(-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2-2*c*d*f+c*e^2) / f^{(1/2)}) / (x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})) * c*e-4*f^2 / (-e+(-4*d*f+e^2)^{(1/2)})^2 / (-4*d*f+e^2)^{(1/2)} * 2^{(1/2)} / ((-(-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2-2*c*d*f+c*e^2) / f^{(1/2)})^{(1/2)} * \ln(((-(-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2-2*c*d*f+c*e^2) / f^{(1/2)})^{(1/2)} * \ln(((-(-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2-2*c*d*f+c*e^2) / f^{(1/2)})^{(1/2)} * (4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})) / f)^2 * c-4*c*(e-(-4*d*f+e^2)^{(1/2)}) / f * (x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})) / f + 2*(-(-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2-2*c*d*f+c*e^2) / f^{(1/2)}) / (x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})) * a+4*f / (-e+(-4*d*f+e^2)^{(1/2)})^2 / (-4*d*f+e^2)^{(1/2)} * 2^{(1/2)} / ((-(-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2-2*c*d*f+c*e^2) / f^{(1/2)})^{(1/2)} * \ln(((-(-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2-2*c*d*f+c*e^2) / f^{(1/2)})^{(1/2)} * \ln(((-(-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2-2*c*d*f+c*e^2) / f^{(1/2)})^{(1/2)} * (4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})) / f)^2 * c-4*c*(e-(-4*d*f+e^2)^{(1/2)}) / f * (x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})) / f + 2*(-(-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2-2*c*d*f+c*e^2) / f^{(1/2)}) / (x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})) * c*d-2 / (-e+(-4*d*f+e^2)^{(1/2)})^2 / (-4*d*f+e^2)^{(1/2)} * 2^{(1/2)} / ((-(-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2-2*c*d*f+c*e^2) / f^{(1/2)})^{(1/2)} * \ln(((-(-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2-2*c*d*f+c*e^2) / f^{(1/2)})^{(1/2)} * \ln(((-(-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2-2*c*d*f+c*e^2) / f^{(1/2)})^{(1/2)} * (4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})) / f)^2 * c-4*c*(e-(-4*d*f+e^2)^{(1/2)}) / f * (x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})) / f + 2*(-(-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2-2*c*d*f+c*e^2) / f^{(1/2)}) / (x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})) * c*e^2+16*f^2*e / (-e+(-4*d*f+e^2)^{(1/2)})^2 / (e+(-4*d*f+e^2)^{(1/2)})^2 * a^{(1/2)} * \ln((2*a+2*a^{(1/2)} * (c*x^2+a)^{(1/2)}) / x) - 16*f^2*e / (-e+(-4*d*f+e^2)^{(1/2)})^2 / (e+(-4*d*f+e^2)^{(1/2)})^2 * (c*x^2+a)^{(1/2)}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + a}}{(fx^2 + ex + d)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] integrate(sqrt(c*x^2 + a)/((f*x^2 + e*x + d)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^2 + a}}{x^2 (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + c*x^2)^(1/2)/(x^2*(d + e*x + f*x^2)), x)`

[Out] `int((a + c*x^2)^(1/2)/(x^2*(d + e*x + f*x^2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + cx^2}}{x^2(d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+a)**(1/2)/x**2/(f*x**2+e*x+d), x)`

[Out] `Integral(sqrt(a + c*x**2)/(x**2*(d + e*x + f*x**2)), x)`

$$3.57 \quad \int \frac{\sqrt{a+cx^2}}{x^3(d+ex+fx^2)} dx$$

Optimal. Leaf size=507

$$\frac{\sqrt{a} (e^2 - df) \tanh^{-1} \left(\frac{\sqrt{a+cx^2}}{\sqrt{a}} \right) + \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{f(a(e^2\sqrt{e^2-4df} - df\sqrt{e^2-4df} - 3def + e^3) + cd^2(\sqrt{e^2-4df} + e))}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}}{d^3}$$

[Out] $-1/2*c*\operatorname{arctanh}((c*x^2+a)^{(1/2)}/a^{(1/2)})/d/a^{(1/2)}-(-d*f+e^2)*\operatorname{arctanh}((c*x^2+a)^{(1/2)}/a^{(1/2)})*a^{(1/2)}/d^3-1/2*(c*x^2+a)^{(1/2)}/d/x^2+e*(c*x^2+a)^{(1/2)}/d^2/x+1/2*f*\operatorname{arctanh}(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^{(1/2)}))*2^{(1/2)}/(c*x^2+a)^{(1/2)})/(2*a*f^2+(-(-4*d*f+e^2)^{(1/2)})*e-2*d*f+e^2)*c^{(1/2)})*(c*d^2*(e+(-4*d*f+e^2)^{(1/2)})+a*(e^3-3*d*e*f+e^2*(-4*d*f+e^2)^{(1/2)}-d*f*(-4*d*f+e^2)^{(1/2)}))/d^3*2^{(1/2)}/(-4*d*f+e^2)^{(1/2)}/(2*a*f^2+(-(-4*d*f+e^2)^{(1/2)})*e-2*d*f+e^2)*c^{(1/2)}-1/2*f*\operatorname{arctanh}(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^{(1/2)}))*2^{(1/2)}/(c*x^2+a)^{(1/2)})/(2*a*f^2+((-4*d*f+e^2)^{(1/2)})*e-2*d*f+e^2)*c^{(1/2)})*(c*d^2*(e-(-4*d*f+e^2)^{(1/2)})+a*(e^3-3*d*e*f-e^2*(-4*d*f+e^2)^{(1/2)}+d*f*(-4*d*f+e^2)^{(1/2)}))/d^3*2^{(1/2)}/(-4*d*f+e^2)^{(1/2)}/(2*a*f^2+((-4*d*f+e^2)^{(1/2)})*e-2*d*f+e^2)*c^{(1/2)}$

Rubi [A] time = 1.88, antiderivative size = 507, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.482$, Rules used = {6728, 266, 47, 63, 208, 277, 217, 206, 50, 1020, 1080, 1034, 725}

$$\frac{f(a(e^2\sqrt{e^2-4df} - df\sqrt{e^2-4df} - 3def + e^3) + cd^2(\sqrt{e^2-4df} + e)) \tanh^{-1} \left(\frac{2af - cx(e - \sqrt{e^2-4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}} \right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + c*x^2]/(x^3*(d + e*x + f*x^2)), x]

[Out] $-\operatorname{Sqrt}[a + c*x^2]/(2*d*x^2) + (e*\operatorname{Sqrt}[a + c*x^2])/(d^2*x) + (f*(c*d^2*(e + \operatorname{Sqrt}[e^2 - 4*d*f]) + a*(e^3 - 3*d*e*f + e^2*\operatorname{Sqrt}[e^2 - 4*d*f] - d*f*\operatorname{Sqrt}[e^2 - 4*d*f]))*\operatorname{ArcTanh}[(2*a*f - c*(e - \operatorname{Sqrt}[e^2 - 4*d*f])*x)/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[2*a*f^2 + c*(e^2 - 2*d*f - e*\operatorname{Sqrt}[e^2 - 4*d*f])])*\operatorname{Sqrt}[a + c*x^2]])/(\operatorname{Sqrt}[2]*d^3*\operatorname{Sqrt}[e^2 - 4*d*f]*\operatorname{Sqrt}[2*a*f^2 + c*(e^2 - 2*d*f - e*\operatorname{Sqrt}[e^2 - 4*d*f])]) - (f*(c*d^2*(e - \operatorname{Sqrt}[e^2 - 4*d*f]) + a*(e^3 - 3*d*e*f - e^2*\operatorname{Sqrt}[e^2 - 4*d*f] + d*f*\operatorname{Sqrt}[e^2 - 4*d*f]))*\operatorname{ArcTanh}[(2*a*f - c*(e + \operatorname{Sqrt}[e^2 - 4*d*f])*x)/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[2*a*f^2 + c*(e^2 - 2*d*f + e*\operatorname{Sqrt}[e^2 - 4*d*f])])*\operatorname{Sqrt}[a + c*x^2]])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[2*a*f^2 + c*(e^2 - 2*d*f + e*\operatorname{Sqrt}[e^2 - 4*d*f])])*\operatorname{Sqrt}[a + c*x^2]$

$$\frac{x^2)}{(\sqrt{2}d^3\sqrt{e^2 - 4df})\sqrt{2af^2 + c(e^2 - 2df + e\sqrt{e^2 - 4df})}} - (c\text{ArcTanh}[\sqrt{a + cx^2}/\sqrt{a}])/(2\sqrt{a}d) - (\sqrt{a}(e^2 - df)\text{ArcTanh}[\sqrt{a + cx^2}/\sqrt{a}])/d^3$$

Rule 47

$$\text{Int}[(a + b \cdot x)^m \cdot (c + d \cdot x)^n, x_Symbol] \rightarrow \text{Simp}[(a + b \cdot x)^{m+1} \cdot (c + d \cdot x)^n / (b \cdot (m+1)), x] - \text{Dist}[(d \cdot n) / (b \cdot (m+1)), \text{Int}[(a + b \cdot x)^{m+1} \cdot (c + d \cdot x)^{n-1}, x], x] /;$$

$$\text{FreeQ}\{a, b, c, d, x\} \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ !(\text{IntegerQ}[n] \ \&\& \ !\text{IntegerQ}[m]) \ \&\& \ !(\text{ILeQ}[m + n + 2, 0] \ \&\& \ (\text{FractionQ}[m] \ || \ \text{GeQ}[2 \cdot n + m + 1, 0])) \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$$

Rule 50

$$\text{Int}[(a + b \cdot x)^m \cdot (c + d \cdot x)^n, x_Symbol] \rightarrow \text{Simp}[(a + b \cdot x)^{m+1} \cdot (c + d \cdot x)^n / (b \cdot (m + n + 1)), x] + \text{Dist}[(n \cdot (b \cdot c - a \cdot d)) / (b \cdot (m + n + 1)), \text{Int}[(a + b \cdot x)^m \cdot (c + d \cdot x)^{n-1}, x], x] /;$$

$$\text{FreeQ}\{a, b, c, d, x\} \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{NeQ}[m + n + 1, 0] \ \&\& \ !(\text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \ || \ (\text{GtQ}[m, 0] \ \&\& \ \text{LtQ}[m - n, 0])) \ \&\& \ !\text{ILtQ}[m + n + 2, 0] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$$

Rule 63

$$\text{Int}[(a + b \cdot x)^m \cdot (c + d \cdot x)^n, x_Symbol] \rightarrow \text{With}\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{p \cdot (m+1) - 1} \cdot (c - (a \cdot d)/b + (d \cdot x^p)/b)^n, x], x, (a + b \cdot x)^{1/p}], x] /;$$

$$\text{FreeQ}\{a, b, c, d, x\} \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$$

Rule 206

$$\text{Int}[(a + b \cdot x^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1 \cdot \text{ArcTanh}[(\text{Rt}[-b, 2] \cdot x) / \text{Rt}[a, 2]]) / (\text{Rt}[a, 2] \cdot \text{Rt}[-b, 2]), x] /;$$

$$\text{FreeQ}\{a, b, x\} \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$$

Rule 208

$$\text{Int}[(a + b \cdot x^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2] \cdot \text{ArcTanh}[x / \text{Rt}[-(a/b), 2]]) / a, x] /;$$

$$\text{FreeQ}\{a, b, x\} \ \&\& \ \text{NegQ}[a/b]$$

Rule 217

$$\text{Int}[1/\sqrt{(a + b \cdot x^2)}, x_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(1 - b \cdot x^2), x], x, x/\sqrt{a + b \cdot x^2}] /;$$

$$\text{FreeQ}\{a, b, x\} \ \&\& \ !\text{GtQ}[a, 0]$$

Rule 266

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b,
m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 277

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c
x)^(m + 1)(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]

Rule 725

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (c_)*(x_)^2]), x_Symbol] := -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]

Rule 1020

Int[((g_) + (h_)*(x_))*((a_) + (c_)*(x_)^2)^(p_)*((d_) + (e_)*(x_) + (f
)*(x)^2)^(q_), x_Symbol] := Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q +
1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]

Rule 1034

Int[((g_) + (h_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (f
)*(x)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rule 1080

Int[((A_) + (B_)*(x_) + (C_)*(x_)^2)/((a_) + (b_)*(x_) + (c_)*(x_)^2)
Sqrt[(d_) + (f_)(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + f*x^2],
x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a

*c, 0]

Rule 6728

Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
 {v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
 mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{a+cx^2}}{x^3(d+ex+fx^2)} dx &= \int \left(\frac{\sqrt{a+cx^2}}{dx^3} - \frac{e\sqrt{a+cx^2}}{d^2x^2} + \frac{(e^2-df)\sqrt{a+cx^2}}{d^3x} + \frac{(-e(e^2-2df)-f(e^2-df)x)\sqrt{a+cx^2}}{d^3(d+ex+fx^2)} \right) dx \\
 &= \frac{\int \frac{(-e(e^2-2df)-f(e^2-df)x)\sqrt{a+cx^2}}{d+ex+fx^2} dx}{d^3} + \frac{\int \frac{\sqrt{a+cx^2}}{x^3} dx}{d} - \frac{e \int \frac{\sqrt{a+cx^2}}{x^2} dx}{d^2} + \frac{(e^2-df) \int \frac{\sqrt{a+cx^2}}{x} dx}{d^3} \\
 &= -\frac{(e^2-df)\sqrt{a+cx^2}}{d^3} + \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x^2} dx, x, x^2\right)}{2d} - \frac{(ce) \int \frac{1}{\sqrt{a+cx^2}} dx}{d^2} + \dots \\
 &= -\frac{\sqrt{a+cx^2}}{2dx^2} + \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{c \text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}} dx, x, x^2\right)}{4d} + \frac{(ce) \int \frac{1}{\sqrt{a+cx^2}} dx}{d^2} - \frac{(ce) \text{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}} dx, x, x^2\right)}{d^2} \\
 &= -\frac{\sqrt{a+cx^2}}{2dx^2} + \frac{e\sqrt{a+cx^2}}{d^2x} - \frac{\sqrt{c} e \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{d^2} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}} dx, x, \sqrt{a+cx^2}\right)}{2d} \\
 &= -\frac{\sqrt{a+cx^2}}{2dx^2} + \frac{e\sqrt{a+cx^2}}{d^2x} - \frac{c \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}d} - \frac{\sqrt{a}(e^2-df) \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3} + \dots \\
 &= -\frac{\sqrt{a+cx^2}}{2dx^2} + \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{f(cd^2(e+\sqrt{e^2-4df})+a(e^3-3def+e^2\sqrt{e^2-4df}-d\sqrt{2d^3\sqrt{e^2-4df}}\sqrt{2af^2+c}))}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c}}
 \end{aligned}$$

Mathematica [A] time = 2.61, size = 642, normalized size = 1.27

$$\frac{2d^2 \left(cx^2 \sqrt{\frac{cx^2}{a} + 1} \tanh^{-1} \left(\sqrt{\frac{cx^2}{a} + 1} \right) + a + cx^2 \right)}{x^2 \sqrt{a + cx^2}} + \frac{\left(\frac{e(e^2 - 3df)}{\sqrt{e^2 - 4df}} - df + e^2 \right) \left(\sqrt{4af^2 - 2c(e\sqrt{e^2 - 4df} + 2df - e^2)} \tanh^{-1} \left(\frac{2af + cx(\sqrt{e^2 - 4df} - e)}{\sqrt{a + cx^2} \sqrt{4af^2 - 2c(e\sqrt{e^2 - 4df} + 2df - e^2)}} \right) \right)}{f}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + c*x^2]/(x^3*(d + e*x + f*x^2)),x]

[Out] $(-2*(e^2 - d*f - (e*(e^2 - 3*d*f))/\text{Sqrt}[e^2 - 4*d*f])*\text{Sqrt}[a + c*x^2] - 2*(e^2 - d*f + (e*(e^2 - 3*d*f))/\text{Sqrt}[e^2 - 4*d*f])*\text{Sqrt}[a + c*x^2] + (4*d*e*(a + c*x^2 - \text{Sqrt}[a]*\text{Sqrt}[c]*x*\text{Sqrt}[1 + (c*x^2)/a]*\text{ArcSinh}[(\text{Sqrt}[c]*x)/\text{Sqrt}[a]]))/x*\text{Sqrt}[a + c*x^2]) + ((e^2 - d*f + (e*(e^2 - 3*d*f))/\text{Sqrt}[e^2 - 4*d*f])*(-(\text{Sqrt}[c]*(-e + \text{Sqrt}[e^2 - 4*d*f])*\text{ArcTanh}[(\text{Sqrt}[c]*x)/\text{Sqrt}[a + c*x^2]]) + \text{Sqrt}[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]])*\text{ArcTanh}[(2*a*f + c*(-e + \text{Sqrt}[e^2 - 4*d*f])*x)/(\text{Sqrt}[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]])*\text{Sqrt}[a + c*x^2])])))/f + ((e^2 - d*f - (e*(e^2 - 3*d*f))/\text{Sqrt}[e^2 - 4*d*f])*(\text{Sqrt}[c]*(e + \text{Sqrt}[e^2 - 4*d*f])*\text{ArcTanh}[(\text{Sqrt}[c]*x)/\text{Sqrt}[a + c*x^2]]) + \text{Sqrt}[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]])*\text{ArcTanh}[(2*a*f - c*(e + \text{Sqrt}[e^2 - 4*d*f])*x)/(\text{Sqrt}[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]])*\text{Sqrt}[a + c*x^2])])))/f + 4*(e^2 - d*f)*(\text{Sqrt}[a + c*x^2] - \text{Sqrt}[a]*\text{ArcTanh}[\text{Sqrt}[a + c*x^2]/\text{Sqrt}[a]]) - (2*d^2*(a + c*x^2 + c*x^2*\text{Sqrt}[1 + (c*x^2)/a]*\text{ArcTanh}[\text{Sqrt}[1 + (c*x^2)/a]]))/(x^2*\text{Sqrt}[a + c*x^2]))/(4*d^3)$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(1/2)/x^3/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(1/2)/x^3/(f*x^2+e*x+d),x, algorithm="giac")

$$\begin{aligned}
& *a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}*(4*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c-4* \\
& c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)+2*((-4*d*f+e^2) \\
& ^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)})/(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}) \\
& /f))*c*e^2+16*f^2*e/(-e+(-4*d*f+e^2)^{(1/2)})^2/(e+(-4*d*f+e^2)^{(1/2)})^2/a/x* \\
& (c*x^2+a)^{(3/2)}-16*f^2*e/(-e+(-4*d*f+e^2)^{(1/2)})^2/(e+(-4*d*f+e^2)^{(1/2)})^2 \\
& *c/a*x*(c*x^2+a)^{(1/2)}-16*f^2*e/(-e+(-4*d*f+e^2)^{(1/2)})^2/(e+(-4*d*f+e^2)^{(1/2)})^2 \\
& *c^{(1/2)}*\ln(c^{(1/2)}*x+(c*x^2+a)^{(1/2)})+2*f/(-e+(-4*d*f+e^2)^{(1/2)})/(\\
& e+(-4*d*f+e^2)^{(1/2)})/a/x^2*(c*x^2+a)^{(3/2)}+2*f/(-e+(-4*d*f+e^2)^{(1/2)})/(e+ \\
& (-4*d*f+e^2)^{(1/2)})*c/a^{(1/2)}*\ln((2*a+2*a^{(1/2)}*(c*x^2+a)^{(1/2)})/x)-2*f/(-e \\
& +(-4*d*f+e^2)^{(1/2)})/(e+(-4*d*f+e^2)^{(1/2)})*c/a*(c*x^2+a)^{(1/2)}+4*f^3/(-e+ \\
& (-4*d*f+e^2)^{(1/2)})^3/(-4*d*f+e^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f \\
&)^2*c-4*c*(e-(-4*d*f+e^2)^{(1/2)})/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)+2*(- \\
& (-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}+4*f^2/(-e+(-4*d*f+e^2) \\
& ^{(1/2)})^3*c^{(1/2)}*\ln((-1/2*c*(e-(-4*d*f+e^2)^{(1/2)}))/f+c*(x-1/2*(-e+(-4*d* \\
& f+e^2)^{(1/2)}))/f)/c^{(1/2)}+((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c-c*(e-(-4*d \\
& *f+e^2)^{(1/2)})/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)+1/2*(-(-4*d*f+e^2)^{(1/2)} \\
& *c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}-4*f^2/(-e+(-4*d*f+e^2)^{(1/2)})^3/(-4 \\
& *d*f+e^2)^{(1/2)}*c^{(1/2)}*\ln((-1/2*c*(e-(-4*d*f+e^2)^{(1/2)}))/f+c*(x-1/2*(-e+ \\
& (-4*d*f+e^2)^{(1/2)}))/f)/c^{(1/2)}+((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c-c*(e- \\
& (-4*d*f+e^2)^{(1/2)})/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)+1/2*(-(-4*d*f+e^2)^{(1/2)} \\
& *c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)})*e+4*f/(-e+(-4*d*f+e^2)^{(1/2)})^3 \\
& *2^{(1/2)}/((-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}*\ln(((\\
& (-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^{(1/2)})/ \\
& f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)+1/2*2^{(1/2)}*((-4*d*f+e^2)^{(1/2)}*c*e+2 \\
& *a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c-4 \\
& *c*(e-(-4*d*f+e^2)^{(1/2)})/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)+2*(-(-4*d*f+e \\
& ^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)})/(x-1/2*(-e+(-4*d*f+e^2)^{(1 \\
& /2))/f))*c*e-8*f^3/(-e+(-4*d*f+e^2)^{(1/2)})^3/(-4*d*f+e^2)^{(1/2)}*2^{(1/2)}/((- \\
& (-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}*\ln(((\\
& (-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^{(1/2)})/f*(x-1/2*(-e \\
& +(-4*d*f+e^2)^{(1/2)}))/f)+1/2*2^{(1/2)}*((-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d \\
& *f+c*e^2)/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c-4*c*(e-(-4*d* \\
& f+e^2)^{(1/2)})/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)+2*(-(-4*d*f+e^2)^{(1/2)}*c \\
& e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)})/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f))*a+8 \\
& *f^2/(-e+(-4*d*f+e^2)^{(1/2)})^3/(-4*d*f+e^2)^{(1/2)}*2^{(1/2)}/((-4*d*f+e^2)^{(1/2)} \\
& *c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}*\ln(((\\
& (-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^{(1/2)})/f*(x-1/2*(-e+ \\
& (-4*d*f+e^2)^{(1/2)}))/f)+1/2*2^{(1/2)}*((-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c \\
& *e^2)/f^2-c*(e-(-4*d*f+e^2)^{(1/2)})/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)+1/2*2 \\
& ^{(1/2)}*((-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}*(4*(x-1
\end{aligned}$$

$$\begin{aligned} & /2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-4*c*(e-(-4*d*f+e^2)^{(1/2)})/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)+2*(-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2 \\ &)^{(1/2)})/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f))*c*e^2-64*f^4/(-e+(-4*d*f+e^2)^{(1/2)})^3/(e+(-4*d*f+e^2)^{(1/2)})^3*a^{(1/2)}*\ln((2*a+2*a^{(1/2)}*(c*x^2+a)^{(1/2)}) \\ & /x)*d+64*f^3/(-e+(-4*d*f+e^2)^{(1/2)})^3/(e+(-4*d*f+e^2)^{(1/2)})^3*a^{(1/2)}*\ln(\\ & (2*a+2*a^{(1/2)}*(c*x^2+a)^{(1/2)})/x)*e^2+64*f^4/(-e+(-4*d*f+e^2)^{(1/2)})^3/(e+ \\ & (-4*d*f+e^2)^{(1/2)})^3*(c*x^2+a)^{(1/2)}*d-64*f^3/(-e+(-4*d*f+e^2)^{(1/2)})^3/(e \\ & +(-4*d*f+e^2)^{(1/2)})^3*(c*x^2+a)^{(1/2)}*e^2 \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + a}}{(fx^2 + ex + d)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(1/2)/x^3/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] integrate(sqrt(c*x^2 + a)/((f*x^2 + e*x + d)*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^2 + a}}{x^3 (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c*x^2)^(1/2)/(x^3*(d + e*x + f*x^2)),x)

[Out] int((a + c*x^2)^(1/2)/(x^3*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + cx^2}}{x^3 (d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+a)**(1/2)/x**3/(f*x**2+e*x+d),x)

[Out] Integral(sqrt(a + c*x**2)/(x**3*(d + e*x + f*x**2)), x)

$$3.58 \quad \int \frac{x^2(a+cx^2)^{3/2}}{d+ex+fx^2} dx$$

Optimal. Leaf size=795

$$\frac{(4e-3fx)(cx^2+a)^{3/2}}{12f^2} - \frac{(8e(af^2+c(e^2-2df)) - f(3af^2+4c(e^2-df))x)\sqrt{cx^2+a}}{8f^4} + \frac{(3a^2f^4+12ac(e^2-df))\sqrt{cx^2+a}}{8f^4}$$

[Out] $-1/12*(-3*f*x+4*e)*(c*x^2+a)^(3/2)/f^2+1/8*(3*a^2*f^4+12*a*c*f^2*(-d*f+e^2)+8*c^2*(d^2*f^2-3*d*e^2*f+e^4))*\operatorname{arctanh}(x*c^(1/2)/(c*x^2+a)^(1/2))/f^5/c^(1/2)-1/8*(8*e*(a*f^2+c*(-2*d*f+e^2))-f*(3*a*f^2+4*c*(-d*f+e^2))*x)*(c*x^2+a)^(1/2)/f^4-1/2*\operatorname{arctanh}(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(a^2*f^4*(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)+2*a*c*f^2*(e^4-4*d*e^2*f+2*d^2*f^2-e^3*(-4*d*f+e^2)^(1/2)+2*d*e*f*(-4*d*f+e^2)^(1/2))+c^2*(e^6-6*d*e^4*f+9*d^2*e^2*f^2-2*d^3*f^3-e^5*(-4*d*f+e^2)^(1/2)+4*d*e^3*f*(-4*d*f+e^2)^(1/2)-3*d^2*e*f^2*(-4*d*f+e^2)^(1/2)))/f^5*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)+1/2*\operatorname{arctanh}(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)*(a^2*f^4*(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)+2*a*c*f^2*(e^4-4*d*e^2*f+2*d^2*f^2+e^3*(-4*d*f+e^2)^(1/2)-2*d*e*f*(-4*d*f+e^2)^(1/2))+c^2*(e^6-6*d*e^4*f+9*d^2*e^2*f^2-2*d^3*f^3+e^5*(-4*d*f+e^2)^(1/2)-4*d*e^3*f*(-4*d*f+e^2)^(1/2)+3*d^2*e*f^2*(-4*d*f+e^2)^(1/2)))/f^5*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)$

Rubi [A] time = 4.26, antiderivative size = 795, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {1069, 1068, 1080, 217, 206, 1034, 725}

$$\frac{(4e-3fx)(cx^2+a)^{3/2}}{12f^2} - \frac{(8e(af^2+c(e^2-2df)) - f(3af^2+4c(e^2-df))x)\sqrt{cx^2+a}}{8f^4} + \frac{(3a^2f^4+12ac(e^2-df))\sqrt{cx^2+a}}{8f^4}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(x^2*(a+c*x^2)^(3/2))/(d+e*x+f*x^2),x]$

[Out] $-((8*e*(a*f^2+c*(e^2-2*d*f))-f*(3*a*f^2+4*c*(e^2-d*f))*x)*\operatorname{Sqrt}[a+c*x^2]/(8*f^4)-((4*e-3*f*x)*(a+c*x^2)^(3/2))/(12*f^2)+((3*a^2*f^4+12*a*c*f^2*(e^2-d*f)+8*c^2*(e^4-3*d*e^2*f+d^2*f^2))*\operatorname{ArcTanh}[(\operatorname{Sqrt}[c]*x)/\operatorname{Sqrt}[a+c*x^2]])/(8*\operatorname{Sqrt}[c]*f^5)-((a^2*f^4*(e^2-2*d*f-e*\operatorname{Sqrt}[c]))*(a+c*x^2)^(3/2))/(12*f^2)+((3*a^2*f^4+12*a*c*f^2*(e^2-d*f)+8*c^2*(e^4-3*d*e^2*f+d^2*f^2))*\operatorname{ArcTanh}[(\operatorname{Sqrt}[c]*x)/\operatorname{Sqrt}[a+c*x^2]])/(8*\operatorname{Sqrt}[c]*f^5)$

```
t[e^2 - 4*d*f]) + 2*a*c*f^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2 - e^3*Sqrt[e^2 - 4*d*f] + 2*d*e*f*Sqrt[e^2 - 4*d*f]) + c^2*(e^6 - 6*d*e^4*f + 9*d^2*e^2*f^2 - 2*d^3*f^3 - e^5*Sqrt[e^2 - 4*d*f] + 4*d*e^3*f*Sqrt[e^2 - 4*d*f] - 3*d^2*e*f^2*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2])]/(Sqrt[2]*f^5*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]])] + ((a^2*f^4*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + 2*a*c*f^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2 + e^3*Sqrt[e^2 - 4*d*f] - 2*d*e*f*Sqrt[e^2 - 4*d*f]) + c^2*(e^6 - 6*d*e^4*f + 9*d^2*e^2*f^2 - 2*d^3*f^3 + e^5*Sqrt[e^2 - 4*d*f] - 4*d*e^3*f*Sqrt[e^2 - 4*d*f] + 3*d^2*e*f^2*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2])]/(Sqrt[2]*f^5*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])])
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (c_)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]
```

Rule 1034

```
Int[((g_) + (h_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (f_)*(x_)^2], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1068

```
Int[((a_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x_)^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((B*c*f*(2*p + 2*q + 3) + C*(-(c*e*(2*p + q + 2))) + 2*c*C*f*(p + q + 1)*x)*(a + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^(q + 1)], x]
```

```

q*Simp[p*(-(a*e))*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p +
q + 1)*(a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3
))) + (2*p*(c*d - a*f)*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(C*e*f*p*(-4*a*c)))*x + (p*(c*e)*(C*(c*e)*(q + 1) - c*(C*e - B*
f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(-4*a*c) - c^2*(C*(e^2 - 4*d*f)*
(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x]
/; FreeQ[{a, c, d, e, f, A, B, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0
] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q
, 0]

```

Rule 1069

```

Int[((a_) + (c_)*(x_)^2)^(p_)*((A_) + (C_)*(x_)^2)*((d_) + (e_)*(x_) +
(f_)*(x_)^2)^(q_), x_Symbol] := Simp[((C*(-(c*e*(2*p + q + 2))) + 2*c*C*f*
(p + q + 1)*x)*(a + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1
)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[
(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(-(a*e))*(C*(c*e)*(q + 1) -
c*(C*e)*(2*p + 2*q + 3)) + (p + q + 1)*(a*c*(C*(2*d*f - e^2*(2*p + q + 2))
+ f*(-2*A*f)*(2*p + 2*q + 3)) + (2*p*(c*d - a*f)*(C*(c*e)*(q + 1) - c*(C*e
)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(-4*a*c)))*x + (p*(c*e)*(C*(c*e)*
(q + 1) - c*(C*e)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(-4*a*c) - c^2*(C
*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3))))*x^2, x]
, x], x] /; FreeQ[{a, c, d, e, f, A, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && Gt
Q[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] && !IGtQ[p, 0] && !
IGtQ[q, 0]

```

Rule 1080

```

Int[((A_) + (B_)*(x_) + (C_)*(x_)^2)/(((a_) + (b_)*(x_) + (c_)*(x_)^2)
*Sqrt[(d_) + (f_)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + f*x^2],
x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 (a + cx^2)^{3/2}}{d + ex + fx^2} dx &= -\frac{(4e - 3fx)(a + cx^2)^{3/2}}{12f^2} - \frac{\int \frac{\sqrt{a+cx^2} (3acd f - 3ce(4cd-af)x - 3c(3af^2+4c(e^2-df))x^2)}{d+ex+fx^2} dx}{12cf^2} \\
&= -\frac{(8e(af^2 + c(e^2 - 2df)) - f(3af^2 + 4c(e^2 - df))x) \sqrt{a + cx^2}}{8f^4} - \frac{(4e - 3fx)(a + cx^2)^{3/2}}{12f^2} \\
&= -\frac{(8e(af^2 + c(e^2 - 2df)) - f(3af^2 + 4c(e^2 - df))x) \sqrt{a + cx^2}}{8f^4} - \frac{(4e - 3fx)(a + cx^2)^{3/2}}{12f^2} \\
&= -\frac{(8e(af^2 + c(e^2 - 2df)) - f(3af^2 + 4c(e^2 - df))x) \sqrt{a + cx^2}}{8f^4} - \frac{(4e - 3fx)(a + cx^2)^{3/2}}{12f^2} \\
&= -\frac{(8e(af^2 + c(e^2 - 2df)) - f(3af^2 + 4c(e^2 - df))x) \sqrt{a + cx^2}}{8f^4} - \frac{(4e - 3fx)(a + cx^2)^{3/2}}{12f^2} \\
&= -\frac{(8e(af^2 + c(e^2 - 2df)) - f(3af^2 + 4c(e^2 - df))x) \sqrt{a + cx^2}}{8f^4} - \frac{(4e - 3fx)(a + cx^2)^{3/2}}{12f^2}
\end{aligned}$$

Mathematica [A] time = 3.45, size = 793, normalized size = 1.00

$$3f\sqrt{a + cx^2} \left(\frac{3a^{3/2} \sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{c} \sqrt{\frac{cx^2}{a} + 1}} + 5ax + 2cx^3 \right) - \frac{3 \left(\frac{2df - e^2}{\sqrt{e^2 - 4df}} + e \right) \left(2 \left(2af^2 + c \left(-e \sqrt{e^2 - 4df} - 2df + e^2 \right) \right) - \sqrt{4af^2 - 2ce \sqrt{e^2 - 4df} - 4cdf + 2ce^2} \tanh^{-1} \left(\frac{\sqrt{a+cx^2}}{\sqrt{a+c}} \right) \right)}{\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x]

[Out] (-4*(e + (e^2 - 2*d*f)/Sqrt[e^2 - 4*d*f])*(a + c*x^2)^(3/2) - 4*(e + (-e^2 + 2*d*f)/Sqrt[e^2 - 4*d*f])*(a + c*x^2)^(3/2) + 3*f*Sqrt[a + c*x^2]*(5*a*x

$$\begin{aligned}
& + 2*c*x^3 + (3*a^{(3/2)}*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[c]*Sqrt[1 + (c*x^2)/a]) - (3*(e + (-e^2 + 2*d*f)/Sqrt[e^2 - 4*d*f])*((2*Sqrt[c]*(-e + Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/a] + Sqrt[a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]]))/Sqrt[1 + (c*x^2)/a] + (2*(2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))*(2*f*Sqrt[a + c*x^2] + Sqrt[c]*(-e + Sqrt[e^2 - 4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] - Sqrt[2*c*e^2 - 4*c*d*f + 4*a*f^2 - 2*c*e*Sqrt[e^2 - 4*d*f]]*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])]*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2])))/f^2)/(2*f) + (3*(e + (e^2 - 2*d*f)/Sqrt[e^2 - 4*d*f])*((2*Sqrt[c]*(e + Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/a] + Sqrt[a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]]))/Sqrt[1 + (c*x^2)/a] + (2*(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*(-2*f*Sqrt[a + c*x^2] + Sqrt[c]*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] + Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])]*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2])))/f^2)/(2*f))/(24*f^2)
\end{aligned}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage₂

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] sage2

maple [B] time = 0.03, size = 19148, normalized size = 24.09

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)

[Out] result too large to display

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details)Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 (cx^2 + a)^{3/2}}{fx^2 + ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x)

[Out] int((x^2*(a + c*x^2)^(3/2))/(d + e*x + f*x^2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)

[Out] Timed out

$$3.59 \quad \int \frac{x(a+cx^2)^{3/2}}{d+ex+fx^2} dx$$

Optimal. Leaf size=553

$$\frac{(2cdef(2af^2 + c(e^2 - 2df)) - (e - \sqrt{e^2 - 4df})(a^2f^4 + 2acf^2(e^2 - df) + c^2(d^2f^2 - 3de^2f + e^4))) \tanh^{-1}\left(\frac{\sqrt{2}f^4\sqrt{e^2 - 4df}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}{\dots}\right)}{\dots}$$

[Out] $1/3*(c*x^2+a)^{(3/2)}/f-1/2*e*(3*a*f^2+2*c*(-2*d*f+e^2))*\operatorname{arctanh}(x*c^{(1/2)}/(c*x^2+a)^{(1/2)})*c^{(1/2)}/f^4+1/2*(2*a*f^2+2*c*(-d*f+e^2)-c*e*f*x)*(c*x^2+a)^{(1/2)}/f^3-1/2*\operatorname{arctanh}(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^{(1/2)})))*2^{(1/2)}/(c*x^2+a)^{(1/2)}/(2*a*f^2+(-(-4*d*f+e^2)^{(1/2)})*e-2*d*f+e^2)*c^{(1/2)}*(2*c*d*e*f*(2*a*f^2+c*(-2*d*f+e^2))-(a^2*f^4+2*a*c*f^2*(-d*f+e^2)+c^2*(d^2*f^2-3*d*e^2*f+e^4))*(e-(-4*d*f+e^2)^{(1/2)}))/f^4*2^{(1/2)}/(-4*d*f+e^2)^{(1/2)}/(2*a*f^2+(-(-4*d*f+e^2)^{(1/2)})*e-2*d*f+e^2)*c^{(1/2)}+1/2*\operatorname{arctanh}(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^{(1/2)})))*2^{(1/2)}/(c*x^2+a)^{(1/2)}/(2*a*f^2+((-4*d*f+e^2)^{(1/2)})*e-2*d*f+e^2)*c^{(1/2)}*(2*c*d*e*f*(2*a*f^2+c*(-2*d*f+e^2))-(a^2*f^4+2*a*c*f^2*(-d*f+e^2)+c^2*(d^2*f^2-3*d*e^2*f+e^4))*(e+(-4*d*f+e^2)^{(1/2)}))/f^4*2^{(1/2)}/(-4*d*f+e^2)^{(1/2)}/(2*a*f^2+((-4*d*f+e^2)^{(1/2)})*e-2*d*f+e^2)*c^{(1/2)}$

Rubi [A] time = 2.43, antiderivative size = 553, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1020, 1068, 1080, 217, 206, 1034, 725}

$$\frac{(2cdef(2af^2 + c(e^2 - 2df)) - (e - \sqrt{e^2 - 4df})(a^2f^4 + 2acf^2(e^2 - df) + c^2(d^2f^2 - 3de^2f + e^4))) \tanh^{-1}\left(\frac{\sqrt{2}f^4\sqrt{e^2 - 4df}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}{\dots}\right)}{\dots}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(x*(a + c*x^2)^{(3/2)})/(d + e*x + f*x^2), x]$

[Out] $((2*(a*f^2 + c*(e^2 - d*f)) - c*e*f*x)*\operatorname{Sqrt}[a + c*x^2])/(2*f^3) + (a + c*x^2)^{(3/2)}/(3*f) - (\operatorname{Sqrt}[c]*e*(3*a*f^2 + 2*c*(e^2 - 2*d*f))*\operatorname{ArcTanh}[(\operatorname{Sqrt}[c]*x)/\operatorname{Sqrt}[a + c*x^2]])/(2*f^4) - ((2*c*d*e*f*(2*a*f^2 + c*(e^2 - 2*d*f)) - (e - \operatorname{Sqrt}[e^2 - 4*d*f])*(a^2*f^4 + 2*a*c*f^2*(e^2 - d*f) + c^2*(e^4 - 3*d*e^2*f + d^2*f^2)))*\operatorname{ArcTanh}[(2*a*f - c*(e - \operatorname{Sqrt}[e^2 - 4*d*f])*x)/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[2*a*f^2 + c*(e^2 - 2*d*f - e*\operatorname{Sqrt}[e^2 - 4*d*f]])*\operatorname{Sqrt}[a + c*x^2])])/(2*f^4*\operatorname{Sqrt}[e^2 - 4*d*f]*\operatorname{Sqrt}[2*a*f^2 + c*(e^2 - 2*d*f - e*\operatorname{Sqrt}[e^2 - 4*d*f])]) + ((2*c*d*e*f*(2*a*f^2 + c*(e^2 - 2*d*f)) - (e + \operatorname{Sqrt}[e^2 - 4*d*f])*(a^2*f^4 + 2*a*c*f^2*(e^2 - d*f) + c^2*(e^4 - 3*d*e^2*f + d^2*f^2)))*\operatorname{ArcTanh}[\dots])$

$$\frac{(2*af - c*(e + \sqrt{e^2 - 4*df})*x)/(\sqrt{2}*\sqrt{2*af^2 + c*(e^2 - 2*df + e*\sqrt{e^2 - 4*df})}*\sqrt{a + c*x^2})}{(\sqrt{2}*f^4*\sqrt{e^2 - 4*df}*\sqrt{2*af^2 + c*(e^2 - 2*df + e*\sqrt{e^2 - 4*df})})}$$

Rule 206

$$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] \text{ /; FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$$

Rule 217

$$\text{Int}[1/\sqrt{(a_ + (b_)*(x_)^2)}, x_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\sqrt{a + b*x^2}] \text{ /; FreeQ}\{a, b\}, x \ \&\& \ !\text{GtQ}[a, 0]$$

Rule 725

$$\text{Int}[1/(((d_ + (e_)*(x_))*\sqrt{(a_ + (c_)*(x_)^2)}), x_Symbol] \rightarrow -\text{Subst}[\text{Int}[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/\sqrt{a + c*x^2}] \text{ /; FreeQ}\{a, c, d, e\}, x]$$

Rule 1020

$$\text{Int}(((g_ + (h_)*(x_))*((a_ + (c_)*(x_)^2)^{p_})*((d_ + (e_)*(x_ + (f_)*(x_)^2)^{q_}), x_Symbol] \rightarrow \text{Simp}[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^{q+1})]/(2*f*(p + q + 1)), x] + \text{Dist}[1/(2*f*(p + q + 1)), \text{Int}[(a + c*x^2)^{p-1}*(d + e*x + f*x^2)^q*\text{Simp}[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x] \text{ /; FreeQ}\{a, c, d, e, f, g, h, q\}, x \ \&\& \ \text{NeQ}[e^2 - 4*d*f, 0] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{NeQ}[p + q + 1, 0]$$

Rule 1034

$$\text{Int}(((g_ + (h_)*(x_))/((a_ + (b_)*(x_ + (c_)*(x_)^2)*\sqrt{(d_ + (f_)*(x_)^2)}), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(2*c*g - h*(b - q))/q, \text{Int}[1/((b - q + 2*c*x)*\sqrt{d + f*x^2}), x], x] - \text{Dist}[(2*c*g - h*(b + q))/q, \text{Int}[1/((b + q + 2*c*x)*\sqrt{d + f*x^2}), x], x]] \text{ /; FreeQ}\{a, b, c, d, f, g, h\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{PosQ}[b^2 - 4*a*c]$$

Rule 1068

$$\text{Int}(((a_ + (c_)*(x_)^2)^{p_})*((A_ + (B_)*(x_ + (C_)*(x_)^2)*((d_ + (e_)*(x_ + (f_)*(x_)^2)^{q_}), x_Symbol] \rightarrow \text{Simp}[(B*c*f*(2*p + 2*q + 3) + C*(-(c*e*(2*p + q + 2))) + 2*c*C*f*(p + q + 1)*x]*(a + c*x^2)^p*(d + e*x + f*x^2)^{q+1})]/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - \text{Dist}[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), \text{Int}[(a + c*x^2)^{p-1}*(d + e*x + f*x^2)^{q+1}], x]$$

```

q*Simp[p*(-(a*e))*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p +
q + 1)*(a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3
)) + (2*p*(c*d - a*f)*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(C*e*f*p*(-4*a*c)))*x + (p*(c*e)*(C*(c*e)*(q + 1) - c*(C*e - B*
f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(-4*a*c) - c^2*(C*(e^2 - 4*d*f)*
(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3)))]*x^2, x], x], x]
/; FreeQ[{a, c, d, e, f, A, B, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0
] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q
, 0]

```

Rule 1080

```

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + f*x^2],
x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{x(a+cx^2)^{3/2}}{d+ex+fx^2} dx &= \frac{(a+cx^2)^{3/2}}{3f} + \frac{\int \frac{\sqrt{a+cx^2}(-3(cd-af)x-3cex^2)}{d+ex+fx^2} dx}{3f} \\
&= \frac{(2(af^2+c(e^2-df))-cefx)\sqrt{a+cx^2}}{2f^3} + \frac{(a+cx^2)^{3/2}}{3f} - \frac{\int \frac{-3ac^2def+3c(ace^2f+2(cd-af)(ce^2-cd-af))\sqrt{a+cx^2}}{\sqrt{a+cx^2}(d+ex+fx^2)} dx}{6f^3} \\
&= \frac{(2(af^2+c(e^2-df))-cefx)\sqrt{a+cx^2}}{2f^3} + \frac{(a+cx^2)^{3/2}}{3f} - \frac{\int \frac{-3ac^2def^2-3c^2de(3af^2+2c(e^2-2df))\sqrt{a+cx^2}}{\sqrt{a+cx^2}(d+ex+fx^2)} dx}{6f^3} \\
&= \frac{(2(af^2+c(e^2-df))-cefx)\sqrt{a+cx^2}}{2f^3} + \frac{(a+cx^2)^{3/2}}{3f} - \frac{(ce(3af^2+2c(e^2-2df)))\sqrt{a+cx^2}}{2f^4} \\
&= \frac{(2(af^2+c(e^2-df))-cefx)\sqrt{a+cx^2}}{2f^3} + \frac{(a+cx^2)^{3/2}}{3f} - \frac{\sqrt{c}e(3af^2+2c(e^2-2df))\tan^{-1}\left(\frac{\sqrt{a+cx^2}}{d+ex+fx^2}\right)}{2f^4} \\
&= \frac{(2(af^2+c(e^2-df))-cefx)\sqrt{a+cx^2}}{2f^3} + \frac{(a+cx^2)^{3/2}}{3f} - \frac{\sqrt{c}e(3af^2+2c(e^2-2df))\tan^{-1}\left(\frac{\sqrt{a+cx^2}}{d+ex+fx^2}\right)}{2f^4}
\end{aligned}$$

Mathematica [A] time = 2.01, size = 755, normalized size = 1.37

$$8f^3 (a + cx^2)^{5/2} \sqrt{\frac{cx^2}{a} + 1} (\sqrt{e^2 - 4df} - e) + 8f^3 (a + cx^2)^{5/2} \sqrt{\frac{cx^2}{a} + 1} (\sqrt{e^2 - 4df} + e) + 3(e - \sqrt{e^2 - 4df}) \left(2\sqrt{\frac{cx^2}{a} + 1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x*(a + c*x^2)^(3/2))/(d + e*x + f*x^2), x]

[Out] (8*f^3*(-e + Sqrt[e^2 - 4*d*f])*(a + c*x^2)^(5/2)*Sqrt[1 + (c*x^2)/a] + 8*f^3*(e + Sqrt[e^2 - 4*d*f])*(a + c*x^2)^(5/2)*Sqrt[1 + (c*x^2)/a] + 3*(e - Sqrt[e^2 - 4*d*f])*(2*Sqrt[c]*f^2*(e - Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2]*(a*Sqrt[c]*x*(1 + (c*x^2)/a)^(3/2) + Sqrt[a]*(a + c*x^2)*ArcSinh[(Sqrt[c]*x)/Sqrt[a]]) - a*(4*a*f^2 + c*(e - Sqrt[e^2 - 4*d*f])^2)*(1 + (c*x^2)/a)^(3/2)*(2*f*Sqrt[a + c*x^2] + Sqrt[c]*(-e + Sqrt[e^2 - 4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] - Sqrt[2*c*e^2 - 4*c*d*f + 4*a*f^2 - 2*c*e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2])]) - 3*(e + Sqrt[e^2 - 4*d*f])*(2*Sqrt[c]*f^2*(e + Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2]*(a*Sqrt[c]*x*(1 + (c*x^2)/a)^(3/2) + Sqrt[a]*(a + c*x^2)*ArcSinh[(Sqrt[c]*x)/Sqrt[a]]) - a*(4*a*f^2 + c*(e + Sqrt[e^2 - 4*d*f])^2)*(1 + (c*x^2)/a)^(3/2)*(2*f*Sqrt[a + c*x^2] - Sqrt[c]*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] - Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2])]))/(48*a*f^4*Sqrt[e^2 - 4*d*f]*(1 + (c*x^2)/a)^(3/2))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^2+a)^(3/2)/(f*x^2+e*x+d), x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage₂

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^2+a)^(3/2)/(f*x^2+e*x+d), x, algorithm="giac")

[Out] sage2

maple [B] time = 0.02, size = 14709, normalized size = 26.60

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(c*x^2+a)^(3/2)/(f*x^2+e*x+d), x)`

[Out] result too large to display

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(c*x^2+a)^(3/2)/(f*x^2+e*x+d), x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details)Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x(c x^2 + a)^{3/2}}{f x^2 + e x + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*(a + c*x^2)^(3/2))/(d + e*x + f*x^2), x)`

[Out] `int((x*(a + c*x^2)^(3/2))/(d + e*x + f*x^2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + c x^2)^{\frac{3}{2}}}{d + e x + f x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(c*x**2+a)**(3/2)/(f*x**2+e*x+d), x)`

[Out] `Integral(x*(a + c*x**2)**(3/2)/(d + e*x + f*x**2), x)`

$$3.60 \quad \int \frac{(a+cx^2)^{3/2}}{d+ex+fx^2} dx$$

Optimal. Leaf size=484

$$\frac{(ce(e - \sqrt{e^2 - 4df})(2af^2 + c(e^2 - 2df)) - 2f(-a^2f^3 + 2acdf^2 + c^2d(e^2 - df))) \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2} \sqrt{a+cx^2} \sqrt{2af^2 + c(e^2 - 2df)}}\right)}{\sqrt{2} f^3 \sqrt{e^2 - 4df} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}$$

[Out] $\frac{1}{2}*(3*a*f^2+2*c*(-d*f+e^2))*\operatorname{arctanh}(x*c^{(1/2)}/(c*x^2+a)^{(1/2)})*c^{(1/2)}/f^3 - 1/2*c*(-f*x+2*e)*(c*x^2+a)^{(1/2)}/f^2 - 1/2*\operatorname{arctanh}(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^{(1/2)})))^2^{(1/2)}/(c*x^2+a)^{(1/2)}/(2*a*f^2+(-(-4*d*f+e^2)^{(1/2)})*e-2*d*f+e^2)*c^{(1/2)}*(-2*f*(2*a*c*d*f^2-a^2*f^3+c^2*d*(-d*f+e^2))+c*e*(2*a*f^2+c*(-2*d*f+e^2))*(e-(-4*d*f+e^2)^{(1/2)}))/f^3*2^{(1/2)}/(-4*d*f+e^2)^{(1/2)}/(2*a*f^2+(-(-4*d*f+e^2)^{(1/2)})*e-2*d*f+e^2)*c^{(1/2)}+1/2*\operatorname{arctanh}(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^{(1/2)})))^2^{(1/2)}/(c*x^2+a)^{(1/2)}/(2*a*f^2+((-4*d*f+e^2)^{(1/2)})*e-2*d*f+e^2)*c^{(1/2)}*(-2*f*(2*a*c*d*f^2-a^2*f^3+c^2*d*(-d*f+e^2))+c*e*(2*a*f^2+c*(-2*d*f+e^2))*(e+(-4*d*f+e^2)^{(1/2)}))/f^3*2^{(1/2)}/(-4*d*f+e^2)^{(1/2)}/(2*a*f^2+((-4*d*f+e^2)^{(1/2)})*e-2*d*f+e^2)*c^{(1/2)}$

Rubi [A] time = 4.24, antiderivative size = 482, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {979, 1080, 217, 206, 1034, 725}

$$\frac{(-2a^2f^4 - ce(e - \sqrt{e^2 - 4df})(2af^2 + c(e^2 - 2df)) + 4acdf^3 + 2c^2df(e^2 - df)) \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2} \sqrt{a+cx^2} \sqrt{2af^2 + c(e^2 - 2df)}}\right)}{\sqrt{2} f^3 \sqrt{e^2 - 4df} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^2)^(3/2)/(d + e*x + f*x^2), x]

[Out] $-(c*(2*e - f*x)*\operatorname{Sqrt}[a + c*x^2])/(2*f^2) + (\operatorname{Sqrt}[c]*(3*a*f^2 + 2*c*(e^2 - d*f))*\operatorname{ArcTanh}[(\operatorname{Sqrt}[c]*x)/\operatorname{Sqrt}[a + c*x^2]])/(2*f^3) + ((4*a*c*d*f^3 - 2*a^2*f^4 + 2*c^2*d*f*(e^2 - d*f) - c*e*(e - \operatorname{Sqrt}[e^2 - 4*d*f])*(2*a*f^2 + c*(e^2 - 2*d*f)))*\operatorname{ArcTanh}[(2*a*f - c*(e - \operatorname{Sqrt}[e^2 - 4*d*f])*x)/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[2*a*f^2 + c*(e^2 - 2*d*f - e*\operatorname{Sqrt}[e^2 - 4*d*f])])*\operatorname{Sqrt}[a + c*x^2]])/(\operatorname{Sqrt}[2]*f^3*\operatorname{Sqrt}[e^2 - 4*d*f]*\operatorname{Sqrt}[2*a*f^2 + c*(e^2 - 2*d*f - e*\operatorname{Sqrt}[e^2 - 4*d*f])]) - ((4*a*c*d*f^3 - 2*a^2*f^4 + 2*c^2*d*f*(e^2 - d*f) - c*e*(e + \operatorname{Sqrt}[e^2 - 4*d*f])*(2*a*f^2 + c*(e^2 - 2*d*f)))*\operatorname{ArcTanh}[(2*a*f - c*(e + \operatorname{Sqrt}[e^2 - 4*d*f])*x)/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[2*a*f^2 + c*(e^2 - 2*d*f + e*\operatorname{Sqrt}[e^2 - 4*d*f])])*\operatorname{Sqrt}$

$$\frac{[a + c*x^2])]}{(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])}$$

Rule 206

$$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \text{ :> } \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] \text{ /; } \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$$

Rule 217

$$\text{Int}[1/\text{Sqrt}[(a_ + (b_)*(x_)^2)], x_Symbol] \text{ :> } \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] \text{ /; } \text{FreeQ}[\{a, b\}, x] \ \&\& \ !\text{GtQ}[a, 0]$$

Rule 725

$$\text{Int}[1/(((d_ + (e_)*(x_))*\text{Sqrt}[(a_ + (c_)*(x_)^2)]), x_Symbol] \text{ :> } -\text{Subst}[\text{Int}[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/\text{Sqrt}[a + c*x^2]] \text{ /; } \text{FreeQ}[\{a, c, d, e\}, x]$$

Rule 979

$$\text{Int}[(a_ + (c_)*(x_)^2)^{(p_)}*((d_ + (e_)*(x_)) + (f_)*(x_)^2)^{(q_)}, x_Symbol] \text{ :> } -\text{Simp}[(c*(e*(2*p + q) - 2*f*(p + q)*x)*(a + c*x^2)^{(p - 1)}*(d + e*x + f*x^2)^{(q + 1)})/(2*f^2*(p + q)*(2*p + 2*q + 1)), x] - \text{Dist}[1/(2*f^2*(p + q)*(2*p + 2*q + 1)), \text{Int}[(a + c*x^2)^{(p - 2)}*(d + e*x + f*x^2)^q*\text{Simp}[-(a*c*e^2*(1 - p)*(2*p + q)) + a*(p + q)*(-2*a*f^2*(2*p + 2*q + 1) + c*(2*d*f - e^2*(2*p + q)) + (2*(c*d - a*f)*(c*e)*(1 - p)*(2*p + q) + 4*a*c*e*f*(1 - p)*(p + q))*x + (p*c^2*e^2*(1 - p) - c*(p + q)*(2*a*f^2*(4*p + 2*q - 1) + c*(2*d*f*(1 - 2*p) + e^2*(3*p + q - 1)))]*x^2, x], x] \text{ /; } \text{FreeQ}[\{a, c, d, e, f, q\}, x] \ \&\& \ \text{NeQ}[e^2 - 4*d*f, 0] \ \&\& \ \text{GtQ}[p, 1] \ \&\& \ \text{NeQ}[p + q, 0] \ \&\& \ \text{NeQ}[2*p + 2*q + 1, 0] \ \&\& \ !\text{IGtQ}[p, 0] \ \&\& \ !\text{IGtQ}[q, 0]$$

Rule 1034

$$\text{Int}[(g_ + (h_)*(x_))/(((a_ + (b_)*(x_)) + (c_)*(x_)^2)*\text{Sqrt}[(d_ + (f_)*(x_)^2)]), x_Symbol] \text{ :> } \text{With}[\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(2*c*g - h*(b - q))/q, \text{Int}[1/((b - q + 2*c*x)*\text{Sqrt}[d + f*x^2]), x], x] - \text{Dist}[(2*c*g - h*(b + q))/q, \text{Int}[1/((b + q + 2*c*x)*\text{Sqrt}[d + f*x^2]), x], x]] \text{ /; } \text{FreeQ}[\{a, b, c, d, f, g, h\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{PosQ}[b^2 - 4*a*c]$$

Rule 1080

$$\text{Int}[(A_ + (B_)*(x_)) + (C_)*(x_)^2)/(((a_ + (b_)*(x_)) + (c_)*(x_)^2)*\text{Sqrt}[(d_ + (f_)*(x_)^2)]), x_Symbol] \text{ :> } \text{Dist}[C/c, \text{Int}[1/\text{Sqrt}[d + f*x^2], x], x] + \text{Dist}[1/c, \text{Int}[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*\text{Sqrt}$$

$[d + f*x^2]), x], x] /; \text{FreeQ}[\{a, b, c, d, f, A, B, C\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rubi steps

$$\begin{aligned} \int \frac{(a + cx^2)^{3/2}}{d + ex + fx^2} dx &= -\frac{c(2e - fx)\sqrt{a + cx^2}}{2f^2} - \frac{\int \frac{af(cd-2af) - ce(2cd-af)x - c(3af^2 + 2c(e^2-df))x^2}{\sqrt{a+cx^2}(d+ex+fx^2)} dx}{2f^2} \\ &= -\frac{c(2e - fx)\sqrt{a + cx^2}}{2f^2} - \frac{\int \frac{af^2(cd-2af) + cd(3af^2 + 2c(e^2-df)) + (-cef(2cd-af) + ce(3af^2 + 2c(e^2-df)))x}{\sqrt{a+cx^2}(d+ex+fx^2)} dx}{2f^3} + \dots \\ &= -\frac{c(2e - fx)\sqrt{a + cx^2}}{2f^2} + \frac{(c(3af^2 + 2c(e^2 - df))) \text{Subst}\left(\int \frac{1}{1-cx^2} dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{2f^3} - \frac{(2f(af^2 + \dots))}{2f^3} \\ &= -\frac{c(2e - fx)\sqrt{a + cx^2}}{2f^2} + \frac{\sqrt{c}(3af^2 + 2c(e^2 - df)) \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2f^3} + \frac{(2f(af^2(cd - 2af) + \dots))}{2f^3} \\ &= -\frac{c(2e - fx)\sqrt{a + cx^2}}{2f^2} + \frac{\sqrt{c}(3af^2 + 2c(e^2 - df)) \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2f^3} - \frac{(ce(e - \sqrt{e^2 - 4df}))}{2f^3} \end{aligned}$$

Mathematica [A] time = 1.07, size = 603, normalized size = 1.25

$$\frac{2(2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)) \left(-\sqrt{4af^2 - 2ce\sqrt{e^2 - 4df} - 4cdf + 2ce^2} \tanh^{-1}\left(\frac{2af + cx(\sqrt{e^2 - 4df} - e)}{\sqrt{a+cx^2}\sqrt{4af^2 - 2c(e\sqrt{e^2 - 4df} + 2df - e^2)}}\right) + \sqrt{c}(\sqrt{e^2 - 4df} - e) \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right) \right)}{f^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^2)^(3/2)/(d + e*x + f*x^2), x]

[Out] ((2*sqrt[c]*(-e + sqrt[e^2 - 4*d*f])*sqrt[a + c*x^2]*(sqrt[c]*x*sqrt[1 + (c*x^2)/a] + sqrt[a]*ArcSinh[(sqrt[c]*x)/sqrt[a]]))/sqrt[1 + (c*x^2)/a] + (2*sqrt[c]*(e + sqrt[e^2 - 4*d*f])*sqrt[a + c*x^2]*(sqrt[c]*x*sqrt[1 + (c*x^2)/a] + sqrt[a]*ArcSinh[(sqrt[c]*x)/sqrt[a]]))/sqrt[1 + (c*x^2)/a] + (2*(2*a*

$$f^2 + c*(e^2 - 2*d*f - e*\sqrt{e^2 - 4*d*f}))*((2*f*\sqrt{a + c*x^2} + \sqrt{c}*(-e + \sqrt{e^2 - 4*d*f}))*\text{ArcTanh}[(\sqrt{c}*x)/\sqrt{a + c*x^2}] - \sqrt{2*c*e^2 - 4*c*d*f + 4*a*f^2 - 2*c*e*\sqrt{e^2 - 4*d*f}}*\text{ArcTanh}[(2*a*f + c*(-e + \sqrt{e^2 - 4*d*f}))*x]/(\sqrt{4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*\sqrt{e^2 - 4*d*f}})]*\sqrt{a + c*x^2}))/f^2 + (2*(2*a*f^2 + c*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f}))*(-2*f*\sqrt{a + c*x^2} + \sqrt{c}*(e + \sqrt{e^2 - 4*d*f}))*\text{ArcTanh}[(\sqrt{c}*x)/\sqrt{a + c*x^2}] + \sqrt{4*a*f^2 + 2*c*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f}})]*\text{ArcTanh}[(2*a*f - c*(e + \sqrt{e^2 - 4*d*f}))*x]/(\sqrt{4*a*f^2 + 2*c*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f}})]*\sqrt{a + c*x^2}))/f^2)/(8*f*\sqrt{e^2 - 4*d*f})$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage₂

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] sage₂

maple [B] time = 0.01, size = 8954, normalized size = 18.50

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)

[Out] result too large to display

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see 'assume?' for more details) Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^2 + a)^{3/2}}{fx^2 + ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c*x^2)^(3/2)/(d + e*x + f*x^2), x)

[Out] int((a + c*x^2)^(3/2)/(d + e*x + f*x^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + cx^2)^{3/2}}{d + ex + fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+a)**(3/2)/(f*x**2+e*x+d), x)

[Out] Integral((a + c*x**2)**(3/2)/(d + e*x + f*x**2), x)

$$3.61 \quad \int \frac{(a+cx^2)^{3/2}}{x(d+ex+fx^2)} dx$$

Optimal. Leaf size=496

$$\frac{a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right) \left(2ef(c^2d^2 - a^2f^2) - (e - \sqrt{e^2 - 4df})(c^2de^2 - f(cd - af)^2)\right) \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2} \sqrt{a+cx^2} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{d \sqrt{2} df^2 \sqrt{e^2 - 4df} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}$$

[Out] $-c^{(3/2)} * e * \operatorname{arctanh}(x * c^{(1/2)} / (c * x^2 + a)^{(1/2)}) / f^2 - a^{(3/2)} * \operatorname{arctanh}((c * x^2 + a)^{(1/2)} / a^{(1/2)}) / d + a * (c * x^2 + a)^{(1/2)} / d + (-a * f + c * d) * (c * x^2 + a)^{(1/2)} / d / f - 1/2 * \operatorname{arctanh}(1/2 * (2 * a * f - c * x * (e - (-4 * d * f + e^2)^{(1/2)}))) * 2^{(1/2)} / (c * x^2 + a)^{(1/2)} / (2 * a * f^2 + (-(-4 * d * f + e^2)^{(1/2)} * e - 2 * d * f + e^2) * c)^{(1/2)}) * (2 * e * f * (-a^2 * f^2 + c^2 * d^2) - (c^2 * d * e^2 - f * (-a * f + c * d)^2) * (e - (-4 * d * f + e^2)^{(1/2)})) / d / f^2 * 2^{(1/2)} / (-4 * d * f + e^2)^{(1/2)} / (2 * a * f^2 + (-(-4 * d * f + e^2)^{(1/2)} * e - 2 * d * f + e^2) * c)^{(1/2)} + 1/2 * \operatorname{arctanh}(1/2 * (2 * a * f - c * x * (e + (-4 * d * f + e^2)^{(1/2)}))) * 2^{(1/2)} / (c * x^2 + a)^{(1/2)} / (2 * a * f^2 + ((-4 * d * f + e^2)^{(1/2)} * e - 2 * d * f + e^2) * c)^{(1/2)}) * (2 * e * f * (-a^2 * f^2 + c^2 * d^2) - (c^2 * d * e^2 - f * (-a * f + c * d)^2) * (e + (-4 * d * f + e^2)^{(1/2)})) / d / f^2 * 2^{(1/2)} / (-4 * d * f + e^2)^{(1/2)} / (2 * a * f^2 + ((-4 * d * f + e^2)^{(1/2)} * e - 2 * d * f + e^2) * c)^{(1/2)}$

Rubi [A] time = 2.57, antiderivative size = 496, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {6728, 266, 50, 63, 208, 1020, 1080, 217, 206, 1034, 725}

$$\frac{\left(2ef(c^2d^2 - a^2f^2) - (e - \sqrt{e^2 - 4df})(c^2de^2 - f(cd - af)^2)\right) \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2} \sqrt{a+cx^2} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2} df^2 \sqrt{e^2 - 4df} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}} +$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^2)^(3/2)/(x*(d + e*x + f*x^2)),x]

[Out] $(a * \operatorname{Sqrt}[a + c * x^2]) / d + ((c * d - a * f) * \operatorname{Sqrt}[a + c * x^2]) / (d * f) - (c^{(3/2)} * e * \operatorname{ArcTanh}[(\operatorname{Sqrt}[c] * x) / \operatorname{Sqrt}[a + c * x^2]]) / f^2 - ((2 * e * f * (c^2 * d^2 - a^2 * f^2) - (c^2 * d * e^2 - f * (c * d - a * f)^2) * (e - \operatorname{Sqrt}[e^2 - 4 * d * f])) * \operatorname{ArcTanh}[(2 * a * f - c * (e - \operatorname{Sqrt}[e^2 - 4 * d * f]) * x) / (\operatorname{Sqrt}[2] * \operatorname{Sqrt}[2 * a * f^2 + c * (e^2 - 2 * d * f - e * \operatorname{Sqrt}[e^2 - 4 * d * f])]) * \operatorname{Sqrt}[a + c * x^2]]) / ((\operatorname{Sqrt}[2] * d * f^2 * \operatorname{Sqrt}[e^2 - 4 * d * f]) * \operatorname{Sqrt}[2 * a * f^2 + c * (e^2 - 2 * d * f - e * \operatorname{Sqrt}[e^2 - 4 * d * f])]) + ((2 * e * f * (c^2 * d^2 - a^2 * f^2) - (c^2 * d * e^2 - f * (c * d - a * f)^2) * (e + \operatorname{Sqrt}[e^2 - 4 * d * f])) * \operatorname{ArcTanh}[(2 * a * f - c * (e + \operatorname{Sqrt}[e^2 - 4 * d * f]) * x) / (\operatorname{Sqrt}[2] * \operatorname{Sqrt}[2 * a * f^2 + c * (e^2 - 2 * d * f + e * \operatorname{Sqrt}[e^2 - 4 * d * f])]) * \operatorname{Sqrt}[a + c * x^2]]) / ((\operatorname{Sqrt}[2] * d * f^2 * \operatorname{Sqrt}[e^2 - 4 * d * f]) * \operatorname{Sqrt}[2 * a * f^2 + c * (e^2 - 2 * d * f + e * \operatorname{Sqrt}[e^2 - 4 * d * f])])$

$f^2 + c(e^2 - 2df + e\sqrt{e^2 - 4df})) - (a^{3/2} \operatorname{ArcTanh}[\sqrt{a + cx^2}/\sqrt{a}])/d$

Rule 50

$\operatorname{Int}[(a_.) + (b_.)x^{(m_)}((c_.) + (d_.)x^{(n_)}), x_Symbol] \rightarrow \operatorname{Simp}[(a + bx)^{(m+1)}(c + dx)^n/(b(m+n+1)), x] + \operatorname{Dist}[(n(b*c - a*d))/(b(m+n+1)), \operatorname{Int}[(a + bx)^m(c + dx)^{(n-1)}, x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d\}, x$ && $\operatorname{NeQ}[b*c - a*d, 0]$ && $\operatorname{GtQ}[n, 0]$ && $\operatorname{NeQ}[m + n + 1, 0]$ && $!(\operatorname{IGtQ}[m, 0] \&\& (!\operatorname{IntegerQ}[n] \mid\mid (\operatorname{GtQ}[m, 0] \&\& \operatorname{LtQ}[m - n, 0])))$ && $!\operatorname{ILtQ}[m + n + 2, 0]$ && $\operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 63

$\operatorname{Int}[(a_.) + (b_.)x^{(m_)}((c_.) + (d_.)x^{(n_)}), x_Symbol] \rightarrow \operatorname{With}\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p(m+1)-1)}(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + bx)^{(1/p)}], x] /;$ $\operatorname{FreeQ}\{a, b, c, d\}, x$ && $\operatorname{NeQ}[b*c - a*d, 0]$ && $\operatorname{LtQ}[-1, m, 0]$ && $\operatorname{LeQ}[-1, n, 0]$ && $\operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]]$ && $\operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 206

$\operatorname{Int}[(a_.) + (b_.)x^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*x]/\operatorname{Rt}[a, 2])]/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /;$ $\operatorname{FreeQ}\{a, b\}, x$ && $\operatorname{NegQ}[a/b]$ && $(\operatorname{GtQ}[a, 0] \mid\mid \operatorname{LtQ}[b, 0])$

Rule 208

$\operatorname{Int}[(a_.) + (b_.)x^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /;$ $\operatorname{FreeQ}\{a, b\}, x$ && $\operatorname{NegQ}[a/b]$

Rule 217

$\operatorname{Int}[1/\sqrt{(a_.) + (b_.)x^2}], x_Symbol] \rightarrow \operatorname{Subst}[\operatorname{Int}[1/(1 - bx^2), x], x, x/\sqrt{a + bx^2}] /;$ $\operatorname{FreeQ}\{a, b\}, x$ && $!\operatorname{GtQ}[a, 0]$

Rule 266

$\operatorname{Int}[x^{(m_)}((a_.) + (b_.)x^{(n_)})^{(p_)}], x_Symbol] \rightarrow \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[x^{(\operatorname{Simplify}[(m+1)/n] - 1)*(a + bx)^p}, x], x, x^n], x] /;$ $\operatorname{FreeQ}\{a, b, m, n, p\}, x$ && $\operatorname{IntegerQ}[\operatorname{Simplify}[(m+1)/n]]$

Rule 725

$\operatorname{Int}[1/(((d_.) + (e_.)x)*\sqrt{(a_.) + (c_.)x^2})], x_Symbol] \rightarrow -\operatorname{Subst}[\operatorname{Int}[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/\sqrt{a + cx^2}] /;$ FreeQ

[{a, c, d, e}, x]

Rule 1020

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x] /; FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1080

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (f_.)*(x_)^2], x_Symbol] := Dist[C/c, Int[1/Sqrt[d + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+cx^2)^{3/2}}{x(d+ex+fx^2)} dx &= \int \left(\frac{(a+cx^2)^{3/2}}{dx} + \frac{(-e-fx)(a+cx^2)^{3/2}}{d(d+ex+fx^2)} \right) dx \\
&= \frac{\int \frac{(a+cx^2)^{3/2}}{x} dx}{d} + \frac{\int \frac{(-e-fx)(a+cx^2)^{3/2}}{d+ex+fx^2} dx}{d} \\
&= -\frac{(a+cx^2)^{3/2}}{3d} + \frac{\text{Subst}\left(\int \frac{(a+cx)^{3/2}}{x} dx, x, x^2\right)}{2d} + \frac{\int \frac{(-3aef+3f(cd-af)x)\sqrt{a+cx^2}}{d+ex+fx^2} dx}{3df} \\
&= \frac{(cd-af)\sqrt{a+cx^2}}{df} + \frac{a \text{Subst}\left(\int \frac{\sqrt{a+cx}}{x} dx, x, x^2\right)}{2d} + \frac{\int \frac{-3a^2ef^2-3f(cd-af)^2x-3c^2defx^2}{\sqrt{a+cx^2}(d+ex+fx^2)} dx}{3df^2} \\
&= \frac{a\sqrt{a+cx^2}}{d} + \frac{(cd-af)\sqrt{a+cx^2}}{df} + \frac{a^2 \text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}} dx, x, x^2\right)}{2d} + \frac{\int \frac{3c^2d^2ef-3a^2ef^3+(3c^2d^2ef-3a^2ef^3+3c^2d^2ef-3a^2ef^3)}{\sqrt{a+cx^2}(d+ex+fx^2)} dx}{3d} \\
&= \frac{a\sqrt{a+cx^2}}{d} + \frac{(cd-af)\sqrt{a+cx^2}}{df} + \frac{a^2 \text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}} dx, x, \sqrt{a+cx^2}\right)}{cd} - \frac{(c^2e) \text{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}} dx, x, \sqrt{a+cx^2}\right)}{d} \\
&= \frac{a\sqrt{a+cx^2}}{d} + \frac{(cd-af)\sqrt{a+cx^2}}{df} - \frac{c^{3/2}e \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f^2} - \frac{a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d} - \frac{(c^2e) \text{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}} dx, x, \sqrt{a+cx^2}\right)}{d} \\
&= \frac{a\sqrt{a+cx^2}}{d} + \frac{(cd-af)\sqrt{a+cx^2}}{df} - \frac{c^{3/2}e \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f^2} - \frac{(2ef(c^2d^2-a^2f^2)-(c^2de) \text{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}} dx, x, \sqrt{a+cx^2}\right))}{d}
\end{aligned}$$

Mathematica [A] time = 1.58, size = 746, normalized size = 1.50

$$\frac{a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d} - \frac{c^{3/2}e \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f^2} + \frac{a\sqrt{af^2 + \frac{1}{2}c(e\sqrt{e^2-4df}-2df+e^2)} \tanh^{-1}\left(\frac{2af-cx(\sqrt{e^2-4df}-2df+e^2)}{\sqrt{a+cx^2}\sqrt{4af^2+2c(e\sqrt{e^2-4df}-2df+e^2)}}\right)}{2df}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^2)^(3/2)/(x*(d + e*x + f*x^2)), x]

[Out] (c*Sqrt[a + c*x^2])/f - (c^(3/2)*e*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/f^2 - ((c*d*(-e + Sqrt[e^2 - 4*d*f]) - a*f*(e + Sqrt[e^2 - 4*d*f]))*Sqrt[4*a*

$$f^2 - 2*c*(-e^2 + 2*d*f + e*\sqrt{e^2 - 4*d*f}))*\text{ArcTanh}[(2*a*f + c*(-e + \sqrt{e^2 - 4*d*f})*x)/(\sqrt{4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*\sqrt{e^2 - 4*d*f})})*\sqrt{a + c*x^2}]]/(4*d*f^2*\sqrt{e^2 - 4*d*f}) - (c*\sqrt{a*f^2 + (c*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f}))}/2)*\text{ArcTanh}[(2*a*f - c*(e + \sqrt{e^2 - 4*d*f})*x)/(\sqrt{4*a*f^2 + 2*c*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f})})*\sqrt{a + c*x^2}]]/(2*f^2) + (a*\sqrt{a*f^2 + (c*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f}))}/2)*\text{ArcTanh}[(2*a*f - c*(e + \sqrt{e^2 - 4*d*f})*x)/(\sqrt{4*a*f^2 + 2*c*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f})})*\sqrt{a + c*x^2}]]/(2*d*f) - (c*e*\sqrt{4*a*f^2 + 2*c*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f})})*\text{ArcTanh}[(2*a*f - c*(e + \sqrt{e^2 - 4*d*f})*x)/(\sqrt{4*a*f^2 + 2*c*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f})})*\sqrt{a + c*x^2}]]/(4*f^2*\sqrt{e^2 - 4*d*f}) - (a*e*\sqrt{4*a*f^2 + 2*c*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f})})*\text{ArcTanh}[(2*a*f - c*(e + \sqrt{e^2 - 4*d*f})*x)/(\sqrt{4*a*f^2 + 2*c*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f})})*\sqrt{a + c*x^2}]]/(4*d*f*\sqrt{e^2 - 4*d*f}) - (a^(3/2)*\text{ArcTanh}[\sqrt{a + c*x^2}/\sqrt{a}])/d$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(3/2)/x/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(3/2)/x/(f*x^2+e*x+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);;OUTPUT:Error: Bad Argument Type

maple [B] time = 0.02, size = 9728, normalized size = 19.61

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+a)^(3/2)/x/(f*x^2+e*x+d),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2 + a)^{\frac{3}{2}}}{(fx^2 + ex + d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(3/2)/x/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] integrate((c*x^2 + a)^(3/2)/((f*x^2 + e*x + d)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^2 + a)^{3/2}}{x(fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c*x^2)^(3/2)/(x*(d + e*x + f*x^2)),x)

[Out] int((a + c*x^2)^(3/2)/(x*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + cx^2)^{\frac{3}{2}}}{x(d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+a)**(3/2)/x/(f*x**2+e*x+d),x)

[Out] Integral((a + c*x**2)**(3/2)/(x*(d + e*x + f*x**2)), x)

$$3.62 \quad \int \frac{(a+cx^2)^{3/2}}{x^2(d+ex+fx^2)} dx$$

Optimal. Leaf size=604

$$\frac{a^{3/2}e \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right) \left(a^2f^2(e\sqrt{e^2-4df}-2df+e^2) + 4acd^2f^2 + c^2d^2(-e\sqrt{e^2-4df}-2df+e^2)\right) \tanh^{-1}\left(\frac{\sqrt{2af^2+cx(-e\sqrt{e^2-4df}-2df+e^2)}}{\sqrt{2}d^2f\sqrt{e^2-4df}}\right)}{d^2}$$

[Out] $-(c*x^2+a)^{(3/2)}/d/x+a^{(3/2)}*e*\operatorname{arctanh}((c*x^2+a)^{(1/2)}/a^{(1/2)})/d^2+3/2*a*a$
 $\operatorname{rctanh}(x*c^{(1/2)}/(c*x^2+a)^{(1/2)})*c^{(1/2)}/d+1/2*(-3*a*f+2*c*d)*\operatorname{arctanh}(x*c^{(1/2)}/(c*x^2+a)^{(1/2)})*c^{(1/2)}/d/f-a*e*(c*x^2+a)^{(1/2)}/d^2+3/2*c*x*(c*x^2+a)^{(1/2)}/d+1/2*(-c*d*x+2*a*e)*(c*x^2+a)^{(1/2)}/d^2-1/2*\operatorname{arctanh}(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^{(1/2)}))*2^{(1/2)}/(c*x^2+a)^{(1/2)}/(2*a*f^2+(-(-4*d*f+e^2)^{(1/2)})*e-2*d*f+e^2)*c)^{(1/2)}*(4*a*c*d^2*f^2+c^2*d^2*(-(-4*d*f+e^2)^{(1/2)})*e-2*d*f+e^2)+a^2*f^2*((-4*d*f+e^2)^{(1/2)})*e-2*d*f+e^2)/d^2/f*2^{(1/2)}/(-4*d*f+e^2)^{(1/2)}/(2*a*f^2+(-(-4*d*f+e^2)^{(1/2)})*e-2*d*f+e^2)*c)^{(1/2)}+1/2*\operatorname{arctanh}(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^{(1/2)}))*2^{(1/2)}/(c*x^2+a)^{(1/2)}/(2*a*f^2+(-4*d*f+e^2)^{(1/2)})*e-2*d*f+e^2)*c)^{(1/2)}*(4*a*c*d^2*f^2+a^2*f^2*(-(-4*d*f+e^2)^{(1/2)})*e-2*d*f+e^2)+c^2*d^2*((-4*d*f+e^2)^{(1/2)})*e-2*d*f+e^2)/d^2/f*2^{(1/2)}/(-4*d*f+e^2)^{(1/2)}/(2*a*f^2+(-(-4*d*f+e^2)^{(1/2)})*e-2*d*f+e^2)*c)^{(1/2)}$

Rubi [A] time = 2.81, antiderivative size = 604, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.518$, Rules used = {6728, 277, 195, 217, 206, 266, 50, 63, 208, 1020, 1068, 1080, 1034, 725}

$$\frac{\left(a^2f^2(e\sqrt{e^2-4df}-2df+e^2) + 4acd^2f^2 + c^2d^2(-e\sqrt{e^2-4df}-2df+e^2)\right) \tanh^{-1}\left(\frac{2af-cx(e-\sqrt{e^2-4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}d^2f\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^2)^(3/2)/(x^2*(d + e*x + f*x^2)),x]

[Out] $-((a*e*\operatorname{Sqrt}[a + c*x^2])/d^2) + (3*c*x*\operatorname{Sqrt}[a + c*x^2])/(2*d) + ((2*a*e - c*d*x)*\operatorname{Sqrt}[a + c*x^2])/(2*d^2) - (a + c*x^2)^{(3/2)}/(d*x) + (3*a*\operatorname{Sqrt}[c]*\operatorname{ArcTanh}[(\operatorname{Sqrt}[c]*x)/\operatorname{Sqrt}[a + c*x^2]])/(2*d) + (\operatorname{Sqrt}[c]*(2*c*d - 3*a*f)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[c]*x)/\operatorname{Sqrt}[a + c*x^2]])/(2*d*f) - ((4*a*c*d^2*f^2 + c^2*d^2*(e^2 - 2*d*f - e*\operatorname{Sqrt}[e^2 - 4*d*f]) + a^2*f^2*(e^2 - 2*d*f + e*\operatorname{Sqrt}[e^2 - 4*d*f]))*\operatorname{ArcTanh}[(2*a*f - c*(e - \operatorname{Sqrt}[e^2 - 4*d*f]))*x]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[2*a*f^2 + c*(e^2 - 2*d*f + e*\operatorname{Sqrt}[e^2 - 4*d*f])])$

$$\frac{-2*d*f - e*\sqrt{e^2 - 4*d*f}}{\sqrt{2}*d^2*f*\sqrt{e^2 - 4*d*f}}*\sqrt{a + c*x^2}}{(\sqrt{2}*d^2*f*\sqrt{e^2 - 4*d*f}*\sqrt{2*a*f^2 + c*(e^2 - 2*d*f - e*\sqrt{e^2 - 4*d*f})}) + ((4*a*c*d^2*f^2 + a^2*f^2*(e^2 - 2*d*f - e*\sqrt{e^2 - 4*d*f}) + c^2*d^2*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f}))*\text{ArcTanh}[(2*a*f - c*(e + \sqrt{e^2 - 4*d*f})*x)/(\sqrt{2}*d*\sqrt{2*a*f^2 + c*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f})}*\sqrt{a + c*x^2})])}/(\sqrt{2}*d^2*f*\sqrt{e^2 - 4*d*f}*\sqrt{2*a*f^2 + c*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f})}) + (a^{3/2}*e*\text{ArcTanh}[\sqrt{a + c*x^2}/\sqrt{a}])/d^2$$

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p
+ 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 266

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 277

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 725

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (c_)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 1020

Int[((g_) + (h_)*(x_))*((a_) + (c_)*(x_)^2)^(p_)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x] /; FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]

Rule 1034

Int[((g_) + (h_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (f_)*(x_)^2], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rule 1068

Int[((a_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x_)^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[(B*c*f*(2*p + 2*q + 3)

```

+ C*(-(c*e*(2*p + q + 2))) + 2*c*C*f*(p + q + 1)*x*(a + c*x^2)^p*(d + e*x
+ f*x^2)^(q + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f
^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)
^q* Simp[p*(-(a*e))*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p +
q + 1)*(a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3
))] + (2*p*(c*d - a*f)*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(C*e*f*p*(-4*a*c)))*x + (p*(c*e)*(C*(c*e)*(q + 1) - c*(C*e - B*
f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(-4*a*c) - c^2*(C*(e^2 - 4*d*f)*
(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3)))]*x^2, x], x]
/; FreeQ[{a, c, d, e, f, A, B, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0
] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q
, 0]

```

Rule 1080

```

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + f*x^2],
x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]

```

Rule 6728

```

Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{(a+cx^2)^{3/2}}{x^2(d+ex+fx^2)} dx &= \int \left(\frac{(a+cx^2)^{3/2}}{dx^2} - \frac{e(a+cx^2)^{3/2}}{d^2x} + \frac{(e^2-df+efx)(a+cx^2)^{3/2}}{d^2(d+ex+fx^2)} \right) dx \\
&= \frac{\int \frac{(e^2-df+efx)(a+cx^2)^{3/2}}{d+ex+fx^2} dx}{d^2} + \frac{\int \frac{(a+cx^2)^{3/2}}{x^2} dx}{d} - \frac{e \int \frac{(a+cx^2)^{3/2}}{x} dx}{d^2} \\
&= \frac{e(a+cx^2)^{3/2}}{3d^2} - \frac{(a+cx^2)^{3/2}}{dx} + \frac{(3c) \int \sqrt{a+cx^2} dx}{d} - \frac{e \operatorname{Subst} \left(\int \frac{(a+cx)^{3/2}}{x} dx, x, x^2 \right)}{2d^2} + \\
&= \frac{3cx\sqrt{a+cx^2}}{2d} + \frac{(2ae-cdx)\sqrt{a+cx^2}}{2d^2} - \frac{(a+cx^2)^{3/2}}{dx} + \frac{(3ac) \int \frac{1}{\sqrt{a+cx^2}} dx}{2d} - \frac{(ae) \operatorname{Subst} \left(\int \frac{1}{x} dx, x, x^2 \right)}{2d^2} \\
&= -\frac{ae\sqrt{a+cx^2}}{d^2} + \frac{3cx\sqrt{a+cx^2}}{2d} + \frac{(2ae-cdx)\sqrt{a+cx^2}}{2d^2} - \frac{(a+cx^2)^{3/2}}{dx} + \frac{(3ac) \operatorname{Subst} \left(\int \frac{1}{\sqrt{a+cx^2}} dx, x, x^2 \right)}{2d} \\
&= -\frac{ae\sqrt{a+cx^2}}{d^2} + \frac{3cx\sqrt{a+cx^2}}{2d} + \frac{(2ae-cdx)\sqrt{a+cx^2}}{2d^2} - \frac{(a+cx^2)^{3/2}}{dx} + \frac{3a\sqrt{c} \operatorname{tanh}^{-1} \left(\frac{\sqrt{cx^2+a}}{\sqrt{a}} \right)}{2d} \\
&= -\frac{ae\sqrt{a+cx^2}}{d^2} + \frac{3cx\sqrt{a+cx^2}}{2d} + \frac{(2ae-cdx)\sqrt{a+cx^2}}{2d^2} - \frac{(a+cx^2)^{3/2}}{dx} + \frac{3a\sqrt{c} \operatorname{tanh}^{-1} \left(\frac{\sqrt{cx^2+a}}{\sqrt{a}} \right)}{2d} \\
&= -\frac{ae\sqrt{a+cx^2}}{d^2} + \frac{3cx\sqrt{a+cx^2}}{2d} + \frac{(2ae-cdx)\sqrt{a+cx^2}}{2d^2} - \frac{(a+cx^2)^{3/2}}{dx} + \frac{3a\sqrt{c} \operatorname{tanh}^{-1} \left(\frac{\sqrt{cx^2+a}}{\sqrt{a}} \right)}{2d}
\end{aligned}$$

Mathematica [C] time = 4.16, size = 885, normalized size = 1.47

$$-x \left(2\sqrt{a} \sqrt{c} df \sqrt{e^2 - 4df} \sqrt{cx^2 + a} \sinh^{-1} \left(\frac{\sqrt{cx}}{\sqrt{a}} \right) + \sqrt{\frac{cx^2}{a} + 1} \left(-2c \sqrt{4af^2 + 2c(e^2 + \sqrt{e^2 - 4df} e - 2df)} \operatorname{tanh}^{-1} \left(\frac{\sqrt{cx^2 + a}}{\sqrt{a}} \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^2)^(3/2)/(x^2*(d + e*x + f*x^2)), x]

```
[Out] 
$$\begin{aligned} & \left( -\left( x \left( 2\sqrt{a} \sqrt{c} d f \sqrt{e^2 - 4df} \sqrt{a + cx^2} \operatorname{ArcSinh} \left[ \frac{\sqrt{c} x}{\sqrt{a}} \right] + \sqrt{1 + \frac{cx^2}{a}} \left( 2cd f \sqrt{e^2 - 4df} x \sqrt{a + cx^2} - 4\sqrt{c} d (cd - af) \sqrt{e^2 - 4df} \operatorname{ArcTanh} \left[ \frac{\sqrt{c} x}{\sqrt{a + cx^2}} \right] + (2cd^2 + a(e^2 - 2df + e\sqrt{e^2 - 4df})) \sqrt{4af^2 - 2c(-e^2 + 2df + e\sqrt{e^2 - 4df})} \operatorname{ArcTanh} \left[ \frac{2af + c(-e + \sqrt{e^2 - 4df}) x}{\sqrt{4af^2 - 2c(-e^2 + 2df + e\sqrt{e^2 - 4df})}} \right] \right) \sqrt{a + cx^2} \right) + \sqrt{2} a e \sqrt{e^2 - 4df} \sqrt{2af^2 + c(e^2 - 2df + e\sqrt{e^2 - 4df})} \operatorname{ArcTanh} \left[ \frac{(2af - c(e + \sqrt{e^2 - 4df})) x}{\sqrt{4af^2 + 2c(e^2 - 2df + e\sqrt{e^2 - 4df})}} \right] \sqrt{a + cx^2} \right) - 2cd^2 \sqrt{4af^2 + 2c(e^2 - 2df + e\sqrt{e^2 - 4df})} \operatorname{ArcTanh} \left[ \frac{(2af - c(e + \sqrt{e^2 - 4df})) x}{\sqrt{4af^2 + 2c(e^2 - 2df + e\sqrt{e^2 - 4df})}} \right] \sqrt{a + cx^2} \right) - a e^2 \sqrt{4af^2 + 2c(e^2 - 2df + e\sqrt{e^2 - 4df})} \operatorname{ArcTanh} \left[ \frac{(2af - c(e + \sqrt{e^2 - 4df})) x}{\sqrt{4af^2 + 2c(e^2 - 2df + e\sqrt{e^2 - 4df})}} \right] \sqrt{a + cx^2} \right) + 2a d f \sqrt{4af^2 + 2c(e^2 - 2df + e\sqrt{e^2 - 4df})} \operatorname{ArcTanh} \left[ \frac{(2af - c(e + \sqrt{e^2 - 4df})) x}{\sqrt{4af^2 + 2c(e^2 - 2df + e\sqrt{e^2 - 4df})}} \right] \sqrt{a + cx^2} \right) - 4a^{3/2} e f \sqrt{e^2 - 4df} \operatorname{ArcTanh} \left[ \frac{\sqrt{a + cx^2}}{\sqrt{a}} \right] \right) - 4a d f \sqrt{e^2 - 4df} \sqrt{a + cx^2} \operatorname{Hypergeometric2F1} \left[ -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, -\left( \frac{cx^2}{a} \right) \right] / (4d^2 f \sqrt{e^2 - 4df} x \sqrt{1 + \frac{cx^2}{a}} \right) \end{aligned}$$

```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x^2/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage₂

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x^2/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage2
```

maple [B] time = 0.02, size = 9912, normalized size = 16.41

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)/x^2/(f*x^2+e*x+d),x)
```

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2 + a)^{\frac{3}{2}}}{(fx^2 + ex + d)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(3/2)/x^2/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] integrate((c*x^2 + a)^(3/2)/((f*x^2 + e*x + d)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^2 + a)^{3/2}}{x^2 (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c*x^2)^(3/2)/(x^2*(d + e*x + f*x^2)),x)

[Out] int((a + c*x^2)^(3/2)/(x^2*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + cx^2)^{\frac{3}{2}}}{x^2 (d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+a)**(3/2)/x**2/(f*x**2+e*x+d),x)

[Out] Integral((a + c*x**2)**(3/2)/(x**2*(d + e*x + f*x**2)), x)

$$3.63 \quad \int \frac{(a+cx^2)^{3/2}}{x^3(d+ex+fx^2)} dx$$

Optimal. Leaf size=668

$$\frac{a^{3/2}(e^2-df) \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right) (a^2f(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3)+2acd^2f(\sqrt{e^2-4df}+e)+c^2d^3(e-\sqrt{e^2-4df}))}{d^3} + \frac{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}$$

[Out] $-1/2*(c*x^2+a)^{(3/2)}/d/x^2+e*(c*x^2+a)^{(3/2)}/d^2/x-a^{(3/2)}*(-d*f+e^2)*\arctan\left(\frac{(c*x^2+a)^{(1/2)}/a^{(1/2)}}{d^3-3/2*c*\arctanh((c*x^2+a)^{(1/2)}/a^{(1/2)})}\right)*a^{(1/2)}/d+3/2*c*(c*x^2+a)^{(1/2)}/d+a*(-d*f+e^2)*(c*x^2+a)^{(1/2)}/d^3-3/2*c*e*x*(c*x^2+a)^{(1/2)}/d^2-1/2*(2*c*d^2+2*a*(-d*f+e^2)-c*d*e*x)*(c*x^2+a)^{(1/2)}/d^3+1/2*\arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^{(1/2)})))*2^{(1/2)}/(c*x^2+a)^{(1/2)}/(2*a*f^2+(-(-4*d*f+e^2)^{(1/2)}*e-2*d*f+e^2)*c)^{(1/2)}*(c^2*d^3*(e-(-4*d*f+e^2)^{(1/2)}))+2*a*c*d^2*f*(e+(-4*d*f+e^2)^{(1/2)}))+a^2*f*(e^3-3*d*e*f+e^2*(-4*d*f+e^2)^{(1/2)}-d*f*(-4*d*f+e^2)^{(1/2)})/d^3*2^{(1/2)}/(-4*d*f+e^2)^{(1/2)}/(2*a*f^2+(-(-4*d*f+e^2)^{(1/2)}*e-2*d*f+e^2)*c)^{(1/2)}-1/2*\arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^{(1/2)})))*2^{(1/2)}/(c*x^2+a)^{(1/2)}/(2*a*f^2+((-4*d*f+e^2)^{(1/2)}*e-2*d*f+e^2)*c)^{(1/2)}*(2*a*c*d^2*f*(e-(-4*d*f+e^2)^{(1/2)}))+c^2*d^3*(e+(-4*d*f+e^2)^{(1/2)}))+a^2*f*(e^3-3*d*e*f-e^2*(-4*d*f+e^2)^{(1/2)}+d*f*(-4*d*f+e^2)^{(1/2)})/d^3*2^{(1/2)}/(-4*d*f+e^2)^{(1/2)}/(2*a*f^2+((-4*d*f+e^2)^{(1/2)}*e-2*d*f+e^2)*c)^{(1/2)}$

Rubi [A] time = 3.46, antiderivative size = 668, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 15, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6728, 266, 47, 50, 63, 208, 277, 195, 217, 206, 1020, 1068, 1080, 1034, 725}

$$\frac{(a^2f(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3)+2acd^2f(\sqrt{e^2-4df}+e)+c^2d^3(e-\sqrt{e^2-4df}))\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^2)^(3/2)/(x^3*(d + e*x + f*x^2)), x]

[Out] $(3*c*\text{Sqrt}[a + c*x^2])/(2*d) + (a*(e^2 - d*f)*\text{Sqrt}[a + c*x^2])/d^3 - (3*c*e*x*\text{Sqrt}[a + c*x^2])/(2*d^2) - ((2*(c*d^2 + a*(e^2 - d*f)) - c*d*e*x)*\text{Sqrt}[a + c*x^2])/(2*d^3) - (a + c*x^2)^{(3/2)}/(2*d*x^2) + (e*(a + c*x^2)^{(3/2)})/(d^2*x) + ((c^2*d^3*(e - \text{Sqrt}[e^2 - 4*d*f]) + 2*a*c*d^2*f*(e + \text{Sqrt}[e^2 - 4*d*f])$


```
f]) + a^2*f*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f])
)*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(
e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[2]*d^3*Sqrt[e^
2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]])] - ((2*a*c
*d^2*f*(e - Sqrt[e^2 - 4*d*f]) + c^2*d^3*(e + Sqrt[e^2 - 4*d*f]) + a^2*f*(e
^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f] + d*f*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a
*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f +
e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqr
t[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])] - (3*Sqrt[a]*c*ArcTanh[
Sqrt[a + c*x^2]/Sqrt[a]])/(2*d) - (a^(3/2)*(e^2 - d*f)*ArcTanh[Sqrt[a + c*x
^2]/Sqrt[a]])/d^3
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 195

```
Int[((a_.) + (b_.)*(x_))^(n_)]^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p
+ 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
Denominator[p]])
```

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 277

Int[((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 725

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 1020

Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x] /; FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N

eQ[p + q + 1, 0]

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1068

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((B*c*f*(2*p + 2*q + 3) + C*(-(c*e*(2*p + q + 2))) + 2*c*C*f*(p + q + 1)*x*(a + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(-(a*e))*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))] + (2*p*(c*d - a*f)*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(-4*a*c)))*x + (p*(c*e)*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(-4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3)))]*x^2, x], x] /; FreeQ[{a, c, d, e, f, A, B, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]
```

Rule 1080

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+cx^2)^{3/2}}{x^3(d+ex+fx^2)} dx &= \int \left(\frac{(a+cx^2)^{3/2}}{dx^3} - \frac{e(a+cx^2)^{3/2}}{d^2x^2} + \frac{(e^2-df)(a+cx^2)^{3/2}}{d^3x} + \frac{(-e(e^2-2df)-f(e^2-df))}{d^3(d+ex+fx^2)} \right) dx \\
&= \frac{\int \frac{(-e(e^2-2df)-f(e^2-df)x)(a+cx^2)^{3/2}}{d+ex+fx^2} dx}{d^3} + \frac{\int \frac{(a+cx^2)^{3/2}}{x^3} dx}{d} - \frac{e \int \frac{(a+cx^2)^{3/2}}{x^2} dx}{d^2} + \frac{(e^2-df) \int \frac{(a+cx^2)^{3/2}}{x} dx}{d^3} \\
&= -\frac{(e^2-df)(a+cx^2)^{3/2}}{3d^3} + \frac{e(a+cx^2)^{3/2}}{d^2x} + \frac{\text{Subst}\left(\int \frac{(a+cx)^{3/2}}{x^2} dx, x, x^2\right)}{2d} - \frac{(3ce) \int \sqrt{a+cx^2} dx}{d^2} \\
&= -\frac{3cex\sqrt{a+cx^2}}{2d^2} - \frac{(2(cd^2+a(e^2-df))-cdex)\sqrt{a+cx^2}}{2d^3} - \frac{(a+cx^2)^{3/2}}{2dx^2} + \frac{e(a+cx^2)^{3/2}}{d^2x} \\
&= \frac{3c\sqrt{a+cx^2}}{2d} + \frac{a(e^2-df)\sqrt{a+cx^2}}{d^3} - \frac{3cex\sqrt{a+cx^2}}{2d^2} - \frac{(2(cd^2+a(e^2-df))-cdex)\sqrt{a+cx^2}}{2d^3} \\
&= \frac{3c\sqrt{a+cx^2}}{2d} + \frac{a(e^2-df)\sqrt{a+cx^2}}{d^3} - \frac{3cex\sqrt{a+cx^2}}{2d^2} - \frac{(2(cd^2+a(e^2-df))-cdex)\sqrt{a+cx^2}}{2d^3} \\
&= \frac{3c\sqrt{a+cx^2}}{2d} + \frac{a(e^2-df)\sqrt{a+cx^2}}{d^3} - \frac{3cex\sqrt{a+cx^2}}{2d^2} - \frac{(2(cd^2+a(e^2-df))-cdex)\sqrt{a+cx^2}}{2d^3} \\
&= \frac{3c\sqrt{a+cx^2}}{2d} + \frac{a(e^2-df)\sqrt{a+cx^2}}{d^3} - \frac{3cex\sqrt{a+cx^2}}{2d^2} - \frac{(2(cd^2+a(e^2-df))-cdex)\sqrt{a+cx^2}}{2d^3}
\end{aligned}$$

Mathematica [C] time = 3.22, size = 904, normalized size = 1.35

$$\frac{6cd^2 {}_2F_1\left(2, \frac{5}{2}; \frac{7}{2}; \frac{cx^2}{a} + 1\right)(cx^2+a)^{5/2}}{a^2} - 5 \left(e^2 - \frac{(e^2-3df)e}{\sqrt{e^2-4df}} - df \right) (cx^2+a)^{3/2} - 5 \left(e^2 + \frac{(e^2-3df)e}{\sqrt{e^2-4df}} - df \right) (cx^2+a)^{3/2} + \frac{30ade {}_2F_1\left(-\frac{3}{2}, \frac{5}{2}; \frac{7}{2}; \frac{cx^2}{a} + 1\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(3/2)/(x^3*(d + e*x + f*x^2)),x]
```

```
[Out] (-5*(e^2 - d*f - (e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*f])*(a + c*x^2)^(3/2) - 5
*(e^2 - d*f + (e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*f])*(a + c*x^2)^(3/2) + (15*
(-e^2 + d*f - (e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*f])*((2*Sqrt[c]*(-e + Sqrt[e
^2 - 4*d*f])*Sqrt[a + c*x^2]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/a] + Sqrt[a]*ArcSi
nh[(Sqrt[c]*x)/Sqrt[a]]))/Sqrt[1 + (c*x^2)/a] + (2*(2*a*f^2 + c*(e^2 - 2*d*
f - e*Sqrt[e^2 - 4*d*f]))*(2*f*Sqrt[a + c*x^2] + Sqrt[c]*(-e + Sqrt[e^2 - 4
*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] - Sqrt[2*c*e^2 - 4*c*d*f + 4*a*
f^2 - 2*c*e*Sqrt[e^2 - 4*d*f]]*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*
x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2
]]))/f^2))/(8*f) - (15*(-e^2 + d*f + (e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*f])*
((2*Sqrt[c]*(e + Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2]*(Sqrt[c]*x*Sqrt[1 + (c*
x^2)/a] + Sqrt[a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]]))/Sqrt[1 + (c*x^2)/a] + (2*(
2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*(-2*f*Sqrt[a + c*x^2] + Sq
rt[c]*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] + Sqrt[4
*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*ArcTanh[(2*a*f - c*(e + S
qrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]
)]*Sqrt[a + c*x^2]]))/f^2))/(8*f) + 10*(e^2 - d*f)*(Sqrt[a + c*x^2]*(4*a +
c*x^2) - 3*a^(3/2)*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]) + (30*a*d*e*Sqrt[a +
c*x^2]*Hypergeometric2F1[-3/2, -1/2, 1/2, -((c*x^2)/a)]/(x*Sqrt[1 + (c*x^2
)/a]) + (6*c*d^2*(a + c*x^2)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 + (c*x^
2)/a])/a^2)/(30*d^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x^3/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage₂

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x^3/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage2
```

maple [B] time = 0.02, size = 10298, normalized size = 15.42

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+a)^(3/2)/x^3/(f*x^2+e*x+d),x)`

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2 + a)^{\frac{3}{2}}}{(fx^2 + ex + d)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+a)^(3/2)/x^3/(f*x^2+e*x+d),x, algorithm="maxima")`

[Out] `integrate((c*x^2 + a)^(3/2)/((f*x^2 + e*x + d)*x^3), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^2 + a)^{3/2}}{x^3 (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + c*x^2)^(3/2)/(x^3*(d + e*x + f*x^2)),x)`

[Out] `int((a + c*x^2)^(3/2)/(x^3*(d + e*x + f*x^2)), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+a)**(3/2)/x**3/(f*x**2+e*x+d),x)`

[Out] Timed out

$$3.64 \quad \int \frac{x^3}{\sqrt{a+cx^2}(d+ex+fx^2)} dx$$

Optimal. Leaf size=380

$$\frac{(2def - (e^2 - df)(e - \sqrt{e^2 - 4df})) \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}} + \frac{(2def - (e^2 - df)(\sqrt{e^2 - 4df})) \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}$$

[Out] $-e \cdot \operatorname{arctanh}\left(\frac{x \cdot c^{1/2}}{(c \cdot x^2 + a)^{1/2}}\right) / f^2 / c^{1/2} + (c \cdot x^2 + a)^{1/2} / c / f - 1/2 \cdot a \cdot \operatorname{rctanh}\left(\frac{1/2 \cdot (2 \cdot a \cdot f - c \cdot x \cdot (e - (-4 \cdot d \cdot f + e^2)^{1/2})) \cdot 2^{1/2}}{(c \cdot x^2 + a)^{1/2}}\right) / (2 \cdot a \cdot f^2 + (-4 \cdot d \cdot f + e^2)^{1/2} \cdot e - 2 \cdot d \cdot f + e^2) \cdot c^{1/2} \cdot (2 \cdot d \cdot e \cdot f - (-d \cdot f + e^2) \cdot (e - (-4 \cdot d \cdot f + e^2)^{1/2})) / f^2 \cdot 2^{1/2} / (-4 \cdot d \cdot f + e^2)^{1/2} / (2 \cdot a \cdot f^2 + (-4 \cdot d \cdot f + e^2)^{1/2} \cdot e - 2 \cdot d \cdot f + e^2) \cdot c^{1/2} + 1/2 \cdot \operatorname{arctanh}\left(\frac{1/2 \cdot (2 \cdot a \cdot f - c \cdot x \cdot (e + (-4 \cdot d \cdot f + e^2)^{1/2})) \cdot 2^{1/2}}{(c \cdot x^2 + a)^{1/2}}\right) / (2 \cdot a \cdot f^2 + ((-4 \cdot d \cdot f + e^2)^{1/2} \cdot e - 2 \cdot d \cdot f + e^2) \cdot c^{1/2}) \cdot (2 \cdot d \cdot e \cdot f - (-d \cdot f + e^2) \cdot (e + (-4 \cdot d \cdot f + e^2)^{1/2})) / f^2 \cdot 2^{1/2} / (-4 \cdot d \cdot f + e^2)^{1/2} / (2 \cdot a \cdot f^2 + ((-4 \cdot d \cdot f + e^2)^{1/2} \cdot e - 2 \cdot d \cdot f + e^2) \cdot c^{1/2})$

Rubi [A] time = 1.17, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6728, 217, 206, 261, 1034, 725}

$$\frac{(2def - (e^2 - df)(e - \sqrt{e^2 - 4df})) \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}} + \frac{(2def - (e^2 - df)(\sqrt{e^2 - 4df})) \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^3 / (\operatorname{Sqrt}[a + c \cdot x^2] \cdot (d + e \cdot x + f \cdot x^2)), x]$

[Out] $\operatorname{Sqrt}[a + c \cdot x^2] / (c \cdot f) - (e \cdot \operatorname{ArcTanh}[(\operatorname{Sqrt}[c] \cdot x) / \operatorname{Sqrt}[a + c \cdot x^2]]) / (\operatorname{Sqrt}[c] \cdot f^2) - ((2 \cdot d \cdot e \cdot f - (e^2 - d \cdot f) \cdot (e - \operatorname{Sqrt}[e^2 - 4 \cdot d \cdot f])) \cdot \operatorname{ArcTanh}[(2 \cdot a \cdot f - c \cdot (e - \operatorname{Sqrt}[e^2 - 4 \cdot d \cdot f]) \cdot x) / (\operatorname{Sqrt}[2] \cdot \operatorname{Sqrt}[2 \cdot a \cdot f^2 + c \cdot (e^2 - 2 \cdot d \cdot f - e \cdot \operatorname{Sqrt}[e^2 - 4 \cdot d \cdot f])]) \cdot \operatorname{Sqrt}[a + c \cdot x^2]]) / (\operatorname{Sqrt}[2] \cdot f^2 \cdot \operatorname{Sqrt}[e^2 - 4 \cdot d \cdot f] \cdot \operatorname{Sqrt}[2 \cdot a \cdot f^2 + c \cdot (e^2 - 2 \cdot d \cdot f - e \cdot \operatorname{Sqrt}[e^2 - 4 \cdot d \cdot f])]) + ((2 \cdot d \cdot e \cdot f - (e^2 - d \cdot f) \cdot (e + \operatorname{Sqrt}[e^2 - 4 \cdot d \cdot f])) \cdot \operatorname{ArcTanh}[(2 \cdot a \cdot f - c \cdot (e + \operatorname{Sqrt}[e^2 - 4 \cdot d \cdot f]) \cdot x) / (\operatorname{Sqrt}[2] \cdot \operatorname{Sqrt}[2 \cdot a \cdot f^2 + c \cdot (e^2 - 2 \cdot d \cdot f + e \cdot \operatorname{Sqrt}[e^2 - 4 \cdot d \cdot f])]) \cdot \operatorname{Sqrt}[a + c \cdot x^2]]) / (\operatorname{Sqrt}[2] \cdot f^2 \cdot \operatorname{Sqrt}[e^2 - 4 \cdot d \cdot f] \cdot \operatorname{Sqrt}[2 \cdot a \cdot f^2 + c \cdot (e^2 - 2 \cdot d \cdot f + e \cdot \operatorname{Sqrt}[e^2 - 4 \cdot d \cdot f])])$

Rule 206

$\operatorname{Int}[(a + (b \cdot x)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1 \cdot \operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2] \cdot x) / \operatorname{Rt}[a, 2]]) / (\operatorname{Rt}[a, 2] \cdot \operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b, x\} \ \&\& \ \operatorname{NegQ}[a/b] \ \&\& \ (\operatorname{Gt}$

$Q[a, 0] \parallel \text{Lt}Q[b, 0]$

Rule 217

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] \text{ /; FreeQ}\{a, b\}, x] \ \&\& \ !\text{Gt}Q[a, 0]$

Rule 261

$\text{Int}[(x_)^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x^n)^{(p+1)}/(b*n*(p+1)), x] \text{ /; FreeQ}\{a, b, m, n, p\}, x] \ \&\& \ \text{Eq}Q[m, n-1] \ \&\& \ \text{Ne}Q[p, -1]$

Rule 725

$\text{Int}[1/(((d_) + (e_)*(x_))*\text{Sqrt}[(a_) + (c_)*(x_)^2]), x_Symbol] \rightarrow -\text{Subst}[\text{Int}[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/\text{Sqrt}[a + c*x^2]] \text{ /; FreeQ}\{a, c, d, e\}, x]$

Rule 1034

$\text{Int}[(g_ + (h_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2)*\text{Sqrt}[(d_) + (f_)*(x_)^2], x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(2*c*g - h*(b - q))/q, \text{Int}[1/((b - q + 2*c*x)*\text{Sqrt}[d + f*x^2]), x], x] - \text{Dist}[(2*c*g - h*(b + q))/q, \text{Int}[1/((b + q + 2*c*x)*\text{Sqrt}[d + f*x^2]), x], x]] \text{ /; FreeQ}\{a, b, c, d, f, g, h\}, x] \ \&\& \ \text{Ne}Q[b^2 - 4*a*c, 0] \ \&\& \ \text{Pos}Q[b^2 - 4*a*c]$

Rule 6728

$\text{Int}[(u_)/((a_) + (b_)*(x_)^{(n_)} + (c_)*(x_)^{(n2_)}), x_Symbol] \rightarrow \text{With}\{v = \text{RationalFunctionExpand}[u/(a + b*x^n + c*x^{(2*n)}), x]\}, \text{Int}[v, x] \text{ /; Sum}Q[v]] \text{ /; FreeQ}\{a, b, c\}, x] \ \&\& \ \text{Eq}Q[n2, 2*n] \ \&\& \ \text{IGt}Q[n, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{a+cx^2}(d+ex+fx^2)} dx &= \int \left(-\frac{e}{f^2\sqrt{a+cx^2}} + \frac{x}{f\sqrt{a+cx^2}} + \frac{de+(e^2-df)x}{f^2\sqrt{a+cx^2}(d+ex+fx^2)} \right) dx \\
&= \frac{\int \frac{de+(e^2-df)x}{\sqrt{a+cx^2}(d+ex+fx^2)} dx}{f^2} - \frac{e \int \frac{1}{\sqrt{a+cx^2}} dx}{f^2} + \frac{\int \frac{x}{\sqrt{a+cx^2}} dx}{f} \\
&= \frac{\sqrt{a+cx^2}}{cf} - \frac{e \operatorname{Subst}\left(\int \frac{1}{1-cx^2} dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{f^2} + \frac{(2def - (e^2 - df)(e - \sqrt{e^2 - 4df})) \operatorname{Subst}\left(\int \frac{1}{\sqrt{e^2 - 4df - cx^2}} dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{f^2\sqrt{e^2 - 4df}} \\
&= \frac{\sqrt{a+cx^2}}{cf} - \frac{e \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}f^2} - \frac{(2def - (e^2 - df)(e - \sqrt{e^2 - 4df})) \operatorname{Subst}\left(\int \frac{1}{\sqrt{e^2 - 4df - cx^2}} dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{f^2\sqrt{e^2 - 4df}} \\
&= \frac{\sqrt{a+cx^2}}{cf} - \frac{e \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}f^2} - \frac{(2def - (e^2 - df)(e - \sqrt{e^2 - 4df})) \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{2}f^2\sqrt{e^2 - 4df}\sqrt{2af^2 + c(e^2 - 4df)}}
\end{aligned}$$

Mathematica [A] time = 1.33, size = 378, normalized size = 0.99

$$\frac{\sqrt{2}((e^2-df)(\sqrt{e^2-4df}-e)+2def) \tanh^{-1}\left(\frac{2af+cx(\sqrt{e^2-4df}-e)}{\sqrt{a+cx^2}\sqrt{4af^2-2c(e\sqrt{e^2-4df}+2df-e^2)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}} + \frac{\sqrt{2}(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3) \tanh^{-1}\left(\frac{2af+cx(\sqrt{e^2-4df}-e)}{\sqrt{a+cx^2}\sqrt{4af^2-2c(e\sqrt{e^2-4df}+2df-e^2)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}}}{2f^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]

[Out]
$$\begin{aligned}
& -1/2*((-2*f*\operatorname{Sqrt}[a + c*x^2])/c + (2*e*\operatorname{ArcTanh}[(\operatorname{Sqrt}[c]*x)/\operatorname{Sqrt}[a + c*x^2]]) \\
& / \operatorname{Sqrt}[c] + (\operatorname{Sqrt}[2]*(2*d*e*f + (e^2 - d*f)*(-e + \operatorname{Sqrt}[e^2 - 4*d*f]))*\operatorname{ArcTan} \\
& h[(2*a*f + c*(-e + \operatorname{Sqrt}[e^2 - 4*d*f])*x)/(\operatorname{Sqrt}[4*a*f^2 - 2*c*(-e^2 + 2*d*f \\
& + e*\operatorname{Sqrt}[e^2 - 4*d*f])]*\operatorname{Sqrt}[a + c*x^2])]) / (\operatorname{Sqrt}[e^2 - 4*d*f]*\operatorname{Sqrt}[2*a*f^2 \\
& + c*(e^2 - 2*d*f - e*\operatorname{Sqrt}[e^2 - 4*d*f])]) + (\operatorname{Sqrt}[2]*(e^3 - 3*d*e*f + e^2*\operatorname{Sqrt}[e^2 - 4*d*f] \\
& - d*f*\operatorname{Sqrt}[e^2 - 4*d*f])*\operatorname{ArcTanh}[(2*a*f - c*(e + \operatorname{Sqrt}[e^2 - 4*d*f])*x) / (\operatorname{Sqrt}[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*\operatorname{Sqrt}[e^2 - 4*d*f])]*\operatorname{Sqrt}[a + c*x^2])]) / (\operatorname{Sqrt}[e^2 - 4*d*f]*\operatorname{Sqrt}[2*a*f^2 + c*(e^2 - 2*d*f + e*\operatorname{Sqrt}[e^2 - 4*d*f])]) / f^2
\end{aligned}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Evaluation time: 11.34sym2poly/r2sym(const g
en & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.03, size = 2397, normalized size = 6.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] (c*x^2+a)^(1/2)/c/f-1/f^2*e*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)+1/2/f^2*
^(1/2)/((( -4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((( -4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((( -4*d*f+e^2)^(1/2)*c*e+2*a*f^2
-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+
(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)
*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*d
-1/2/f^3*2^(1/2)/((( -4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)
*ln((( -4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(
1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((( -4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*
c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1
/2))/f))*e^2+3/2/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((( -4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((( -4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f
+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2
*2^(1/2)*((( -4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1
```

$$\begin{aligned} & /2*(e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-4*c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x+1/2*(e+(-4 \\ & *d*f+e^2)^{(1/2)})/f)+2*((-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)} \\ & /((x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f))*d*e-1/2/f^3/(-4*d*f+e^2)^{(1/2)}*2^{(1/2)} \\ & /(((-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}*\ln(((-4*d*f \\ & +e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x+1/ \\ & 2*(e+(-4*d*f+e^2)^{(1/2)})/f)+1/2*2^{(1/2)}*((-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2* \\ & c*d*f+c*e^2)/f^2)^{(1/2)}*(4*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-4*c*(e+(-4* \\ & d*f+e^2)^{(1/2)})/f*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)+2*((-4*d*f+e^2)^{(1/2)}*c* \\ & e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)})/(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f))*e^3+ \\ & 1/2/f^2*2^{(1/2)}/((-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)} \\ & *ln(((-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^{(1/2)})/ \\ & f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)+1/2*2^{(1/2)}*((-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/ \\ & f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-4*c*(e+(-4*d*f+e^2)^{(1/2)})/ \\ & f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)+2*((-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/ \\ & f^2)^{(1/2)})/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f))*d-1/2/f^3*2^{(1/2)}/((-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f \\ & +c*e^2)/f^2)^{(1/2)}*\ln(((-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c* \\ & (e+(-4*d*f+e^2)^{(1/2)})/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)+1/2*2^{(1/2)}*((\\ & -4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4 \\ & *d*f+e^2)^{(1/2)})/f)^2*c-4*c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x-1/2*(-e+(-4*d*f+e^2) \\ &)^{(1/2)})/f)+2*((-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)})/(\\ & x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f))*e^2-3/2/f^2/(-4*d*f+e^2)^{(1/2)}*2^{(1/2)}/(((\\ & -4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}*\ln(((-4*d*f+e^2) \\ &)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x-1/2*(- \\ & e+(-4*d*f+e^2)^{(1/2)})/f)+1/2*2^{(1/2)}*((-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c* \\ & d*f+c*e^2)/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-4*c*(e+(-4*d \\ & *f+e^2)^{(1/2)})/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)+2*((-4*d*f+e^2)^{(1/2)}*c \\ & *e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)})/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f))*d* \\ & e+1/2/f^3/(-4*d*f+e^2)^{(1/2)}*2^{(1/2)}/(((-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c* \\ & d*f+c*e^2)/f^2)^{(1/2)}*\ln(((-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f \\ & ^2-c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)+1/2*2^{(1/2)} \\ & *((-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}*(4*(x-1/2*(-e+ \\ & (-4*d*f+e^2)^{(1/2)})/f)^2*c-4*c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x-1/2*(-e+(-4*d*f+ \\ & e^2)^{(1/2)})/f)+2*((-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)} \\ &)/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f))*e^3 \end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* h

elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see 'assume?' for more details) Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{cx^2 + a} (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)

[Out] int(x^3/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a + cx^2} (d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(f*x**2+e*x+d)/(c*x**2+a)**(1/2), x)

[Out] Integral(x**3/(sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)

$$3.65 \quad \int \frac{x^2}{\sqrt{a+cx^2}(d+ex+fx^2)} dx$$

Optimal. Leaf size=344

$$\frac{(-e\sqrt{e^2-4df}-2df+e^2)\tanh^{-1}\left(\frac{2af-cx(e-\sqrt{e^2-4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}} - \frac{(2df-e(\sqrt{e^2-4df}+e))\tanh^{-1}\left(\frac{2af-cx(e-\sqrt{e^2-4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}$$

[Out] arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/f/c^(1/2)-1/2*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)/f*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)-1/2*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*((2*d*f-e*(e+(-4*d*f+e^2)^(1/2)))/f*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))

Rubi [A] time = 0.54, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1081, 217, 206, 1034, 725}

$$\frac{(-e\sqrt{e^2-4df}-2df+e^2)\tanh^{-1}\left(\frac{2af-cx(e-\sqrt{e^2-4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}} - \frac{(2df-e(\sqrt{e^2-4df}+e))\tanh^{-1}\left(\frac{2af-cx(e-\sqrt{e^2-4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]

[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/(Sqrt[c]*f) - ((e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x]/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]))/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]]) - ((2*d*f - e*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x]/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]))/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

$Q[a, 0] \parallel \text{LtQ}[b, 0]$)

Rule 217

$\text{Int}[1/\text{Sqrt}[(a_) + (b_.)*(x_)^2], x_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{!GtQ}[a, 0]$

Rule 725

$\text{Int}[1/(((d_) + (e_.)*(x_))*\text{Sqrt}[(a_) + (c_.)*(x_)^2]), x_Symbol] \rightarrow -\text{Subst}[\text{Int}[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/\text{Sqrt}[a + c*x^2]] /; \text{FreeQ}[\{a, c, d, e\}, x]$

Rule 1034

$\text{Int}(((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*\text{Sqrt}[(d_) + (f_.)*(x_)^2]), x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(2*c*g - h*(b - q))/q, \text{Int}[1/((b - q + 2*c*x)*\text{Sqrt}[d + f*x^2]), x], x] - \text{Dist}[(2*c*g - h*(b + q))/q, \text{Int}[1/((b + q + 2*c*x)*\text{Sqrt}[d + f*x^2]), x], x]] /; \text{FreeQ}[\{a, b, c, d, f, g, h\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{PosQ}[b^2 - 4*a*c]$

Rule 1081

$\text{Int}(((A_.) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*\text{Sqrt}[(d_.) + (f_.)*(x_)^2]), x_Symbol] \rightarrow \text{Dist}[C/c, \text{Int}[1/\text{Sqrt}[d + f*x^2], x], x] + \text{Dist}[1/c, \text{Int}[(A*c - a*C - b*C*x)/((a + b*x + c*x^2)*\text{Sqrt}[d + f*x^2]), x], x] /; \text{FreeQ}[\{a, b, c, d, f, A, C\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{a+cx^2}(d+ex+fx^2)} dx &= \frac{\int \frac{1}{\sqrt{a+cx^2}} dx}{f} + \frac{\int \frac{-d-ex}{\sqrt{a+cx^2}(d+ex+fx^2)} dx}{f} \\
&= \frac{\text{Subst}\left(\int \frac{1}{1-cx^2} dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{f} + \frac{(e^2 - 2df - e\sqrt{e^2 - 4df}) \int \frac{1}{(e - \sqrt{e^2 - 4df} + 2fx)\sqrt{a+cx^2}} dx}{f\sqrt{e^2 - 4df}} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}f} - \frac{(e^2 - 2df - e\sqrt{e^2 - 4df}) \text{Subst}\left(\int \frac{1}{4af^2 + c(e - \sqrt{e^2 - 4df})^2 - x^2} dx\right)}{f\sqrt{e^2 - 4df}} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}f} - \frac{(e^2 - 2df - e\sqrt{e^2 - 4df}) \tanh^{-1}\left(\frac{2af - c(e - \sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{2af^2 + c(e^2 - 2df - e\sqrt{e^2 - 4df})}}\right)}{\sqrt{2}f\sqrt{e^2 - 4df}\sqrt{2af^2 + c(e^2 - 2df - e\sqrt{e^2 - 4df})}}
\end{aligned}$$

Mathematica [A] time = 0.70, size = 334, normalized size = 0.97

$$\frac{\sqrt{2}(e\sqrt{e^2-4df}+2df-e^2)\tanh^{-1}\left(\frac{2af+cx(\sqrt{e^2-4df}-e)}{\sqrt{a+cx^2}\sqrt{4af^2-2c(e\sqrt{e^2-4df}+2df-e^2)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}} + \frac{\sqrt{2}(e\sqrt{e^2-4df}-2df+e^2)\tanh^{-1}\left(\frac{2af-cx(\sqrt{e^2-4df}+e)}{\sqrt{a+cx^2}\sqrt{4af^2+2c(e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}}}{2f}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]

[Out] ((2*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/Sqrt[c] + (Sqrt[2]*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]]))/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (Sqrt[2]*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]]))/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]))/(2*f)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

```
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Evaluation time: 6.03Error index.cc index_gc
d Error: Bad Argument Value
```

```
maple [B] time = 0.02, size = 1796, normalized size = 5.22
```

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/f*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)+1/2/f^2*2^(1/2)/((2*a*f^2-2*c*d*f
+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-e+(-4*d*f+e^2)^(1/2))*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c
*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1
/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x+1/2
*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*
c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*e-1/(-4*d*f+e^2)^(1/2)/f
*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-e
+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+
c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d
*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e
+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f
+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)
)*d+1/2/(-4*d*f+e^2)^(1/2)/f^2*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)
^(1/2)*c*e)/f^2)^(1/2)*ln((-e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(
1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)
*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-
4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1
/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*e^2+1/2/f^2*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2
-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-e+(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+
(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f
^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*
```



```

4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e
)/f^2)^(1/2)/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e+1/(-4*d*f+e^2)^(1/2)/f*2
^(1/2)/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-e-(-
4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c
*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*
f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-
(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*
f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/
f))*d-1/2/(-4*d*f+e^2)^(1/2)/f^2*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^
2)^(1/2)*c*e)/f^2)^(1/2)*ln((-e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2
)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1
/2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-
e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^
2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2
))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e^2

```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details)Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{c x^2 + a} (f x^2 + e x + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)

[Out] int(x^2/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a + c x^2} (d + e x + f x^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

$$3.66 \quad \int \frac{x}{\sqrt{a+cx^2}(d+ex+fx^2)} dx$$

Optimal. Leaf size=294

$$\frac{(e - \sqrt{e^2 - 4df}) \tanh^{-1} \left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2} \sqrt{a+cx^2} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}} \right)}{\sqrt{2} \sqrt{e^2 - 4df} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}} - \frac{(\sqrt{e^2 - 4df} + e) \tanh^{-1} \left(\frac{2af - cx(\sqrt{e^2 - 4df} + e)}{\sqrt{2} \sqrt{a+cx^2} \sqrt{2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)}} \right)}{\sqrt{2} \sqrt{e^2 - 4df} \sqrt{2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)}}$$

[Out] $1/2 \cdot \operatorname{arctanh}(1/2 \cdot (2 \cdot a \cdot f - c \cdot x \cdot (e - (-4 \cdot d \cdot f + e^2)^{1/2}))) \cdot 2^{1/2} / (c \cdot x^2 + a)^{1/2} / (2 \cdot a \cdot f^2 + (-(-4 \cdot d \cdot f + e^2)^{1/2} \cdot e - 2 \cdot d \cdot f + e^2) \cdot c)^{1/2} \cdot (e - (-4 \cdot d \cdot f + e^2)^{1/2}) \cdot 2^{1/2} / (-4 \cdot d \cdot f + e^2)^{1/2} / (2 \cdot a \cdot f^2 + (-(-4 \cdot d \cdot f + e^2)^{1/2} \cdot e - 2 \cdot d \cdot f + e^2) \cdot c)^{1/2} - 1/2 \cdot \operatorname{arctanh}(1/2 \cdot (2 \cdot a \cdot f - c \cdot x \cdot (e + (-4 \cdot d \cdot f + e^2)^{1/2}))) \cdot 2^{1/2} / (c \cdot x^2 + a)^{1/2} / (2 \cdot a \cdot f^2 + ((-4 \cdot d \cdot f + e^2)^{1/2} \cdot e - 2 \cdot d \cdot f + e^2) \cdot c)^{1/2} \cdot (e + (-4 \cdot d \cdot f + e^2)^{1/2}) \cdot 2^{1/2} / (-4 \cdot d \cdot f + e^2)^{1/2} / (2 \cdot a \cdot f^2 + ((-4 \cdot d \cdot f + e^2)^{1/2} \cdot e - 2 \cdot d \cdot f + e^2) \cdot c)^{1/2}$

Rubi [A] time = 0.24, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1034, 725, 206}

$$\frac{(e - \sqrt{e^2 - 4df}) \tanh^{-1} \left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2} \sqrt{a+cx^2} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}} \right)}{\sqrt{2} \sqrt{e^2 - 4df} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}} - \frac{(\sqrt{e^2 - 4df} + e) \tanh^{-1} \left(\frac{2af - cx(\sqrt{e^2 - 4df} + e)}{\sqrt{2} \sqrt{a+cx^2} \sqrt{2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)}} \right)}{\sqrt{2} \sqrt{e^2 - 4df} \sqrt{2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x/(\operatorname{Sqrt}[a + c \cdot x^2] \cdot (d + e \cdot x + f \cdot x^2)), x]$

[Out] $((e - \operatorname{Sqrt}[e^2 - 4 \cdot d \cdot f]) \cdot \operatorname{ArcTanh}[(2 \cdot a \cdot f - c \cdot (e - \operatorname{Sqrt}[e^2 - 4 \cdot d \cdot f]) \cdot x) / (\operatorname{Sqrt}[2] \cdot \operatorname{Sqrt}[2 \cdot a \cdot f^2 + c \cdot (e^2 - 2 \cdot d \cdot f - e \cdot \operatorname{Sqrt}[e^2 - 4 \cdot d \cdot f])] \cdot \operatorname{Sqrt}[a + c \cdot x^2])]) / (\operatorname{Sqrt}[2] \cdot \operatorname{Sqrt}[e^2 - 4 \cdot d \cdot f] \cdot \operatorname{Sqrt}[2 \cdot a \cdot f^2 + c \cdot (e^2 - 2 \cdot d \cdot f - e \cdot \operatorname{Sqrt}[e^2 - 4 \cdot d \cdot f])]) - ((e + \operatorname{Sqrt}[e^2 - 4 \cdot d \cdot f]) \cdot \operatorname{ArcTanh}[(2 \cdot a \cdot f - c \cdot (e + \operatorname{Sqrt}[e^2 - 4 \cdot d \cdot f]) \cdot x) / (\operatorname{Sqrt}[2] \cdot \operatorname{Sqrt}[2 \cdot a \cdot f^2 + c \cdot (e^2 - 2 \cdot d \cdot f + e \cdot \operatorname{Sqrt}[e^2 - 4 \cdot d \cdot f])] \cdot \operatorname{Sqrt}[a + c \cdot x^2])]) / (\operatorname{Sqrt}[2] \cdot \operatorname{Sqrt}[e^2 - 4 \cdot d \cdot f] \cdot \operatorname{Sqrt}[2 \cdot a \cdot f^2 + c \cdot (e^2 - 2 \cdot d \cdot f + e \cdot \operatorname{Sqrt}[e^2 - 4 \cdot d \cdot f])])$

Rule 206

$\operatorname{Int}[(a + (b \cdot x)^2)^{-1}, x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(1 \cdot \operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2] \cdot x) / \operatorname{Rt}[a, 2]]) / (\operatorname{Rt}[a, 2] \cdot \operatorname{Rt}[-b, 2]), x] / ; \operatorname{FreeQ}\{a, b\}, x \ \&\& \ \operatorname{NegQ}[a/b] \ \&\& \ (\operatorname{Gt}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 725

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (c_)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1034

```
Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (f_
_)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+cx^2} (d+ex+fx^2)} dx = - \left(\left(-1 - \frac{e}{\sqrt{e^2-4df}} \right) \int \frac{1}{(e + \sqrt{e^2-4df} + 2fx) \sqrt{a+cx^2}} dx \right) + \left(1 - \frac{e}{\sqrt{e^2-4df}} \right) \int \frac{1}{(e - \sqrt{e^2-4df} + 2fx) \sqrt{a+cx^2}} dx$$

$$= \left(-1 + \frac{e}{\sqrt{e^2-4df}} \right) \text{Subst} \left(\int \frac{1}{4af^2 + c(e - \sqrt{e^2-4df})^2 - x^2} dx, x, \frac{2af - c(e - \sqrt{e^2-4df})}{\sqrt{a+cx^2}} \right) - \left(1 - \frac{e}{\sqrt{e^2-4df}} \right) \text{Subst} \left(\int \frac{1}{4af^2 + c(e + \sqrt{e^2-4df})^2 - x^2} dx, x, \frac{2af - c(e + \sqrt{e^2-4df})}{\sqrt{a+cx^2}} \right)$$

$$= \frac{\left(1 - \frac{e}{\sqrt{e^2-4df}} \right) \tanh^{-1} \left(\frac{2af - c(e - \sqrt{e^2-4df})x}{\sqrt{2} \sqrt{2af^2 + c(e^2 - 2df - e\sqrt{e^2-4df})} \sqrt{a+cx^2}} \right)}{\sqrt{2} \sqrt{2af^2 + c(e^2 - 2df - e\sqrt{e^2-4df})}} - \frac{\left(1 + \frac{e}{\sqrt{e^2-4df}} \right) \tanh^{-1} \left(\frac{2af - c(e + \sqrt{e^2-4df})x}{\sqrt{2} \sqrt{2af^2 + c(e^2 - 2df + e\sqrt{e^2-4df})} \sqrt{a+cx^2}} \right)}{\sqrt{2} \sqrt{2af^2 + c(e^2 - 2df + e\sqrt{e^2-4df})}}$$

Mathematica [A] time = 0.32, size = 275, normalized size = 0.94

$$\sqrt{2} \left(\frac{(\sqrt{e^2-4df}-e) \tanh^{-1} \left(\frac{2af+cx(\sqrt{e^2-4df}-e)}{\sqrt{a+cx^2} \sqrt{4af^2-2c(e\sqrt{e^2-4df}+2df-e^2)}} \right)}{2\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}} - \frac{(\sqrt{e^2-4df}+e) \tanh^{-1} \left(\frac{2af-cx(\sqrt{e^2-4df}+e)}{\sqrt{a+cx^2} \sqrt{4af^2+2c(e\sqrt{e^2-4df}-2df+e^2)}} \right)}{2\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}} \right) \frac{1}{\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)), x]
```

```
[Out] (Sqrt[2]*(-1/2*((-e + Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x]/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2])))/Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])] - ((e + Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x]/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2])))/(2*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]))/Sqrt[e^2 - 4*d*f]
```

fricas [B] time = 3.46, size = 5085, normalized size = 17.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f))*log((4*a*c*d^2*e*x - 2*a^2*d*e^2 + sqrt(2)*(a^2*e^4 - 4*a^2*d*e^2*f - (2*c^3*d^4*e^2 + 3*a*c^2*d^2*e^4 + a^2*c*e^6 + 8*a^3*d^2*f^4 - 6*(4*a^2*c*d^3 + a^3*d*e^2)*f^3 + (24*a*c^2*d^4 + 22*a^2*c*d^2*e^2 + a^3*e^4)*f^2 - 2*(4*c^3*d^5 + 9*a*c^2*d^3*e^2 + 4*a^2*c*d*e^4)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))*sqrt(c*x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)) + 2*(a*c^2*d^3*e^2 + a^2*c*d*e^4 - 4*a^3*d^2*f^3 + (8*a^2*c*d^3 + a^3*d*e^2)*f^2 - 2*(2*a*c^2*d^4 + 3*a^2*c*d^2*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/x + 1/4*sqrt(2)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f))
```

$$\begin{aligned} & + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)) * \log((4*a*c*d^2 * e * x - 2*a^2*d*e^2 - \sqrt{2}*(a^2*e^4 - 4*a^2*d*e^2*f - (2*c^3*d^4*e^2 + 3*a*c^2*d^2*e^4 + a^2*c*e^6 + 8*a^3*d^2*f^4 - 6*(4*a^2*c*d^3 + a^3*d*e^2)*f^3 + (24*a*c^2*d^4 + 22*a^2*c*d^2*e^2 + a^3*e^4)*f^2 - 2*(4*c^3*d^5 + 9*a*c^2*d^3*e^2 + 4*a^2*c*d*e^4)*f)*\sqrt{a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f})) * \sqrt{c*x^2 + a} * \sqrt{((2*c*d^2 + a*e^2 - 2*a*d*f + (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*\sqrt{a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f})))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)) + 2*(a*c^2*d^3*e^2 + a^2*c*d*e^4 - 4*a^3*d^2*f^3 + (8*a^2*c*d^3 + a^3*d*e^2)*f^2 - 2*(2*a*c^2*d^4 + 3*a^2*c*d^2*e^2)*f)*\sqrt{a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/x) - 1/4*\sqrt{2}*\sqrt{((2*c*d^2 + a*e^2 - 2*a*d*f - (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*\sqrt{a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)) * \log((4*a*c*d^2 * e * x - 2*a^2*d*e^2 + \sqrt{2}*(a^2*e^4 - 4*a^2*d*e^2*f + (2*c^3*d^4*e^2 + 3*a*c^2*d^2*e^4 + a^2*c*e^6 + 8*a^3*d^2*f^4 - 6*(4*a^2*c*d^3 + a^3*d*e^2)*f^3 + (24*a*c^2*d^4 + 22*a^2*c*d^2*e^2 + a^3*e^4)*f^2 - 2*(4*c^3*d^5 + 9*a*c^2*d^3*e^2 + 4*a^2*c*d*e^4)*f)*\sqrt{a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f})) * \sqrt{c*x^2 + a} * \sqrt{((2*c*d^2 + a*e^2 - 2*a*d*f - (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*\sqrt{a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)) - 2*(a*c^2*d^3*e^2 + a^2*c*d*e^4 - 4*a^3*d^2*f^3 + (8*a^2*c*d^3 + a^3*d*e^2)*f^2 - 2*(2*a*c^2*d^4 + 3*a^2*c*d^2*e^2)*f)*\sqrt{a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/x) + 1/4*\sqrt{2}*\sqrt{((2*c*d^2$$

$$\begin{aligned}
 &+ a^2 e^2 - 2 a d f - (c^2 d^2 e^2 + a c e^4 - 4 a^2 d f^3 + (8 a c d^2 + a^2 e^2) f^2 - 2 (2 c^2 d^3 + 3 a c d e^2) f) \sqrt{a^2 e^2 / (c^4 d^4 e^2 + 2 a c^3 d^2 e^4 + a^2 c^2 e^6 - 4 a^4 d f^5 + (16 a^3 c d^2 + a^4 e^2) f^4 - 12 (2 a^2 c^2 d^3 + a^3 c d e^2) f^3 + 2 (8 a c^3 d^4 + 11 a^2 c^2 d^2 e^2 + a^3 c e^4) f^2 - 4 (c^4 d^5 + 3 a c^3 d^3 e^2 + 2 a^2 c^2 d e^4) f)} \\
 &/ (c^2 d^2 e^2 + a c e^4 - 4 a^2 d f^3 + (8 a c d^2 + a^2 e^2) f^2 - 2 (2 c^2 d^3 + 3 a c d e^2) f) \log((4 a c d^2 e^2 x - 2 a^2 d e^2 - \sqrt{2} (a^2 e^4 - 4 a^2 d e^2 f + (2 c^3 d^4 e^2 + 3 a c^2 d^2 e^4 + a^2 c e^6 + 8 a^3 d^2 f^4 - 6 (4 a^2 c d^3 + a^3 d e^2) f^3 + (24 a c^2 d^4 + 22 a^2 c d^2 e^2 + a^3 e^4) f^2 - 2 (4 c^3 d^5 + 9 a c^2 d^3 e^2 + 4 a^2 c d e^4) f) \sqrt{a^2 e^2 / (c^4 d^4 e^2 + 2 a c^3 d^2 e^4 + a^2 c^2 e^6 - 4 a^4 d f^5 + (16 a^3 c d^2 + a^4 e^2) f^4 - 12 (2 a^2 c^2 d^3 + a^3 c d e^2) f^3 + 2 (8 a c^3 d^4 + 11 a^2 c^2 d^2 e^2 + a^3 c e^4) f^2 - 4 (c^4 d^5 + 3 a c^3 d^3 e^2 + 2 a^2 c^2 d e^4) f})) \sqrt{c x^2 + a} \sqrt{(2 c d^2 + a e^2 - 2 a d f - (c^2 d^2 e^2 + a c e^4 - 4 a^2 d f^3 + (8 a c d^2 + a^2 e^2) f^2 - 2 (2 c^2 d^3 + 3 a c d e^2) f) \sqrt{a^2 e^2 / (c^4 d^4 e^2 + 2 a c^3 d^2 e^4 + a^2 c^2 e^6 - 4 a^4 d f^5 + (16 a^3 c d^2 + a^4 e^2) f^4 - 12 (2 a^2 c^2 d^3 + a^3 c d e^2) f^3 + 2 (8 a c^3 d^4 + 11 a^2 c^2 d^2 e^2 + a^3 c e^4) f^2 - 4 (c^4 d^5 + 3 a c^3 d^3 e^2 + 2 a^2 c^2 d e^4) f))} / (c^2 d^2 e^2 + a c e^4 - 4 a^2 d f^3 + (8 a c d^2 + a^2 e^2) f^2 - 2 (2 c^2 d^3 + 3 a c d e^2) f) - 2 (a c^2 d^3 e^2 + a^2 c d e^4 - 4 a^3 d^2 f^3 + (8 a^2 c d^3 + a^3 d e^2) f^2 - 2 (2 a c^2 d^4 + 3 a^2 c d^2 e^2) f) \sqrt{a^2 e^2 / (c^4 d^4 e^2 + 2 a c^3 d^2 e^4 + a^2 c^2 e^6 - 4 a^4 d f^5 + (16 a^3 c d^2 + a^4 e^2) f^4 - 12 (2 a^2 c^2 d^3 + a^3 c d e^2) f^3 + 2 (8 a c^3 d^4 + 11 a^2 c^2 d^2 e^2 + a^3 c e^4) f^2 - 4 (c^4 d^5 + 3 a c^3 d^3 e^2 + 2 a^2 c^2 d e^4) f)) / x)
 \end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 1.77index.cc index_m operat
 or + Error: Bad Argument Value

maple [B] time = 0.01, size = 1172, normalized size = 3.99

$$\sqrt{2} e \ln \left(\frac{\left(e - \sqrt{-4df+e^2} \right) \left(x - \frac{-e + \sqrt{-4df+e^2}}{2f} \right)^c}{f} + \frac{2af^2 - 2cdf + ce^2 - \sqrt{-4df+e^2} ce}{f^2} + \frac{\sqrt{2} \sqrt{\frac{2af^2 - 2cdf + ce^2 - \sqrt{-4df+e^2} ce}{f^2}}}{\sqrt{4 \left(x - \frac{-e + \sqrt{-4df+e^2}}{2f} \right)^2}} \frac{4 \left(e - \sqrt{-4df+e^2} \right)^c}{2} \right) \frac{1}{x - \frac{-e + \sqrt{-4df+e^2}}{2f}}$$

$$2\sqrt{-4df+e^2} \sqrt{\frac{2af^2 - 2cdf + ce^2 - \sqrt{-4df+e^2} ce}{f^2}} f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)`

[Out]
$$\begin{aligned} & -1/2/(-4*d*f+e^2)^{(1/2)}/f*2^{(1/2)}/((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)} \\ &)*c*e)/f^2)^{(1/2)}*\ln((-e+(-4*d*f+e^2)^{(1/2)})*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)} \\ &)/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(1/2)}*((2* \\ & a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(x+1/2*(e+(-4*d*f \\ & +e^2)^{(1/2)}))/f)^2*c-4*(e+(-4*d*f+e^2)^{(1/2)})*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/ \\ & f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)})/(x+1/2* \\ & (e+(-4*d*f+e^2)^{(1/2)}))/f)*e-1/2/f*2^{(1/2)}/((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+ \\ & e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*\ln((-e+(-4*d*f+e^2)^{(1/2)})*(x+1/2*(e+(-4*d*f+e^ \\ & 2)^{(1/2)}))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(\\ & 1/2)}*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(x+1/2*(\\ & e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c-4*(e+(-4*d*f+e^2)^{(1/2)})*(x+1/2*(e+(-4*d*f+e^2 \\ &)^{(1/2)}))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)} \\ &)/(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)-1/2/f*2^{(1/2)}/((2*a*f^2-2*c*d*f+c*e^2- \\ & -4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*\ln((-e+(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(- \\ & 4*d*f+e^2)^{(1/2)}))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2 \\ & +1/2*2^{(1/2)}*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(\\ & x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c-4*(e+(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+ \\ & -4*d*f+e^2)^{(1/2)}))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/ \\ & f^2)^{(1/2)})/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)+1/2/(-4*d*f+e^2)^{(1/2)}/f*2^{(\\ & 1/2)}/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*\ln((-e+(-4 \\ & *d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)*c/f+(2*a*f^2-2*c*d*f+c*e \\ & ^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(1/2)}*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+ \\ & e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c-4*(e+(- \\ & 4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)*c/f+2*(2*a*f^2-2*c*d*f+ \\ & c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)})/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f \\ &)*e \end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details)Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{cx^2+a} (fx^2+ex+d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)

[Out] int(x/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+cx^2} (d+ex+fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)

[Out] Integral(x/(sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)

$$3.67 \quad \int \frac{1}{\sqrt{a+cx^2} (d+ex+fx^2)} dx$$

Optimal. Leaf size=266

$$\frac{\sqrt{2} f \tanh^{-1} \left(\frac{2af - cx(\sqrt{e^2 - 4df} + e)}{\sqrt{2} \sqrt{a+cx^2} \sqrt{2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)}} \right)}{\sqrt{e^2 - 4df} \sqrt{2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)}} - \frac{\sqrt{2} f \tanh^{-1} \left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2} \sqrt{a+cx^2} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}} \right)}{\sqrt{e^2 - 4df} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}$$

[Out] $-f \cdot \operatorname{arctanh} \left(\frac{1}{2} \cdot (2af - cx \cdot (e - \sqrt{e^2 - 4df})) \cdot 2^{1/2} / (cx^2 + a)^{1/2} / (2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2))^{1/2} \right) \cdot 2^{1/2} / (-4df + e^2)^{1/2} / (2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2))^{1/2} + f \cdot \operatorname{arctanh} \left(\frac{1}{2} \cdot (2af - cx \cdot (e + \sqrt{e^2 - 4df})) \cdot 2^{1/2} / (cx^2 + a)^{1/2} / (2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2))^{1/2} \right) \cdot 2^{1/2} / (-4df + e^2)^{1/2} / (2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2))^{1/2}$

Rubi [A] time = 0.15, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {985, 725, 206}

$$\frac{\sqrt{2} f \tanh^{-1} \left(\frac{2af - cx(\sqrt{e^2 - 4df} + e)}{\sqrt{2} \sqrt{a+cx^2} \sqrt{2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)}} \right)}{\sqrt{e^2 - 4df} \sqrt{2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)}} - \frac{\sqrt{2} f \tanh^{-1} \left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2} \sqrt{a+cx^2} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}} \right)}{\sqrt{e^2 - 4df} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]

[Out] $-\left(\frac{\sqrt{2} f \operatorname{ArcTanh} \left[\frac{(2af - cx(e - \sqrt{e^2 - 4df})) \cdot 2^{1/2}}{(cx^2 + a)^{1/2} \sqrt{2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)}} \right]}{\sqrt{e^2 - 4df} \sqrt{2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)}} \right) + \left(\frac{\sqrt{2} f \operatorname{ArcTanh} \left[\frac{(2af - cx(e + \sqrt{e^2 - 4df})) \cdot 2^{1/2}}{(cx^2 + a)^{1/2} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}} \right]}{\sqrt{e^2 - 4df} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}} \right)$

Rule 206

Int[((a_) + (b_.)(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 985

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]), x_Sym
bol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[1/((b - q + 2*c*x)
*Sqrt[d + f*x^2]), x], x] - Dist[(2*c)/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, f}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ
[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+cx^2}(d+ex+fx^2)} dx = \frac{(2f) \int \frac{1}{(e-\sqrt{e^2-4df}+2fx)\sqrt{a+cx^2}} dx}{\sqrt{e^2-4df}} - \frac{(2f) \int \frac{1}{(e+\sqrt{e^2-4df}+2fx)\sqrt{a+cx^2}} dx}{\sqrt{e^2-4df}}$$

$$= -\frac{(2f) \text{Subst}\left(\int \frac{1}{4af^2+c(e-\sqrt{e^2-4df})^2-x^2} dx, x, \frac{2af-c(e-\sqrt{e^2-4df})x}{\sqrt{a+cx^2}}\right)}{\sqrt{e^2-4df}} + \frac{(2f) \text{Subst}\left(\int \frac{1}{4af^2+c(e+\sqrt{e^2-4df})^2-x^2} dx, x, \frac{2af+c(e+\sqrt{e^2-4df})x}{\sqrt{a+cx^2}}\right)}{\sqrt{e^2-4df}}$$

$$= -\frac{\sqrt{2} f \tanh^{-1}\left(\frac{2af-c(e-\sqrt{e^2-4df})x}{\sqrt{2}\sqrt{2af^2+c(e^2-2df-e\sqrt{e^2-4df})}\sqrt{a+cx^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c(e^2-2df-e\sqrt{e^2-4df})}} + \frac{\sqrt{2} f \tanh^{-1}\left(\frac{2af+c(e+\sqrt{e^2-4df})x}{\sqrt{2}\sqrt{2af^2+c(e^2-2df+e\sqrt{e^2-4df})}\sqrt{a+cx^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c(e^2-2df+e\sqrt{e^2-4df})}}$$

Mathematica [A] time = 0.29, size = 247, normalized size = 0.93

$$\frac{2\sqrt{2} f \left(\frac{\tanh^{-1}\left(\frac{2af-cx(\sqrt{e^2-4df}+e)}{\sqrt{a+cx^2}\sqrt{4af^2+2c(e\sqrt{e^2-4df}-2df+e^2)}}\right)}{2\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}} - \frac{\tanh^{-1}\left(\frac{2af+cx(\sqrt{e^2-4df}-e)}{\sqrt{a+cx^2}\sqrt{4af^2-2c(e\sqrt{e^2-4df}+2df-e^2)}}\right)}{2\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}} \right)}{\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)), x]
```


$$\begin{aligned}
& *d^2 + a^2e^2)*f^2 - 2*(2c^2d^3 + 3a*c*d*e^2)*f)) * \log((4c^2d*e*f*x - \\
& 2a*c*e^2*f - \sqrt{2}*(c^2d*e^3 + 4a*c*d*e*f^2 - (4c^2d^2*e + a*c*e^3)* \\
& f - (c^3d^3*e^3 + a*c^2d*e^5 - 4a^3d*e*f^4 + (4a^2c*d^2*e + a^3e^3)* \\
& f^3 + (4a*c^2d^3*e - 5a^2c*d*e^3)*f^2 - (4c^3d^4*e + 5a*c^2d^2*e^3 \\
& - a^2c*e^5)*f) * \sqrt{c^2e^2/(c^4d^4e^2 + 2a*c^3d^2e^4 + a^2c^2e^6 - \\
& 4a^4d*f^5 + (16a^3c*d^2 + a^4e^2)*f^4 - 12*(2a^2c^2d^3 + a^3c*d*e \\
& ^2)*f^3 + 2*(8a*c^3d^4 + 11a^2c^2d^2e^2 + a^3c*e^4)*f^2 - 4*(c^4d^5 \\
& + 3a*c^3d^3e^2 + 2a^2c^2d*e^4)*f)) * \sqrt{c*x^2 + a} * \sqrt{(c*e^2 - 2 \\
& c*d*f + 2a*f^2 + (c^2d^2e^2 + a*c*e^4 - 4a^2d*f^3 + (8a*c*d^2 + a^2e \\
& ^2)*f^2 - 2*(2c^2d^3 + 3a*c*d*e^2)*f) * \sqrt{c^2e^2/(c^4d^4e^2 + 2a*c^ \\
& 3d^2e^4 + a^2c^2e^6 - 4a^4d*f^5 + (16a^3c*d^2 + a^4e^2)*f^4 - 12*(\\
& 2a^2c^2d^3 + a^3c*d*e^2)*f^3 + 2*(8a*c^3d^4 + 11a^2c^2d^2e^2 + a^ \\
& 3c*e^4)*f^2 - 4*(c^4d^5 + 3a*c^3d^3e^2 + 2a^2c^2d*e^4)*f)))/(c^2d^ \\
& 2e^2 + a*c*e^4 - 4a^2d*f^3 + (8a*c*d^2 + a^2e^2)*f^2 - 2*(2c^2d^3 + \\
& 3a*c*d*e^2)*f)) + 2*(4a^3d*f^4 - (8a^2c*d^2 + a^3e^2)*f^3 + 2*(2a*c^ \\
& 2d^3 + 3a^2c*d*e^2)*f^2 - (a*c^2d^2e^2 + a^2c*e^4)*f) * \sqrt{c^2e^2/(c \\
& ^4d^4e^2 + 2a*c^3d^2e^4 + a^2c^2e^6 - 4a^4d*f^5 + (16a^3c*d^2 + \\
& a^4e^2)*f^4 - 12*(2a^2c^2d^3 + a^3c*d*e^2)*f^3 + 2*(8a*c^3d^4 + 11a \\
& ^2c^2d^2e^2 + a^3c*e^4)*f^2 - 4*(c^4d^5 + 3a*c^3d^3e^2 + 2a^2c^2 \\
& d*e^4)*f)))/x - 1/4*\sqrt{2}*\sqrt{((c*e^2 - 2c*d*f + 2a*f^2 - (c^2d^2e^2 \\
& + a*c*e^4 - 4a^2d*f^3 + (8a*c*d^2 + a^2e^2)*f^2 - 2*(2c^2d^3 + 3a*c \\
& *d*e^2)*f) * \sqrt{c^2e^2/(c^4d^4e^2 + 2a*c^3d^2e^4 + a^2c^2e^6 - 4a^ \\
& 4d*f^5 + (16a^3c*d^2 + a^4e^2)*f^4 - 12*(2a^2c^2d^3 + a^3c*d*e^2)*f \\
& ^3 + 2*(8a*c^3d^4 + 11a^2c^2d^2e^2 + a^3c*e^4)*f^2 - 4*(c^4d^5 + 3a \\
& *c^3d^3e^2 + 2a^2c^2d*e^4)*f)))/(c^2d^2e^2 + a*c*e^4 - 4a^2d*f^3 \\
& + (8a*c*d^2 + a^2e^2)*f^2 - 2*(2c^2d^3 + 3a*c*d*e^2)*f)) * \log((4c^2d* \\
& e*f*x - 2a*c*e^2*f + \sqrt{2}*(c^2d*e^3 + 4a*c*d*e*f^2 - (4c^2d^2*e + a \\
& *c*e^3)*f + (c^3d^3*e^3 + a*c^2d*e^5 - 4a^3d*e*f^4 + (4a^2c*d^2*e + a \\
& ^3e^3)*f^3 + (4a*c^2d^3*e - 5a^2c*d*e^3)*f^2 - (4c^3d^4*e + 5a*c^2d \\
& ^2e^3 - a^2c*e^5)*f) * \sqrt{c^2e^2/(c^4d^4e^2 + 2a*c^3d^2e^4 + a^2c^ \\
& ^2e^6 - 4a^4d*f^5 + (16a^3c*d^2 + a^4e^2)*f^4 - 12*(2a^2c^2d^3 + a \\
& ^3c*d*e^2)*f^3 + 2*(8a*c^3d^4 + 11a^2c^2d^2e^2 + a^3c*e^4)*f^2 - 4* \\
& (c^4d^5 + 3a*c^3d^3e^2 + 2a^2c^2d*e^4)*f)) * \sqrt{c*x^2 + a} * \sqrt{(c \\
& e^2 - 2c*d*f + 2a*f^2 - (c^2d^2e^2 + a*c*e^4 - 4a^2d*f^3 + (8a*c*d^2 \\
& + a^2e^2)*f^2 - 2*(2c^2d^3 + 3a*c*d*e^2)*f) * \sqrt{c^2e^2/(c^4d^4e^2 \\
& + 2a*c^3d^2e^4 + a^2c^2e^6 - 4a^4d*f^5 + (16a^3c*d^2 + a^4e^2)*f^ \\
& 4 - 12*(2a^2c^2d^3 + a^3c*d*e^2)*f^3 + 2*(8a*c^3d^4 + 11a^2c^2d^2 \\
& e^2 + a^3c*e^4)*f^2 - 4*(c^4d^5 + 3a*c^3d^3e^2 + 2a^2c^2d*e^4)*f)) \\
& / (c^2d^2e^2 + a*c*e^4 - 4a^2d*f^3 + (8a*c*d^2 + a^2e^2)*f^2 - 2*(2c^ \\
& 2d^3 + 3a*c*d*e^2)*f)) - 2*(4a^3d*f^4 - (8a^2c*d^2 + a^3e^2)*f^3 + 2 \\
& *(2a*c^2d^3 + 3a^2c*d*e^2)*f^2 - (a*c^2d^2e^2 + a^2c*e^4)*f) * \sqrt{c^ \\
& 2e^2/(c^4d^4e^2 + 2a*c^3d^2e^4 + a^2c^2e^6 - 4a^4d*f^5 + (16a^3c \\
& *d^2 + a^4e^2)*f^4 - 12*(2a^2c^2d^3 + a^3c*d*e^2)*f^3 + 2*(8a*c^3d^ \\
& 4 + 11a^2c^2d^2e^2 + a^3c*e^4)*f^2 - 4*(c^4d^5 + 3a*c^3d^3e^2 + 2a \\
& ^2c^2d*e^4)*f)))/x + 1/4*\sqrt{2}*\sqrt{((c*e^2 - 2c*d*f + 2a*f^2 - (c^2
\end{aligned}$$

```

*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3
+ 3*a*c*d*e^2)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^
6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*
d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*
d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^
2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f))*log((
4*c^2*d*e*f*x - 2*a*c*e^2*f - sqrt(2)*(c^2*d*e^3 + 4*a*c*d*e*f^2 - (4*c^2*d
^2*e + a*c*e^3)*f + (c^3*d^3*e^3 + a*c^2*d*e^5 - 4*a^3*d*e*f^4 + (4*a^2*c*d
^2*e + a^3*e^3)*f^3 + (4*a*c^2*d^3*e - 5*a^2*c*d*e^3)*f^2 - (4*c^3*d^4*e +
5*a*c^2*d^2*e^3 - a^2*c*e^5)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4
+ a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2
*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*
f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))*sqrt(c*x^2 + a)*
sqrt((c*e^2 - 2*c*d*f + 2*a*f^2 - (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8
*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(c^2*e^2/(c^4*
d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4
*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*
c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e
^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 -
2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)) - 2*(4*a^3*d*f^4 - (8*a^2*c*d^2 + a^3*e^2)
*f^3 + 2*(2*a*c^2*d^3 + 3*a^2*c*d*e^2)*f^2 - (a*c^2*d^2*e^2 + a^2*c*e^4)*f)
)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 +
(16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*
a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*
e^2 + 2*a^2*c^2*d*e^4)*f)))/x)

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Evaluation time: 1.45Error index.cc index_gc
d Error: Bad Argument Value
```

maple [B] time = 0.01, size = 589, normalized size = 2.21

$$\sqrt{2} \ln \left(\frac{\left(e^{-\sqrt{-4df+e^2}} \right) \left(x - \frac{-e+\sqrt{-4df+e^2}}{2f} \right)^c + \frac{2af^2-2cdf+ce^2-\sqrt{-4df+e^2}ce}{f^2} + \sqrt{2} \sqrt{\frac{2af^2-2cdf+ce^2-\sqrt{-4df+e^2}ce}{f^2}} \sqrt{4 \left(x - \frac{-e+\sqrt{-4df+e^2}}{2f} \right)^2 - \frac{4(e-\sqrt{-4df+e^2})^2}{c}}}{x - \frac{-e+\sqrt{-4df+e^2}}{2f}} \right)$$

$$\sqrt{-4df+e^2} \sqrt{\frac{2af^2-2cdf+ce^2-\sqrt{-4df+e^2}ce}{f^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)`

[Out] $\frac{1}{(-4df+e^2)^{1/2} 2^{1/2} \left((2af^2-2cdf+ce^2+(-4df+e^2)^{1/2}ce) / f^2 \right)^{1/2} \ln \left(\left(e + (-4df+e^2)^{1/2} \right) \left(x + \frac{1}{2} \left(e + (-4df+e^2)^{1/2} \right) / f \right) \right) / \left(f + \frac{2af^2-2cdf+ce^2+(-4df+e^2)^{1/2}ce}{f^2} + \frac{1}{2} 2^{1/2} \left((2af^2-2cdf+ce^2+(-4df+e^2)^{1/2}ce) / f^2 \right)^{1/2} \right) \left(4 \left(x + \frac{1}{2} \left(e + (-4df+e^2)^{1/2} \right) / f \right)^2 - \frac{4(e+(-4df+e^2)^{1/2})^2}{c} \right) / \left(e + (-4df+e^2)^{1/2} \right) \left(x + \frac{1}{2} \left(e + (-4df+e^2)^{1/2} \right) / f \right) \right) / \left(f + \frac{2af^2-2cdf+ce^2+(-4df+e^2)^{1/2}ce}{f^2} \right)^{1/2} \left(x + \frac{1}{2} \left(e + (-4df+e^2)^{1/2} \right) / f \right) - \frac{1}{(-4df+e^2)^{1/2} 2^{1/2} \left((2af^2-2cdf+ce^2-(-4df+e^2)^{1/2}ce) / f^2 \right)^{1/2} \ln \left(\left(e - (-4df+e^2)^{1/2} \right) \left(x - \frac{1}{2} \left(e + (-4df+e^2)^{1/2} \right) / f \right) \right) / \left(f + \frac{2af^2-2cdf+ce^2-(-4df+e^2)^{1/2}ce}{f^2} + \frac{1}{2} 2^{1/2} \left((2af^2-2cdf+ce^2-(-4df+e^2)^{1/2}ce) / f^2 \right)^{1/2} \right) \left(4 \left(x - \frac{1}{2} \left(e + (-4df+e^2)^{1/2} \right) / f \right)^2 - \frac{4(e-(-4df+e^2)^{1/2})^2}{c} \right) / \left(e - (-4df+e^2)^{1/2} \right) \left(x - \frac{1}{2} \left(e + (-4df+e^2)^{1/2} \right) / f \right) \right) / \left(f + \frac{2af^2-2cdf+ce^2-(-4df+e^2)^{1/2}ce}{f^2} \right)^{1/2} \left(x - \frac{1}{2} \left(e + (-4df+e^2)^{1/2} \right) / f \right) \right)}$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details)Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{cx^2+a} (fx^2+ex+d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)`

[Out] `int(1/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + cx^2} (d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)`

[Out] `Integral(1/(sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)`

$$3.68 \quad \int \frac{1}{x \sqrt{a+cx^2} (d+ex+fx^2)} dx$$

Optimal. Leaf size=330

$$\frac{f(\sqrt{e^2-4df}+e) \tanh^{-1}\left(\frac{2af-cx(e-\sqrt{e^2-4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}} - \frac{f(e-\sqrt{e^2-4df}) \tanh^{-1}\left(\frac{2af-cx(\sqrt{e^2-4df}+e)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}}$$

[Out] $-\operatorname{arctanh}\left(\frac{(c*x^2+a)^{1/2}/a^{1/2}}{d/a^{1/2}+1/2*f*\operatorname{arctanh}\left(\frac{1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^{1/2}))}{(c*x^2+a)^{1/2}/(2*a*f^2+(-(-4*d*f+e^2)^{1/2})*e-2*d*f+e^2)*c}\right)^{1/2}}\right)}{d*2^{1/2}/(-4*d*f+e^2)^{1/2}} - \frac{1/2*f*\operatorname{arctanh}\left(\frac{1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^{1/2}))}{(c*x^2+a)^{1/2}/(2*a*f^2+((-4*d*f+e^2)^{1/2})*e-2*d*f+e^2)*c}\right)^{1/2}}{d*2^{1/2}/(-4*d*f+e^2)^{1/2}}\right)$

Rubi [A] time = 0.82, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {6728, 266, 63, 208, 1034, 725, 206}

$$\frac{f(\sqrt{e^2-4df}+e) \tanh^{-1}\left(\frac{2af-cx(e-\sqrt{e^2-4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}} - \frac{f(e-\sqrt{e^2-4df}) \tanh^{-1}\left(\frac{2af-cx(\sqrt{e^2-4df}+e)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}\left[\frac{1}{(x*\operatorname{Sqrt}[a+c*x^2])*(d+e*x+f*x^2)}, x\right]$

[Out] $(f*(e+\operatorname{Sqrt}[e^2-4*d*f])*ArcTanh\left[\frac{(2*a*f-c*(e-\operatorname{Sqrt}[e^2-4*d*f]))*x}{(Sqrt[2]*Sqrt[2*a*f^2+c*(e^2-2*d*f-e*\operatorname{Sqrt}[e^2-4*d*f]])*Sqrt[a+c*x^2]}\right])/(Sqrt[2]*d*Sqrt[e^2-4*d*f]*Sqrt[2*a*f^2+c*(e^2-2*d*f-e*\operatorname{Sqrt}[e^2-4*d*f])]) - (f*(e-\operatorname{Sqrt}[e^2-4*d*f])*ArcTanh\left[\frac{(2*a*f-c*(e+\operatorname{Sqrt}[e^2-4*d*f]))*x}{(Sqrt[2]*Sqrt[2*a*f^2+c*(e^2-2*d*f+e*\operatorname{Sqrt}[e^2-4*d*f])]*Sqrt[a+c*x^2])}\right])/(Sqrt[2]*d*Sqrt[e^2-4*d*f]*Sqrt[2*a*f^2+c*(e^2-2*d*f+e*\operatorname{Sqrt}[e^2-4*d*f])]) - ArcTanh[Sqrt[a+c*x^2]/Sqrt[a]]/(Sqrt[a]*d)$

Rule 63

$\operatorname{Int}[(a_. + (b_.)*(x_))^{(m_)}*((c_.) + (d_.)*(x_))^{(n_)}, x_Symbol] := \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}$

$[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 206

$\text{Int}[(a_) + (b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rule 208

$\text{Int}[(a_) + (b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{NegQ}[a/b]$

Rule 266

$\text{Int}[(x_)^{(m_.)}*((a_) + (b_.)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p}, x], x, x^n], x] /; \text{FreeQ}[\{a, b, m, n, p\}, x] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 725

$\text{Int}[1/(((d_) + (e_.)*(x_))*\text{Sqrt}[(a_) + (c_.)*(x_)^2]), x_Symbol] \rightarrow -\text{Subst}[\text{Int}[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/\text{Sqrt}[a + c*x^2]] /; \text{FreeQ}[\{a, c, d, e\}, x]$

Rule 1034

$\text{Int}[(g_.) + (h_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*\text{Sqrt}[(d_) + (f_.)*(x_)^2]), x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(2*c*g - h*(b - q))/q, \text{Int}[1/((b - q + 2*c*x)*\text{Sqrt}[d + f*x^2]), x], x] - \text{Dist}[(2*c*g - h*(b + q))/q, \text{Int}[1/((b + q + 2*c*x)*\text{Sqrt}[d + f*x^2]), x], x]] /; \text{FreeQ}[\{a, b, c, d, f, g, h\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{PosQ}[b^2 - 4*a*c]$

Rule 6728

$\text{Int}[(u_)/((a_.) + (b_.)*(x_)^{(n_.)} + (c_.)*(x_)^{(2*n_.)}), x_Symbol] \rightarrow \text{With}[\{v = \text{RationalFunctionExpand}[u/(a + b*x^n + c*x^{(2*n)})], x\}, \text{Int}[v, x] /; \text{SumQ}[v]] /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{EqQ}[n2, 2*n] \&\& \text{IGtQ}[n, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a+cx^2}(d+ex+fx^2)} dx &= \int \left(\frac{1}{dx\sqrt{a+cx^2}} + \frac{-e-fx}{d\sqrt{a+cx^2}(d+ex+fx^2)} \right) dx \\
&= \frac{\int \frac{1}{x\sqrt{a+cx^2}} dx}{d} + \frac{\int \frac{-e-fx}{\sqrt{a+cx^2}(d+ex+fx^2)} dx}{d} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}} dx, x, x^2\right)}{2d} - \frac{\left(f\left(1 - \frac{e}{\sqrt{e^2-4df}}\right)\right) \int \frac{1}{(e+\sqrt{e^2-4df}+2fx)\sqrt{a+cx^2}} dx}{d} \\
&= \frac{\text{Subst}\left(\int \frac{1}{\frac{-a}{c} + \frac{x^2}{c}} dx, x, \sqrt{a+cx^2}\right)}{cd} + \frac{\left(f\left(1 - \frac{e}{\sqrt{e^2-4df}}\right)\right) \text{Subst}\left(\int \frac{1}{4af^2+c(e+\sqrt{e^2-4df})}\right)}{d} \\
&= \frac{f\left(1 + \frac{e}{\sqrt{e^2-4df}}\right) \tanh^{-1}\left(\frac{2af-c(e-\sqrt{e^2-4df})x}{\sqrt{2}\sqrt{2af^2+c(e^2-2df-e\sqrt{e^2-4df})}\sqrt{a+cx^2}}\right)}{\sqrt{2}d\sqrt{2af^2+c(e^2-2df-e\sqrt{e^2-4df})}} + \frac{f\left(1 - \frac{e}{\sqrt{e^2-4df}}\right)}{\sqrt{2}d}
\end{aligned}$$

Mathematica [A] time = 0.74, size = 319, normalized size = 0.97

$$\frac{\sqrt{2}f(\sqrt{e^2-4df}+e) \tanh^{-1}\left(\frac{2af+cx(\sqrt{e^2-4df}-e)}{\sqrt{a+cx^2}\sqrt{4af^2-2c(e\sqrt{e^2-4df}+2df-e^2)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}} + \frac{\sqrt{2}f(\sqrt{e^2-4df}-e) \tanh^{-1}\left(\frac{2af-cx(\sqrt{e^2-4df}+e)}{\sqrt{a+cx^2}\sqrt{4af^2+2c(e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}} - \frac{2 \tanh^{-1}\left(\frac{e}{\sqrt{e^2-4df}}\right)}{\sqrt{2}d}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]

[Out] ((Sqrt[2]*f*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f]))*x]/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]))*Sqrt[a + c*x^2]))/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])) + (Sqrt[2]*f*(-e + Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f]))*x]/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*Sqrt[a + c*x^2]))/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])) - (2*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/Sqrt[a])/(2*d)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.02, size = 681, normalized size = 2.06

$$2\sqrt{2} f \ln \left(\frac{\left(e - \sqrt{-4df + e^2} \right) \left(x - \frac{-e + \sqrt{-4df + e^2}}{2f} \right) c}{f} + \frac{2af^2 - 2cdf + ce^2 - \sqrt{-4df + e^2} ce}{f^2} + \frac{\sqrt{2} \sqrt{\frac{2af^2 - 2cdf + ce^2 - \sqrt{-4df + e^2} ce}{f^2}}}{x - \frac{-e + \sqrt{-4df + e^2}}{2f}} \sqrt{\frac{\left(x - \frac{-e + \sqrt{-4df + e^2}}{2f} \right)^2 c - \frac{4(e - \sqrt{-4df + e^2})^2}{2}}{2}} \right)$$

$$\frac{(-e + \sqrt{-4df + e^2}) \sqrt{-4df + e^2} \sqrt{\frac{2af^2 - 2cdf + ce^2 - \sqrt{-4df + e^2} ce}{f^2}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)

[Out]
$$\frac{-2*f/(e+(-4*d*f+e^2)^{(1/2)})/(-4*d*f+e^2)^{(1/2)}*2^{(1/2)}/((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*\ln((-e+(-4*d*f+e^2)^{(1/2)})*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(1/2)}*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-4*(e+(-4*d*f+e^2)^{(1/2)})*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)})/(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f))-2*f/(-e+(-4*d*f+e^2)^{(1/2)})/(-4*d*f+e^2)^{(1/2)}*2^{(1/2)}/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*\ln((-e+(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(1/2)}*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-4*(e+(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)})/(x-1/2*(-e$$

$+(-4*d*f+e^2)^{(1/2)}/f))+4*f/(-e+(-4*d*f+e^2)^{(1/2)})/(e+(-4*d*f+e^2)^{(1/2)})$
 $/a^{(1/2)}*\ln((2*a+2*a^{(1/2)}*(c*x^2+a)^{(1/2)})/x)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + a} (fx^2 + ex + d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x \sqrt{cx^2 + a} (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)

[Out] int(1/(x*(a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a + cx^2} (d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)

[Out] Integral(1/(x*sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)

$$3.69 \quad \int \frac{1}{x^2 \sqrt{a+cx^2} (d+ex+fx^2)} dx$$

Optimal. Leaf size=367

$$\frac{f(e\sqrt{e^2-4df}-2df+e^2)\tanh^{-1}\left(\frac{2af-cx(e-\sqrt{e^2-4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}} + \frac{f(-e\sqrt{e^2-4df}-2df+e^2)\tanh^{-1}\left(\frac{2af-cx(e-\sqrt{e^2-4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}$$

[Out] e*arctanh((c*x^2+a)^(1/2)/a^(1/2))/d^2/a^(1/2)-(c*x^2+a)^(1/2)/a/d/x-1/2*f*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)/d^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)+1/2*f*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)/d^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)

Rubi [A] time = 1.20, antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {6728, 264, 266, 63, 208, 1034, 725, 206}

$$\frac{f(e\sqrt{e^2-4df}-2df+e^2)\tanh^{-1}\left(\frac{2af-cx(e-\sqrt{e^2-4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}} + \frac{f(-e\sqrt{e^2-4df}-2df+e^2)\tanh^{-1}\left(\frac{2af-cx(e-\sqrt{e^2-4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]

[Out] -(Sqrt[a + c*x^2]/(a*d*x)) - (f*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]]) + (f*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]]) + (e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^2)

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +

$(d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{NeQ}\{b*c - a*d, 0\} \&\& \text{LtQ}\{-1, m, 0\} \&\& \text{LeQ}\{-1, n, 0\} \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}\{a, b, c, d, m, n, x\}$

Rule 206

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{NegQ}\{a/b\} \&\& (\text{GtQ}\{a, 0\} \parallel \text{LtQ}\{b, 0\})$

Rule 208

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{NegQ}\{a/b\}$

Rule 264

$\text{Int}[(c_)*(x_)^{(m_)}*(a_ + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(c*x)^{(m+1)}*(a + b*x^n)^{(p+1)}/(a*c*(m+1)), x] /; \text{FreeQ}\{a, b, c, m, n, p\}, x\} \&\& \text{EqQ}\{(m+1)/n + p + 1, 0\} \&\& \text{NeQ}\{m, -1\}$

Rule 266

$\text{Int}[(x_)^{(m_)}*(a_ + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n) - 1}*(a + b*x)^p, x], x, x^n], x] /; \text{FreeQ}\{a, b, m, n, p\}, x\} \&\& \text{IntegerQ}[\text{Simplify}[(m+1)/n]]$

Rule 725

$\text{Int}[1/(((d_ + (e_)*(x_))*\text{Sqrt}[(a_ + (c_)*(x_)^2])), x_Symbol] \rightarrow -\text{Subst}[\text{Int}[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/\text{Sqrt}[a + c*x^2]] /; \text{FreeQ}\{a, c, d, e\}, x\}$

Rule 1034

$\text{Int}[(g_ + (h_)*(x_))/((a_ + (b_)*(x_ + (c_)*(x_)^2))*\text{Sqrt}[(d_ + (f_)*(x_)^2])), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(2*c*g - h*(b - q))/q, \text{Int}[1/((b - q + 2*c*x)*\text{Sqrt}[d + f*x^2]), x], x] - \text{Dist}[(2*c*g - h*(b + q))/q, \text{Int}[1/((b + q + 2*c*x)*\text{Sqrt}[d + f*x^2]), x], x]] /; \text{FreeQ}\{a, b, c, d, f, g, h\}, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{PosQ}[b^2 - 4*a*c]$

Rule 6728

$\text{Int}[(u_)/((a_ + (b_)*(x_)^{(n_)} + (c_)*(x_)^{(n2_)}), x_Symbol] \rightarrow \text{With}\{v = \text{RationalFunctionExpand}[u/(a + b*x^n + c*x^{(2*n)}), x]\}, \text{Int}[v, x] /; \text{Su}$

mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^2 \sqrt{a+cx^2} (d+ex+fx^2)} dx &= \int \left(\frac{1}{dx^2 \sqrt{a+cx^2}} - \frac{e}{d^2 x \sqrt{a+cx^2}} + \frac{e^2 - df + efx}{d^2 \sqrt{a+cx^2} (d+ex+fx^2)} \right) dx \\
 &= \frac{\int \frac{e^2 - df + efx}{\sqrt{a+cx^2} (d+ex+fx^2)} dx}{d^2} + \frac{\int \frac{1}{x^2 \sqrt{a+cx^2}} dx}{d} - \frac{e \int \frac{1}{x \sqrt{a+cx^2}} dx}{d^2} \\
 &= -\frac{\sqrt{a+cx^2}}{adx} - \frac{e \operatorname{Subst}\left(\int \frac{1}{x \sqrt{a+cx}} dx, x, x^2\right)}{2d^2} - \frac{(f(e^2 - 2df - e\sqrt{e^2 - 4df})) \int}{d^2 \sqrt{e^2 - 4df}} \\
 &= -\frac{\sqrt{a+cx^2}}{adx} - \frac{e \operatorname{Subst}\left(\int \frac{1}{\frac{-a}{c} + \frac{x^2}{c}} dx, x, \sqrt{a+cx^2}\right)}{cd^2} + \frac{(f(e^2 - 2df - e\sqrt{e^2 - 4df})) \int}{d^2 \sqrt{e^2 - 4df}} \\
 &= -\frac{\sqrt{a+cx^2}}{adx} - \frac{f(e^2 - 2df + e\sqrt{e^2 - 4df}) \tanh^{-1}\left(\frac{2af - c(e - \sqrt{e^2 - 4df})x}{\sqrt{2}\sqrt{2af^2 + c(e^2 - 2df - e\sqrt{e^2 - 4df})}}\right)}{\sqrt{2} d^2 \sqrt{e^2 - 4df} \sqrt{2af^2 + c(e^2 - 2df - e\sqrt{e^2 - 4df})}}
 \end{aligned}$$

Mathematica [A] time = 0.84, size = 356, normalized size = 0.97

$$\frac{\sqrt{2} f (e \sqrt{e^2 - 4df} - 2df + e^2) \tanh^{-1}\left(\frac{2af + cx(\sqrt{e^2 - 4df} - e)}{\sqrt{a+cx^2} \sqrt{4af^2 - 2c(e\sqrt{e^2 - 4df} + 2df - e^2)}}\right)}{\sqrt{e^2 - 4df} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}} + \frac{\sqrt{2} f (e \sqrt{e^2 - 4df} + 2df - e^2) \tanh^{-1}\left(\frac{2af - cx(\sqrt{e^2 - 4df} + e)}{\sqrt{a+cx^2} \sqrt{4af^2 + 2c(e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{e^2 - 4df} \sqrt{2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)}}}{2d^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*Sqrt[a + c*x^2]*(d + e*x + f*x^2)), x]

[Out] -1/2*((2*d*Sqrt[a + c*x^2])/(a*x) + (Sqrt[2]*f*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x]/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]))/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (Sqrt[2]*f*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x]/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]))/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e\sqrt{e^2 - 4*d*f} - 2*d*f + e^2)]))

))/((Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
) - (2*e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/Sqrt[a])/d^2

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x);OUTPUT:Evaluation time: 9.45Error index.cc index_gc
 d Error: Bad Argument Value

maple [B] time = 0.02, size = 736, normalized size = 2.01

$$\frac{16e f^2 \ln\left(\frac{2a+2\sqrt{cx^2+a}\sqrt{a}}{x}\right)}{(-e + \sqrt{-4df + e^2})^2 (e + \sqrt{-4df + e^2})^2 \sqrt{a}} \left[\frac{4\sqrt{2} f^2 \ln\left(\frac{\left(e - \sqrt{-4df + e^2}\right) \left(x - \frac{-e + \sqrt{-4df + e^2}}{2f}\right)^c}{f} + \frac{2af^2 - 2cdf + ce^2 - \sqrt{-4df + e^2} ce}{f^2} + \frac{\sqrt{2}}{f}\right)}{(-e + \sqrt{-4df + e^2})^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)

[Out] $4*f^2/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-2*c*d*f$
 $+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*\ln((-e+(-4*d*f+e^2)^(1/2))*(x+1/$
 $2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c$
 $*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1$
 $/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x+1/2$

$$\begin{aligned} & * (e + (-4*d*f + e^2)^{(1/2)}) / f * c / f + 2 * (2*a*f^2 - 2*c*d*f + c*e^2 + (-4*d*f + e^2)^{(1/2)} * \\ & c*e) / f^2)^{(1/2)} / (x + 1/2 * (e + (-4*d*f + e^2)^{(1/2)}) / f) + 4*f / (-e + (-4*d*f + e^2)^{(1/2)}) \\ &) / (e + (-4*d*f + e^2)^{(1/2)}) / a / x * (c*x^2 + a)^{(1/2)} - 4*f^2 / (-e + (-4*d*f + e^2)^{(1/2)}) \\ &)^2 / (-4*d*f + e^2)^{(1/2)} * 2^{(1/2)} / ((2*a*f^2 - 2*c*d*f + c*e^2 - (-4*d*f + e^2)^{(1/2)} * c \\ & *e) / f^2)^{(1/2)} * \ln((-e - (-4*d*f + e^2)^{(1/2)}) * (x - 1/2 * (-e + (-4*d*f + e^2)^{(1/2)}) / f) \\ &) * c / f + (2*a*f^2 - 2*c*d*f + c*e^2 - (-4*d*f + e^2)^{(1/2)} * c*e) / f^2 + 1/2 * 2^{(1/2)} * ((2*a \\ & f^2 - 2*c*d*f + c*e^2 - (-4*d*f + e^2)^{(1/2)} * c*e) / f^2)^{(1/2)} * (4 * (x - 1/2 * (-e + (-4*d*f + \\ & e^2)^{(1/2)}) / f)^2 * c - 4 * (e - (-4*d*f + e^2)^{(1/2)}) * (x - 1/2 * (-e + (-4*d*f + e^2)^{(1/2)}) / \\ & f) * c / f + 2 * (2*a*f^2 - 2*c*d*f + c*e^2 - (-4*d*f + e^2)^{(1/2)} * c*e) / f^2)^{(1/2)} / (x - 1/2 * \\ & (-e + (-4*d*f + e^2)^{(1/2)}) / f) + 16*f^2 * e / (-e + (-4*d*f + e^2)^{(1/2)})^2 / (e + (-4*d*f + e \\ & ^2)^{(1/2)})^2 / a^{(1/2)} * \ln((2*a + 2 * (c*x^2 + a)^{(1/2)} * a^{(1/2)}) / x) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + a} (fx^2 + ex + d)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2 \sqrt{cx^2 + a} (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)

[Out] int(1/(x^2*(a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{a + cx^2} (d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)

[Out] Integral(1/(x**2*sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)

$$3.70 \quad \int \frac{1}{x^3 \sqrt{a+cx^2} (d+ex+fx^2)} dx$$

Optimal. Leaf size=457

$$\frac{c \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d} - \frac{(e^2 - df) \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^3} + \frac{f(-e^2 - df)(e - \sqrt{e^2 - 4df}) - 4def + 2e^3}{\sqrt{2}d^3\sqrt{e^2 - 4df}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}} \tanh^{-1}\left(\frac{\sqrt{2} \sqrt{a+cx^2}}{\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)$$

[Out] 1/2*c*arctanh((c*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/d-(-d*f+e^2)*arctanh((c*x^2+a)^(1/2)/a^(1/2))/d^3/a^(1/2)-1/2*(c*x^2+a)^(1/2)/a/d/x^2+e*(c*x^2+a)^(1/2)/a/d^2/x+1/2*f*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(2*e^3-4*d*e*f-(-d*f+e^2)*(e-(-4*d*f+e^2)^(1/2)))/d^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)-1/2*f*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(2*e^3-4*d*e*f-(-d*f+e^2)*(e+(-4*d*f+e^2)^(1/2)))/d^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)

Rubi [A] time = 1.86, antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6728, 266, 51, 63, 208, 264, 1034, 725, 206}

$$\frac{c \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d} + \frac{f(-e^2 - df)(e - \sqrt{e^2 - 4df}) - 4def + 2e^3}{\sqrt{2}d^3\sqrt{e^2 - 4df}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}} \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2} \sqrt{a+cx^2} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]

[Out] -Sqrt[a + c*x^2]/(2*a*d*x^2) + (e*Sqrt[a + c*x^2])/(a*d^2*x) + (f*(2*e^3 - 4*d*e*f - (e^2 - d*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]])]*Sqrt[a + c*x^2]))/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (f*(2*e^3 - 4*d*e*f - (e^2 - d*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])]*Sqrt[a + c*x^2]))/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) + (c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*a^(3/2)*d) - ((e^2 - d*f)*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^3)

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

[{a, c, d, e}, x]

Rule 1034

Int[((g_.) + (h_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rule 6728

Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^3 \sqrt{a+cx^2} (d+ex+fx^2)} dx &= \int \left(\frac{1}{dx^3 \sqrt{a+cx^2}} - \frac{e}{d^2 x^2 \sqrt{a+cx^2}} + \frac{e^2-df}{d^3 x \sqrt{a+cx^2}} + \frac{-e(e^2-2df)-f(e^2-df)}{d^3 \sqrt{a+cx^2} (d+ex+fx^2)} \right) dx \\
 &= \frac{\int \frac{-e(e^2-2df)-f(e^2-df)x}{\sqrt{a+cx^2} (d+ex+fx^2)} dx}{d^3} + \frac{\int \frac{1}{x^3 \sqrt{a+cx^2}} dx}{d} - \frac{e \int \frac{1}{x^2 \sqrt{a+cx^2}} dx}{d^2} + \frac{(e^2-df) \int \frac{1}{x \sqrt{a+cx^2}} dx}{d^3} \\
 &= \frac{e\sqrt{a+cx^2}}{ad^2x} + \frac{\text{Subst}\left(\int \frac{1}{x^2 \sqrt{a+cx}} dx, x, x^2\right)}{2d} + \frac{(e^2-df) \text{Subst}\left(\int \frac{1}{x \sqrt{a+cx}} dx, x, x^2\right)}{2d^3} \\
 &= -\frac{\sqrt{a+cx^2}}{2adx^2} + \frac{e\sqrt{a+cx^2}}{ad^2x} - \frac{c \text{Subst}\left(\int \frac{1}{x \sqrt{a+cx}} dx, x, x^2\right)}{4ad} + \frac{(e^2-df) \text{Subst}\left(\int \frac{1}{x \sqrt{a+cx}} dx, x, x^2\right)}{2d^3} \\
 &= -\frac{\sqrt{a+cx^2}}{2adx^2} + \frac{e\sqrt{a+cx^2}}{ad^2x} + \frac{f(2e^3-4def-(e^2-df)(e-\sqrt{e^2-4df})) \tan^{-1}\left(\frac{e-\sqrt{e^2-4df}}{\sqrt{2}d^3\sqrt{e^2-4df}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c(e^2-df)}} \\
 &= -\frac{\sqrt{a+cx^2}}{2adx^2} + \frac{e\sqrt{a+cx^2}}{ad^2x} + \frac{f(2e^3-4def-(e^2-df)(e-\sqrt{e^2-4df})) \tan^{-1}\left(\frac{e-\sqrt{e^2-4df}}{\sqrt{2}d^3\sqrt{e^2-4df}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c(e^2-df)}}
 \end{aligned}$$

Mathematica [A] time = 1.57, size = 460, normalized size = 1.01

$$\frac{cd^2\sqrt{a+cx^2}\left(\frac{a}{cx^2}-\frac{\tanh^{-1}\left(\sqrt{\frac{cx^2}{a}+1}\right)}{\sqrt{\frac{cx^2}{a}+1}}\right)}{a^2} + \frac{2(e^2-df)\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{2}f(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3)\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-\sqrt{a+cx^2}\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}-\sqrt{a+cx^2}\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}}{2d^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]

[Out]
$$-1/2*((-2*d*e*Sqrt[a + c*x^2])/(a*x) - (Sqrt[2]*f*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f]))*x]/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2])))/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (Sqrt[2]*f*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f] + d*f*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f]))*x]/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2])))/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) + (2*(e^2 - d*f)*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/Sqrt[a] + (c*d^2*Sqrt[a + c*x^2]*(a/(c*x^2) - ArcTanh[Sqrt[1 + (c*x^2)/a]]/Sqrt[1 + (c*x^2)/a]))/a^2)/d^3$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.02, size = 911, normalized size = 1.99

$$\frac{64d f^4 \ln\left(\frac{2a+2\sqrt{cx^2+a}\sqrt{a}}{x}\right)}{\left(-e+\sqrt{-4df+e^2}\right)^3\left(e+\sqrt{-4df+e^2}\right)^3\sqrt{a}} + \frac{64e^2 f^3 \ln\left(\frac{2a+2\sqrt{cx^2+a}\sqrt{a}}{x}\right)}{\left(-e+\sqrt{-4df+e^2}\right)^3\left(e+\sqrt{-4df+e^2}\right)^3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2), x)`

[Out]
$$\begin{aligned} & -8*f^3/(e+(-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*\ln((-e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))+16*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+e^2)^(1/2))^2/a/x*(c*x^2+a)^(1/2)+2*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a/x^2*(c*x^2+a)^(1/2)-2*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*c/a^(3/2)*\ln((2*a+2*(c*x^2+a)^(1/2)*a^(1/2))/x)-8*f^3/(-e+(-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*\ln((-e+(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))-64*f^4/(-e+(-4*d*f+e^2)^(1/2))^3/(e+(-4*d*f+e^2)^(1/2))^3/a^(1/2)*\ln((2*a+2*(c*x^2+a)^(1/2)*a^(1/2))/x)*d+64*f^3/(-e+(-4*d*f+e^2)^(1/2))^3/(e+(-4*d*f+e^2)^(1/2))^3/a^(1/2)*\ln((2*a+2*(c*x^2+a)^(1/2)*a^(1/2))/x)*e^2 \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+a}(fx^2+ex+d)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3 \sqrt{cx^2 + a} (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)

[Out] int(1/(x^3*(a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \sqrt{a + cx^2} (d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)

[Out] Integral(1/(x**3*sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)

$$3.71 \quad \int \frac{x^3}{(a+cx^2)^{3/2}(d+ex+fx^2)} dx$$

Optimal. Leaf size=499

$$\frac{cex(a(e^2-2df)+cd^2)+af(a(e^2-df)+cd^2)}{af^2\sqrt{a+cx^2}((cd-af)^2+ace^2)} \frac{(2adef-(e-\sqrt{e^2-4df})(a(e^2-df)+cd^2))\tanh^{-1}\left(\frac{\dots}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{e^2-4df}((cd-af)^2+ace^2)\sqrt{2af^2+c}(-e\sqrt{e^2-4df})}$$

[Out] $-1/c/f/(c*x^2+a)^{(1/2)}-e*x/a/f^2/(c*x^2+a)^{(1/2)}+(a*f*(c*d^2+a*(-d*f+e^2))+c*e*(c*d^2+a*(-2*d*f+e^2))*x)/a/f^2/(a*c*e^2+(-a*f+c*d)^2)/(c*x^2+a)^{(1/2)}-1/2*\operatorname{arctanh}(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^{(1/2)})))*2^{(1/2)}/(c*x^2+a)^{(1/2)}/(2*a*f^2+(-(-4*d*f+e^2)^{(1/2)}*e-2*d*f+e^2)*c)^{(1/2)}*(2*a*d*e*f-(c*d^2+a*(-d*f+e^2))*(e-(-4*d*f+e^2)^{(1/2)}))/(a*c*e^2+(-a*f+c*d)^2)*2^{(1/2)}/(-4*d*f+e^2)^{(1/2)}/(2*a*f^2+(-(-4*d*f+e^2)^{(1/2)}*e-2*d*f+e^2)*c)^{(1/2)}+1/2*\operatorname{arctanh}(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^{(1/2)})))*2^{(1/2)}/(c*x^2+a)^{(1/2)}/(2*a*f^2+((-4*d*f+e^2)^{(1/2)}*e-2*d*f+e^2)*c)^{(1/2)}*(2*a*d*e*f-(c*d^2+a*(-d*f+e^2))*(e+(-4*d*f+e^2)^{(1/2)}))/(a*c*e^2+(-a*f+c*d)^2)*2^{(1/2)}/(-4*d*f+e^2)^{(1/2)}/(2*a*f^2+((-4*d*f+e^2)^{(1/2)}*e-2*d*f+e^2)*c)^{(1/2)}$

Rubi [A] time = 2.11, antiderivative size = 499, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {6728, 191, 261, 1017, 1034, 725, 206}

$$\frac{cex(a(e^2-2df)+cd^2)+af(a(e^2-df)+cd^2)}{af^2\sqrt{a+cx^2}((cd-af)^2+ace^2)} \frac{(2adef-(e-\sqrt{e^2-4df})(a(e^2-df)+cd^2))\tanh^{-1}\left(\frac{\dots}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{e^2-4df}((cd-af)^2+ace^2)\sqrt{2af^2+c}(-e\sqrt{e^2-4df})}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^3/((a+cx^2)^{(3/2)}*(d+ex+fx^2)),x]$

[Out] $-(1/(c*f*\operatorname{Sqrt}[a+cx^2]))-(e*x)/(a*f^2*\operatorname{Sqrt}[a+cx^2])+(a*f*(c*d^2+a*(e^2-df))+c*e*(c*d^2+a*(e^2-2*d*f))*x)/(a*f^2*(a*c*e^2+(c*d-a*f)^2)*\operatorname{Sqrt}[a+cx^2])-\frac{((2*a*d*e*f-(e-\operatorname{Sqrt}[e^2-4*d*f]))*(c*d^2+a*(e^2-df)))*\operatorname{ArcTanh}[(2*a*f-c*(e-\operatorname{Sqrt}[e^2-4*d*f]))*x]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[2*a*f^2+c*(e^2-2*d*f-e*\operatorname{Sqrt}[e^2-4*d*f]))*\operatorname{Sqrt}[a+cx^2])]}{(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[e^2-4*d*f]*(a*c*e^2+(c*d-a*f)^2)*\operatorname{Sqrt}[2*a*f^2+c*(e^2-2*d*f-e*\operatorname{Sqrt}[e^2-4*d*f]))}+\frac{((2*a*d*e*f-(e+\operatorname{Sqrt}[e^2-4*d*f]))*(c*d^2+a*(e^2-df)))*\operatorname{ArcTanh}[(2*a*f-c*(e+\operatorname{Sqrt}[e^2-4*d*f]))*x]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[2*a*f^2+c*(e^2-2*d*f+e*\operatorname{Sqrt}[e^2-4*d*f]))*\operatorname{Sqrt}[a+cx^2])]}{(\operatorname{Sqrt}[$

$2) * \text{Sqrt}[e^2 - 4*d*f] * (a*c*e^2 + (c*d - a*f)^2) * \text{Sqrt}[2*a*f^2 + c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f])]$)

Rule 191

$\text{Int}[(a_) + (b_)*(x_)^(n_)]^(p_), x_Symbol] \rightarrow \text{Simp}[(x*(a + b*x^n)^(p + 1))/a, x] /;$ FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 206

$\text{Int}[(a_) + (b_)*(x_)^2]^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 261

$\text{Int}[(x_)^(m_)*((a_) + (b_)*(x_)^(n_)]^(p_), x_Symbol] \rightarrow \text{Simp}[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /;$ FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 725

$\text{Int}[1/(((d_) + (e_)*(x_))*\text{Sqrt}[(a_) + (c_)*(x_)^2]), x_Symbol] \rightarrow -\text{Subst}[\text{Int}[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/\text{Sqrt}[a + c*x^2]] /;$ FreeQ[{a, c, d, e}, x]

Rule 1017

$\text{Int}[(g_) + (h_)*(x_)]*((a_) + (c_)*(x_)^2)^(p_)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] \rightarrow \text{Simp}[(a + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e) + (-a*h)*(2*c^2*d - c*(2*a*f)) + c*(g*(2*c^2*d - c*(2*a*f)) - h*(-2*a*c*e))*x)/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), x] + \text{Dist}[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), \text{Int}[(a + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*\text{Simp}[(-2*g*c)*((c*d - a*f)^2 - (-a*e)*(c*e))^(p + 1) + (2*(g*c*(c*d - a*f) - a*(-h*c*e)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e) + (-a*h)*(2*c^2*d - c*(Plus[2])*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*a*c*e) + (-a*h)*(2*c^2*d - c*(Plus[2])*a*f)))*(p + q + 2) - (2*(g*c*(c*d - a*f) - a*(-h*c*e)))*(-c*e*(2*p + q + 4)))*x - c*f*(2*(g*c*(c*d - a*f) - a*(-h*c*e)))*(2*p + 2*q + 5)*x^2, x], x] /;$ FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[a*c*e^2 + (c*d - a*f)^2, 0] && !(IntegerQ[p] && ILtQ[q, -1])

Rule 1034

$\text{Int}[(g_) + (h_)*(x_)]/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*\text{Sqrt}[(d_) + (f_)*(x_)^2]), x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(2*c*g - h*($

$(b - q)/q, \text{Int}[1/((b - q + 2*c*x)*\text{Sqrt}[d + f*x^2]), x], x] - \text{Dist}[(2*c*g - h*(b + q))/q, \text{Int}[1/((b + q + 2*c*x)*\text{Sqrt}[d + f*x^2]), x], x] /; \text{FreeQ}\{a, b, c, d, f, g, h\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{PosQ}[b^2 - 4*a*c]$

Rule 6728

$\text{Int}[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] \text{:>} \text{With}[\{v = \text{RationalFunctionExpand}[u/(a + b*x^n + c*x^(2*n)), x]\}, \text{Int}[v, x] /; \text{SumQ}[v]] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{EqQ}[n2, 2*n] \&\& \text{IGtQ}[n, 0]$

Rubi steps

$$\begin{aligned} \int \frac{x^3}{(a+cx^2)^{3/2}(d+ex+fx^2)} dx &= \int \left(-\frac{e}{f^2(a+cx^2)^{3/2}} + \frac{x}{f(a+cx^2)^{3/2}} + \frac{de+(e^2-df)x}{f^2(a+cx^2)^{3/2}(d+ex+fx^2)} \right) dx \\ &= \frac{\int \frac{de+(e^2-df)x}{(a+cx^2)^{3/2}(d+ex+fx^2)} dx}{f^2} - \frac{e \int \frac{1}{(a+cx^2)^{3/2}} dx}{f^2} + \frac{\int \frac{x}{(a+cx^2)^{3/2}} dx}{f} \\ &= -\frac{1}{cf\sqrt{a+cx^2}} - \frac{ex}{af^2\sqrt{a+cx^2}} + \frac{af(cd^2+a(e^2-df))+ce(cd^2+a(e^2-2df))}{af^2(ace^2+(cd-af)^2)\sqrt{a+cx^2}} \\ &= -\frac{1}{cf\sqrt{a+cx^2}} - \frac{ex}{af^2\sqrt{a+cx^2}} + \frac{af(cd^2+a(e^2-df))+ce(cd^2+a(e^2-2df))}{af^2(ace^2+(cd-af)^2)\sqrt{a+cx^2}} \\ &= -\frac{1}{cf\sqrt{a+cx^2}} - \frac{ex}{af^2\sqrt{a+cx^2}} + \frac{af(cd^2+a(e^2-df))+ce(cd^2+a(e^2-2df))}{af^2(ace^2+(cd-af)^2)\sqrt{a+cx^2}} \\ &= -\frac{1}{cf\sqrt{a+cx^2}} - \frac{ex}{af^2\sqrt{a+cx^2}} + \frac{af(cd^2+a(e^2-df))+ce(cd^2+a(e^2-2df))}{af^2(ace^2+(cd-af)^2)\sqrt{a+cx^2}} \end{aligned}$$

Mathematica [A] time = 2.70, size = 577, normalized size = 1.16

$$\frac{\left(-\frac{e(e^2-3df)}{\sqrt{e^2-4df}} - df + e^2\right)(2af + cx(e - \sqrt{e^2-4df}))}{af^2\sqrt{a+cx^2}(4af^2 + c(e - \sqrt{e^2-4df})^2)} + \frac{\left(\frac{e(e^2-3df)}{\sqrt{e^2-4df}} - df + e^2\right)(2af + cx(\sqrt{e^2-4df} + e))}{af^2\sqrt{a+cx^2}(4af^2 + c(\sqrt{e^2-4df} + e)^2)} + \frac{\sqrt{2}(-e^2)}{af^2\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] -(1/(c*f*Sqrt[a + c*x^2])) - (e*x)/(a*f^2*Sqrt[a + c*x^2]) + ((e^2 - d*f -
(e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*f])*(2*a*f + c*(e - Sqrt[e^2 - 4*d*f])*x))
/(a*f^2*(4*a*f^2 + c*(e - Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + c*x^2]) + ((e^2 -
d*f + (e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*f])*(2*a*f + c*(e + Sqrt[e^2 - 4*d*f]
)*x))/(a*f^2*(4*a*f^2 + c*(e + Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + c*x^2]) + (S
qrt[2]*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f] + d*f*Sqrt[e^2 - 4*d*f])*ArcT
anh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*
f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]))/(Sqrt[e^2 - 4*d*f]*(2*a*f^2 +
c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))^(3/2)) - (Sqrt[2]*(e^3 - 3*d*e*f + e
^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[
e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*S
qrt[a + c*x^2]))/(Sqrt[e^2 - 4*d*f]*(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2
- 4*d*f]))^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage₂

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage2
```

maple [B] time = 0.03, size = 6124, normalized size = 12.27

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details)Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{(cx^2 + a)^{3/2} (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)

[Out] int(x^3/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a + cx^2)^{\frac{3}{2}} (d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)

[Out] Integral(x**3/((a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)

$$3.72 \quad \int \frac{x^2}{(a+cx^2)^{3/2}(d+ex+fx^2)} dx$$

Optimal. Leaf size=410

$$\frac{f(2d(cd-af) + ae(e - \sqrt{e^2 - 4df})) \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2 + ace^2\right)\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}} + \frac{f(2d(cd-af) + ae(\sqrt{e^2 - 4df}))}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2 + ace^2\right)}$$

[Out] $(-a*e - (-a*f + c*d)*x)/(a*c*e^2 + (-a*f + c*d)^2)/(c*x^2 + a)^{(1/2)} - 1/2*f*\operatorname{arctanh}(1/2*(2*a*f - c*x*(e - (-4*d*f + e^2)^{(1/2)})))*2^{(1/2)}/(c*x^2 + a)^{(1/2)}/(2*a*f^2 + (-(-4*d*f + e^2)^{(1/2)}*e - 2*d*f + e^2)*c)^{(1/2)})*(2*d*(-a*f + c*d) + a*e*(e - (-4*d*f + e^2)^{(1/2)}))/((a*c*e^2 + (-a*f + c*d)^2)*2^{(1/2)}/(-4*d*f + e^2)^{(1/2)})/(2*a*f^2 + (-(-4*d*f + e^2)^{(1/2)}*e - 2*d*f + e^2)*c)^{(1/2)} + 1/2*f*\operatorname{arctanh}(1/2*(2*a*f - c*x*(e + (-4*d*f + e^2)^{(1/2)})))*2^{(1/2)}/(c*x^2 + a)^{(1/2)}/(2*a*f^2 + ((-4*d*f + e^2)^{(1/2)}*e - 2*d*f + e^2)*c)^{(1/2)})*(2*d*(-a*f + c*d) + a*e*(e + (-4*d*f + e^2)^{(1/2)}))/((a*c*e^2 + (-a*f + c*d)^2)*2^{(1/2)}/(-4*d*f + e^2)^{(1/2)})/(2*a*f^2 + ((-4*d*f + e^2)^{(1/2)}*e - 2*d*f + e^2)*c)^{(1/2)}$

Rubi [A] time = 0.71, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1063, 1034, 725, 206}

$$\frac{f(2d(cd-af) + ae(e - \sqrt{e^2 - 4df})) \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2 + ace^2\right)\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}} + \frac{f(2d(cd-af) + ae(\sqrt{e^2 - 4df}))}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2 + ace^2\right)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^2/((a + c*x^2)^{(3/2)}*(d + e*x + f*x^2)), x]$

[Out] $-((a*e + (c*d - a*f)*x)/((a*c*e^2 + (c*d - a*f)^2)*\operatorname{Sqrt}[a + c*x^2])) - (f*(2*d*(c*d - a*f) + a*e*(e - \operatorname{Sqrt}[e^2 - 4*d*f]))*\operatorname{ArcTanh}[(2*a*f - c*(e - \operatorname{Sqrt}[e^2 - 4*d*f]))*x]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[2*a*f^2 + c*(e^2 - 2*d*f - e*\operatorname{Sqrt}[e^2 - 4*d*f])])*\operatorname{Sqrt}[a + c*x^2]))/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*\operatorname{Sqrt}[2*a*f^2 + c*(e^2 - 2*d*f - e*\operatorname{Sqrt}[e^2 - 4*d*f])]) + (f*(2*d*(c*d - a*f) + a*e*(e + \operatorname{Sqrt}[e^2 - 4*d*f]))*\operatorname{ArcTanh}[(2*a*f - c*(e + \operatorname{Sqrt}[e^2 - 4*d*f]))*x]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[2*a*f^2 + c*(e^2 - 2*d*f + e*\operatorname{Sqrt}[e^2 - 4*d*f])])*\operatorname{Sqrt}[a + c*x^2]))/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*\operatorname{Sqrt}[2*a*f^2 + c*(e^2 - 2*d*f + e*\operatorname{Sqrt}[e^2 - 4*d*f])])$

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (c_)*(x_)^2]), x_Symbol] := -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1034

```
Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (f
_)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1063

```
Int[((a_) + (c_)*(x_)^2)^(p_)*((A_) + (C_)*(x_)^2)*((d_) + (e_)*(x_) +
(f_)*(x_)^2)^(q_), x_Symbol] := Simp[((a + c*x^2)^(p + 1)*(d + e*x + f*x^2
)^(q + 1)*((A*c - a*C)*(2*a*c*e) + c*(A*(2*c^2*d - c*(2*a*f)) + C*(-2*a*(c*
d - a*f)))*x))/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((
-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^(p + 1)*(d + e*
x + f*x^2)^q*Simp[(-2*A*c - 2*a*C)*((c*d - a*f)^2 - (-(a*e))*(c*e))*(p + 1)
+ (2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) -
e*((A*c - a*C)*(2*a*c*e))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e))*(p + q
+ 2) - (2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(-(c*e*(2*p + q + 4)))]*x
- c*f*(2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x
] /; FreeQ[{a, c, d, e, f, A, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p
, -1] && NeQ[a*c*e^2 + (c*d - a*f)^2, 0] && !( !IntegerQ[p] && ILtQ[q, -1]
) && !IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(a+cx^2)^{3/2}(d+ex+fx^2)} dx &= -\frac{ae+(cd-af)x}{(ace^2+(cd-af)^2)\sqrt{a+cx^2}} + \frac{\int \frac{2acd(cd-af)-2a^2cex}{\sqrt{a+cx^2}(d+ex+fx^2)} dx}{2ac(ace^2+(cd-af)^2)} \\
&= -\frac{ae+(cd-af)x}{(ace^2+(cd-af)^2)\sqrt{a+cx^2}} + \frac{(f(2d(cd-af)+ae(e-\sqrt{e^2-4df}))) \int \frac{1}{\sqrt{e^2-4df}(ace^2+(cd-af)^2)} dx}{\sqrt{e^2-4df}(ace^2+(cd-af)^2)} \\
&= -\frac{ae+(cd-af)x}{(ace^2+(cd-af)^2)\sqrt{a+cx^2}} - \frac{(f(2d(cd-af)+ae(e-\sqrt{e^2-4df}))) \operatorname{Subst}(\int \frac{1}{\sqrt{e^2-4df}} dx, \sqrt{e^2-4df}, ae+cx)}{\sqrt{e^2-4df}} \\
&= -\frac{ae+(cd-af)x}{(ace^2+(cd-af)^2)\sqrt{a+cx^2}} - \frac{f(2d(cd-af)+ae(e-\sqrt{e^2-4df})) \operatorname{tanh}^{-1}\left(\frac{ae+cx}{\sqrt{e^2-4df}}\right)}{\sqrt{2}\sqrt{e^2-4df}(ace^2+(cd-af)^2)\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 2.07, size = 509, normalized size = 1.24

$$\frac{\left(\frac{2df-e^2}{\sqrt{e^2-4df}}+e\right)(2af+cx(e-\sqrt{e^2-4df}))}{a\sqrt{a+cx^2}(4af^2+c(e-\sqrt{e^2-4df})^2)} - \frac{\left(\frac{e^2-2df}{\sqrt{e^2-4df}}+e\right)(2af+cx(\sqrt{e^2-4df}+e))}{a\sqrt{a+cx^2}(4af^2+c(\sqrt{e^2-4df}+e)^2)} + \frac{\sqrt{2}f^2(e\sqrt{e^2-4df}+2df-e^2)\operatorname{tanh}^{-1}\left(\frac{2af+cx(\sqrt{e^2-4df}-e)}{\sqrt{a+cx^2}\sqrt{4af^2-2c(e\sqrt{e^2-4df}+e)}}\right)}{\sqrt{e^2-4df}(2af^2+c(-e\sqrt{e^2-4df}-2df+e^2))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

[Out] (x/(a*Sqrt[a + c*x^2]) - ((e + (-e^2 + 2*d*f)/Sqrt[e^2 - 4*d*f])*(2*a*f + c*(e - Sqrt[e^2 - 4*d*f])*x))/(a*(4*a*f^2 + c*(e - Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + c*x^2]) - ((e + (e^2 - 2*d*f)/Sqrt[e^2 - 4*d*f])*(2*a*f + c*(e + Sqrt[e^2 - 4*d*f])*x))/(a*(4*a*f^2 + c*(e + Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + c*x^2]) + (Sqrt[2]*f^2*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]]))/(Sqrt[e^2 - 4*d*f]*(2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))^(3/2)) + (Sqrt[2]*f^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]]))/(Sqrt[e^2 - 4*d*f]*(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))^(3/2))/f

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x²/(c*x²+a)^(3/2)/(f*x²+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage₂

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x²/(c*x²+a)^(3/2)/(f*x²+e*x+d),x, algorithm="giac")

[Out] sage₂

maple [B] time = 0.02, size = 4752, normalized size = 11.59

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x²/(c*x²+a)^(3/2)/(f*x²+e*x+d),x)

[Out]
$$\frac{1}{f} \frac{x}{a} \frac{1}{(c x^2 + a)^{1/2}} - \frac{1}{(2 a^2 f^2 - 2 c d f + c^2 e^2 + (-4 d^2 f + e^2)^{1/2} c e)} \left(\left(x + \frac{1}{2} (e + (-4 d^2 f + e^2)^{1/2}) / f \right)^2 c - (e + (-4 d^2 f + e^2)^{1/2}) \left(x + \frac{1}{2} (e + (-4 d^2 f + e^2)^{1/2}) / f \right) c / f + \frac{1}{2} (2 a^2 f^2 - 2 c d f + c^2 e^2 + (-4 d^2 f + e^2)^{1/2} c e) / f^2 \right)^{1/2} e + \frac{2}{(-4 d^2 f + e^2)^{1/2}} \frac{f}{(2 a^2 f^2 - 2 c d f + c^2 e^2 + (-4 d^2 f + e^2)^{1/2} c e)} \left(\left(x + \frac{1}{2} (e + (-4 d^2 f + e^2)^{1/2}) / f \right)^2 c - (e + (-4 d^2 f + e^2)^{1/2}) \left(x + \frac{1}{2} (e + (-4 d^2 f + e^2)^{1/2}) / f \right) c / f + \frac{1}{2} (2 a^2 f^2 - 2 c d f + c^2 e^2 + (-4 d^2 f + e^2)^{1/2} c e) / f^2 \right)^{1/2} d - \frac{1}{(-4 d^2 f + e^2)^{1/2}} \frac{1}{(2 a^2 f^2 - 2 c d f + c^2 e^2 + (-4 d^2 f + e^2)^{1/2} c e)} \left(\left(x + \frac{1}{2} (e + (-4 d^2 f + e^2)^{1/2}) / f \right)^2 c - (e + (-4 d^2 f + e^2)^{1/2}) \left(x + \frac{1}{2} (e + (-4 d^2 f + e^2)^{1/2}) / f \right) c / f + \frac{1}{2} (2 a^2 f^2 - 2 c d f + c^2 e^2 + (-4 d^2 f + e^2)^{1/2} c e) / f^2 \right)^{1/2} e^2 - 2 \frac{(-4 d^2 f + e^2)^{1/2}}{f} \frac{c^2}{(2 a^2 f^2 - 2 c d f + c^2 e^2 + (-4 d^2 f + e^2)^{1/2} c e)} \left(\frac{4 a^2 c - 4 c^2 / f d + c^2 / f^2 e^2 - c^2 / f^2 (-4 d^2 f + e^2)}{(x + \frac{1}{2} (e + (-4 d^2 f + e^2)^{1/2}) / f)^2 c - (e + (-4 d^2 f + e^2)^{1/2}) \left(x + \frac{1}{2} (e + (-4 d^2 f + e^2)^{1/2}) / f \right) c / f + \frac{1}{2} (2 a^2 f^2 - 2 c d f + c^2 e^2 + (-4 d^2 f + e^2)^{1/2} c e) / f^2} \right)^{1/2} x e + 4 \frac{c^2}{(2 a^2 f^2 - 2 c d f + c^2 e^2 + (-4 d^2 f + e^2)^{1/2} c e)} \left(\frac{4 a^2 c - 4 c^2 / f d + c^2 / f^2 e^2 - c^2 / f^2 (-4 d^2 f + e^2)}{(x + \frac{1}{2} (e + (-4 d^2 f + e^2)^{1/2}) / f)^2 c - (e + (-4 d^2 f + e^2)^{1/2}) \left(x + \frac{1}{2} (e + (-4 d^2 f + e^2)^{1/2}) / f \right) c / f + \frac{1}{2} (2 a^2 f^2 - 2 c d f + c^2 e^2 + (-4 d^2 f + e^2)^{1/2} c e) / f^2} \right)^{1/2} x d - 4 \frac{f c^2}{(2 a^2 f^2 - 2 c d f + c^2 e^2 + (-4 d^2 f + e^2)^{1/2} c e)} \left(\frac{4 a^2 c - 4 c^2 / f d + c^2 / f^2 e^2 - c^2 / f^2 (-4 d^2 f + e^2)}{(x + \frac{1}{2} (e + (-4 d^2 f + e^2)^{1/2}) / f)^2 c - (e + (-4 d^2 f + e^2)^{1/2}) \left(x + \frac{1}{2} (e + (-4 d^2 f + e^2)^{1/2}) / f \right) c / f + \frac{1}{2} (2 a^2 f^2 - 2 c d f + c^2 e^2 + (-4 d^2 f + e^2)^{1/2} c e) / f^2} \right)^{1/2} x$$

$$\begin{aligned} & *(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)} \\ & *c*e)/f^2)^{(1/2)}*x*d-4/f*c^2/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c* \\ & e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c- \\ & (e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2- \\ & (-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*x*e^2-4/(-4*d*f+e^2)^{(1/2)}*c^2/(2*a*f^2-2*c*d*f+c*e^2- \\ & (-4*d*f+e^2)^{(1/2)}*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c- \\ & (e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2- \\ & (-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*e*x*d+2/(-4*d*f+e^2)^{(1/2)}/f*c^2/(2*a*f^2-2*c*d*f+c*e^2- \\ & (-4*d*f+e^2)^{(1/2)}*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c- \\ & (e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2- \\ & (-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*e^3*x+1/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)*2^{(1/2)}/ \\ & ((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*\ln((-e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+ \\ & (2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(1/2)}*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}* \\ & (4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-4*(e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2- \\ & (-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)})/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f))*e+2/(-4*d*f+e^2)^{(1/2)}*f/(2*a*f^2-2*c*d*f+c*e^2- \\ & (-4*d*f+e^2)^{(1/2)}*c*e)*2^{(1/2)}/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*\ln((-e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+ \\ & (2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(1/2)}*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}* \\ & (4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-4*(e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2- \\ & (-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)})/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f))*d-1/(-4*d*f+e^2)^{(1/2)}/(2*a*f^2-2*c*d*f+c*e^2- \\ & (-4*d*f+e^2)^{(1/2)}*c*e)*2^{(1/2)}/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*\ln((-e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+ \\ & (2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(1/2)}*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}* \\ & (4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-4*(e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2- \\ & (-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)})/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f))*e^2 \end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* h

elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details) Is $4*d*f-e^2$ positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{(cx^2 + a)^{3/2} (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)

[Out] int(x^2/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a + cx^2)^{\frac{3}{2}} (d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(c*x**2+a)**(3/2)/(f*x**2+e*x+d), x)

[Out] Integral(x**2/((a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)

$$3.73 \quad \int \frac{x}{(a+cx^2)^{3/2} (d+ex+fx^2)} dx$$

Optimal. Leaf size=411

$$\frac{f(2cde - (e - \sqrt{e^2 - 4df})(cd - af)) \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2} \sqrt{a+cx^2} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2} \sqrt{e^2 - 4df} ((cd - af)^2 + ace^2) \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}} \frac{f(2cde - (\sqrt{e^2 - 4df} + e)(cd - af)) \tanh^{-1}\left(\frac{2af - cx(e + \sqrt{e^2 - 4df})}{\sqrt{2} \sqrt{a+cx^2} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2} \sqrt{e^2 - 4df} ((cd - af)^2 + ace^2) \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}$$

[Out] (c*e*x+a*f-c*d)/(a*c*e^2+(-a*f+c*d)^2)/(c*x^2+a)^(1/2)+1/2*f*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(2*c*d*e-(-a*f+c*d)*(e-(-4*d*f+e^2)^(1/2))))/(a*c*e^2+(-a*f+c*d)^2)*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)-1/2*f*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(2*c*d*e-(-a*f+c*d)*(e+(-4*d*f+e^2)^(1/2)))/(a*c*e^2+(-a*f+c*d)^2)*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)

Rubi [A] time = 0.83, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1017, 1034, 725, 206}

$$\frac{f(2cde - (e - \sqrt{e^2 - 4df})(cd - af)) \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2} \sqrt{a+cx^2} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2} \sqrt{e^2 - 4df} ((cd - af)^2 + ace^2) \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}} \frac{f(2cde - (\sqrt{e^2 - 4df} + e)(cd - af)) \tanh^{-1}\left(\frac{2af - cx(e + \sqrt{e^2 - 4df})}{\sqrt{2} \sqrt{a+cx^2} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2} \sqrt{e^2 - 4df} ((cd - af)^2 + ace^2) \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}$$

Antiderivative was successfully verified.

[In] Int[x/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

[Out] -((c*d - a*f - c*e*x)/((a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2])) + (f*(2*c*d*e - (c*d - a*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]])]*Sqrt[a + c*x^2]))/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]]) - (f*(2*c*d*e - (c*d - a*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])]*Sqrt[a + c*x^2]))/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1017

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + c*x^2)^(p + 1)*(d + e*x + f*x^2)^
(q + 1)*(g*c*(2*a*c*e) + (-a*h)*(2*c^2*d - c*(2*a*f)) + c*(g*(2*c^2*d - c
*(2*a*f)) - h*(-2*a*c*e)*x))/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)),
x] + Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^
(p + 1)*(d + e*x + f*x^2)^q*Simp[(-2*g*c)*((c*d - a*f)^2 - (-a*e)*(c*e))*
(p + 1) + (2*(g*c*(c*d - a*f) - a*(-h*c*e)))*(a*f*(p + 1) - c*d*(p + 2))
- e*((g*c)*(2*a*c*e) + (-a*h)*(2*c^2*d - c*(Plus[2])*a*f)))*(p + q + 2)
- (2*f*((g*c)*(2*a*c*e) + (-a*h)*(2*c^2*d - c*(Plus[2])*a*f)))*(p + q +
2) - (2*(g*c*(c*d - a*f) - a*(-h*c*e)))*(-c*e*(2*p + q + 4)))*x - c*f*(
2*(g*c*(c*d - a*f) - a*(-h*c*e)))*(2*p + 2*q + 5)*x^2, x], x] /; Free
Q[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[
a*c*e^2 + (c*d - a*f)^2, 0] && !( !IntegerQ[p] && !LtQ[q, -1])
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(a+cx^2)^{3/2}(d+ex+fx^2)} dx &= -\frac{cd-af-cex}{(ace^2+(cd-af)^2)\sqrt{a+cx^2}} + \frac{\int \frac{-2ac^2de-2acf(cd-af)x}{\sqrt{a+cx^2}(d+ex+fx^2)} dx}{2ac(ace^2+(cd-af)^2)} \\
&= -\frac{cd-af-cex}{(ace^2+(cd-af)^2)\sqrt{a+cx^2}} - \frac{(f(2cde-(cd-af)(e-\sqrt{e^2-4df})))}{\sqrt{e^2-4df}(ace^2+(cd-af)^2)} \\
&= -\frac{cd-af-cex}{(ace^2+(cd-af)^2)\sqrt{a+cx^2}} + \frac{(f(2cde-(cd-af)(e-\sqrt{e^2-4df})))}{\sqrt{e^2-4df}(ace^2+(cd-af)^2)} \\
&= -\frac{cd-af-cex}{(ace^2+(cd-af)^2)\sqrt{a+cx^2}} + \frac{f(2cde-(cd-af)(e-\sqrt{e^2-4df})) \tan^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{e^2-4df}}\right)}{\sqrt{2}\sqrt{e^2-4df}(ace^2+(cd-af)^2)\sqrt{a+cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.81, size = 457, normalized size = 1.11

$$\frac{\left(1 - \frac{e}{\sqrt{e^2-4df}}\right)(2af+cx(e-\sqrt{e^2-4df}))}{a\sqrt{a+cx^2}(4af^2+c(e-\sqrt{e^2-4df})^2)} + \frac{\left(\frac{e}{\sqrt{e^2-4df}}+1\right)(2af+cx(\sqrt{e^2-4df}+e))}{a\sqrt{a+cx^2}(4af^2+c(\sqrt{e^2-4df}+e)^2)} + \frac{\sqrt{2}f^2(e-\sqrt{e^2-4df})}{\sqrt{e^2-4df}(2af^2+c(e-\sqrt{e^2-4df})^2)}$$

Antiderivative was successfully verified.

[In] Integrate[x/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

[Out] ((1 - e/Sqrt[e^2 - 4*d*f])*(2*a*f + c*(e - Sqrt[e^2 - 4*d*f])*x))/(a*(4*a*f^2 + c*(e - Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + c*x^2]) + ((1 + e/Sqrt[e^2 - 4*d*f])*(2*a*f + c*(e + Sqrt[e^2 - 4*d*f])*x))/(a*(4*a*f^2 + c*(e + Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + c*x^2]) + (Sqrt[2]*f^2*(e - Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[e^2 - 4*d*f]*(2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))^(3/2)) - (Sqrt[2]*f^2*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[e^2 - 4*d*f]*(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))^(3/2))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage₂

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage2
```

maple [B] time = 0.02, size = 3000, normalized size = 7.30

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] 1/(-4*d*f+e^2)^(1/2)*f*e/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/((x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f
+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)
^(1/2)+4*e*c^2/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/
f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c
-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2
*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*x+2/(-4*d*f+e^2)^(1/2)*c^2/
(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2
-c^2/f^2*(-4*d*f+e^2))/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2)
)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-
4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*e^2*x-1/(-4*d*f+e^2)^(1/2)*f*e/(2*a*f^2-2*
c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f
+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e
^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^
(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^
2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)
)/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))+f/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(
1/2)*c*e)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2)^(1/2))*(x+
1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(
1/2)*c*e)/f^2)^(1/2)+2*(-4*d*f+e^2)^(1/2)*c^2/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*
f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^
```


$$\begin{aligned}
& 2)^{(1/2)})/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)} \\
& *x-f/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)*2^{(1/2)}/((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*\ln((-e+(-4*d*f+e^2)^{(1/2)}) \\
& *(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(1/2)}*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)} \\
& *(4*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-4*(e+(-4*d*f+e^2)^{(1/2)})*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}) \\
& /((x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)+f/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-(e-(-4*d*f+e^2)^{(1/2)}) \\
& *(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}-2*(-4*d*f+e^2)^{(1/2)}*c^2/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e) \\
& /((4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-(e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*x+4*c^2/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e) \\
& /((4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-(e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*e*x-f \\
& /((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)*2^{(1/2)}/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*\ln((-e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+ \\
& (2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(1/2)}*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-4*(e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}) \\
& /((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f))-1/(-4*d*f+e^2)^{(1/2)}*f*e/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-(e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}-2/(-4*d*f+e^2)^{(1/2)}*c^2/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e) \\
& /((4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-(e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*e^2*x+1/(-4*d*f+e^2)^{(1/2)}*f*e/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)*2^{(1/2)}/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*\ln((-e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+ \\
& (2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(1/2)}*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-4*(e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}) \\
& /((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f))
\end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details)Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{(cx^2 + a)^{3/2} (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)

[Out] int(x/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + cx^2)^{\frac{3}{2}} (d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)

[Out] Integral(x/((a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)

$$3.74 \quad \int \frac{1}{(a+cx^2)^{3/2}(d+ex+fx^2)} dx$$

Optimal. Leaf size=416

$$\frac{f(2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)) \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2} \sqrt{a+cx^2} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2} \sqrt{e^2 - 4df} ((cd - af)^2 + ace^2) \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}} + \frac{f(2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)) \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2} \sqrt{a+cx^2} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2} \sqrt{e^2 - 4df} ((cd - af)^2 + ace^2) \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}$$

[Out] $c*(a*e+(-a*f+c*d)*x)/a/(a*c*e^2+(-a*f+c*d)^2)/(c*x^2+a)^{(1/2)}-1/2*f*\operatorname{arctanh}(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^{(1/2)})))*2^{(1/2)}/(c*x^2+a)^{(1/2)}/(2*a*f^2+(-4*d*f+e^2)^{(1/2)}*e-2*d*f+e^2)*c)^{(1/2)}*(2*a*f^2+((-4*d*f+e^2)^{(1/2)}*e-2*d*f+e^2)*c)/(a*c*e^2+(-a*f+c*d)^2)*2^{(1/2)}/(-4*d*f+e^2)^{(1/2)}/(2*a*f^2+(-4*d*f+e^2)^{(1/2)}*e-2*d*f+e^2)*c)^{(1/2)}+1/2*f*\operatorname{arctanh}(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^{(1/2)})))*2^{(1/2)}/(c*x^2+a)^{(1/2)}/(2*a*f^2+((-4*d*f+e^2)^{(1/2)}*e-2*d*f+e^2)*c)^{(1/2)}*(2*a*f^2+((-4*d*f+e^2)^{(1/2)}*e-2*d*f+e^2)*c)/(a*c*e^2+(-a*f+c*d)^2)*2^{(1/2)}/(-4*d*f+e^2)^{(1/2)}/(2*a*f^2+((-4*d*f+e^2)^{(1/2)}*e-2*d*f+e^2)*c)^{(1/2)}$

Rubi [A] time = 0.62, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {976, 1034, 725, 206}

$$\frac{f(2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)) \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2} \sqrt{a+cx^2} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2} \sqrt{e^2 - 4df} ((cd - af)^2 + ace^2) \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}} + \frac{f(2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)) \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2} \sqrt{a+cx^2} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2} \sqrt{e^2 - 4df} ((cd - af)^2 + ace^2) \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

[Out] $(c*(a*e + (c*d - a*f)*x))/(a*(a*c*e^2 + (c*d - a*f)^2)*\operatorname{Sqrt}[a + c*x^2]) - (f*(2*a*f^2 + c*(e^2 - 2*d*f + e*\operatorname{Sqrt}[e^2 - 4*d*f]))*\operatorname{ArcTanh}[(2*a*f - c*(e - \operatorname{Sqrt}[e^2 - 4*d*f])*x)/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[2*a*f^2 + c*(e^2 - 2*d*f - e*\operatorname{Sqrt}[e^2 - 4*d*f])])*\operatorname{Sqrt}[a + c*x^2]])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*\operatorname{Sqrt}[2*a*f^2 + c*(e^2 - 2*d*f - e*\operatorname{Sqrt}[e^2 - 4*d*f])]) + (f*(2*a*f^2 + c*(e^2 - 2*d*f - e*\operatorname{Sqrt}[e^2 - 4*d*f]))*\operatorname{ArcTanh}[(2*a*f - c*(e + \operatorname{Sqrt}[e^2 - 4*d*f])*x)/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[2*a*f^2 + c*(e^2 - 2*d*f + e*\operatorname{Sqrt}[e^2 - 4*d*f])])*\operatorname{Sqrt}[a + c*x^2]])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*\operatorname{Sqrt}[2*a*f^2 + c*(e^2 - 2*d*f + e*\operatorname{Sqrt}[e^2 - 4*d*f])])$

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 976

```
Int[((a_.) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x
_Symbol] := Simp[((2*a*c^2*e + c*(2*c^2*d - c*(2*a*f))*x)*(a + c*x^2)^(p +
1)*(d + e*x + f*x^2)^(q + 1))/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)),
x] - Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^(
p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - (-a*e)*(c*e))*(p +
1) - (2*c^2*d - c*(2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(-2*a*c^2*e)*(p
+ q + 2) + (2*f*(2*a*c^2*e)*(p + q + 2) - (2*c^2*d - c*(2*a*f))*(-(c*e*(2*p
+ q + 4)))]*x + c*f*(2*c^2*d - c*(2*a*f))*(2*p + 2*q + 5)*x^2, x], x]
/; FreeQ[{a, c, d, e, f, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ
[a*c*e^2 + (c*d - a*f)^2, 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[
q, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+cx^2)^{3/2}(d+ex+fx^2)} dx &= \frac{c(ae+(cd-af)x)}{a(ace^2+(cd-af)^2)\sqrt{a+cx^2}} - \frac{\int \frac{-2ac(af^2+c(e^2-df))-2ac^2efx}{\sqrt{a+cx^2}(d+ex+fx^2)} dx}{2ac(ace^2+(cd-af)^2)} \\
&= \frac{c(ae+(cd-af)x)}{a(ace^2+(cd-af)^2)\sqrt{a+cx^2}} - \frac{(f(2af^2+c(e^2-2df-e\sqrt{e^2-4df})))}{\sqrt{e^2-4df}(ace^2+(cd-af)^2)} \\
&= \frac{c(ae+(cd-af)x)}{a(ace^2+(cd-af)^2)\sqrt{a+cx^2}} + \frac{(f(2af^2+c(e^2-2df-e\sqrt{e^2-4df})))}{\sqrt{e^2-4df}} \\
&= \frac{c(ae+(cd-af)x)}{a(ace^2+(cd-af)^2)\sqrt{a+cx^2}} - \frac{f(2af^2+c(e^2-2df+e\sqrt{e^2-4df}))}{\sqrt{2}\sqrt{e^2-4df}(ace^2+(cd-af)^2)}
\end{aligned}$$

Mathematica [A] time = 2.06, size = 320, normalized size = 0.77

$$\frac{c(a(e-fx)+cdx)}{a\sqrt{a+cx^2}(a^2f^2+ac(e^2-2df)+c^2d^2)} - \frac{2\sqrt{2}f^3 \tanh^{-1}\left(\frac{2af+cx(\sqrt{e^2-4df}-e)}{\sqrt{a+cx^2}\sqrt{4af^2-2c(e\sqrt{e^2-4df}+2df-e^2)}}\right)}{\sqrt{e^2-4df}(2af^2+c(-e\sqrt{e^2-4df}-2df+e^2))^{3/2}} + \frac{2\sqrt{2}f^3 \tanh^{-1}\left(\frac{2af+cx(\sqrt{e^2-4df}+e)}{\sqrt{a+cx^2}\sqrt{4af^2-2c(e\sqrt{e^2-4df}-2df-e^2)}}\right)}{\sqrt{e^2-4df}(2af^2+c(e\sqrt{e^2-4df}-2df+e^2))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

[Out] (c*(c*d*x + a*(e - f*x)))/(a*(c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))*Sqrt[a + c*x^2] - (2*Sqrt[2]*f^3*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f]))x]/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]))/(Sqrt[e^2 - 4*d*f]*(2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))^(3/2)) + (2*Sqrt[2]*f^3*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f]))x]/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]))/(Sqrt[e^2 - 4*d*f]*(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))^(3/2))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage₂

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] sage2

maple [B] time = 0.02, size = 1713, normalized size = 4.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)

[Out]
$$\begin{aligned} & -2/(-4*d*f+e^2)^{(1/2)}/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)*f^2/((\\ & x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-(e+(-4*d*f+e^2)^{(1/2)})*(x+1/2*(e+(-4*d*f \\ & +e^2)^{(1/2)})/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2 \\ &)^{(1/2)}-4*c^2*f/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/(4*a*c-4*c^2 \\ & /f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)^2* \\ & c-(e+(-4*d*f+e^2)^{(1/2)})*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+1/2*(2*a*f^2- \\ & 2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*x-4/(-4*d*f+e^2)^{(1/2)}*c^2 \\ & *f/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2* \\ & e^2-c^2/f^2*(-4*d*f+e^2))/((x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-(e+(-4*d*f+ \\ & e^2)^{(1/2)})*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2 \\ & +(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*e*x+2/(-4*d*f+e^2)^{(1/2)}/(2*a*f^2-2*c*d \\ & *f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)*f^2*2^{(1/2)}/((2*a*f^2-2*c*d*f+c*e^2+(-4*d* \\ & f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*ln((-e+(-4*d*f+e^2)^{(1/2)})*(x+1/2*(e+(-4*d*f+ \\ & e^2)^{(1/2)})/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2 \\ & ^{(1/2)}*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(x+1/2 \\ & *(e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-4*(e+(-4*d*f+e^2)^{(1/2)})*(x+1/2*(e+(-4*d*f+e \\ & ^2)^{(1/2)})/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/ \\ & 2)}/(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f))+2/(-4*d*f+e^2)^{(1/2)}/(2*a*f^2-2*c*d*f \\ & +c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)*f^2/((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-(\\ & e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+1/2*(2*a*f^2-2* \\ & c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}-4*c^2*f/(2*a*f^2-2*c*d*f+c*e \\ & ^2-(-4*d*f+e^2)^{(1/2)}*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2 \\ &))/((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-(e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e \\ & +(-4*d*f+e^2)^{(1/2)})/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c \\ & *e)/f^2)^{(1/2)}*x+4/(-4*d*f+e^2)^{(1/2)}*c^2*f/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+ \\ & e^2)^{(1/2)}*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x-1/2* \\ & (-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-(e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^ \\ & \end{aligned}$$

$$2)^{(1/2)}/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*e*x-2/(-4*d*f+e^2)^{(1/2)}/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)*f^2*2^{(1/2)}/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*\ln((-e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(1/2)}*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c-4*(e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f))$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details)Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(cx^2 + a)^{3/2} (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)

[Out] int(1/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + cx^2)^{\frac{3}{2}} (d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)

[Out] Integral(1/((a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)

$$3.75 \quad \int \frac{1}{x(a+cx^2)^{3/2}(d+ex+fx^2)} dx$$

Optimal. Leaf size=526

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{a(af^2 + c(e^2 - df)) + c^2dex}{ad\sqrt{a+cx^2}((cd - af)^2 + ace^2)} + \frac{f(2e(af^2 + c(e^2 - 2df)) - (e - \sqrt{e^2 - 4df})(af^2 + c(e^2 - 2df)))}{\sqrt{2}d\sqrt{e^2 - 4df}((cd - af)^2 + ace^2)\sqrt{2a}}$$

[Out] $-\operatorname{arctanh}\left(\frac{(c*x^2+a)^{1/2}/a^{1/2}}{a^{3/2}/d+1/a/d/(c*x^2+a)^{1/2}}\right)+(-a*(a*f^2+2*c*(-d*f+e^2))-c^2*d*e*x)/a/d/(a*c*e^2+(-a*f+c*d)^2)/(c*x^2+a)^{1/2}+1/2*f*\operatorname{arctanh}\left(\frac{1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^{1/2}))}{2^{1/2}/(c*x^2+a)^{1/2}}\right)/(2*a*f^2+(-(-4*d*f+e^2)^{1/2}*e-2*d*f+e^2)*c)^{1/2})*(2*e*(a*f^2+c*(-2*d*f+e^2))-(a*f^2+c*(-d*f+e^2))*(e-(-4*d*f+e^2)^{1/2}))/d/(a*c*e^2+(-a*f+c*d)^2)^{1/2}/(-4*d*f+e^2)^{1/2}/(2*a*f^2+(-(-4*d*f+e^2)^{1/2}*e-2*d*f+e^2)*c)^{1/2})-1/2*f*\operatorname{arctanh}\left(\frac{1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^{1/2}))}{2^{1/2}/(c*x^2+a)^{1/2}}\right)/(2*a*f^2+((-4*d*f+e^2)^{1/2}*e-2*d*f+e^2)*c)^{1/2})*(2*e*(a*f^2+c*(-2*d*f+e^2))-(a*f^2+c*(-d*f+e^2))*(e+(-4*d*f+e^2)^{1/2}))/d/(a*c*e^2+(-a*f+c*d)^2)^{1/2}/(-4*d*f+e^2)^{1/2}/(2*a*f^2+((-4*d*f+e^2)^{1/2}*e-2*d*f+e^2)*c)^{1/2})$

Rubi [A] time = 2.18, antiderivative size = 526, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6728, 266, 51, 63, 208, 1017, 1034, 725, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{a(af^2 + c(e^2 - df)) + c^2dex}{ad\sqrt{a+cx^2}((cd - af)^2 + ace^2)} + \frac{f(2e(af^2 + c(e^2 - 2df)) - (e - \sqrt{e^2 - 4df})(af^2 + c(e^2 - 2df)))}{\sqrt{2}d\sqrt{e^2 - 4df}((cd - af)^2 + ace^2)\sqrt{2a}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}\left[\frac{1}{x*(a + c*x^2)^{3/2}*(d + e*x + f*x^2)}, x\right]$

[Out] $\frac{1}{a*d*\operatorname{Sqrt}[a + c*x^2]} - \frac{(a*(a*f^2 + c*(e^2 - d*f)) + c^2*d*e*x)/(a*d*(a*c*e^2 + (c*d - a*f)^2)*\operatorname{Sqrt}[a + c*x^2]) + (f*(2*e*(a*f^2 + c*(e^2 - 2*d*f)) - (e - \operatorname{Sqrt}[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - d*f))))*\operatorname{ArcTanh}\left[\frac{(2*a*f - c*(e - \operatorname{Sqrt}[e^2 - 4*d*f])*x)}{(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[2*a*f^2 + c*(e^2 - 2*d*f - e*\operatorname{Sqrt}[e^2 - 4*d*f]])*\operatorname{Sqrt}[a + c*x^2])}\right]}{(\operatorname{Sqrt}[2]*d*\operatorname{Sqrt}[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*\operatorname{Sqrt}[2*a*f^2 + c*(e^2 - 2*d*f - e*\operatorname{Sqrt}[e^2 - 4*d*f]])} - (f*(2*e*(a*f^2 + c*(e^2 - 2*d*f)) - (e + \operatorname{Sqrt}[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - d*f))))*\operatorname{ArcTanh}\left[\frac{(2*a*f - c*(e + \operatorname{Sqrt}[e^2 - 4*d*f])*x)}{(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[2*a*f^2 + c*(e^2 - 2*d*f + e*\operatorname{Sqrt}[e^2 - 4*d*f]])*\operatorname{Sqrt}[a + c*x^2])}\right]}{(\operatorname{Sqrt}[2]*d*\operatorname{Sqrt}[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*\operatorname{Sqrt}[2*a*f^2 + c*(e^2 - 2*d*f + e*\operatorname{Sqrt}[e^2 - 4*d*f]])}$

$4*d*f*(a*c*e^2 + (c*d - a*f)^2)*\text{Sqrt}[2*a*f^2 + c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f])] - \text{ArcTanh}[\text{Sqrt}[a + c*x^2]/\text{Sqrt}[a]]/(a^{(3/2)*d})$

Rule 51

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*(c + d*x)^{(n + 1)} / ((b*c - a*d)*(m + 1)), x] - \text{Dist}[(d*(m + n + 2)) / ((b*c - a*d)*(m + 1)), \text{Int}[(a + b*x)^{(m + 1)}*(c + d*x)^n, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m + 1) - 1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x]] /;$ FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 206

$\text{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x] / \text{Rt}[a, 2]) / (\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

$\text{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x / \text{Rt}[-(a/b), 2]]) / a, x] /;$ FreeQ[{a, b}, x] && NegQ[a/b]

Rule 266

$\text{Int}[(x_.)^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p}, x], x, x^n], x] /;$ FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 725

$\text{Int}[1/(((d_.) + (e_.)*(x_.))*\text{Sqrt}[(a_.) + (c_.)*(x_.)^2]), x_Symbol] \rightarrow -\text{Subst}[\text{Int}[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x) / \text{Sqrt}[a + c*x^2]] /;$ FreeQ[{a, c, d, e}, x]

Rule 1017

```

Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e) + (-a*h)*(2*c^2*d - c*(2*a*f)) + c*(g*(2*c^2*d - c*(2*a*f)) - h*(-2*a*c*e))*x)/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(-2*g*c)*((c*d - a*f)^2 - (-a*e)*(c*e))^(p + 1) + (2*(g*c*(c*d - a*f) - a*(-h*c*e)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e) + (-a*h)*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*a*c*e) + (-a*h)*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q + 2) - (2*(g*c*(c*d - a*f) - a*(-h*c*e)))*(-c*e*(2*p + q + 4)))]*x - c*f*(2*(g*c*(c*d - a*f) - a*(-h*c*e)))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[a*c*e^2 + (c*d - a*f)^2, 0] && !(IntegerQ[p] && ILtQ[q, -1])

```

Rule 1034

```

Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

```

Rule 6728

```

Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(a+cx^2)^{3/2}(d+ex+fx^2)} dx &= \int \left(\frac{1}{dx(a+cx^2)^{3/2}} + \frac{-e-fx}{d(a+cx^2)^{3/2}(d+ex+fx^2)} \right) dx \\
&= \frac{\int \frac{1}{x(a+cx^2)^{3/2}} dx}{d} + \frac{\int \frac{-e-fx}{(a+cx^2)^{3/2}(d+ex+fx^2)} dx}{d} \\
&= \frac{a(af^2+c(e^2-df))+c^2dex}{ad(ace^2+(cd-af)^2)\sqrt{a+cx^2}} + \frac{\text{Subst}\left(\int \frac{1}{x(a+cx)^{3/2}} dx, x, x^2\right)}{2d} + \frac{\int \frac{-2ax}{(a+cx)^2} dx}{2d} \\
&= \frac{1}{ad\sqrt{a+cx^2}} - \frac{a(af^2+c(e^2-df))+c^2dex}{ad(ace^2+(cd-af)^2)\sqrt{a+cx^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}} dx, x, x^2\right)}{2ad} \\
&= \frac{1}{ad\sqrt{a+cx^2}} - \frac{a(af^2+c(e^2-df))+c^2dex}{ad(ace^2+(cd-af)^2)\sqrt{a+cx^2}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}} dx, x, x^2\right)}{acd} \\
&= \frac{1}{ad\sqrt{a+cx^2}} - \frac{a(af^2+c(e^2-df))+c^2dex}{ad(ace^2+(cd-af)^2)\sqrt{a+cx^2}} + \frac{f(2e(af^2+c(e^2-2df))+2cd)}{\sqrt{2}d}
\end{aligned}$$

Mathematica [C] time = 3.50, size = 497, normalized size = 0.94

$$\frac{f\left(\frac{e}{\sqrt{e^2-4df}}+1\right)(2af+cx(e-\sqrt{e^2-4df}))}{a\sqrt{a+cx^2}\left(4af^2+c(e-\sqrt{e^2-4df})^2\right)} - \frac{f\left(1-\frac{e}{\sqrt{e^2-4df}}\right)(2af+cx(\sqrt{e^2-4df}+e))}{a\sqrt{a+cx^2}\left(4af^2+c(\sqrt{e^2-4df}+e)^2\right)} + \frac{\sqrt{2}f^3(\sqrt{e^2-4df}+e)\tanh^{-1}\left(\frac{2af+cx(\sqrt{e^2-4df}-e)}{\sqrt{a+cx^2}\sqrt{4af^2-2c(e\sqrt{e^2-4df}+2df+e^2)}}\right)}{\sqrt{e^2-4df}\left(2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x]

[Out]
$$\frac{-\left(\left(f*(1 + e/\text{Sqrt}[e^2 - 4*d*f])\right)*(2*a*f + c*(e - \text{Sqrt}[e^2 - 4*d*f])*x)\right)/\left(a*(4*a*f^2 + c*(e - \text{Sqrt}[e^2 - 4*d*f])^2)*\text{Sqrt}[a + c*x^2]\right) - \left(f*(1 - e/\text{Sqrt}[e^2 - 4*d*f])\right)*(2*a*f + c*(e + \text{Sqrt}[e^2 - 4*d*f])*x)\right)/\left(a*(4*a*f^2 + c*(e + \text{Sqrt}[e^2 - 4*d*f])^2)*\text{Sqrt}[a + c*x^2]\right) + \left(\text{Sqrt}[2]*f^3*(e + \text{Sqrt}[e^2 - 4*d*f])\right)*\text{ArcTanh}\left[\left(2*a*f + c*(-e + \text{Sqrt}[e^2 - 4*d*f])*x\right)/\left(\text{Sqrt}[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f])]*\text{Sqrt}[a + c*x^2]\right)\right]}{\left(\text{Sqrt}[e^2 - 4*d*f]*(2*a*f + c*(e - \text{Sqrt}[e^2 - 4*d*f])*x)\right)^{3/2}}$$

$$\begin{aligned} &^2 + c*(e^2 - 2*d*f - e*\text{Sqrt}[e^2 - 4*d*f]))^{(3/2)} + (\text{Sqrt}[2]*f^3*(-e + \text{Sqrt}[e^2 - 4*d*f]) * \text{ArcTanh}[(2*a*f - c*(e + \text{Sqrt}[e^2 - 4*d*f])*x)/(\text{Sqrt}[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f])]*\text{Sqrt}[a + c*x^2])]) / (\text{Sqrt}[e^2 - 4*d*f]*(2*a*f^2 + c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]))^{(3/2)} + \text{Hypergeometric2F1}[-1/2, 1, 1/2, 1 + (c*x^2)/a]/(a*\text{Sqrt}[a + c*x^2]))/d \end{aligned}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.02, size = 1945, normalized size = 3.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)

[Out]
$$\begin{aligned} &4*f^3/(e+(-4*d*f+e^2)^{(1/2)})/(-4*d*f+e^2)^{(1/2)}/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/((x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-(e+(-4*d*f+e^2)^{(1/2)})*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}+8*f^2/(e+(-4*d*f+e^2)^{(1/2)})*c^2/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2)/((x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-(e+(-4*d*f+e^2)^{(1/2)})*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*x+8*f^2/(e+(-4*d*f+e^2)^{(1/2)})/(-4*d*f+e^2)^{(1/2)}*c^2/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2)/((x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-(e+(-4*d*f+e^2)^{(1/2)})*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*e*x-4*f^3/(e+(-4*d*f+e^2)^{(1/2)})/(-4*d*f+e^2)^{(1/2)}/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)*2^{(1/2)}/((2*a*f^2-$$

$$\begin{aligned}
& 2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*\ln((-e+(-4*d*f+e^2)^{(1/2)}) \\
&)*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)} \\
&)*(c*e)/f^2+1/2*2^{(1/2)}*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/ \\
& f^2)^{(1/2)}*(4*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-4*(e+(-4*d*f+e^2)^{(1/2)}) \\
&)*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)} \\
&)*(c*e)/f^2)^{(1/2)})/(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f))+4*f^3/(-e+(-4*d*f \\
& +e^2)^{(1/2)})/(-4*d*f+e^2)^{(1/2)}/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c \\
& *e)/((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-(e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e \\
& +(-4*d*f+e^2)^{(1/2)})/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}* \\
& c*e)/f^2)^{(1/2)}-8*f^2/(-e+(-4*d*f+e^2)^{(1/2)})*c^2/(2*a*f^2-2*c*d*f+c*e^2-(- \\
& 4*d*f+e^2)^{(1/2)}*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2)/((\\
& x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-(e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4* \\
& d*f+e^2)^{(1/2)})/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f \\
& ^2)^{(1/2)}*x+8*f^2/(-e+(-4*d*f+e^2)^{(1/2)})/(-4*d*f+e^2)^{(1/2)}*c^2/(2*a*f^2-2 \\
& *c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e \\
& ^2)*c^2/f^2)/((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-(e-(-4*d*f+e^2)^{(1/2)})* \\
& (x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^ \\
& 2)^{(1/2)}*c*e)/f^2)^{(1/2)}*e*x-4*f^3/(-e+(-4*d*f+e^2)^{(1/2)})/(-4*d*f+e^2)^{(1/ \\
& 2)}/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)*2^{(1/2)}/((2*a*f^2-2*c*d*f \\
& +c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*\ln((-e-(-4*d*f+e^2)^{(1/2)})*(x-1/ \\
& 2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}* \\
& c*e)/f^2+1/2*2^{(1/2)}*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(\\
& 1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-4*(e-(-4*d*f+e^2)^{(1/2)})*(x-1 \\
& /2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/ \\
& 2)*c*e)/f^2)^{(1/2)})/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f))-4*f/(-e+(-4*d*f+e^2) \\
& ^{(1/2)})/(e+(-4*d*f+e^2)^{(1/2)})/a/(c*x^2+a)^{(1/2)}+4*f/(-e+(-4*d*f+e^2)^{(1/2)}) \\
&)/(e+(-4*d*f+e^2)^{(1/2)})/a^{(3/2)}*\ln((2*a+2*(c*x^2+a)^{(1/2)}*a^{(1/2)})/x)
\end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + a)^{\frac{3}{2}}(fx^2 + ex + d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] integrate(1/((c*x^2 + a)^(3/2)*(f*x^2 + e*x + d)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x(c x^2 + a)^{3/2} (f x^2 + e x + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(1/(x*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + cx^2)^{\frac{3}{2}}(d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(1/(x*(a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

$$3.76 \quad \int \frac{1}{x^2(a+cx^2)^{3/2}(d+ex+fx^2)} dx$$

Optimal. Leaf size=618

$$\frac{e \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^2} - \frac{2cx}{a^2d\sqrt{a+cx^2}} + \frac{cdx(af^2+c(e^2-df))+ae(af^2+c(e^2-2df))}{ad^2\sqrt{a+cx^2}((cd-af)^2+ace^2)} + \frac{f(e(e-\sqrt{e^2-4df}))(af^2)}{ad^2\sqrt{a+cx^2}((cd-af)^2+ace^2)}$$

[Out] e*arctanh((c*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/d^2-e/a/d^2/(c*x^2+a)^(1/2)-1/a/d/x/(c*x^2+a)^(1/2)-2*c*x/a^2/d/(c*x^2+a)^(1/2)+(a*e*(a*f^2+c*(-2*d*f+e^2))+c*d*(a*f^2+c*(-d*f+e^2))*x)/a/d^2/(a*c*e^2+(-a*f+c*d)^2)/(c*x^2+a)^(1/2)+1/2*f*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(-2*a*f^2*(-d*f+e^2)-2*c*(d^2*f^2-3*d*e^2*f+e^4)+e*(a*f^2+c*(-2*d*f+e^2))*(e-(-4*d*f+e^2)^(1/2)))/d^2/(a*c*e^2+(-a*f+c*d)^2)*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)-1/2*f*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(-2*a*f^2*(-d*f+e^2)-2*c*(d^2*f^2-3*d*e^2*f+e^4)+e*(a*f^2+c*(-2*d*f+e^2))*(e+(-4*d*f+e^2)^(1/2)))/d^2/(a*c*e^2+(-a*f+c*d)^2)*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)

Rubi [A] time = 2.28, antiderivative size = 618, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 27, number of rules / integrand size = 0.407, Rules used = {6728, 271, 191, 266, 51, 63, 208, 1017, 1034, 725, 206}

$$\frac{e \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^2} - \frac{2cx}{a^2d\sqrt{a+cx^2}} + \frac{cdx(af^2+c(e^2-df))+ae(af^2+c(e^2-2df))}{ad^2\sqrt{a+cx^2}((cd-af)^2+ace^2)} + \frac{f(e(e-\sqrt{e^2-4df}))(af^2)}{ad^2\sqrt{a+cx^2}((cd-af)^2+ace^2)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

[Out] -(e/(a*d^2*Sqrt[a + c*x^2])) - 1/(a*d*x*Sqrt[a + c*x^2]) - (2*c*x)/(a^2*d*Sqrt[a + c*x^2]) + (a*e*(a*f^2 + c*(e^2 - 2*d*f)) + c*d*(a*f^2 + c*(e^2 - d*f))*x)/(a*d^2*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2]) + (f*(e*(e - Sqrt[e^2 - 4*d*f]))*(a*f^2 + c*(e^2 - 2*d*f)) - 2*(a*f^2*(e^2 - d*f) + c*(e^4 - 3*d*e^2*f + d^2*f^2)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f]))*x]/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))*Sqrt[a + c*x^2])]/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))] - (f*(e*(e + Sqrt[e^2 - 4*d*f]))*(a*f^2

$$+ c*(e^2 - 2*d*f) - 2*(a*f^2*(e^2 - d*f) + c*(e^4 - 3*d*e^2*f + d^2*f^2))$$

$$)*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]))/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])] + (e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(a^(3/2)*d^2)$$

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[
n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```


Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1017

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + c*x^2)^(p + 1)*(d + e*x + f*x^2)^
(q + 1)*(g*c*(2*a*c*e) + (-a*h)*(2*c^2*d - c*(2*a*f)) + c*(g*(2*c^2*d - c
*(2*a*f)) - h*(-2*a*c*e)*x))/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)),
x] + Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^
(p + 1)*(d + e*x + f*x^2)^q*Simp[(-2*g*c)*((c*d - a*f)^2 - (-a*e))*(c*e))*
(p + 1) + (2*(g*c*(c*d - a*f) - a*(-h*c*e)))*(a*f*(p + 1) - c*d*(p + 2))
- e*((g*c)*(2*a*c*e) + (-a*h)*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q + 2)
- (2*f*((g*c)*(2*a*c*e) + (-a*h)*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q +
2) - (2*(g*c*(c*d - a*f) - a*(-h*c*e)))*(-c*e*(2*p + q + 4)))*x - c*f*(
2*(g*c*(c*d - a*f) - a*(-h*c*e)))*(2*p + 2*q + 5)*x^2, x], x] /; Free
Q[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[
a*c*e^2 + (c*d - a*f)^2, 0] && !(IntegerQ[p] && ILtQ[q, -1])
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 (a + cx^2)^{3/2} (d + ex + fx^2)} dx &= \int \left(\frac{1}{dx^2 (a + cx^2)^{3/2}} - \frac{e}{d^2 x (a + cx^2)^{3/2}} + \frac{e^2 - df + efx}{d^2 (a + cx^2)^{3/2} (d + ex + fx^2)} \right) dx \\
&= \frac{\int \frac{e^2 - df + efx}{(a + cx^2)^{3/2} (d + ex + fx^2)} dx}{d^2} + \frac{\int \frac{1}{x^2 (a + cx^2)^{3/2}} dx}{d} - \frac{e \int \frac{1}{x (a + cx^2)^{3/2}} dx}{d^2} \\
&= -\frac{1}{adx \sqrt{a + cx^2}} + \frac{ae (af^2 + c(e^2 - 2df)) + cd (af^2 + c(e^2 - df)) x}{ad^2 (ace^2 + (cd - af)^2) \sqrt{a + cx^2}} \quad (2c) \\
&= -\frac{e}{ad^2 \sqrt{a + cx^2}} - \frac{1}{adx \sqrt{a + cx^2}} - \frac{2cx}{a^2 d \sqrt{a + cx^2}} + \frac{ae (af^2 + c(e^2 - 2df))}{ad^2 (ace^2 + (cd - af)^2) \sqrt{a + cx^2}} \\
&= -\frac{e}{ad^2 \sqrt{a + cx^2}} - \frac{1}{adx \sqrt{a + cx^2}} - \frac{2cx}{a^2 d \sqrt{a + cx^2}} + \frac{ae (af^2 + c(e^2 - 2df))}{ad^2 (ace^2 + (cd - af)^2) \sqrt{a + cx^2}} \\
&= -\frac{e}{ad^2 \sqrt{a + cx^2}} - \frac{1}{adx \sqrt{a + cx^2}} - \frac{2cx}{a^2 d \sqrt{a + cx^2}} + \frac{ae (af^2 + c(e^2 - 2df))}{ad^2 (ace^2 + (cd - af)^2) \sqrt{a + cx^2}}
\end{aligned}$$

Mathematica [C] time = 3.60, size = 557, normalized size = 0.90

$$\frac{d(a+2cx^2)}{a^2 x \sqrt{a+cx^2}} - \frac{f \left(\frac{e^2 - 2df}{\sqrt{e^2 - 4df}} + e \right) (2af + cx(e - \sqrt{e^2 - 4df}))}{a \sqrt{a+cx^2} (4af^2 + c(e - \sqrt{e^2 - 4df})^2)} - \frac{f \left(\frac{2df - e^2}{\sqrt{e^2 - 4df}} + e \right) (2af + cx(\sqrt{e^2 - 4df} + e))}{a \sqrt{a+cx^2} (4af^2 + c(\sqrt{e^2 - 4df} + e)^2)} + \frac{\sqrt{2} f^3 (e \sqrt{e^2 - 4df} - 2df + e^2) \tanh^{-1} \left(\frac{2af + cx(e - \sqrt{e^2 - 4df})}{\sqrt{a+cx^2} \sqrt{e^2 - 4df}} \right)}{\sqrt{e^2 - 4df} (2af^2 + c(-e \sqrt{e^2 - 4df} + e)^2)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

[Out] -(((f*(e + (e^2 - 2*d*f)/Sqrt[e^2 - 4*d*f])*(2*a*f + c*(e - Sqrt[e^2 - 4*d*f]))*x))/(a*(4*a*f^2 + c*(e - Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + c*x^2])) - (f*(e + (-e^2 + 2*d*f)/Sqrt[e^2 - 4*d*f])*(2*a*f + c*(e + Sqrt[e^2 - 4*d*f]))*x))/(a*(4*a*f^2 + c*(e + Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + c*x^2]) + (d*(a + 2*c*x^2))/(a^2*x*Sqrt[a + c*x^2]) + (Sqrt[2]*f^3*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f]))*x]/(Sqrt[4*a*f^2 - 2*c

$$\frac{(-e^2 + 2df + e\sqrt{e^2 - 4df})\sqrt{a + cx^2}}{\sqrt{e^2 - 4df}(2af^2 + c(e^2 - 2df - e\sqrt{e^2 - 4df}))^{3/2}} + \frac{\sqrt{2}f^3(-e^2 + 2df + e\sqrt{e^2 - 4df})\operatorname{ArcTanh}((2af - c(e + \sqrt{e^2 - 4df}))x)}{\sqrt{4af^2 + 2c(e^2 - 2df + e\sqrt{e^2 - 4df})}\sqrt{a + cx^2}}}{\sqrt{e^2 - 4df}(2af^2 + c(e^2 - 2df + e\sqrt{e^2 - 4df}))^{3/2}} + \frac{e\operatorname{Hypergeometric2F1}[-1/2, 1, 1/2, 1 + (cx^2)/a]}{a\sqrt{a + cx^2}})/d^2$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage₂

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] sage2

maple [B] time = 0.02, size = 2046, normalized size = 3.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)

[Out]
$$\frac{-8f^4}{(e + (-4df + e^2)^{1/2})^2} \frac{(-4df + e^2)^{1/2}}{(2af^2 - 2c*df + ce^2 + (-4df + e^2)^{1/2} * ce) / ((x + 1/2 * (e + (-4df + e^2)^{1/2})) / f)^2} \frac{c}{f + 1/2 * (2af^2 - 2c*df + ce^2 + (-4df + e^2)^{1/2} * ce) / f^2} - \frac{16f^3}{(e + (-4df + e^2)^{1/2})^2} \frac{c^2}{(2af^2 - 2c*df + ce^2 + (-4df + e^2)^{1/2} * ce) / (4ac - 4c^2d/f + c^2e^2/f^2 - (-4df + e^2) * c^2/f^2)} / ((x + 1/2 * (e + (-4df + e^2)^{1/2})) / f)^2} \frac{c}{f + 1/2 * (2af^2 - 2c*df + ce^2 + (-4df + e^2)^{1/2} * ce) / f^2} - \frac{16f^3}{(e + (-4df + e^2)^{1/2})^2} \frac{c^2}{(2af^2 - 2c*df + ce^2 + (-4df + e^2)^{1/2} * ce) / (4ac - 4c^2d/f + c^2e^2/f^2 - (-4df + e^2) * c^2/f^2)} / ((x + 1/2 * (e + (-4df + e^2)^{1/2})) / f)^2} \frac{c}{f + 1/2 * (2af^2 - 2c*df + ce^2 + (-4df + e^2)^{1/2} * ce) / f^2}$$

```

+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*e*x+8*f^4/(e+(-4*d*f+e^2)^(1/2))^(1/2)
/((-4*d*f+e^2)^(1/2)/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)
/((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-e+(-4*d*f
+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-
4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(
1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+
e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(
-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))+4*f/(-
e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a/x/(c*x^2+a)^(1/2)+8*f/(-e+(-
4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*c/a^2*x/(c*x^2+a)^(1/2)+8*f^4/(-e+
(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2
)^(1/2)*c*e)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e-(-4*d*f+e^2)^(1/2))*
(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^
2)^(1/2)*c*e)/f^2)^(1/2)-16*f^3/(-e+(-4*d*f+e^2)^(1/2))^2*c^2/(2*a*f^2-2*c*
d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2)
*c^2/f^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e-(-4*d*f+e^2)^(1/2))*
(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(
1/2)*c*e)/f^2)^(1/2)*x+16*f^3/(-e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)
*c^2/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^
2/f^2-(-4*d*f+e^2)*c^2/f^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e-(-4*d
*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c
*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*e*x-8*f^4/(-e+(-4*d*f+e^2)^(1/2))^2
/(-4*d*f+e^2)^(1/2)/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/
((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-e-(-4*d*f+
e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-
4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(
1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-(-4*d*f
+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2
-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))-16*
f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+e^2)^(1/2))^2/a/(c*x^2+a)^(1/2)+
16*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+e^2)^(1/2))^2/a^(3/2)*ln((2*a
+2*(c*x^2+a)^(1/2)*a^(1/2))/x)

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + a)^{\frac{3}{2}}(fx^2 + ex + d)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] integrate(1/((c*x^2 + a)^(3/2)*(f*x^2 + e*x + d)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2 (cx^2 + a)^{3/2} (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)

[Out] int(1/(x^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + cx^2)^{3/2} (d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(c*x**2+a)**(3/2)/(f*x**2+e*x+d), x)

[Out] Integral(1/(x**2*(a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)

$$3.77 \quad \int \frac{x^3 \sqrt{a+bx+cx^2}}{d-fx^2} dx$$

Optimal. Leaf size=392

$$\frac{b(b^2 - 4ac) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}f} + \frac{b(b+2cx)\sqrt{a+bx+cx^2}}{8c^2f} - \frac{d\sqrt{af+b(-\sqrt{d})}\sqrt{f}+cd \tanh^{-1}\left(\frac{-2a\sqrt{f}+x}{2\sqrt{a+bx+cx^2}}\right)}{2f^{5/2}}$$

[Out] $-1/3*(c*x^2+b*x+a)^{(3/2)}/c/f-1/16*b*(-4*a*c+b^2)*\operatorname{arctanh}(1/2*(2*c*x+b)/c^{(1/2)})/(c*x^2+b*x+a)^{(1/2)}/c^{(5/2)}/f-1/2*b*d*\operatorname{arctanh}(1/2*(2*c*x+b)/c^{(1/2)})/(c*x^2+b*x+a)^{(1/2)}/f^2/c^{(1/2)}-d*(c*x^2+b*x+a)^{(1/2)}/f^2+1/8*b*(2*c*x+b)*(c*x^2+b*x+a)^{(1/2)}/c^2/f-1/2*d*\operatorname{arctanh}(1/2*(b*d^{(1/2)}-2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}-b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)}/(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(1/2)}*(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(1/2)}/f^{(5/2)}+1/2*d*\operatorname{arctanh}(1/2*(b*d^{(1/2)}+2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}+b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)}/(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(1/2)}*(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(1/2)}/f^{(5/2)}$

Rubi [A] time = 0.95, antiderivative size = 392, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {6725, 640, 612, 621, 206, 1021, 1078, 1033, 724}

$$\frac{b(b^2 - 4ac) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}f} + \frac{b(b+2cx)\sqrt{a+bx+cx^2}}{8c^2f} - \frac{d\sqrt{a+bx+cx^2}}{f^2} - \frac{bd \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(x^3*\operatorname{Sqrt}[a + b*x + c*x^2])/(d - f*x^2), x]$

[Out] $-((d*\operatorname{Sqrt}[a + b*x + c*x^2])/f^2) + (b*(b + 2*c*x)*\operatorname{Sqrt}[a + b*x + c*x^2])/(8*c^2*f) - (a + b*x + c*x^2)^{(3/2)}/(3*c*f) - (b*d*\operatorname{ArcTanh}[(b + 2*c*x)/(2*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*\operatorname{Sqrt}[c]*f^2) - (b*(b^2 - 4*a*c)*\operatorname{ArcTanh}[(b + 2*c*x)/(2*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(16*c^{(5/2)}*f) - (d*\operatorname{Sqrt}[c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] - 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] - b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*f^{(5/2)}) + (d*\operatorname{Sqrt}[c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] + 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] + b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*f^{(5/2)})$

Rule 206

$\operatorname{Int}[(a_ + (b_ .)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{Gt}$

$Q[a, 0] \parallel LtQ[b, 0]$

Rule 612

$Int[((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^(p_.), x_Symbol] \rightarrow Simp[((b + 2*c*x) * (a + b*x + c*x^2)^p) / (2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c)) / (2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /;$ FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

$Int[1/Sqrt[(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2], x_Symbol] \rightarrow Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /;$ FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 640

$Int[((d_.) + (e_.)*(x_.)) * ((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^(p_.), x_Symbol] \rightarrow Simp[(e*(a + b*x + c*x^2)^(p + 1)) / (2*c*(p + 1)), x] + Dist[(2*c*d - b*e) / (2*c), Int[(a + b*x + c*x^2)^p, x], x] /;$ FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 724

$Int[1/(((d_.) + (e_.)*(x_.)) * Sqrt[(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2]), x_Symbol] \rightarrow Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x) / Sqrt[a + b*x + c*x^2]], x] /;$ FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1021

$Int[((g_.) + (h_.)*(x_.)) * ((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^(p_.) * ((d_.) + (f_.)*(x_.)^2)^(q_.), x_Symbol] \rightarrow Simp[(h*(a + b*x + c*x^2)^p * (d + f*x^2)^(q + 1)) / (2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2)^(p - 1) * (d + f*x^2)^q * Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q + 1))*x^2, x], x] /;$ FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]

Rule 1033

$Int[((g_.) + (h_.)*(x_.)) / (((a_.) + (c_.)*(x_.)^2) * Sqrt[(d_.) + (e_.)*(x_.) + (f_.)*(x_.)^2]), x_Symbol] \rightarrow With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x) * Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x) * Sqrt[d + e*x + f*x^2]), x], x]] /;$ FreeQ[{a, c, d, e, f

, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]

Rule 1078

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f, 0]

Rule 6725

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^3 \sqrt{a+bx+cx^2}}{d-fx^2} dx &= \int \left(-\frac{x\sqrt{a+bx+cx^2}}{f} + \frac{dx\sqrt{a+bx+cx^2}}{f(d-fx^2)} \right) dx \\
 &= -\frac{\int x\sqrt{a+bx+cx^2} dx}{f} + \frac{d \int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2} dx}{f} \\
 &= -\frac{d\sqrt{a+bx+cx^2}}{f^2} - \frac{(a+bx+cx^2)^{3/2}}{3cf} + \frac{d \int \frac{\frac{bd}{2} + (cd+af)x + \frac{1}{2}bfx^2}{\sqrt{a+bx+cx^2}(d-fx^2)} dx}{f^2} + \frac{b \int \sqrt{a+bx+cx^2}}{2cf} \\
 &= -\frac{d\sqrt{a+bx+cx^2}}{f^2} + \frac{b(b+2cx)\sqrt{a+bx+cx^2}}{8c^2f} - \frac{(a+bx+cx^2)^{3/2}}{3cf} - \frac{d \int \frac{-bdf-f(cd+af)}{\sqrt{a+bx+cx^2}(d-fx^2)} dx}{f^3} \\
 &= -\frac{d\sqrt{a+bx+cx^2}}{f^2} + \frac{b(b+2cx)\sqrt{a+bx+cx^2}}{8c^2f} - \frac{(a+bx+cx^2)^{3/2}}{3cf} - \frac{(bd) \text{Subst}\left(\int \frac{1}{4c-}\right)}{f^3} \\
 &= -\frac{d\sqrt{a+bx+cx^2}}{f^2} + \frac{b(b+2cx)\sqrt{a+bx+cx^2}}{8c^2f} - \frac{(a+bx+cx^2)^{3/2}}{3cf} - \frac{bd \tanh^{-1}\left(\frac{b+}{2\sqrt{c}\sqrt{a}}\right)}{2\sqrt{c}f^2} \\
 &= -\frac{d\sqrt{a+bx+cx^2}}{f^2} + \frac{b(b+2cx)\sqrt{a+bx+cx^2}}{8c^2f} - \frac{(a+bx+cx^2)^{3/2}}{3cf} - \frac{bd \tanh^{-1}\left(\frac{b+}{2\sqrt{c}\sqrt{a}}\right)}{2\sqrt{c}f^2}
 \end{aligned}$$

Mathematica [A] time = 0.89, size = 327, normalized size = 0.83

$$\frac{2\sqrt{f}\sqrt{a+cx}(2cf(4a+bx)-3b^2f+8c^2(3d+fx^2))}{c^2} - \frac{3b\sqrt{f}(-4acf+b^2f+8c^2d)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+cx}}\right)}{c^{5/2}} + 24d\sqrt{af+b\sqrt{d}\sqrt{f}+cd}$$

$48f^{5/2}$

Antiderivative was successfully verified.

[In] Integrate[(x^3*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]

[Out] ((-2*Sqrt[f]*Sqrt[a + x*(b + c*x)]*(-3*b^2*f + 2*c*f*(4*a + b*x) + 8*c^2*(3*d + f*x^2)))/c^2 - (3*b*Sqrt[f]*(8*c^2*d + b^2*f - 4*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/c^(5/2) + 24*d*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] - 24*d*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(48*f^(5/2))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);;OUTPUT:Error: Bad Argument Type

maple [B] time = 0.02, size = 1817, normalized size = 4.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)

```
[Out] -1/3*(c*x^2+b*x+a)^(3/2)/c/f+1/4/f*b/c*x*(c*x^2+b*x+a)^(1/2)+1/8/f*b^2/c^2*
(c*x^2+b*x+a)^(1/2)+1/4/f*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1
/2))*a-1/16/f*b^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/2/f
^2*d*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+
c*d-(d*f)^(1/2)*b)/f)^(1/2)+1/2/f^3*d*ln((1/2/f*(b*f-2*(d*f)^(1/2)*c)+c*(x+
(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*
f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*(d*f)^(1/2)-1/4/f^2
*d*ln((1/2/f*(b*f-2*(d*f)^(1/2)*c)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(
1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*
b)/f)^(1/2))/c^(1/2)*b-1/2/f^3*d/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a
*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c
*d-(d*f)^(1/2)*b)/f)^(1/2))*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+
(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*(d*f)
^(1/2)*b+1/2/f^2*d/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(
1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)
*b)/f)^(1/2))*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)
/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*a+1/2/f^3*d^2/((a*f
+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(
1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*((x+(d*f)^(
1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*
b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*c-1/2/f^2*d*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(
d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)-1/2/f^3*
d*ln((1/2*(b*f+2*(d*f)^(1/2)*c)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1
/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b
)/f)^(1/2))*c^(1/2)*(d*f)^(1/2)-1/4/f^2*d*ln((1/2*(b*f+2*(d*f)^(1/2)*c)/f+c
*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x
-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b+1/2/f^3*d/((a
*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)
^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*((x-(d*f)
^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)
*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*(d*f)^(1/2)*b+1/2/f^2*d/((a*f+c*d+(d*f)^(
1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(
d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*((x-(d*f)^(1/2)/f)^2*c+
(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)
)/(x-(d*f)^(1/2)/f))*a+1/2/f^3*d^2/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(
a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+
c*d+(d*f)^(1/2)*b)/f)^(1/2))*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x
-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see 'assume?' for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2 - (c*((b*sqrt(4*d*f)) / (2*f) + (c*d)/f+a)) /f^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \sqrt{cx^2 + bx + a}}{d - fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*(a + b*x + c*x^2)^(1/2))/(d - f*x^2), x)

[Out] int((x^3*(a + b*x + c*x^2)^(1/2))/(d - f*x^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3 \sqrt{a + bx + cx^2}}{-d + fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(c*x**2+b*x+a)**(1/2)/(-f*x**2+d), x)

[Out] -Integral(x**3*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)

$$3.78 \quad \int \frac{x^2 \sqrt{a+bx+cx^2}}{d-fx^2} dx$$

Optimal. Leaf size=316

$$\frac{(4acf + b^2(-f) + 8c^2d) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) \sqrt{d}\sqrt{af+b(-\sqrt{d})}\sqrt{f} + cd \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f}}\right)}{8c^{3/2}f^2} + \frac{\sqrt{d}\sqrt{af+b(-\sqrt{d})}\sqrt{f} + cd \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f}}\right)}{2f^2}$$

[Out] $-1/8*(4*a*c*f-b^2*f+8*c^2*d)*\operatorname{arctanh}(1/2*(2*c*x+b)/c^{(1/2)/(c*x^2+b*x+a)^{(1/2)})}/c^{(3/2)}/f^2-1/4*(2*c*x+b)*(c*x^2+b*x+a)^{(1/2)}/c/f+1/2*\operatorname{arctanh}(1/2*(b*d^{(1/2)}-2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}-b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)}/(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(1/2)}*d^{(1/2)}*(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(1/2)}/f^2+1/2*\operatorname{arctanh}(1/2*(b*d^{(1/2)}+2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}+b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)}/(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(1/2)}*d^{(1/2)}*(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(1/2)}/f^2$

Rubi [A] time = 0.49, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1071, 1078, 621, 206, 1033, 724}

$$\frac{(4acf + b^2(-f) + 8c^2d) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) \sqrt{d}\sqrt{af+b(-\sqrt{d})}\sqrt{f} + cd \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f}}\right)}{8c^{3/2}f^2} + \frac{\sqrt{d}\sqrt{af+b(-\sqrt{d})}\sqrt{f} + cd \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f}}\right)}{2f^2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(x^2*\operatorname{Sqrt}[a + b*x + c*x^2])/(d - f*x^2), x]$

[Out] $-((b + 2*c*x)*\operatorname{Sqrt}[a + b*x + c*x^2])/(4*c*f) - ((8*c^2*d - b^2*f + 4*a*c*f)*\operatorname{ArcTanh}[(b + 2*c*x)/(2*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(8*c^{(3/2)}*f^2) + (\operatorname{Sqrt}[d]*\operatorname{Sqrt}[c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] - 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] - b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*f^2) + (\operatorname{Sqrt}[d]*\operatorname{Sqrt}[c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] + 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] + b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*f^2)$

Rule 206

$\operatorname{Int}[(a_ + (b_ .)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

Rule 621

```
Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1033

```
Int[((g_) + (h_)*(x_))/(((a_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1071

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (C_)*(x_)^2)*((d_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[(((C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*(C*(-(b*f))*(q + 1)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(-(b*f))*(q + 1)) + (p + q + 1)*(-(b*c*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))))*x + (p*(-(b*f))*(C*(-(b*f))*(q + 1)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))))*x^2, x], x] /; FreeQ[{a, b, c, d, f, A, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]
```

Rule 1078

```
Int[((A_) + (B_)*(x_) + (C_)*(x_)^2)/(((a_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 \sqrt{a+bx+cx^2}}{d-fx^2} dx &= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf} - \frac{\int \frac{-\frac{1}{4}(b^2+4ac)df-2bcdfx-\frac{1}{4}f(8c^2d-b^2f+4acf)x^2}{\sqrt{a+bx+cx^2}(d-fx^2)} dx}{2cf^2} \\
&= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf} + \frac{\int \frac{\frac{1}{4}(b^2+4ac)df^2+\frac{1}{4}df(8c^2d-b^2f+4acf)+2bcd f^2 x}{\sqrt{a+bx+cx^2}(d-fx^2)} dx}{2cf^3} - \frac{(8c^2d-b^2f)}{2cf^2} \\
&= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf} - \frac{(\sqrt{d}(cd-b\sqrt{d}\sqrt{f}+af)) \int \frac{1}{(-\sqrt{d}\sqrt{f}-fx)\sqrt{a+bx+cx^2}} dx}{2f^{3/2}} + \frac{(8c^2d-b^2f+4acf)}{8c^{3/2}f^2} \\
&= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf} - \frac{(8c^2d-b^2f+4acf) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}f^2} + \frac{(\sqrt{d}(cd-b\sqrt{d}\sqrt{f}+af))}{8c^{3/2}f^2} \\
&= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf} - \frac{(8c^2d-b^2f+4acf) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}f^2} + \frac{\sqrt{d}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}}{8c^{3/2}f^2}
\end{aligned}$$

Mathematica [A] time = 0.49, size = 302, normalized size = 0.96

$$\frac{(-4acf + b^2f - 8c^2d) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right) - 2\sqrt{c} \left(-2c\sqrt{d}\sqrt{af+b\sqrt{d}\sqrt{f}+cd} \tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right) \right)}{8c^{3/2}f^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*sqrt[a + b*x + c*x^2])/(d - f*x^2), x]

[Out] ((-8*c^2*d + b^2*f - 4*a*c*f)*ArcTanh[(b + 2*c*x)/(2*sqrt[c]*sqrt[a + x*(b + c*x)])] - 2*sqrt[c]*(f*(b + 2*c*x)*sqrt[a + x*(b + c*x)] - 2*c*sqrt[d]*sqrt[c*d + b*sqrt[d]*sqrt[f] + a*f]*ArcTanh[(b*sqrt[d] + 2*a*sqrt[f] + 2*c*sqrt[d]*x + b*sqrt[f]*x)/(2*sqrt[c*d + b*sqrt[d]*sqrt[f] + a*f]*sqrt[a + x*(b + c*x)])] - 2*c*sqrt[d]*sqrt[c*d - b*sqrt[d]*sqrt[f] + a*f]*ArcTanh[(-2*a*sqrt[f] + 2*c*sqrt[d]*x + b*(sqrt[d] - sqrt[f]*x))/(2*sqrt[c*d - b*sqrt[d]*sqrt[f] + a*f]*sqrt[a + x*(b + c*x)])]))/(8*c^(3/2)*f^2)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

```
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:Error: Bad Argument Type
```

```
maple [B] time = 0.02, size = 1810, normalized size = 5.73
```

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/2/f*x*(c*x^2+b*x+a)^(1/2)-1/4/f/c*(c*x^2+b*x+a)^(1/2)*b-1/2/f/c^(1/2)*ln
((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+1/8/f/c^(3/2)*ln((c*x+1/2*b)/c^(
1/2)+(c*x^2+b*x+a)^(1/2))*b^2+1/2*d/(d*f)^(1/2)/f*((x+(d*f)^(1/2)/f)^2*c+(
b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)-1
/2*d/f^2*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(
d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(
1/2)*b)/f)^(1/2))*c^(1/2)+1/4*d/(d*f)^(1/2)/f*ln(((x+(d*f)^(1/2)/f)*c+1/2*
(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*
c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b+1/2*d/f^
2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*
(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*((x+
(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)
^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*b-1/2*d/(d*f)^(1/2)/f/((a*f+c*d-(d*f)
^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(
x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*((x+(d*f)^(1/2)/f)^2
*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/
2)))/(x+(d*f)^(1/2)/f))*a-1/2*d^2/(d*f)^(1/2)/f^2/((a*f+c*d-(d*f)^(1/2)*b)/f
)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)
)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d
*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)
^(1/2)/f))*c-1/2*d/(d*f)^(1/2)/f*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*
c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)-1/2*d/f^2*ln(((x-(d
*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+
(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))
```

```

*c^(1/2)-1/4*d/(d*f)^(1/2)/f*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)
*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/
f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b+1/2*d/f^2/((a*f+c*d+(d*f)^(
1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-
(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c
+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)
)/(x-(d*f)^(1/2)/f))*b+1/2*d/(d*f)^(1/2)/f/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)
)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f
+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1
/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/((x-(d*f)^(1/2)
/f))*a+1/2*d^2/(d*f)^(1/2)/f^2/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f
+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d
+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d
*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/((x-(d*f)^(1/2)/f))*c

```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2 -c*((b*sqrt(4*d*f)) / (2*f) + (c*d)/f+a) /f^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \sqrt{c x^2 + b x + a}}{d - f x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(a + b*x + c*x^2)^(1/2))/(d - f*x^2),x)

[Out] int((x^2*(a + b*x + c*x^2)^(1/2))/(d - f*x^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{x^2 \sqrt{a + b x + c x^2}}{-d + f x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(x**2*(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x**2*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)
```

$$3.79 \quad \int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2} dx$$

Optimal. Leaf size=282

$$\frac{\sqrt{af+b(-\sqrt{d})}\sqrt{f+cd} \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f+cd}}\right)}{2f^{3/2}} + \frac{\sqrt{af+b\sqrt{d}}\sqrt{f+cd} \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}}\sqrt{f+cd}}\right)}{2f^{3/2}}$$

[Out] $-1/2*b*\operatorname{arctanh}(1/2*(2*c*x+b)/c^{(1/2)}/(c*x^2+b*x+a)^{(1/2)})/f/c^{(1/2)}-(c*x^2+b*x+a)^{(1/2)}/f-1/2*\operatorname{arctanh}(1/2*(b*d^{(1/2)}-2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}-b*f^{(1/2)}))/((c*x^2+b*x+a)^{(1/2)}/(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(1/2)})*(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(1/2)}/f^{(3/2)}+1/2*\operatorname{arctanh}(1/2*(b*d^{(1/2)}+2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}+b*f^{(1/2)}))/((c*x^2+b*x+a)^{(1/2)}/(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(1/2)})*(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(1/2)}/f^{(3/2)}$

Rubi [A] time = 0.29, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1021, 1078, 621, 206, 1033, 724}

$$\frac{\sqrt{af+b(-\sqrt{d})}\sqrt{f+cd} \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f+cd}}\right)}{2f^{3/2}} + \frac{\sqrt{af+b\sqrt{d}}\sqrt{f+cd} \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}}\sqrt{f+cd}}\right)}{2f^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(x*sqrt[a + b*x + c*x^2])/(d - f*x^2), x]

[Out] $-(\operatorname{Sqrt}[a + b*x + c*x^2]/f) - (b*\operatorname{ArcTanh}[(b + 2*c*x)/(2*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*\operatorname{Sqrt}[c]*f) - (\operatorname{Sqrt}[c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] - 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] - b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*f^{(3/2)}) + (\operatorname{Sqrt}[c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] + 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] + b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*f^{(3/2)})$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

```
Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1021

```
Int[((g_) + (h_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q + 1))*x^2, x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1033

```
Int[((g_) + (h_)*(x_))/(((a_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1078

```
Int[((A_) + (B_)*(x_) + (C_)*(x_)^2)/(((a_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2} dx &= -\frac{\sqrt{a+bx+cx^2}}{f} + \frac{\int \frac{\frac{bd}{2}+(cd+af)x+\frac{1}{2}bf x^2}{\sqrt{a+bx+cx^2}(d-fx^2)} dx}{f} \\
&= -\frac{\sqrt{a+bx+cx^2}}{f} - \frac{\int \frac{-bdf-f(cd+af)x}{\sqrt{a+bx+cx^2}(d-fx^2)} dx}{f^2} - \frac{b \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{2f} \\
&= -\frac{\sqrt{a+bx+cx^2}}{f} - \frac{b \operatorname{Subst}\left(\int \frac{1}{4c-x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f} + \frac{(cd-b\sqrt{d}\sqrt{f}+af) \int \frac{1}{(-\sqrt{d}\sqrt{f}-\sqrt{a+bx+cx^2})} dx}{2f} \\
&= -\frac{\sqrt{a+bx+cx^2}}{f} - \frac{b \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f} - \frac{(cd-b\sqrt{d}\sqrt{f}+af) \operatorname{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}\sqrt{f}-x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{2f} \\
&= -\frac{\sqrt{a+bx+cx^2}}{f} - \frac{b \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f} - \frac{\sqrt{cd-b\sqrt{d}\sqrt{f}+af} \tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}}\right)}{2f^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.29, size = 272, normalized size = 0.96

$$\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd} \tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right) - \sqrt{af+b(-\sqrt{d})\sqrt{f}+cd} \tanh^{-1}\left(\frac{-2a\sqrt{f}+b(\sqrt{d}-\sqrt{f}x)}{2\sqrt{a+x(b+cx)}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2f^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*Sqrt[a + b*x + c*x^2])/(d - f*x^2), x]

[Out] (-2*Sqrt[f]*Sqrt[a + x*(b + c*x)] - (b*Sqrt[f]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] - Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(2*f^(3/2))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

```
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:Error: Bad Argument Type
```

```
maple [B] time = 0.01, size = 1667, normalized size = 5.91
```

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/2/f*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*
f+c*d-(d*f)^(1/2)*b)/f)^(1/2)+1/2/f^2*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d
*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*
f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*(d*f)^(1/2)-1/4/f*ln
(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)
)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f
)^(1/2))/c^(1/2)*b-1/2/f^2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d
-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*
f)^(1/2)*b)/f)^(1/2))*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)
^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*(d*f)^(1/2)
*b+1/2/f/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+
(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/
2))*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*
d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*a+1/2/f^2/((a*f+c*d-(d*f)^(1/
2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*
f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*((x+(d*f)^(1/2)/f)^2*c+(b
*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(
x+(d*f)^(1/2)/f))*c*d-1/2/f*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x
-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)-1/2/f^2*ln(((x-(d*f)^(1/
2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*
(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)
*(d*f)^(1/2)-1/4/f*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^
(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f
+c*d+(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b+1/2/f^2/((a*f+c*d+(d*f)^(1/2)*b)/f)
```

$$\begin{aligned} & \frac{1}{f} \ln\left(\frac{2(a f + c d + (d f)^{1/2} b)}{f + (b f + 2(d f)^{1/2} c)} \cdot \frac{x - (d f)^{1/2}}{f}\right) \\ & + \frac{2(a f + c d + (d f)^{1/2} b)}{f} \cdot \frac{(x - (d f)^{1/2})^2 c + (b f + 2(d f)^{1/2} c) \cdot \frac{x - (d f)^{1/2}}{f}}{f + 2 \frac{(a f + c d + (d f)^{1/2} b)}{f} \cdot \frac{x - (d f)^{1/2}}{f}} \\ & + \frac{(d f)^{1/2} b + 1/2 f}{f} \cdot \frac{1}{f} \ln\left(\frac{2(a f + c d + (d f)^{1/2} b)}{f + (b f + 2(d f)^{1/2} c)} \cdot \frac{x - (d f)^{1/2}}{f}\right) \\ & + \frac{2(a f + c d + (d f)^{1/2} b)}{f} \cdot \frac{(x - (d f)^{1/2})^2 c + (b f + 2(d f)^{1/2} c) \cdot \frac{x - (d f)^{1/2}}{f}}{f + 2 \frac{(a f + c d + (d f)^{1/2} b)}{f} \cdot \frac{x - (d f)^{1/2}}{f}} \\ & + \frac{a + 1/2 f^2}{f} \cdot \frac{1}{f} \ln\left(\frac{2(a f + c d + (d f)^{1/2} b)}{f + (b f + 2(d f)^{1/2} c)} \cdot \frac{x - (d f)^{1/2}}{f}\right) \\ & + \frac{2(a f + c d + (d f)^{1/2} b)}{f} \cdot \frac{(x - (d f)^{1/2})^2 c + (b f + 2(d f)^{1/2} c) \cdot \frac{x - (d f)^{1/2}}{f}}{f + 2 \frac{(a f + c d + (d f)^{1/2} b)}{f} \cdot \frac{x - (d f)^{1/2}}{f}} \\ & + \frac{c d}{f} \end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see 'assume?' for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2 - (c*((b*sqrt(4*d*f)))/(2*f) + (c*d)/f+a) /f^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x \sqrt{c x^2 + b x + a}}{d - f x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(a + b*x + c*x^2)^(1/2))/(d - f*x^2),x)

[Out] int((x*(a + b*x + c*x^2)^(1/2))/(d - f*x^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{x \sqrt{a + b x + c x^2}}{-d + f x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)

[Out] -Integral(x*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)

$$3.80 \quad \int \frac{\sqrt{a+bx+cx^2}}{d-fx^2} dx$$

Optimal. Leaf size=266

$$\frac{\sqrt{af+b(-\sqrt{d})}\sqrt{f+cd} \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f+cd}}\right)}{2\sqrt{d}f} + \frac{\sqrt{af+b\sqrt{d}}\sqrt{f+cd} \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}}\sqrt{f+cd}}\right)}{2\sqrt{d}f}$$

[Out] $-\operatorname{arctanh}\left(\frac{1/2*(2*c*x+b)/c^{1/2}/(c*x^2+b*x+a)^{1/2}}{c^{1/2}/f+1/2*\operatorname{arctanh}\left(\frac{1/2*(b*d^{1/2}-2*a*f^{1/2}+x*(2*c*d^{1/2}-b*f^{1/2}))}{(c*x^2+b*x+a)^{1/2}/(c*d+a*f-b*d^{1/2}*f^{1/2})^{1/2}}\right)}{c^{1/2}/f+1/2*\operatorname{arctanh}\left(\frac{1/2*(b*d^{1/2}+2*a*f^{1/2}+x*(2*c*d^{1/2}+b*f^{1/2}))}{(c*x^2+b*x+a)^{1/2}/(c*d+a*f+b*d^{1/2}*f^{1/2})^{1/2}}\right)}\right)$

Rubi [A] time = 0.23, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {990, 621, 206, 1033, 724}

$$\frac{\sqrt{af+b(-\sqrt{d})}\sqrt{f+cd} \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f+cd}}\right)}{2\sqrt{d}f} + \frac{\sqrt{af+b\sqrt{d}}\sqrt{f+cd} \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}}\sqrt{f+cd}}\right)}{2\sqrt{d}f}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[a + b*x + c*x^2]/(d - f*x^2), x]`

[Out] $-\left(\frac{\operatorname{ArcTanh}\left[\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right]}{\sqrt{c}}\right)/f + \left(\frac{\operatorname{ArcTanh}\left[\frac{b\sqrt{d}-2a\sqrt{f}+(2c\sqrt{d}-b\sqrt{f})x}{2\sqrt{c*d-b}\sqrt{d}\sqrt{f+af}}\right]}{2\sqrt{d}\sqrt{f}} + \frac{\operatorname{ArcTanh}\left[\frac{b\sqrt{d}+2a\sqrt{f}+(2c\sqrt{d}+b\sqrt{f})x}{2\sqrt{c*d+b}\sqrt{d}\sqrt{f+af}}\right]}{2\sqrt{d}\sqrt{f}}\right)$

Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 621

`Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,`

b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 990

Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (f_.)*(x_)^2), x_Symbol] :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*f - b*f*x)/(Sqrt[a + b*x + c*x^2]*(d + f*x^2)), x], x] /; FreeQ[{a, b, c, d, f}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1033

Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{a+bx+cx^2}}{d-fx^2} dx &= \frac{\int \frac{cd+af+bf x}{\sqrt{a+bx+cx^2}(d-fx^2)} dx}{f} - \frac{c \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{f} \\
 &= -\frac{(2c) \text{Subst}\left(\int \frac{1}{4c-x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f} + \frac{1}{2} \left(b - \frac{cd+af}{\sqrt{d}\sqrt{f}}\right) \int \frac{1}{(-\sqrt{d}\sqrt{f}-fx)\sqrt{a+bx+cx^2}} dx \\
 &= -\frac{\sqrt{c} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f} + \left(-b - \frac{cd+af}{\sqrt{d}\sqrt{f}}\right) \text{Subst}\left(\int \frac{1}{4cdf+4b\sqrt{d}f^{3/2}+4af^2-x^2} dx, x, \frac{b\sqrt{d}-2a\sqrt{f}+(2c\sqrt{d}-b\sqrt{f})x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right) \\
 &= -\frac{\sqrt{c} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f} + \frac{\sqrt{cd-b\sqrt{d}\sqrt{f}+af} \tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+(2c\sqrt{d}-b\sqrt{f})x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}f}
 \end{aligned}$$

Mathematica [A] time = 0.17, size = 253, normalized size = 0.95

$$\frac{\sqrt{af + b(-\sqrt{d})}\sqrt{f} + cd \tanh^{-1}\left(\frac{-2a\sqrt{f} + b\sqrt{d} - b\sqrt{f}x + 2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af + b(-\sqrt{d})}\sqrt{f} + cd}\right) + \sqrt{af + b\sqrt{d}}\sqrt{f} + cd \tanh^{-1}\left(\frac{2a\sqrt{f} + b\sqrt{d} + b\sqrt{f}x + 2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af + b\sqrt{d}}\sqrt{f} + cd}\right)}{2\sqrt{d}f}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*x + c*x^2]/(d - f*x^2), x]

[Out] (-2*Sqrt[c]*Sqrt[d]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + 2*c*Sqrt[d]*x - b*Sqrt[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(2*Sqrt[d]*f)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(1/2)/(-f*x^2+d), x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(1/2)/(-f*x^2+d), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INP UT:sage2:=int(sage0,x);;OUTPUT:Error: Bad Argument Type

maple [B] time = 0.01, size = 1669, normalized size = 6.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x+a)^(1/2)/(-f*x^2+d), x)

[Out] 1/2/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^(1/2)*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)-1/2/f*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b

$$\begin{aligned}
& *f-2*(d*f)^{(1/2)*c}/f)/c^{(1/2)}+((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)*c} \\
& *(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2))*c^{(1/2)}+1/4/(d*f)^{(1/2)} \\
& *ln(((x+(d*f)^{(1/2)}/f)*c+1/2*(b*f-2*(d*f)^{(1/2)*c})/f)/c^{(1/2)}+((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)*c} \\
& *(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)))/c^{(1/2)}*b+1/2/f/((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*ln((2*(a*f+ \\
& c*d-(d*f)^{(1/2)*b})/f+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+2*((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)))/(x+(d*f)^{(1/2)}/f))*b-1/2/(d*f)^{(1/2)}/((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*ln((2*(a*f+c*d-(d*f)^{(1/2)*b})/f+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+2*((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)))/(x+(d*f)^{(1/2)}/f))*a-1/2/(d*f)^{(1/2)}/f/((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*ln((2*(a*f+c*d-(d*f)^{(1/2)*b})/f+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+2*((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)))/(x+(d*f)^{(1/2)}/f))*c*d-1/2/(d*f)^{(1/2)}*((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}-1/2/f*ln(((x-(d*f)^{(1/2)}/f)*c+1/2*(b*f+2*(d*f)^{(1/2)*c})/f)/c^{(1/2)}+((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2))*c^{(1/2)}-1/4/(d*f)^{(1/2)}*ln(((x-(d*f)^{(1/2)}/f)*c+1/2*(b*f+2*(d*f)^{(1/2)*c})/f)/c^{(1/2)}+((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)))/c^{(1/2)}*b+1/2/f/((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*ln((2*(a*f+c*d+(d*f)^{(1/2)*b})/f+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+2*((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)))/(x-(d*f)^{(1/2)}/f))*b+1/2/(d*f)^{(1/2)}/((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*ln((2*(a*f+c*d+(d*f)^{(1/2)*b})/f+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+2*((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)))/(x-(d*f)^{(1/2)}/f))*a+1/2/(d*f)^{(1/2)}/f/((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*ln((2*(a*f+c*d+(d*f)^{(1/2)*b})/f+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+2*((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)))/(x-(d*f)^{(1/2)}/f))*c*d
\end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see 'assume?' for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2

$$-\frac{c \left((b \sqrt{4df}) \right)}{f^2} \text{ positive, negative or zero?} \quad + \frac{c \cdot d}{f+a}$$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{d - fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x + c*x^2)^(1/2)/(d - f*x^2), x)

[Out] int((a + b*x + c*x^2)^(1/2)/(d - f*x^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{a + bx + cx^2}}{-d + fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x+a)**(1/2)/(-f*x**2+d), x)

[Out] -Integral(sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)

$$3.81 \quad \int \frac{\sqrt{a+bx+cx^2}}{x(d-fx^2)} dx$$

Optimal. Leaf size=267

$$\frac{\sqrt{af+b(-\sqrt{d})}\sqrt{f}+cd \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f}+cd}\right)}{2d\sqrt{f}} + \frac{\sqrt{af+b\sqrt{d}}\sqrt{f}+cd \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}}\sqrt{f}+cd}\right)}{2d\sqrt{f}}$$

[Out] $-\operatorname{arctanh}\left(\frac{1/2*(b*x+2*a)/a^{(1/2)}/(c*x^2+b*x+a)^{(1/2)}}{a^{(1/2)}/d-1/2*\operatorname{arctanh}\left(\frac{1/2*(b*d^{(1/2)}-2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}-b*f^{(1/2)}))}{(c*x^2+b*x+a)^{(1/2)}/(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(1/2)}*(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(1/2)}/d/f^{(1/2)}\right)+1/2*\operatorname{arctanh}\left(\frac{1/2*(b*d^{(1/2)}+2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}+b*f^{(1/2)}))}{(c*x^2+b*x+a)^{(1/2)}/(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(1/2)}*(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(1/2)}/d/f^{(1/2)}\right)}{d/f^{(1/2)}}\right)$

Rubi [A] time = 0.78, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {6725, 734, 843, 621, 206, 724, 1021, 1078, 1033}

$$\frac{\sqrt{af+b(-\sqrt{d})}\sqrt{f}+cd \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f}+cd}\right)}{2d\sqrt{f}} + \frac{\sqrt{af+b\sqrt{d}}\sqrt{f}+cd \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}}\sqrt{f}+cd}\right)}{2d\sqrt{f}}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[a + b*x + c*x^2]/(x*(d - f*x^2)),x]`

[Out] $-\left(\frac{\operatorname{ArcTanh}\left[\frac{2*a + b*x}{2*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[a + b*x + c*x^2]}\right]}{d}\right) - \left(\frac{\operatorname{ArcTanh}\left[\frac{b*\operatorname{Sqrt}[d] - 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] - b*\operatorname{Sqrt}[f])*x}{2*\operatorname{Sqrt}[c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]*\operatorname{Sqrt}[a + b*x + c*x^2]}\right]}{2*d*\operatorname{Sqrt}[f]}\right) + \left(\frac{\operatorname{ArcTanh}\left[\frac{b*\operatorname{Sqrt}[d] + 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] + b*\operatorname{Sqrt}[f])*x}{2*\operatorname{Sqrt}[c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]*\operatorname{Sqrt}[a + b*x + c*x^2]}\right]}{2*d*\operatorname{Sqrt}[f]}\right)$

Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 621

```
Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && (!RationalQ[m] || LtQ[m, 1]) && !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1021

```
Int[((g_) + (h_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1033

```
Int[((g_) + (h_)*(x_))/(((a_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
```

, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]

Rule 1078

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f, 0]

Rule 6725

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{a+bx+cx^2}}{x(d-fx^2)} dx &= \int \left(\frac{\sqrt{a+bx+cx^2}}{dx} - \frac{fx\sqrt{a+bx+cx^2}}{d(-d+fx^2)} \right) dx \\
 &= \frac{\int \frac{\sqrt{a+bx+cx^2}}{x} dx}{d} - \frac{f \int \frac{x\sqrt{a+bx+cx^2}}{-d+fx^2} dx}{d} \\
 &= -\frac{\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}} dx}{2d} + \frac{\int \frac{-\frac{bd}{2}-(cd+af)x-\frac{1}{2}bfx^2}{\sqrt{a+bx+cx^2}(-d+fx^2)} dx}{d} \\
 &= \frac{a \int \frac{1}{x\sqrt{a+bx+cx^2}} dx}{d} + \frac{\int \frac{-bdf+f(-cd-af)x}{\sqrt{a+bx+cx^2}(-d+fx^2)} dx}{df} \\
 &= -\frac{(2a) \text{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d} - \frac{(cd-b\sqrt{d}\sqrt{f}+af) \int \frac{1}{(\sqrt{d}\sqrt{f}+fx)\sqrt{a+bx+cx^2}} dx}{2d} \\
 &= -\frac{\sqrt{a} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d} + \frac{(cd-b\sqrt{d}\sqrt{f}+af) \text{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2} dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d} \\
 &= -\frac{\sqrt{a} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d} - \frac{\sqrt{cd-b\sqrt{d}\sqrt{f}+af} \tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+(2c\sqrt{d}-b\sqrt{f})x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d\sqrt{f}}
 \end{aligned}$$

Mathematica [A] time = 0.30, size = 255, normalized size = 0.96

$$\frac{\sqrt{af + b(-\sqrt{d})}\sqrt{f} + cd \tanh^{-1}\left(\frac{-2a\sqrt{f} + b\sqrt{d} - b\sqrt{f}x + 2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b(-\sqrt{d})}\sqrt{f} + cd}\right) - \sqrt{af + b\sqrt{d}}\sqrt{f} + cd \tanh^{-1}\left(\frac{2a\sqrt{f} + b\sqrt{d} + b\sqrt{f}x + 2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}}\sqrt{f} + cd}\right)}{2d\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*x + c*x^2]/(x*(d - f*x^2)), x]

[Out]
$$-1/2*(2*\text{Sqrt}[a]*\text{Sqrt}[f]*\text{ArcTanh}[(2*a + b*x)/(2*\text{Sqrt}[a]*\text{Sqrt}[a + x*(b + c*x)])] + \text{Sqrt}[c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{ArcTanh}[(b*\text{Sqrt}[d] - 2*a*\text{Sqrt}[f] + 2*c*\text{Sqrt}[d]*x - b*\text{Sqrt}[f]*x)/(2*\text{Sqrt}[c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{Sqrt}[a + x*(b + c*x)])] - \text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{ArcTanh}[(b*\text{Sqrt}[d] + 2*a*\text{Sqrt}[f] + 2*c*\text{Sqrt}[d]*x + b*\text{Sqrt}[f]*x)/(2*\text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{Sqrt}[a + x*(b + c*x)])])/(d*\text{Sqrt}[f])$$

fricas [B] time = 26.88, size = 1253, normalized size = 4.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(1/2)/x/(-f*x^2+d), x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/4*(d*\text{sqrt}((d^2*f*\text{sqrt}(b^2/(d^3*f)) + c*d + a*f)/(d^2*f))*\log((2*\text{sqrt}(c*x^2 + b*x + a)*d^2*f*\text{sqrt}(b^2/(d^3*f)))*\text{sqrt}((d^2*f*\text{sqrt}(b^2/(d^3*f)) + c*d + a*f)/(d^2*f)) + 2*b*c*x + b^2 + (b*d*f*x + 2*a*d*f)*\text{sqrt}(b^2/(d^3*f))))/x) \\ & - d*\text{sqrt}((d^2*f*\text{sqrt}(b^2/(d^3*f)) + c*d + a*f)/(d^2*f))*\log(-(2*\text{sqrt}(c*x^2 + b*x + a)*d^2*f*\text{sqrt}(b^2/(d^3*f)))*\text{sqrt}((d^2*f*\text{sqrt}(b^2/(d^3*f)) + c*d + a*f)/(d^2*f)) - 2*b*c*x - b^2 - (b*d*f*x + 2*a*d*f)*\text{sqrt}(b^2/(d^3*f))))/x) - d \\ & *\text{sqrt}(-(d^2*f*\text{sqrt}(b^2/(d^3*f)) - c*d - a*f)/(d^2*f))*\log((2*\text{sqrt}(c*x^2 + b*x + a)*d^2*f*\text{sqrt}(b^2/(d^3*f)))*\text{sqrt}(-(d^2*f*\text{sqrt}(b^2/(d^3*f)) - c*d - a*f)/(d^2*f)) + 2*b*c*x + b^2 - (b*d*f*x + 2*a*d*f)*\text{sqrt}(b^2/(d^3*f))))/x) + d*\text{sqrt} \\ & (-(d^2*f*\text{sqrt}(b^2/(d^3*f)) - c*d - a*f)/(d^2*f))*\log(-(2*\text{sqrt}(c*x^2 + b*x + a)*d^2*f*\text{sqrt}(b^2/(d^3*f)))*\text{sqrt}(-(d^2*f*\text{sqrt}(b^2/(d^3*f)) - c*d - a*f)/(d^2*f)) - 2*b*c*x - b^2 + (b*d*f*x + 2*a*d*f)*\text{sqrt}(b^2/(d^3*f))))/x) + 2*\text{sqrt} \\ & (a)*\log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*\text{sqrt}(c*x^2 + b*x + a)*(b*x + 2*a)*\text{sqrt}(a) + 8*a^2)/x^2))/d, 1/4*(d*\text{sqrt}((d^2*f*\text{sqrt}(b^2/(d^3*f)) + c*d + a*f)/(d^2*f))*\log((2*\text{sqrt}(c*x^2 + b*x + a)*d^2*f*\text{sqrt}(b^2/(d^3*f)))*\text{sqrt}((d^2*f*\text{sqrt}(b^2/(d^3*f)) + c*d + a*f)/(d^2*f)) + 2*b*c*x + b^2 + (b*d*f*x + 2*a*d*f)*\text{sqrt}(b^2/(d^3*f))))/x) - d*\text{sqrt}((d^2*f*\text{sqrt}(b^2/(d^3*f)) + c*d + a*f)/(d^2*f))*\log(-(2*\text{sqrt}(c*x^2 + b*x + a)*d^2*f*\text{sqrt}(b^2/(d^3*f)))*\text{sqrt}((d^2*f*\text{sqrt}(b^2/(d^3*f)) + c*d + a*f)/(d^2*f)) - 2*b*c*x - b^2 - (b*d*f*x + 2*a*d*f)*\text{sqrt}(b^2/(d^3*f))))/x) - d*\text{sqrt}(-(d^2*f*\text{sqrt}(b^2/(d^3*f)) - c*d - a*f)/(d^2*f))*\log((2*\text{sqrt}(c*x^2 + b*x + a)*d^2*f*\text{sqrt}(b^2/(d^3*f)))*\text{sqrt}(-(d^2*f*\text{sqrt}(b^2/(d^3*f)) - c*d - a*f)/(d^2*f)) + 2*b*c*x + b^2 - (b*d*f*x + 2*a*d*f)*\text{sqrt}(b^2/(d^3*f))))/x) + d*\text{sqrt} \\ & (-(d^2*f*\text{sqrt}(b^2/(d^3*f)) - c*d - a*f)/(d^2*f))*\log(-(2*\text{sqrt}(c*x^2 + b*x + a)*d^2*f*\text{sqrt}(b^2/(d^3*f)))*\text{sqrt}(-(d^2*f*\text{sqrt}(b^2/(d^3*f)) - c*d - a*f)/(d^2*f)) - 2*b*c*x - b^2 + (b*d*f*x + 2*a*d*f)*\text{sqrt}(b^2/(d^3*f))))/x) + 2*\text{sqrt} \\ & (a)*\log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*\text{sqrt}(c*x^2 + b*x + a)*(b*x + 2*a)*\text{sqrt}(a) + 8*a^2)/x^2))/d \end{aligned}$$

$$\begin{aligned} &^2*f)) * \log((2*\sqrt{c*x^2 + b*x + a}*d^2*f*\sqrt{b^2/(d^3*f)}) * \sqrt{-(d^2*f*\sqrt{b^2/(d^3*f)}) - c*d - a*f}/(d^2*f)) + 2*b*c*x + b^2 - (b*d*f*x + 2*a*d*f) * \sqrt{b^2/(d^3*f)})/x) + d*\sqrt{-(d^2*f*\sqrt{b^2/(d^3*f)}) - c*d - a*f}/(d^2*f)) * \log(-(2*\sqrt{c*x^2 + b*x + a}*d^2*f*\sqrt{b^2/(d^3*f)}) * \sqrt{-(d^2*f*\sqrt{b^2/(d^3*f)}) - c*d - a*f}/(d^2*f)) - 2*b*c*x - b^2 + (b*d*f*x + 2*a*d*f) * \sqrt{b^2/(d^3*f)})/x) + 4*\sqrt{-a}*\arctan(1/2*\sqrt{c*x^2 + b*x + a}*(b*x + 2*a)*\sqrt{-a}/(a*c*x^2 + a*b*x + a^2)))/d] \end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage₂

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(1/2)/x/(-f*x^2+d),x, algorithm="giac")

[Out] sage2

maple [B] time = 0.02, size = 1764, normalized size = 6.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x+a)^(1/2)/x/(-f*x^2+d),x)

[Out]
$$\begin{aligned} &-1/2/d*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)+1/2/d/f*\ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*(d*f)^(1/2)-1/4/d*\ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b-1/2/d/f/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*\ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*(d*f)^(1/2)*b+1/2/d/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*\ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*c-1/2/d*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)-1/2/d/f*\ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*(d \end{aligned}$$

$$\begin{aligned} & *f)^{(1/2)} - 1/4/d * \ln\left(\frac{(x - (d*f)^{(1/2)})/f * c + 1/2 * (b*f + 2*(d*f)^{(1/2)} * c)/f}{c^{(1/2)}}\right) \\ & + \left(\frac{(x - (d*f)^{(1/2)})/f}{f} * c + (b*f + 2*(d*f)^{(1/2)} * c) * \frac{(x - (d*f)^{(1/2)})/f}{f} + (a*f + c*d + (d*f)^{(1/2)} * b)/f\right)^{(1/2)} / c^{(1/2)} * b + 1/2/d/f / \left(\frac{(a*f + c*d + (d*f)^{(1/2)} * b)/f}{f}\right)^{(1/2)} \\ & * \ln\left(\frac{2 * (a*f + c*d + (d*f)^{(1/2)} * b)/f + (b*f + 2*(d*f)^{(1/2)} * c) * \frac{(x - (d*f)^{(1/2)})/f}{f}}{f + 2 * \left(\frac{(a*f + c*d + (d*f)^{(1/2)} * b)/f}{f}\right)^{(1/2)} * \left(\frac{(x - (d*f)^{(1/2)})/f}{f}\right)^2 * c + (b*f + 2*(d*f)^{(1/2)} * c) * \frac{(x - (d*f)^{(1/2)})/f}{f} + (a*f + c*d + (d*f)^{(1/2)} * b)/f}\right)^{(1/2)} \\ & / \left(\frac{(x - (d*f)^{(1/2)})/f}{f}\right) * (d*f)^{(1/2)} * b + 1/2/d / \left(\frac{(a*f + c*d + (d*f)^{(1/2)} * b)/f}{f}\right)^{(1/2)} * \ln\left(\frac{2 * (a*f + c*d + (d*f)^{(1/2)} * b)/f + (b*f + 2*(d*f)^{(1/2)} * c) * \frac{(x - (d*f)^{(1/2)})/f}{f}}{f + 2 * \left(\frac{(a*f + c*d + (d*f)^{(1/2)} * b)/f}{f}\right)^{(1/2)} * \left(\frac{(x - (d*f)^{(1/2)})/f}{f}\right)^2 * c + (b*f + 2*(d*f)^{(1/2)} * c) * \frac{(x - (d*f)^{(1/2)})/f}{f} + (a*f + c*d + (d*f)^{(1/2)} * b)/f}\right)^{(1/2)} \\ & / \left(\frac{(x - (d*f)^{(1/2)})/f}{f}\right) * a + 1/2/f / \left(\frac{(a*f + c*d + (d*f)^{(1/2)} * b)/f}{f}\right)^{(1/2)} * \ln\left(\frac{2 * (a*f + c*d + (d*f)^{(1/2)} * b)/f + (b*f + 2*(d*f)^{(1/2)} * c) * \frac{(x - (d*f)^{(1/2)})/f}{f}}{f + 2 * \left(\frac{(a*f + c*d + (d*f)^{(1/2)} * b)/f}{f}\right)^{(1/2)} * \left(\frac{(x - (d*f)^{(1/2)})/f}{f}\right)^2 * c + (b*f + 2*(d*f)^{(1/2)} * c) * \frac{(x - (d*f)^{(1/2)})/f}{f} + (a*f + c*d + (d*f)^{(1/2)} * b)/f}\right)^{(1/2)} \\ & / \left(\frac{(x - (d*f)^{(1/2)})/f}{f}\right) * c + 1/d * (c*x^2 + b*x + a)^{(1/2)} + 1/2/d * b * \ln\left(\frac{c*x + 1/2 * b}{c^{(1/2)} + (c*x^2 + b*x + a)^{(1/2)}}\right) / c^{(1/2)} - 1/d * a^{(1/2)} * \ln\left(\frac{2 * a + b*x + 2 * a^{(1/2)} * (c*x^2 + b*x + a)^{(1/2)}}{x}\right) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{\sqrt{cx^2 + bx + a}}{(fx^2 - d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(1/2)/x/(-f*x^2+d),x, algorithm="maxima")

[Out] -integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 - d)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{x(d - fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x + c*x^2)^(1/2)/(x*(d - f*x^2)),x)

[Out] int((a + b*x + c*x^2)^(1/2)/(x*(d - f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{\sqrt{a + bx + cx^2}}{-dx + fx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/x/(-f*x**2+d),x)
```

```
[Out] -Integral(sqrt(a + b*x + c*x**2)/(-d*x + f*x**3), x)
```

$$3.82 \quad \int \frac{\sqrt{a+bx+cx^2}}{x^2(d-fx^2)} dx$$

Optimal. Leaf size=286

$$\frac{\sqrt{af+b(-\sqrt{d})}\sqrt{f}+cd \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f}+cd}\right)}{2d^{3/2}} + \frac{\sqrt{af+b\sqrt{d}}\sqrt{f}+cd \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}}\sqrt{f}+cd}\right)}{2d^{3/2}}$$

[Out] $-1/2*b*\operatorname{arctanh}(1/2*(b*x+2*a)/a^{(1/2)/(c*x^2+b*x+a)^{(1/2)})/d/a^{(1/2)}-(c*x^2+b*x+a)^{(1/2)/d/x+1/2*\operatorname{arctanh}(1/2*(b*d^{(1/2)}-2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}-b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)/(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(1/2)}*(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(1/2)}/d^{(3/2)}+1/2*\operatorname{arctanh}(1/2*(b*d^{(1/2)}+2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}+b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)/(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(1/2)}*(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(1/2)}/d^{(3/2)}$

Rubi [A] time = 0.71, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6725, 732, 843, 621, 206, 724, 990, 1033}

$$\frac{\sqrt{af+b(-\sqrt{d})}\sqrt{f}+cd \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f}+cd}\right)}{2d^{3/2}} + \frac{\sqrt{af+b\sqrt{d}}\sqrt{f}+cd \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}}\sqrt{f}+cd}\right)}{2d^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sqrt}[a + b*x + c*x^2]/(x^2*(d - f*x^2)), x]$

[Out] $-(\operatorname{Sqrt}[a + b*x + c*x^2]/(d*x)) - (b*\operatorname{ArcTanh}[(2*a + b*x)/(2*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*\operatorname{Sqrt}[a]*d) + (\operatorname{Sqrt}[c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] - 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] - b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*d^{(3/2)}) + (\operatorname{Sqrt}[c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] + 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] + b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*d^{(3/2)})$

Rule 206

$\operatorname{Int}[(a_0 + (b_0*x)^2)^{-1}, x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 621

```
Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Dist[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1]) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 990

```
Int[Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]/((d_) + (f_)*(x_)^2), x_Symbol] := Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*f - b*f*x)/(Sqrt[a + b*x + c*x^2]*(d + f*x^2)), x], x] /; FreeQ[{a, b, c, d, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1033

```
Int[((g_) + (h_)*(x_))/((a_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2], x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 6725

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{a+bx+cx^2}}{x^2(d-fx^2)} dx &= \int \left(\frac{\sqrt{a+bx+cx^2}}{x^2} + \frac{f\sqrt{a+bx+cx^2}}{d(d-fx^2)} \right) dx \\
 &= \frac{\int \frac{\sqrt{a+bx+cx^2}}{x^2} dx}{d} + \frac{f \int \frac{\sqrt{a+bx+cx^2}}{d-fx^2} dx}{d} \\
 &= -\frac{\sqrt{a+bx+cx^2}}{dx} + \frac{\int \frac{b+2cx}{x\sqrt{a+bx+cx^2}} dx}{2d} + \frac{\int \frac{cd+af+bf x}{\sqrt{a+bx+cx^2}(d-fx^2)} dx}{d} - \frac{c \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{d} \\
 &= -\frac{\sqrt{a+bx+cx^2}}{dx} + \frac{b \int \frac{1}{x\sqrt{a+bx+cx^2}} dx}{2d} + \frac{c \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{d} - \frac{(2c) \text{Subst} \left(\int \frac{1}{4c-x^2} dx, x, \frac{\sqrt{a+bx+cx^2}}{c} \right)}{d} \\
 &= -\frac{\sqrt{a+bx+cx^2}}{dx} - \frac{\sqrt{c} \tanh^{-1} \left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}} \right)}{d} - \frac{b \text{Subst} \left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}} \right)}{d} + \dots \\
 &= -\frac{\sqrt{a+bx+cx^2}}{dx} - \frac{b \tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}} \right)}{2\sqrt{a}d} + \frac{\sqrt{cd-b\sqrt{d}}\sqrt{f} + af \tanh^{-1} \left(\frac{b\sqrt{d}-2a\sqrt{f}}{2\sqrt{cd-b\sqrt{d}}\sqrt{a+bx+cx^2}} \right)}{2d^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.43, size = 275, normalized size = 0.96

$$\frac{\sqrt{af+b\sqrt{d}}\sqrt{f}+cd \tanh^{-1} \left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}}\sqrt{f}+cd} \right) + \sqrt{af+b(-\sqrt{d})}\sqrt{f}+cd \tanh^{-1} \left(\frac{-2a\sqrt{f}+b(\sqrt{d}-\sqrt{f})}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}}\sqrt{f}+cd} \right)}{2d^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*x + c*x^2]/(x^2*(d - f*x^2)),x]

[Out] ((-2*Sqrt[d]*Sqrt[a + x*(b + c*x)])/x - (b*Sqrt[d]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])))/Sqrt[a] + Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]

$d*\text{Sqrt}[f + a*f]*\text{ArcTanh}[(-2*a*\text{Sqrt}[f] + 2*c*\text{Sqrt}[d]*x + b*(\text{Sqrt}[d] - \text{Sqrt}[f]*x))/(2*\text{Sqrt}[c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{Sqrt}[a + x*(b + c*x)])]/(2*d^{(3/2)})$

fricas [B] time = 38.76, size = 1094, normalized size = 3.83

$$\left[\text{adx} \sqrt{\frac{d^3 \sqrt{\frac{b^2 f}{d^5} + cd + af}}{d^3}} \log \left(\frac{2bcx + 2\sqrt{cx^2 + bx + a}bd \sqrt{\frac{d^3 \sqrt{\frac{b^2 f}{d^5} + cd + af}}{d^3}} + b^2 + (bd^2 x + 2ad^2) \sqrt{\frac{b^2 f}{d^5}}}{x} \right) - \text{adx} \sqrt{\frac{d^3 \sqrt{\frac{b^2 f}{d^5} + cd + af}}{d^3}} \log \left(\frac{2bcx - 2\sqrt{cx^2 + bx + a}bd \sqrt{\frac{d^3 \sqrt{\frac{b^2 f}{d^5} + cd + af}}{d^3}} + b^2 + (bd^2 x + 2ad^2) \sqrt{\frac{b^2 f}{d^5}}}{x} \right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(1/2)/x^2/(-f*x^2+d),x, algorithm="fricas")

[Out] [1/4*(a*d*x*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3)*log((2*b*c*x + 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3) + b^2 + (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x) - a*d*x*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3)*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3) + b^2 + (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x) + a*d*x*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3)*log((2*b*c*x + 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3) + b^2 - (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x) - a*d*x*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3)*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3) + b^2 - (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x) + sqrt(a)*b*x*log(-(8*a*b*x + (b^2 + 4*a*c))*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*sqrt(c*x^2 + b*x + a)*a/(a*d*x), 1/4*(a*d*x*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3)*log((2*b*c*x + 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3) + b^2 + (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x) - a*d*x*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3)*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3) + b^2 + (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x) + a*d*x*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3)*log((2*b*c*x + 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3) + b^2 - (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x) - a*d*x*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3)*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3) + b^2 - (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x) + 2*sqrt(-a)*b*x*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - 4*sqrt(c*x^2 + b*x + a)*a/(a*d*x)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(1/2)/x^2/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Evaluation time: 0.87sym2poly/r2sym(const ge
n & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [B] time = 0.02, size = 1819, normalized size = 6.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x+a)^(1/2)/x^2/(-f*x^2+d),x)

[Out] $\frac{1}{2} \frac{f}{d} \frac{1}{(d*f)^{1/2}} * ((x + (d*f)^{1/2} / f)^2 * c + (b*f - 2*(d*f)^{1/2} * c) * (x + (d*f)^{1/2} / f) / f + (a*f + c*d - (d*f)^{1/2} * b) / f)^{1/2} - \frac{1}{2} \frac{d * \ln(((x + (d*f)^{1/2} / f) * c + 1/2 * (b*f - 2*(d*f)^{1/2} * c) / f) / c^{1/2} + ((x + (d*f)^{1/2} / f)^2 * c + (b*f - 2*(d*f)^{1/2} * c) * (x + (d*f)^{1/2} / f) / f + (a*f + c*d - (d*f)^{1/2} * b) / f)^{1/2}) * c^{1/2} + 1/4 * f / d / (d*f)^{1/2} * \ln(((x + (d*f)^{1/2} / f) * c + 1/2 * (b*f - 2*(d*f)^{1/2} * c) / f) / c^{1/2} + ((x + (d*f)^{1/2} / f)^2 * c + (b*f - 2*(d*f)^{1/2} * c) * (x + (d*f)^{1/2} / f) / f + (a*f + c*d - (d*f)^{1/2} * b) / f)^{1/2}) / c^{1/2} * b + 1/2 * d / ((a*f + c*d - (d*f)^{1/2} * b) / f)^{1/2} * \ln((2 * (a*f + c*d - (d*f)^{1/2} * b) / f + (b*f - 2*(d*f)^{1/2} * c) * (x + (d*f)^{1/2} / f) / f + 2 * ((a*f + c*d - (d*f)^{1/2} * b) / f)^{1/2}) * ((x + (d*f)^{1/2} / f)^2 * c + (b*f - 2*(d*f)^{1/2} * c) * (x + (d*f)^{1/2} / f) / f + (a*f + c*d - (d*f)^{1/2} * b) / f)^{1/2}) / (x + (d*f)^{1/2} / f) * a - 1/2 / (d*f)^{1/2} / ((a*f + c*d - (d*f)^{1/2} * b) / f)^{1/2} * \ln((2 * (a*f + c*d - (d*f)^{1/2} * b) / f + (b*f - 2*(d*f)^{1/2} * c) * (x + (d*f)^{1/2} / f) / f + 2 * ((a*f + c*d - (d*f)^{1/2} * b) / f)^{1/2}) * ((x + (d*f)^{1/2} / f)^2 * c + (b*f - 2*(d*f)^{1/2} * c) * (x + (d*f)^{1/2} / f) / f + (a*f + c*d - (d*f)^{1/2} * b) / f)^{1/2}) / (x + (d*f)^{1/2} / f) * c - 1/d/a/x * (c*x^2 + b*x + a)^{3/2} + 1/d*b/a * (c*x^2 + b*x + a)^{1/2} - 1/2 * d*b/a^{1/2} * \ln((b*x + 2*a + 2 * (c*x^2 + b*x + a)^{1/2}) * a^{1/2} / x) + 1/d*c/a * (c*x^2 + b*x + a)^{1/2} * x + 1/d*c^{1/2} * \ln((c*x + 1/2*b) / c^{1/2} + (c*x^2 + b*x + a)^{1/2}) - 1/2 * f / d / (d*f)^{1/2} * ((x - (d*f)^{1/2} / f)^2 * c + (b*f + 2*(d*f)^{1/2} * c) * (x - (d*f)^{1/2} / f) / f + (a*f + c*d + (d*f)^{1/2} * b) / f)^{1/2} - 1/2 * d * \ln(((x - (d*f)^{1/2} / f) * c + 1/2 * (b*f + 2*(d*f)^{1/2} * c) / f) / c^{1/2} + ((x - (d*f)^{1/2} / f)^2 * c + (b*f + 2*(d*f)^{1/2} * c) * (x - (d*f)^{1/2} / f) / f + (a*f + c*d + (d*f)^{1/2} * b) / f)^{1/2}) * c^{1/2} - 1/4 * f / d / (d*f)^{1/2} * \ln(((x - (d*f)^{1/2} / f) * c + 1/2 * (b*f + 2*(d*f)^{1/2} * c) / f) / c^{1/2} + ((x - (d*f)^{1/2} / f)^2 * c + (b*f + 2*(d*f)^{1/2} * c) * (x - (d*f)^{1/2} / f) / f + (a*f + c*d + (d*f)^{1/2} * b) / f)^{1/2}) * c^{1/2} + ((x - (d*f)^{1/2} / f)^2 * c + (b*f + 2*(d*f)^{1/2} * c) * (x - (d*f)^{1/2} / f) / f + (a*f + c*d + (d*f)^{1/2} * b) / f)^{1/2}$

$$\frac{2)/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2))/c^{(1/2)*b+1/2}/d/((a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*\ln((2*(a*f+c*d+(d*f)^{(1/2)*b)/f+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f)/f+2*((a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*((x-(d*f)^{(1/2)/f)^2*c+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)))/(x-(d*f)^{(1/2)/f)))*b+1/2*f/d/(d*f)^{(1/2)/((a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*\ln((2*(a*f+c*d+(d*f)^{(1/2)*b)/f+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f)/f+2*((a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*((x-(d*f)^{(1/2)/f)^2*c+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)))/(x-(d*f)^{(1/2)/f)))*a+1/2/(d*f)^{(1/2)/((a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*\ln((2*(a*f+c*d+(d*f)^{(1/2)*b)/f+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f)/f+2*((a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*((x-(d*f)^{(1/2)/f)^2*c+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)))/(x-(d*f)^{(1/2)/f)))*c$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{cx^2 + bx + a}}{(fx^2 - d)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(1/2)/x^2/(-f*x^2+d),x, algorithm="maxima")

[Out] -integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 - d)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{x^2 (d - fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x + c*x^2)^(1/2)/(x^2*(d - f*x^2)),x)

[Out] int((a + b*x + c*x^2)^(1/2)/(x^2*(d - f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{a + bx + cx^2}}{-dx^2 + fx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x+a)**(1/2)/x**2/(-f*x**2+d),x)

[Out] -Integral(sqrt(a + b*x + c*x**2)/(-d*x**2 + f*x**4), x)

$$3.83 \quad \int \frac{\sqrt{a+bx+cx^2}}{x^3(d-fx^2)} dx$$

Optimal. Leaf size=353

$$\frac{(b^2 - 4ac) \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d} - \frac{\sqrt{a} f \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2} - \frac{\sqrt{f}\sqrt{af+b(-\sqrt{d})}\sqrt{f} + cd \tanh^{-1}\left(\frac{-2a\sqrt{f}}{2\sqrt{a+bx}}\right)}{2d^2}$$

[Out] 1/8*(-4*a*c+b^2)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(3/2)/d-f*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))*a^(1/2)/d^2-1/4*(b*x+2*a)*(c*x^2+b*x+a)^(1/2)/a/d/x^2-1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*f^(1/2)*(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2)/d^2+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*f^(1/2)*(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2)/d^2

Rubi [A] time = 0.88, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6725, 720, 724, 206, 734, 843, 621, 1021, 1078, 1033}

$$\frac{(b^2 - 4ac) \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d} - \frac{\sqrt{a} f \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2} - \frac{\sqrt{f}\sqrt{af+b(-\sqrt{d})}\sqrt{f} + cd \tanh^{-1}\left(\frac{-2a\sqrt{f}}{2\sqrt{a+bx}}\right)}{2d^2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*x + c*x^2]/(x^3*(d - f*x^2)), x]

[Out] -((2*a + b*x)*Sqrt[a + b*x + c*x^2])/(4*a*d*x^2) + ((b^2 - 4*a*c)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*a^(3/2)*d) - (Sqrt[a]*f*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d^2 - (Sqrt[f]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2) + (Sqrt[f]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2)

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

```
Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 720

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && (!RationalQ[m] || LtQ[m, 1]) && !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1021

```
Int[((g_) + (h_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^q +
```

```

1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]

```

Rule 1033

```

Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]

```

Rule 1078

```

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f, 0]

```

Rule 6725

```

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx+cx^2}}{x^3(d-fx^2)} dx &= \int \left(\frac{\sqrt{a+bx+cx^2}}{dx^3} + \frac{f\sqrt{a+bx+cx^2}}{d^2x} + \frac{f^2x\sqrt{a+bx+cx^2}}{d^2(d-fx^2)} \right) dx \\
&= \frac{\int \frac{\sqrt{a+bx+cx^2}}{x^3} dx}{d} + \frac{f \int \frac{\sqrt{a+bx+cx^2}}{x} dx}{d^2} + \frac{f^2 \int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2} dx}{d^2} \\
&= -\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2} - \frac{(b^2-4ac) \int \frac{1}{x\sqrt{a+bx+cx^2}} dx}{8ad} - \frac{f \int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}} dx}{2d^2} + \frac{f \int \frac{\frac{bd}{2} + (ca)}{\sqrt{a+bx+cx^2}} dx}{d^2} \\
&= -\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2} - \frac{\int \frac{-bdf-f(cd+af)x}{\sqrt{a+bx+cx^2}(d-fx^2)} dx}{d^2} + \frac{(b^2-4ac) \text{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{2a}{\sqrt{a+bx+cx^2}}\right)}{4ad} \\
&= -\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2} + \frac{(b^2-4ac) \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d} - \frac{(2af) \text{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{2a}{\sqrt{a+bx+cx^2}}\right)}{d^2} \\
&= -\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2} + \frac{(b^2-4ac) \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d} - \frac{\sqrt{a} f \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2} \\
&= -\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2} + \frac{(b^2-4ac) \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d} - \frac{\sqrt{a} f \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2}
\end{aligned}$$

Mathematica [A] time = 0.52, size = 316, normalized size = 0.90

$$x^2 (b^2 d - 4a(2af + cd)) \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right) - 2\sqrt{a} \left(-2a\sqrt{f} x^2 \sqrt{af + b\sqrt{d}\sqrt{f} + cd} \tanh^{-1}\left(\frac{2a\sqrt{f} + b\sqrt{d} + b\sqrt{f}}{2\sqrt{a+x(b+cx)}\sqrt{af + b\sqrt{d}\sqrt{f} + cd}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*x + c*x^2]/(x^3*(d - f*x^2)), x]

[Out] ((b^2*d - 4*a*(c*d + 2*a*f))*x^2*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])] - 2*Sqrt[a]*(d*(2*a + b*x)*Sqrt[a + x*(b + c*x)] - 2*a*Sqrt[f]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])*x^2*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + 2*a*Sqrt[f]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f])*x^2*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(8*a^(3/2)*d^2*x^2)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^3/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^3/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Evaluation time: 1.36sym2poly/r2sym(const ge
n & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.02, size = 1953, normalized size = 5.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/x^3/(-f*x^2+d),x)
```

```
[Out] -1/2*f/d^2*((x+(d*f)^(1/2)/f)^(1/2)*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f
+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)+1/2/d^2*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-
2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^(1/2)*c+(b*f-2*(d*f)^(1/2)*c)*(x
+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*(d*f)^(1/2)-1/4
*f/d^2*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*
f)^(1/2)/f)^(1/2)*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1
/2)*b)/f)^(1/2))/c^(1/2)*b-1/2/d^2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*
(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f
+c*d-(d*f)^(1/2)*b)/f)^(1/2))*((x+(d*f)^(1/2)/f)^(1/2)*c+(b*f-2*(d*f)^(1/2)*c)*(
x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*(d*
f)^(1/2)*b+1/2*f/d^2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)
^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/
2)*b)/f)^(1/2))*((x+(d*f)^(1/2)/f)^(1/2)*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)
/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*a+1/2/d/((a*f+c*d
-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)
*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*((x+(d*f)^(1/2)
/f)^(1/2)*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f
```

$$\begin{aligned} &)^{(1/2)})/(x+(d*f)^{(1/2)/f))*c-1/2*f/d^2*((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f) \\ &^{(1/2)*c)*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)-1/2/d^2*\ln((\\ &(x-(d*f)^{(1/2)/f})*c+1/2*(b*f+2*(d*f)^{(1/2)*c)/f)/c^{(1/2)}+((x-(d*f)^{(1/2)/f}) \\ &^2*c+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(\\ &1/2))*c^{(1/2)*(d*f)^{(1/2)-1/4*f/d^2*\ln(((x-(d*f)^{(1/2)/f})*c+1/2*(b*f+2*(d*f) \\ &)^{(1/2)*c)/f)/c^{(1/2)}+((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f) \\ &^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)})/c^{(1/2)*b+1/2/d^2/((a*f+c*d+(\\ &d*f)^{(1/2)*b)/f)^{(1/2)*\ln((2*(a*f+c*d+(d*f)^{(1/2)*b)/f+(b*f+2*(d*f)^{(1/2)*c} \\ &)*(x-(d*f)^{(1/2)/f})/f+2*((a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2))*((x-(d*f)^{(1/2)/f} \\ &)^2*c+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(\\ &1/2)})/(x-(d*f)^{(1/2)/f))*c-1/2*f/d^2/((a*f+c*d+(d*f)^{(1/2)*b)/ \\ &f)^{(1/2)*\ln((2*(a*f+c*d+(d*f)^{(1/2)*b)/f+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/ \\ &2)/f)/f+2*((a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2))*((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(\\ &d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)})/(x-(d*f) \\ &)^{(1/2)/f))*a+1/2/d/((a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*\ln((2*(a*f+c*d+(d*f)^{(\\ &1/2)*b)/f+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f})/f+2*((a*f+c*d+(d*f)^{(1/2) \\ &)*b)/f)^{(1/2))*((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f} \\ &)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)})/(x-(d*f)^{(1/2)/f))*c-1/2/d/a/x^2*(c*x \\ &^2+b*x+a)^{(3/2)+1/4/d*b/a^2/x*(c*x^2+b*x+a)^{(3/2)-1/4/d*b^2/a^2*(c*x^2+b*x+ \\ &a)^{(1/2)+1/8/d*b^2/a^2*(3/2)*\ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)*a^{(1/2)})/x)-1/ \\ &4/d*b/a^2*c*(c*x^2+b*x+a)^{(1/2)*x+1/2/d*c/a*(c*x^2+b*x+a)^{(1/2)-1/2/d*c/a^{(\\ &1/2)*\ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)*a^{(1/2)})/x)+f/d^2*(c*x^2+b*x+a)^{(1/2} \\ &)+1/2*f/d^2*b*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})/c^{(1/2)-f/d^2*a^{(\\ &1/2)*\ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)*a^{(1/2)})/x} \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{cx^2 + bx + a}}{(fx^2 - d)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(1/2)/x^3/(-f*x^2+d),x, algorithm="maxima")

[Out] -integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 - d)*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{x^3(d - fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x + c*x^2)^(1/2)/(x^3*(d - f*x^2)),x)

[Out] int((a + b*x + c*x^2)^(1/2)/(x^3*(d - f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{a + bx + cx^2}}{-dx^3 + fx^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x+a)**(1/2)/x**3/(-f*x**2+d), x)

[Out] -Integral(sqrt(a + b*x + c*x**2)/(-d*x**3 + f*x**5), x)

$$3.84 \quad \int \frac{x^3(a+bx+cx^2)^{3/2}}{d-fx^2} dx$$

Optimal. Leaf size=501

$$\frac{3b(b^2 - 4ac)^2 \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{7/2}f} - \frac{3b(b^2 - 4ac)(b + 2cx)\sqrt{a + bx + cx^2}}{128c^3f} - \frac{d\sqrt{a + bx + cx^2} (8acf + b^2f + 2bd)}{8cf^3}$$

[Out] $-1/3*d*(c*x^2+b*x+a)^{(3/2)}/f^2+1/16*b*(2*c*x+b)*(c*x^2+b*x+a)^{(3/2)}/c^2/f-1/5*(c*x^2+b*x+a)^{(5/2)}/c/f+3/256*b*(-4*a*c+b^2)^2*\operatorname{arctanh}(1/2*(2*c*x+b)/c^{(1/2)})/(c*x^2+b*x+a)^{(1/2)}/c^{(7/2)}/f-1/16*b*d*(12*a*c*f-b^2*f+24*c^2*d)*\operatorname{arctanh}(1/2*(2*c*x+b)/c^{(1/2)})/(c*x^2+b*x+a)^{(1/2)}/c^{(3/2)}/f^3-1/2*d*\operatorname{arctanh}(1/2*(b*d^{(1/2)}-2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}-b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)}/(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(1/2)}*(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(3/2)}/f^{(7/2)}+1/2*d*\operatorname{arctanh}(1/2*(b*d^{(1/2)}+2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}+b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)}/(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(1/2)}*(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(3/2)}/f^{(7/2)}-3/128*b*(-4*a*c+b^2)*(2*c*x+b)*(c*x^2+b*x+a)^{(1/2)}/c^3/f-1/8*d*(2*b*c*f*x+8*a*c*f+b^2*f+8*c^2*d)*(c*x^2+b*x+a)^{(1/2)}/c/f^3$

Rubi [A] time = 1.41, antiderivative size = 501, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6725, 640, 612, 621, 206, 1021, 1070, 1078, 1033, 724}

$$\frac{d\sqrt{a + bx + cx^2} (8acf + b^2f + 2bcfx + 8c^2d)}{8cf^3} - \frac{bd(12acf + b^2(-f) + 24c^2d) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}f^3} - \frac{3b(b^2 - 4ac)^2}{8cf^3}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(a + b*x + c*x^2)^(3/2))/(d - f*x^2), x]

[Out] $(-3*b*(b^2 - 4*a*c)*(b + 2*c*x)*\operatorname{Sqrt}[a + b*x + c*x^2])/(128*c^3*f) - (d*(8*c^2*d + b^2*f + 8*a*c*f + 2*b*c*f*x)*\operatorname{Sqrt}[a + b*x + c*x^2])/(8*c*f^3) - (d*(a + b*x + c*x^2)^{(3/2)})/(3*f^2) + (b*(b + 2*c*x)*(a + b*x + c*x^2)^{(3/2)})/(16*c^2*f) - (a + b*x + c*x^2)^{(5/2)}/(5*c*f) + (3*b*(b^2 - 4*a*c)^2*\operatorname{ArcTanh}[(b + 2*c*x)/(2*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(256*c^{(7/2)}*f) - (b*d*(24*c^2*d - b^2*f + 12*a*c*f)*\operatorname{ArcTanh}[(b + 2*c*x)/(2*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(16*c^{(3/2)}*f^3) - (d*(c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)^{(3/2)}*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] - 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] - b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*f^{(7/2)}) + (d*(c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)^{(3/2)}*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] + 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] + b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*f^{(7/2)})$

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 640

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 724

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1021

Int[((g_) + (h_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-b*f)) + c*(-2*g*f)*(p + q + 1))*x^2, x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1070

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((B*c*f*(2*p + 2*q + 3)
+ C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1
))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*
(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*
(C*(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*
p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(-(
b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(-(b*c*(C*(-4*d*f
)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))))*x + (p*(-(b*f))*(C*
(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2
- 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3
)))))*x^2, x], x] /; FreeQ[{a, b, c, d, f, A, B, C, q}, x] && NeQ[b^2 -
4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] && !I
GtQ[p, 0] && !IGtQ[q, 0]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
+ f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3 (a + bx + cx^2)^{3/2}}{d - fx^2} dx &= \int \left(-\frac{x (a + bx + cx^2)^{3/2}}{f} + \frac{dx (a + bx + cx^2)^{3/2}}{f(d - fx^2)} \right) dx \\
&= -\frac{\int x (a + bx + cx^2)^{3/2} dx}{f} + \frac{d \int \frac{x(a+bx+cx^2)^{3/2}}{d-fx^2} dx}{f} \\
&= -\frac{d(a + bx + cx^2)^{3/2}}{3f^2} - \frac{(a + bx + cx^2)^{5/2}}{5cf} + \frac{d \int \frac{\sqrt{a+bx+cx^2} \left(\frac{3bd}{2} + 3(cd+af)x + \frac{3}{2}bf x^2 \right)}{d-fx^2} dx}{3f^2} + \\
&= -\frac{d(8c^2d + b^2f + 8acf + 2bcfx) \sqrt{a + bx + cx^2}}{8cf^3} - \frac{d(a + bx + cx^2)^{3/2}}{3f^2} + \frac{b(b + 2cx)}{3f^2} \\
&= -\frac{3b(b^2 - 4ac)(b + 2cx) \sqrt{a + bx + cx^2}}{128c^3f} - \frac{d(8c^2d + b^2f + 8acf + 2bcfx) \sqrt{a + bx}}{8cf^3} \\
&= -\frac{3b(b^2 - 4ac)(b + 2cx) \sqrt{a + bx + cx^2}}{128c^3f} - \frac{d(8c^2d + b^2f + 8acf + 2bcfx) \sqrt{a + bx}}{8cf^3} \\
&= -\frac{3b(b^2 - 4ac)(b + 2cx) \sqrt{a + bx + cx^2}}{128c^3f} - \frac{d(8c^2d + b^2f + 8acf + 2bcfx) \sqrt{a + bx}}{8cf^3} \\
&= -\frac{3b(b^2 - 4ac)(b + 2cx) \sqrt{a + bx + cx^2}}{128c^3f} - \frac{d(8c^2d + b^2f + 8acf + 2bcfx) \sqrt{a + bx}}{8cf^3}
\end{aligned}$$

Mathematica [A] time = 1.49, size = 447, normalized size = 0.89

$$\frac{\sqrt{f} \sqrt{a+x(b+cx)} (24c^2f(16a^2f+7abfx+b^2(10d+fx^2))-30b^2cf^2(10a+bx)+16c^3f(160ad+48afx^2+70bdx+33bf x^3)+45b^4f^2+128c^4(15d^2+5dfx^2+3d^2+3a^2f))}{c^3} \operatorname{ArcTanh}\left[\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right]$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(a + b*x + c*x^2)^(3/2))/(d - f*x^2), x]

[Out] (b*(-384*c^4*d^2 - 192*a*c^3*d*f + 3*b^4*f^2 - 24*a*b^2*c*f^2 + 16*c^2*f*(b^2*d + 3*a^2*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]/(2

$$56*c^{(7/2)}*f^3) + (-((\text{Sqrt}[f]*\text{Sqrt}[a + x*(b + c*x)]*(45*b^4*f^2 - 30*b^2*c*f^2*(10*a + b*x) + 16*c^3*f*(160*a*d + 70*b*d*x + 48*a*f*x^2 + 33*b*f*x^3) + 128*c^4*(15*d^2 + 5*d*f*x^2 + 3*f^2*x^4) + 24*c^2*f*(16*a^2*f + 7*a*b*f*x + b^2*(10*d + f*x^2))))/c^3) + 960*d*(c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f)^{(3/2)}*\text{ArcTanh}[(-b*\text{Sqrt}[d]) + 2*a*\text{Sqrt}[f] - 2*c*\text{Sqrt}[d]*x + b*\text{Sqrt}[f]*x)/(2*\text{Sqrt}[c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{Sqrt}[a + x*(b + c*x)])] - 960*d*(c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f)^{(3/2)}*\text{ArcTanh}[(-2*(a*\text{Sqrt}[f] + c*\text{Sqrt}[d]*x) - b*(\text{Sqrt}[d] + \text{Sqrt}[f]*x))/(2*\text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{Sqrt}[a + x*(b + c*x)])])]/(1920*f^{(7/2)})$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);;OUTPUT:Error: Bad Argument Type

maple [B] time = 0.03, size = 4884, normalized size = 9.75

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)

[Out]
$$-1/5*(c*x^2+b*x+a)^{(5/2)}/c/f+1/16/f*b^2/c^2*(c*x^2+b*x+a)^{(3/2)}-3/128/f*b^4/c^3*(c*x^2+b*x+a)^{(1/2)}+3/256/f*b^5/c^{(7/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})-1/2/f^2*d*((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)}*c)*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)}*b)/f)^{(1/2)}*a-1/2/f^3*d^2*((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)}*c)*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)}*b)/f)^{(1/2)}*c-1/2/f^2*d*((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)}*c)*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)}*b)/f)^{(1/2)}*a-1/2/f^3*d^2*((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)}*c)*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)}*b)/f)^{(1/2)}*c$$

$$\begin{aligned}
& (1/2)*c+1/2/f^4*d^3/((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*\ln((2*(a*f+c*d+(d*f)^{(1/2)*b})/f+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f})/f+2*((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)})/(x-(d*f)^{(1/2)/f})*c^2-1/8/f^2*d*((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*x*b+5/8/f^3*d*((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*b*(d*f)^{(1/2)}-1/16/f^2*d/c*((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*b^2+1/32/f^2*d/c^(3/2)*\ln(((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})/f)/c^(1/2)+((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*b^3-3/4/f^3*d^2*\ln(((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})/f)/c^(1/2)+((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*c^(1/2)*b+1/2/f^4*d^2*\ln(((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})/f)/c^(1/2)+((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*c^(3/2)*(d*f)^{(1/2)}+1/2/f^3*d^2/((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*\ln((2*(a*f+c*d-(d*f)^{(1/2)*b})/f+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+2*((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)})/(x+(d*f)^{(1/2)/f})*b^2+1/2/f^2*d/((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*\ln((2*(a*f+c*d-(d*f)^{(1/2)*b})/f+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+2*((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)})/(x+(d*f)^{(1/2)/f})*a^2+1/2/f^4*d^3/((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*\ln((2*(a*f+c*d-(d*f)^{(1/2)*b})/f+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+2*((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)})/(x+(d*f)^{(1/2)/f})*c^2+1/8/f*b/c*x*(c*x^2+b*x+a)^(3/2)-3/64/f*b^3/c^2*(c*x^2+b*x+a)^(1/2)*x+3/32/f*b^2/c^2*(c*x^2+b*x+a)^(1/2)*a+3/16/f*b/c^(3/2)*\ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2-3/32/f*b^3/c^(5/2)*\ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/4/f^3*d*((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*x*c*(d*f)^{(1/2)}-3/4/f^3*d*\ln(((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*c+1/2*(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*c^(1/2)*(d*f)^{(1/2)}*a-3/8/f^2*d/c^(1/2)*\ln(((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*c+1/2*(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*c^(1/2)+((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*a*b-3/16/f^3*d*\ln(((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*c+1/2*(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*c^(1/2)+((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*c^(1/2)*b^2*(d*f)^{(1/2)}+1/f^3*d^2/((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*\ln((2*(a*f+c*d+(d*f)^{(1/2)*b})/f+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f})/f+2*((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)})/(x-(d*f)^{(1/2)/f})*a*c+3/16/f^3*d*\ln(((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})/f)/c^(1/2)+((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*c^(1/2)
\end{aligned}$$


```
) / f + (a * f + c * d - (d * f)^(1/2) * b) / f)^(1/2)) / (x + (d * f)^(1/2) / f)) * b * (d * f)^(1/2) * c - 1 /
8 / f^2 * d * ((x - (d * f)^(1/2) / f)^2 * c + (b * f + 2 * (d * f)^(1/2) * c) * (x - (d * f)^(1/2) / f) / f + (a
* f + c * d + (d * f)^(1/2) * b) / f)^(1/2) * x * b - 5 / 8 / f^3 * d * ((x - (d * f)^(1/2) / f)^2 * c + (b * f + 2 *
(d * f)^(1/2) * c) * (x - (d * f)^(1/2) / f) / f + (a * f + c * d + (d * f)^(1/2) * b) / f)^(1/2) * b * (d * f)
^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2
-(c*((b*sqrt(4*d*f)) / (2*f) + (c*d)/f+a))
/f^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 (c x^2 + b x + a)^{3/2}}{d - f x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x)
```

```
[Out] int((x^3*(a + b*x + c*x^2)^(3/2))/(d - f*x^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

$$3.85 \quad \int \frac{x^2(a+bx+cx^2)^{3/2}}{d-fx^2} dx$$

Optimal. Leaf size=417

$$\frac{(48c^2f(a^2f+b^2d) - 24ab^2cf^2 + 192ac^3df + 3b^4f^2 + 128c^4d^2) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) \sqrt{a+bx+cx^2} (2cx(1) - \dots)}{128c^{5/2}f^3}$$

[Out] $-1/8*(2*c*x+b)*(c*x^2+b*x+a)^{(3/2)}/c/f-1/128*(128*c^4*d^2+192*a*c^3*d*f+3*b^4*f^2-24*a*b^2*c*f^2+48*c^2*f*(a^2*f+b^2*d))*\arctanh(1/2*(2*c*x+b)/c^{(1/2)})/(c*x^2+b*x+a)^{(1/2)}/c^{(5/2)}/f^3+1/2*\arctanh(1/2*(b*d^{(1/2)}-2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}-b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)}/(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(1/2)})*d^{(1/2)}*(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(3/2)}/f^3+1/2*\arctanh(1/2*(b*d^{(1/2)}+2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}+b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)}/(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(1/2)})*d^{(1/2)}*(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(3/2)}/f^3-1/64*(b*(12*a*c*f-3*b^2*f+80*c^2*d)+2*c*(12*a*c*f-3*b^2*f+16*c^2*d)*x)*(c*x^2+b*x+a)^{(1/2)}/c^2/f^2$

Rubi [A] time = 1.02, antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1071, 1070, 1078, 621, 206, 1033, 724}

$$\frac{(48c^2f(a^2f+b^2d) - 24ab^2cf^2 + 192ac^3df + 3b^4f^2 + 128c^4d^2) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) \sqrt{a+bx+cx^2} (2cx(1) - \dots)}{128c^{5/2}f^3}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(a + b*x + c*x^2)^(3/2))/(d - f*x^2), x]

[Out] $-((b*(80*c^2*d - 3*b^2*f + 12*a*c*f) + 2*c*(16*c^2*d - 3*b^2*f + 12*a*c*f))*x*\text{Sqrt}[a + b*x + c*x^2])/(64*c^2*f^2) - ((b + 2*c*x)*(a + b*x + c*x^2)^{(3/2)})/(8*c*f) - ((128*c^4*d^2 + 192*a*c^3*d*f + 3*b^4*f^2 - 24*a*b^2*c*f^2 + 48*c^2*f*(b^2*d + a^2*f))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/(128*c^{(5/2)}*f^3) + (\text{Sqrt}[d]*(c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f)^{(3/2)}*\text{ArcTanh}[(b*\text{Sqrt}[d] - 2*a*\text{Sqrt}[f] + (2*c*\text{Sqrt}[d] - b*\text{Sqrt}[f])*x)/(2*\text{Sqrt}[c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{Sqrt}[a + b*x + c*x^2])])/(2*f^3) + (\text{Sqrt}[d]*(c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f)^{(3/2)}*\text{ArcTanh}[(b*\text{Sqrt}[d] + 2*a*\text{Sqrt}[f] + (2*c*\text{Sqrt}[d] + b*\text{Sqrt}[f])*x)/(2*\text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{Sqrt}[a + b*x + c*x^2])])/(2*f^3)$

Rule 206


```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Sym
bol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1033

```
Int[((g_) + (h_)*(x_))/(((a_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (
f_)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1070

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x_
)^2)*((d_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((B*c*f*(2*p + 2*q + 3)
+ C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1
))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*
(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*
(C*(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*
p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3)) + (2*p*(c*d - a*f)*(C*(-(
b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(-(b*c*(C*(-4*d*f
)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))))*x + (p*(-(b*f))*(C*
(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2
- 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3
)))))*x^2, x], x] /; FreeQ[{a, b, c, d, f, A, B, C, q}, x] && NeQ[b^2 -
4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] && !I
GtQ[p, 0] && !IGtQ[q, 0]
```

Rule 1071

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (C_)*(x_)^2)*((d_) +
```

```
(f_.)*(x_)^2)^q), x_Symbol] := Simp[((C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*
(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x^
2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*(C*(-(b*f))*(q + 1)) + (p + q + 1)*(b
^2*C*d*f*p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))] + (2*p*(c*d - a*
f)*(C*(-(b*f))*(q + 1)) + (p + q + 1)*(-(b*c*(C*(-4*d*f)*(2*p + q + 2) + f*
(2*C*d + 2*A*f)*(2*p + 2*q + 3)))))*x + (p*(-(b*f))*(C*(-(b*f))*(q + 1)) +
(p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C
*d + 2*A*f)*(2*p + 2*q + 3)))]*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, A,
C, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2
*p + 2*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
+ f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 (a + bx + cx^2)^{3/2}}{d - fx^2} dx &= -\frac{(b + 2cx)(a + bx + cx^2)^{3/2}}{8cf} - \int \frac{\sqrt{a+bx+cx^2} \left(-\frac{3}{4}(3b^2+4ac)df - 12bcdfx - \frac{3}{4}f(16c^2d-3(b^2-4ac)f) \right)}{d-fx^2} dx \\
&= -\frac{(b(80c^2d - 3b^2f + 12acf) + 2c(16c^2d - 3b^2f + 12acf)x) \sqrt{a + bx + cx^2}}{64c^2f^2} - \frac{(b + 2cx)(a + bx + cx^2)^{3/2}}{8cf} \\
&= -\frac{(b(80c^2d - 3b^2f + 12acf) + 2c(16c^2d - 3b^2f + 12acf)x) \sqrt{a + bx + cx^2}}{64c^2f^2} - \frac{(b + 2cx)(a + bx + cx^2)^{3/2}}{8cf} \\
&= -\frac{(b(80c^2d - 3b^2f + 12acf) + 2c(16c^2d - 3b^2f + 12acf)x) \sqrt{a + bx + cx^2}}{64c^2f^2} - \frac{(b + 2cx)(a + bx + cx^2)^{3/2}}{8cf} \\
&= -\frac{(b(80c^2d - 3b^2f + 12acf) + 2c(16c^2d - 3b^2f + 12acf)x) \sqrt{a + bx + cx^2}}{64c^2f^2} - \frac{(b + 2cx)(a + bx + cx^2)^{3/2}}{8cf} \\
&= -\frac{(b(80c^2d - 3b^2f + 12acf) + 2c(16c^2d - 3b^2f + 12acf)x) \sqrt{a + bx + cx^2}}{64c^2f^2} - \frac{(b + 2cx)(a + bx + cx^2)^{3/2}}{8cf}
\end{aligned}$$

Mathematica [A] time = 1.02, size = 395, normalized size = 0.95

$$-\left((48c^2f(a^2f + b^2d) - 24ab^2cf^2 + 192ac^3df + 3b^4f^2 + 128c^4d^2) \tanh^{-1} \left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}} \right) \right) - 2\sqrt{c} \left(f\sqrt{a+x(b+cx)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]

[Out] $-\left((128c^4d^2 + 192aac^3d*f + 3b^4*f^2 - 24ab^2*c*f^2 + 48c^2*f*(b^2*d + a^2*f)) * \text{ArcTanh} \left[\frac{b + 2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}} \right] \right) - 2\sqrt{c} \left(f\sqrt{a+x(b+cx)} \right) - \frac{(b(80c^2d - 3b^2f + 12acf) + 2c(16c^2d - 3b^2f + 12acf)x) \sqrt{a + bx + cx^2}}{64c^2f^2} - \frac{(b + 2cx)(a + bx + cx^2)^{3/2}}{8cf}$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.02, size = 4900, normalized size = 11.75

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] -3/8/f/c^(1/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2-3/128/f/c^(5
/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^4-1/6*d/(d*f)^(1/2)/f*((x
-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f
)^(1/2)*b)/f)^(3/2)-5/8*d/f^2*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*
(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*b-1/2*d^2/f^3*ln(((x-(
d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c
+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)
)*c^(3/2)+1/6*d/(d*f)^(1/2)/f*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*
(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(3/2)-5/8*d/f^2*((x+(d*f)^(1
/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b
)/f)^(1/2)*b-1/2*d^2/f^3*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/
f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f
+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(3/2)-1/8/f/c*(c*x^2+b*x+a)^(3/2)*b-3/
8/f*(c*x^2+b*x+a)^(1/2)*x*a+3/64/f/c^2*(c*x^2+b*x+a)^(1/2)*b^3-d^2/(d*f)^(1
/2)/f^2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(
b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)
)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d
-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*a*c+3/8*d/(d*f)^(1/2)/f/c^(1/2)
```


$$\begin{aligned}
& +1/16*d/(d*f)^{(1/2)}/f/c*((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)*b^2-1/32*d/(d*f)^{(1/2)}/f/c^{(3/2)*\ln(((x+(d*f)^{(1/2)}/f)*c+1/2*(b*f-2*(d*f)^{(1/2)*c})/f)/c^{(1/2)}+((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2))*b^3+3/4*d^2/(d*f)^{(1/2)}/f^2*\ln(((x+(d*f)^{(1/2)}/f)*c+1/2*(b*f-2*(d*f)^{(1/2)*c})/f)/c^{(1/2)}+((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2))*c^{(1/2)*b-1/2*d^2/(d*f)^{(1/2)}/f^2/((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)*\ln((2*(a*f+c*d-(d*f)^{(1/2)*b})/f+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+2*((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2))*((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)))/(x+(d*f)^{(1/2)}/f))*b^2+d/f^2/((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)*\ln((2*(a*f+c*d-(d*f)^{(1/2)*b})/f+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+2*((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2))*((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)))/(x+(d*f)^{(1/2)}/f))*b*a+d^2/(d*f)^{(1/2)}/f^2/((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)*\ln((2*(a*f+c*d+(d*f)^{(1/2)*b})/f+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+2*((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2))*((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)))/(x-(d*f)^{(1/2)}/f))*a*c-1/4/f*x*(c*x^2+b*x+a)^{(3/2)}-3/16*d/f^2*\ln(((x+(d*f)^{(1/2)}/f)*c+1/2*(b*f-2*(d*f)^{(1/2)*c})/f)/c^{(1/2)}+((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2))/c^{(1/2)*b^2+1/2*d/(d*f)^{(1/2)}/f*((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2))*a+1/2*d^2/(d*f)^{(1/2)}/f^2*((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2))*c-1/4*d/f^2*((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2))*x*c-3/4*d/f^2*\ln(((x-(d*f)^{(1/2)}/f)*c+1/2*(b*f+2*(d*f)^{(1/2)*c})/f)/c^{(1/2)}+((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2))*c^{(1/2)*a-3/16*d/f^2*\ln(((x-(d*f)^{(1/2)}/f)*c+1/2*(b*f+2*(d*f)^{(1/2)*c})/f)/c^{(1/2)}+((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2))/c^{(1/2)*b^2-1/2*d/(d*f)^{(1/2)}/f*((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2))*a-1/2*d^2/(d*f)^{(1/2)}/f^2*((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2))*c+3/32/f/c*(c*x^2+b*x+a)^{(1/2))*x*b^2-3/16/f/c*(c*x^2+b*x+a)^{(1/2))*b*a+3/16/f/c^{(3/2)*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2))*b^2*a-1/4*d/f^2*((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2))*x*c-3/4*d/f^2*\ln(((x+(d*f)^{(1/2)}/f)*c+1/2*(b*f-2*(d*f)^{(1/2)*c})/f)/c^{(1/2)}+((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2))*c^{(1/2)*a}
\end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see 'assume?' for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2 -c*((b*sqrt(4*d*f)) / (2*f) + (c*d)/f+a)) /f^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 (c x^2 + b x + a)^{3/2}}{d - f x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x)

[Out] int((x^2*(a + b*x + c*x^2)^(3/2))/(d - f*x^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax^2\sqrt{a+bx+cx^2}}{-d+fx^2} dx - \int \frac{bx^3\sqrt{a+bx+cx^2}}{-d+fx^2} dx - \int \frac{cx^4\sqrt{a+bx+cx^2}}{-d+fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)

[Out] -Integral(a*x**2*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(b*x**3*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(c*x**4*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)

$$3.86 \quad \int \frac{x(a+bx+cx^2)^{3/2}}{d-fx^2} dx$$

Optimal. Leaf size=349

$$\frac{\sqrt{a+bx+cx^2} (8acf + b^2f + 2bcfx + 8c^2d)}{8cf^2} - \frac{b(12acf + b^2(-f) + 24c^2d) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}f^2} (af + b(-\sqrt{a+bx+cx^2}))$$

[Out] $-1/3*(c*x^2+b*x+a)^{(3/2)}/f-1/16*b*(12*a*c*f-b^2*f+24*c^2*d)*\arctanh(1/2*(2*c*x+b)/c^{(1/2)}/(c*x^2+b*x+a)^{(1/2)})/c^{(3/2)}/f^2-1/2*\arctanh(1/2*(b*d^{(1/2)}-2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}-b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)}/(c*d+a*f-b*d^{(1/2)})*f^{(1/2)})^{(1/2)})*(c*d+a*f-b*d^{(1/2)})*f^{(1/2)})^{(3/2)}/f^{(5/2)}+1/2*\arctanh(1/2*(b*d^{(1/2)}+2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}+b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)}/(c*d+a*f+b*d^{(1/2)})*f^{(1/2)})^{(1/2)})*(c*d+a*f+b*d^{(1/2)})*f^{(1/2)})^{(3/2)}/f^{(5/2)}-1/8*(2*b*c*f*x+8*a*c*f+b^2*f+8*c^2*d)*(c*x^2+b*x+a)^{(1/2)}/c/f^2$

Rubi [A] time = 0.52, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1021, 1070, 1078, 621, 206, 1033, 724}

$$\frac{\sqrt{a+bx+cx^2} (8acf + b^2f + 2bcfx + 8c^2d)}{8cf^2} - \frac{b(12acf + b^2(-f) + 24c^2d) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}f^2} (af + b(-\sqrt{a+bx+cx^2}))$$

Antiderivative was successfully verified.

[In] Int[(x*(a + b*x + c*x^2)^(3/2))/(d - f*x^2), x]

[Out] $-((8*c^2*d + b^2*f + 8*a*c*f + 2*b*c*f*x)*\text{Sqrt}[a + b*x + c*x^2])/(8*c*f^2) - (a + b*x + c*x^2)^{(3/2)}/(3*f) - (b*(24*c^2*d - b^2*f + 12*a*c*f)*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/(16*c^{(3/2)}*f^2) - ((c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f)^{(3/2)}*\text{ArcTanh}[(b*\text{Sqrt}[d] - 2*a*\text{Sqrt}[f] + (2*c*\text{Sqrt}[d] - b*\text{Sqrt}[f])*x)/(2*\text{Sqrt}[c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f)*\text{Sqrt}[a + b*x + c*x^2])])/(2*f^{(5/2)}) + ((c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f)^{(3/2)}*\text{ArcTanh}[(b*\text{Sqrt}[d] + 2*a*\text{Sqrt}[f] + (2*c*\text{Sqrt}[d] + b*\text{Sqrt}[f])*x)/(2*\text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f)*\text{Sqrt}[a + b*x + c*x^2])])/(2*f^{(5/2)})$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

```
Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1021

```
Int[((g_) + (h_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q + 1))*x^2, x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1033

```
Int[((g_) + (h_)*(x_))/(((a_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1070

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x_)^2)*((d_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((B*c*f*(2*p + 2*q + 3) + C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*(C*(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(-(b*c*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))))*x + (p*(-(b*f))*(C*(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))))*x^2, x], x] /; FreeQ[{a, b, c, d, f, A, B, C, q}, x] && NeQ[b^2 -
```

4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] && !I
GtQ[p, 0] && !IGtQ[q, 0]

Rule 1078

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
+ f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]

Rubi steps

$$\begin{aligned} \int \frac{x(a+bx+cx^2)^{3/2}}{d-fx^2} dx &= -\frac{(a+bx+cx^2)^{3/2}}{3f} + \frac{\int \frac{\sqrt{a+bx+cx^2} \left(\frac{3bd}{2} + 3(cd+af)x + \frac{3}{2}bfx^2\right)}{d-fx^2} dx}{3f} \\ &= -\frac{(8c^2d + b^2f + 8acf + 2bcfx) \sqrt{a+bx+cx^2}}{8cf^2} - \frac{(a+bx+cx^2)^{3/2}}{3f} - \frac{\int \frac{-\frac{3}{8}bdf(8c^2d+b^2f)}{d-fx^2} dx}{3f} \\ &= -\frac{(8c^2d + b^2f + 8acf + 2bcfx) \sqrt{a+bx+cx^2}}{8cf^2} - \frac{(a+bx+cx^2)^{3/2}}{3f} + \frac{\int \frac{\frac{3}{8}bdf^2(24c^2d-b^2f)}{d-fx^2} dx}{3f} \\ &= -\frac{(8c^2d + b^2f + 8acf + 2bcfx) \sqrt{a+bx+cx^2}}{8cf^2} - \frac{(a+bx+cx^2)^{3/2}}{3f} + \frac{(cd - b\sqrt{d}\sqrt{f})}{3f} \\ &= -\frac{(8c^2d + b^2f + 8acf + 2bcfx) \sqrt{a+bx+cx^2}}{8cf^2} - \frac{(a+bx+cx^2)^{3/2}}{3f} - \frac{b(24c^2d - b^2f)}{3f} \\ &= -\frac{(8c^2d + b^2f + 8acf + 2bcfx) \sqrt{a+bx+cx^2}}{8cf^2} - \frac{(a+bx+cx^2)^{3/2}}{3f} - \frac{b(24c^2d - b^2f)}{3f} \end{aligned}$$

Mathematica [A] time = 0.80, size = 330, normalized size = 0.95

$$\frac{b(-12acf + b^2f - 24c^2d) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right) + \sqrt{f}\sqrt{a+x(b+cx)}(2cf(16a+7bx) + 3b^2f + 8c^2(3d+fx^2))}{16c^{3/2}f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

```
[Out] (b*(-24*c^2*d + b^2*f - 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x
*(b + c*x)])])/(16*c^(3/2)*f^2) - (Sqrt[f]*Sqrt[a + x*(b + c*x)]*(3*b^2*f +
2*c*f*(16*a + 7*b*x) + 8*c^2*(3*d + f*x^2)) - 12*c*(c*d - b*Sqrt[d]*Sqrt[f
+ a*f)^(3/2)*ArcTanh[(-b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt
[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)]) + 12*
c*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*
x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a
+ x*(b + c*x)])])/(24*c*f^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.02, size = 4567, normalized size = 13.09

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] -1/6/f*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*
f+c*d+(d*f)^(1/2)*b)/f)^(3/2)-1/6/f*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/
2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(3/2)-1/2/f^2*((x-(d*f
)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/
2)*b)/f)^(1/2)*c*d+1/2/f/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(
d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)
```


$$\begin{aligned}
& *f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)})/c^{(1/2)*b^2*(d*f)^{(1/2)}-3/ \\
& 4/f^2*\ln(((x-(d*f)^{(1/2)}/f)*c+1/2*(b*f+2*(d*f)^{(1/2)*c)/f)/c^{(1/2)}+((x-(d*f) \\
&)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/ \\
& 2)*b)/f)^{(1/2)})*c^{(1/2)*d*b-1/2}/f^3*\ln(((x-(d*f)^{(1/2)}/f)*c+1/2*(b*f+2*(d*f) \\
&)^{(1/2)*c)/f)/c^{(1/2)}+((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f) \\
&)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)})*c^{(3/2)*(d*f)^{(1/2)*d+1/2}/f^2 \\
& /((a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)}*\ln((2*(a*f+c*d+(d*f)^{(1/2)*b)/f+(b*f+2*(\\
& d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+2*((a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)}*((x-(\\
& d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^ \\
& (1/2)*b)/f)^{(1/2)})/(x-(d*f)^{(1/2)}/f))*d*b^2+1/2/f^3/((a*f+c*d+(d*f)^{(1/2)*b} \\
&)/f)^{(1/2)}*\ln((2*(a*f+c*d+(d*f)^{(1/2)*b)/f+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(\\
& 1/2)}/f)/f+2*((a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)}*((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2 \\
& *(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)})/(x-(d \\
& *f)^{(1/2)}/f))*c^2*d^2+1/4/f^2*((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)*c})* \\
& (x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2)}*x*(d*f)^{(1/2)*c+3/4/f^ \\
& 2*\ln(((x+(d*f)^{(1/2)}/f)*c+1/2*(b*f-2*(d*f)^{(1/2)*c)/f)/c^{(1/2)}+((x+(d*f)^{(1 \\
& /2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b} \\
&)/f)^{(1/2)})*c^{(1/2)*(d*f)^{(1/2)*a-3/8}/f/c^{(1/2)}*\ln(((x+(d*f)^{(1/2)}/f)*c+1/2 \\
& *(b*f-2*(d*f)^{(1/2)*c)/f)/c^{(1/2)}+((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2) \\
& *c})*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2)})*a*b+3/16/f^2*\ln(((\\
& x+(d*f)^{(1/2)}/f)*c+1/2*(b*f-2*(d*f)^{(1/2)*c)/f)/c^{(1/2)}+((x+(d*f)^{(1/2)}/f) \\
& ^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b)/f)^{(\\
& 1/2)})/c^{(1/2)*b^2*(d*f)^{(1/2)}-3/4/f^2*\ln(((x+(d*f)^{(1/2)}/f)*c+1/2*(b*f-2*(d \\
& *f)^{(1/2)*c)/f)/c^{(1/2)}+((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d* \\
& f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2)})*c^{(1/2)*d*b+1/2}/f^3*\ln(((x+ \\
& (d*f)^{(1/2)}/f)*c+1/2*(b*f-2*(d*f)^{(1/2)*c)/f)/c^{(1/2)}+((x+(d*f)^{(1/2)}/f)^2* \\
& c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2) \\
&))*c^{(3/2)*(d*f)^{(1/2)*d+1/2}/f^2/((a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2)}*\ln((2*(a \\
& *f+c*d-(d*f)^{(1/2)*b)/f+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+2*((a*f+c \\
& *d-(d*f)^{(1/2)*b)/f)^{(1/2)}*((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+ \\
& (d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2)})/(x+(d*f)^{(1/2)}/f))*d*b^2 \\
& +1/2/f^3/((a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2)}*\ln((2*(a*f+c*d-(d*f)^{(1/2)*b)/f+ \\
& (b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+2*((a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/ \\
& 2)*((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+(a*f+c* \\
& d-(d*f)^{(1/2)*b)/f)^{(1/2)})/(x+(d*f)^{(1/2)}/f))*c^2*d^2-1/4/f^2*((x-(d*f)^{(1/ \\
& 2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)*b} \\
&)/f)^{(1/2)}*x*(d*f)^{(1/2)*c+1/f^3/((a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)}*\ln((2*(a* \\
& f+c*d+(d*f)^{(1/2)*b)/f+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+2*((a*f+c* \\
& d+(d*f)^{(1/2)*b)/f)^{(1/2)}*((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(\\
& d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)})/(x-(d*f)^{(1/2)}/f))*d*f)^ \\
& (1/2)*b*c*d-1/f^3/((a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2)}*\ln((2*(a*f+c*d-(d*f)^{(1 \\
& /2)*b)/f+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+2*((a*f+c*d-(d*f)^{(1/2)* \\
& b)/f)^{(1/2)}*((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/ \\
& f+(a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2)})/(x+(d*f)^{(1/2)}/f))*d*f)^{(1/2)*b*c*d
\end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see 'assume?' for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2 -c*((b*sqrt(4*d*f)) / (2*f) + (c*d)/f+a) /f^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x(c x^2 + b x + a)^{3/2}}{d - f x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x)

[Out] int((x*(a + b*x + c*x^2)^(3/2))/(d - f*x^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax\sqrt{a+bx+cx^2}}{-d+fx^2} dx - \int \frac{bx^2\sqrt{a+bx+cx^2}}{-d+fx^2} dx - \int \frac{cx^3\sqrt{a+bx+cx^2}}{-d+fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)

[Out] -Integral(a*x*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(b*x**2*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(c*x**3*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)

$$3.87 \quad \int \frac{(a+bx+cx^2)^{3/2}}{d-fx^2} dx$$

Optimal. Leaf size=315

$$\frac{(12acf + 3b^2f + 8c^2d) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (af + b(-\sqrt{d})\sqrt{f} + cd)^{3/2} \tanh^{-1}\left(\frac{-2a\sqrt{f} + x(2c\sqrt{d}-b\sqrt{f}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}}}\right)}{8\sqrt{c}f^2} + \frac{(af + b(-\sqrt{d})\sqrt{f} + cd)^{3/2} \tanh^{-1}\left(\frac{-2a\sqrt{f} + x(2c\sqrt{d}-b\sqrt{f}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}}}\right)}{2\sqrt{d}f^2}$$

[Out] $-1/8*(12*a*c*f+3*b^2*f+8*c^2*d)*\operatorname{arctanh}(1/2*(2*c*x+b)/c^{(1/2)/(c*x^2+b*x+a)^{(1/2)})/f^{(1/2)/c^{(1/2)}+1/2*\operatorname{arctanh}(1/2*(b*d^{(1/2)}-2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}-b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)/(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(1/2)}*(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(3/2)}/f^{(1/2)/d^{(1/2)}+1/2*\operatorname{arctanh}(1/2*(b*d^{(1/2)}+2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}+b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)/(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(1/2)}*(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(3/2)}/f^{(1/2)/d^{(1/2)}-1/4*(2*c*x+5*b)*(c*x^2+b*x+a)^{(1/2)}/f$

Rubi [A] time = 0.52, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {978, 1078, 621, 206, 1033, 724}

$$\frac{(12acf + 3b^2f + 8c^2d) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (af + b(-\sqrt{d})\sqrt{f} + cd)^{3/2} \tanh^{-1}\left(\frac{-2a\sqrt{f} + x(2c\sqrt{d}-b\sqrt{f}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}}}\right)}{8\sqrt{c}f^2} + \frac{(af + b(-\sqrt{d})\sqrt{f} + cd)^{3/2} \tanh^{-1}\left(\frac{-2a\sqrt{f} + x(2c\sqrt{d}-b\sqrt{f}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}}}\right)}{2\sqrt{d}f^2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + b*x + c*x^2)^{(3/2)/(d - f*x^2)}, x]$

[Out] $-((5*b + 2*c*x)*\operatorname{Sqrt}[a + b*x + c*x^2])/(4*f) - ((8*c^2*d + 3*b^2*f + 12*a*c*f)*\operatorname{ArcTanh}[(b + 2*c*x)/(2*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(8*\operatorname{Sqrt}[c]*f^2) + ((c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)^{(3/2)}*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] - 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] - b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*\operatorname{Sqrt}[d]*f^2) + ((c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)^{(3/2)}*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] + 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] + b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*\operatorname{Sqrt}[d]*f^2)$

Rule 206

$\operatorname{Int}[(a + b*x + c*x^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /;$ $\operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{Gt} Q[a, 0] \ || \ \operatorname{Lt} Q[b, 0])$

Rule 621

```
Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 978

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((b*(3*p + 2*q) + 2*c*(p + q)*x)*(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^(q + 1))/(2*f*(p + q)*(2*p + 2*q + 1)), x] - Dist[1/(2*f*(p + q)*(2*p + 2*q + 1)), Int[(a + b*x + c*x^2)^(p - 2)*(d + f*x^2)^q*Simp[b^2*d*(p - 1)*(2*p + q) - (p + q)*(b^2*d*(1 - p) - 2*a*(c*d - a*f*(2*p + 2*q + 1))) - (2*b*(c*d - a*f)*(1 - p)*(2*p + q) - 2*(p + q)*b*(2*c*d*(2*p + q) - (c*d + a*f)*(2*p + 2*q + 1)))*x + (b^2*f*p*(1 - p) + 2*c*(p + q)*(c*d*(2*p - 1) - a*f*(4*p + 2*q - 1)))*x^2, x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*q + 1, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]
```

Rule 1033

```
Int(((g_) + (h_)*(x_))/(((a_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1078

```
Int(((A_) + (B_)*(x_) + (C_)*(x_)^2)/(((a_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx + cx^2)^{3/2}}{d - fx^2} dx &= -\frac{(5b + 2cx)\sqrt{a + bx + cx^2}}{4f} + \frac{\int \frac{\frac{1}{4}(5b^2d + 4a(cd + 2af)) + 4b(cd + af)x + \frac{1}{4}(8c^2d + 3b^2f + 12acf)x^2}{\sqrt{a + bx + cx^2}(d - fx^2)} dx}{2f} \\
&= -\frac{(5b + 2cx)\sqrt{a + bx + cx^2}}{4f} - \frac{\int \frac{-\frac{1}{4}d(8c^2d + 3b^2f + 12acf) - \frac{1}{4}f(5b^2d + 4a(cd + 2af)) - 4bf(cd + af)x}{\sqrt{a + bx + cx^2}(d - fx^2)} dx}{2f^2} \\
&= -\frac{(5b + 2cx)\sqrt{a + bx + cx^2}}{4f} - \frac{(cd - b\sqrt{d}\sqrt{f} + af)^2 \int \frac{1}{(-\sqrt{d}\sqrt{f} - fx)\sqrt{a + bx + cx^2}} dx}{2\sqrt{d}f^{3/2}} + \frac{(cd - b\sqrt{d}\sqrt{f} + af)^2}{2\sqrt{d}f^{3/2}} \\
&= -\frac{(5b + 2cx)\sqrt{a + bx + cx^2}}{4f} - \frac{(8c^2d + 3b^2f + 12acf) \tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + bx + cx^2}}\right)}{8\sqrt{c}f^2} + \frac{(cd - b\sqrt{d}\sqrt{f} + af)^2}{2\sqrt{d}f^{3/2}} \\
&= -\frac{(5b + 2cx)\sqrt{a + bx + cx^2}}{4f} - \frac{(8c^2d + 3b^2f + 12acf) \tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + bx + cx^2}}\right)}{8\sqrt{c}f^2} + \frac{(cd - b\sqrt{d}\sqrt{f} + af)^2}{2\sqrt{d}f^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.81, size = 298, normalized size = 0.95

$$\frac{(12acf + 3b^2f + 8c^2d) \tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + x(b + cx)}}\right)}{\sqrt{c}} + \frac{4(af + b(-\sqrt{d})\sqrt{f} + cd)^{3/2} \tanh^{-1}\left(\frac{2a\sqrt{f} - b\sqrt{d} + b\sqrt{f}x - 2c\sqrt{d}x}{2\sqrt{a + x(b + cx)}\sqrt{af + b(-\sqrt{d})\sqrt{f} + cd}}\right)}{\sqrt{d}} + \frac{4(af + b\sqrt{d}\sqrt{f} + cd)^{3/2} \tanh^{-1}\left(\frac{2a\sqrt{f} + b\sqrt{d} + b\sqrt{f}x + 2c\sqrt{d}x}{2\sqrt{a + x(b + cx)}\sqrt{af + b\sqrt{d}\sqrt{f} + cd}}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d - f*x^2), x]

[Out]
$$-\frac{1}{8} \frac{(2f(5b + 2cx)\sqrt{a + x(b + cx)} + ((8c^2d + 3b^2f + 12acf)\text{ArcTanh}[\frac{b + 2cx}{2\sqrt{c}\sqrt{a + x(b + cx)}}])/\sqrt{c} + (4(cd - b\sqrt{d}\sqrt{f} + af)^{3/2}\text{ArcTanh}[\frac{(-b\sqrt{d}) + 2a\sqrt{f} - 2c\sqrt{d}x}{2\sqrt{a + x(b + cx)}\sqrt{af + b(-\sqrt{d})\sqrt{f} + cd}}] - 2c\sqrt{d}x + b\sqrt{f}x)/(2\sqrt{c}d - b\sqrt{d}\sqrt{f} + af)\sqrt{a + x(b + cx)}))/\sqrt{d} + (4(cd + b\sqrt{d}\sqrt{f} + af)^{3/2}\text{ArcTanh}[\frac{(-2(a\sqrt{f} + c\sqrt{d}x) - b(\sqrt{d} + \sqrt{f}x))/(2\sqrt{c}d + b\sqrt{d}\sqrt{f} + af)\sqrt{a + x(b + cx)}}])/\sqrt{d})}{f^2}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

```
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:Error: Bad Argument Type
```

```
maple [B] time = 0.02, size = 4574, normalized size = 14.52
```

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] -1/6/(d*f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)
)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(3/2)+1/6/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^
2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(3
/2)-1/2/f^2*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((
x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*
f)^(1/2)*b)/f)^(1/2))*c^(3/2)*d-1/2/(d*f)^(1/2)/((a*f+c*d-(d*f)^(1/2)*b)/f)
^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)
)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*
f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(
1/2)/f))*a^2+1/32/(d*f)^(1/2)/c^(3/2)*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(
d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d
*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*b^3-1/2/f^2*ln(((x-(d*f)^(
1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+
2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*c^(3
/2)*d+1/2/(d*f)^(1/2)/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)
)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1
/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)
)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a^2-1/4/f*((x+(d
*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(
1/2)*b)/f)^(1/2)*x*c+1/8/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1
/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*b+1/16/(d*f)^(
1/2)/c*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a
*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b^2-3/4/f*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2
*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+
```



```

*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(
1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f)*c^2*d^2-5/8/f*((x-(d*f)^(1/2)/f)^2*c+(
b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*b
-1/2/(d*f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)
)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*a-5/8/f*((x+(d*f)^(1/2)/f)^2*c+(b*f
-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b+1/
2/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f
)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*a+3/8/(d*f)^(1/2)/c^(1/2)*ln(((x+(d*f)
^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*
f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*a*
b+1/2/(d*f)^(1/2)/f*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(
1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*c*d+1/f/((a*f+c*d-(d*f)^(1/2)*b)
/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1
/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*
(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*
f)^(1/2)/f))*b*a-3/8/(d*f)^(1/2)/c^(1/2)*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2
*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-
(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*a*b-1/2/(d*f)^(1/2)/f*((
x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*
f)^(1/2)*b)/f)^(1/2)*c*d+1/f/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c
*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(
d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)
)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b*a-1/(d*
f)^(1/2)/f/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/
f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(
1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+
c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*a*c*d

```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2 -c*((b*sqrt(4*d*f)) / (2*f) + (c*d)/f+a) /f^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^2 + bx + a)^{3/2}}{d - fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x + c*x^2)^(3/2)/(d - f*x^2), x)`

[Out] `int((a + b*x + c*x^2)^(3/2)/(d - f*x^2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a\sqrt{a+bx+cx^2}}{-d+fx^2} dx - \int \frac{bx\sqrt{a+bx+cx^2}}{-d+fx^2} dx - \int \frac{cx^2\sqrt{a+bx+cx^2}}{-d+fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+b*x+a)**(3/2)/(-f*x**2+d), x)`

[Out] `-Integral(a*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(b*x*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(c*x**2*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)`

$$3.88 \quad \int \frac{(a+bx+cx^2)^{3/2}}{x(d-fx^2)} dx$$

Optimal. Leaf size=469

$$\frac{a^{3/2} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d} - \frac{b(b^2-12ac) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d} - \frac{\sqrt{a+bx+cx^2} (8acf + b^2f + 2bcfx + 8c^2d)}{8cdf}$$

[Out] $-a^{3/2} \operatorname{arctanh}\left(\frac{1/2(b*x+2*a)}{a^{1/2}/(c*x^2+b*x+a)^{1/2}}\right)/d - 1/16*b*(-12*a*c+b^2)*\operatorname{arctanh}\left(\frac{1/2(2*c*x+b)}{c^{1/2}/(c*x^2+b*x+a)^{1/2}}\right)/c^{3/2}/d - 1/16*b*(12*a*c*f-b^2*f+24*c^2*d)*\operatorname{arctanh}\left(\frac{1/2(2*c*x+b)}{c^{1/2}/(c*x^2+b*x+a)^{1/2}}\right)/c^{3/2}/d - 1/2*\operatorname{arctanh}\left(\frac{1/2(b*d^{1/2}-2*a*f^{1/2}+x*(2*c*d^{1/2}-b*f^{1/2}))}{(c*x^2+b*x+a)^{1/2}/(c*d+a*f-b*d^{1/2}*f^{1/2})^{1/2}}\right)*(c*d+a*f-b*d^{1/2}*f^{1/2})^{3/2}/d - 1/2*\operatorname{arctanh}\left(\frac{1/2(b*d^{1/2}+2*a*f^{1/2}+x*(2*c*d^{1/2}+b*f^{1/2}))}{(c*x^2+b*x+a)^{1/2}/(c*d+a*f+b*d^{1/2}*f^{1/2})^{1/2}}\right)*(c*d+a*f+b*d^{1/2}*f^{1/2})^{3/2}/d - 1/8*(2*b*c*x+8*a*c+b^2)*(c*x^2+b*x+a)^{1/2}/c/d - 1/8*(2*b*c*f*x+8*a*c*f+b^2*f+8*c^2*d)*(c*x^2+b*x+a)^{1/2}/c/d/f$

Rubi [A] time = 1.27, antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {6725, 734, 814, 843, 621, 206, 724, 1021, 1070, 1078, 1033}

$$\frac{a^{3/2} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d} - \frac{\sqrt{a+bx+cx^2} (8acf + b^2f + 2bcfx + 8c^2d)}{8cdf} - \frac{b(12acf + b^2(-f) + 24c^2d) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + c*x^2)^(3/2)/(x*(d - f*x^2)), x]

[Out] $((b^2 + 8*a*c + 2*b*c*x)*\operatorname{Sqrt}[a + b*x + c*x^2])/(8*c*d) - ((8*c^2*d + b^2*f + 8*a*c*f + 2*b*c*f*x)*\operatorname{Sqrt}[a + b*x + c*x^2])/(8*c*d*f) - (a^{3/2}*\operatorname{ArcTanh}[(2*a + b*x)/(2*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[a + b*x + c*x^2])])/d - (b*(b^2 - 12*a*c)*\operatorname{ArcTanh}[(b + 2*c*x)/(2*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(16*c^{3/2}*d) - (b*(24*c^2*d - b^2*f + 12*a*c*f)*\operatorname{ArcTanh}[(b + 2*c*x)/(2*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(16*c^{3/2}*d*f) - ((c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)^{3/2}*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] - 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] - b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*d*f^{3/2}) + ((c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)^{3/2}*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] + 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] + b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*d*f^{3/2})$

$\text{Sqrt}[f]) * x) / (2 * \text{Sqrt}[c * d + b * \text{Sqrt}[d] * \text{Sqrt}[f] + a * f] * \text{Sqrt}[a + b * x + c * x^2])]) / (2 * d * f^{(3/2)})$

Rule 206

$\text{Int}[(a_) + (b_) * (x_)^2]^{-1}, x_Symbol] := \text{Simp}[(1 * \text{ArcTanh}[\text{Rt}[-b, 2] * x] / \text{Rt}[a, 2]]) / (\text{Rt}[a, 2] * \text{Rt}[-b, 2]), x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

$\text{Int}[1 / \text{Sqrt}[(a_) + (b_) * (x_) + (c_) * (x_)^2], x_Symbol] := \text{Dist}[2, \text{Subst}[\text{Int}[1 / (4 * c - x^2), x], x, (b + 2 * c * x) / \text{Sqrt}[a + b * x + c * x^2]], x] /;$ FreeQ[{a, b, c}, x] && NeQ[b^2 - 4 * a * c, 0]

Rule 724

$\text{Int}[1 / (((d_) + (e_) * (x_)) * \text{Sqrt}[(a_) + (b_) * (x_) + (c_) * (x_)^2]), x_Symbol] := \text{Dist}[-2, \text{Subst}[\text{Int}[1 / (4 * c * d^2 - 4 * b * d * e + 4 * a * e^2 - x^2), x], x, (2 * a * e - b * d - (2 * c * d - b * e) * x) / \text{Sqrt}[a + b * x + c * x^2]], x] /;$ FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4 * a * c, 0] && NeQ[2 * c * d - b * e, 0]

Rule 734

$\text{Int}[(d_) + (e_) * (x_)]^{(m_)} * ((a_) + (b_) * (x_) + (c_) * (x_)^2)^{(p_)}, x_Symbol] := \text{Simp}[(d + e * x)^{(m + 1)} * (a + b * x + c * x^2)^p / (e * (m + 2 * p + 1)), x] - \text{Dist}[p / (e * (m + 2 * p + 1)), \text{Int}[(d + e * x)^m * \text{Simp}[b * d - 2 * a * e + (2 * c * d - b * e) * x, x] * (a + b * x + c * x^2)^{(p - 1)}, x], x] /;$ FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4 * a * c, 0] && NeQ[c * d^2 - b * d * e + a * e^2, 0] && NeQ[2 * c * d - b * e, 0] && GtQ[p, 0] && NeQ[m + 2 * p + 1, 0] && (!RationalQ[m] || LtQ[m, 1]) && !ILtQ[m + 2 * p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]

Rule 814

$\text{Int}[(d_) + (e_) * (x_)]^{(m_)} * ((f_) + (g_) * (x_)) * ((a_) + (b_) * (x_) + (c_) * (x_)^2)^{(p_)}, x_Symbol] := \text{Simp}[(d + e * x)^{(m + 1)} * (c * e * f * (m + 2 * p + 2) - g * (c * d + 2 * c * d * p - b * e * p) + g * c * e * (m + 2 * p + 1) * x) * (a + b * x + c * x^2)^p / (c * e^2 * (m + 2 * p + 1) * (m + 2 * p + 2)), x] - \text{Dist}[p / (c * e^2 * (m + 2 * p + 1) * (m + 2 * p + 2)), \text{Int}[(d + e * x)^m * (a + b * x + c * x^2)^{(p - 1)} * \text{Simp}[c * e * f * (b * d - 2 * a * e) * (m + 2 * p + 2) + g * (a * e * (b * e - 2 * c * d * m + b * e * m) + b * d * (b * e * p - c * d - 2 * c * d * p)) + (c * e * f * (2 * c * d - b * e) * (m + 2 * p + 2) + g * (b^2 * e^2 * (p + m + 1) - 2 * c^2 * d^2 * (1 + 2 * p) - c * e * (b * d * (m - 2 * p) + 2 * a * e * (m + 2 * p + 1)))] * x, x], x] /;$ FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4 * a * c, 0] && NeQ[c * d^2 - b * d * e + a * e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2 * p, 0] && (IntegerQ[m] || IntegerQ[p])

|| IntegersQ[2*m, 2*p])

Rule 843

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

Rule 1021

Int[((g_.) + (h_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-b*f)) + c*(-2*g*f)*(p + q + 1))*x^2, x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]

Rule 1033

Int[((g_.) + (h_.)*(x_))/(((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]

Rule 1070

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_.) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((B*c*f*(2*p + 2*q + 3) + C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*(C*(-b*f))*(q + 1) - c*(-B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(-b*f))*(q + 1) - c*(-B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(-(b*c*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))]*x + (p*(-b*f))*(C*(-b*f))*(q + 1) - c*(-B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))]*x^2, x], x] /; FreeQ[{a, b, c, d, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]], x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
+ f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx + cx^2)^{3/2}}{x(d - fx^2)} dx &= \int \left(\frac{(a + bx + cx^2)^{3/2}}{dx} - \frac{fx(a + bx + cx^2)^{3/2}}{d(-d + fx^2)} \right) dx \\
&= \frac{\int \frac{(a+bx+cx^2)^{3/2}}{x} dx}{d} - \frac{f \int \frac{x(a+bx+cx^2)^{3/2}}{-d+fx^2} dx}{d} \\
&= \frac{\int \frac{\sqrt{a+bx+cx^2} \left(-\frac{3bd}{2} - 3(cd+af)x - \frac{3}{2}bf x^2 \right)}{-d+fx^2} dx}{3d} - \frac{\int \frac{(-2a-bx)\sqrt{a+bx+cx^2}}{x} dx}{2d} \\
&= \frac{(b^2 + 8ac + 2bcx) \sqrt{a + bx + cx^2}}{8cd} - \frac{(8c^2d + b^2f + 8acf + 2bcfx) \sqrt{a + bx + cx^2}}{8cdf} + \frac{\int}{\dots} \\
&= \frac{(b^2 + 8ac + 2bcx) \sqrt{a + bx + cx^2}}{8cd} - \frac{(8c^2d + b^2f + 8acf + 2bcfx) \sqrt{a + bx + cx^2}}{8cdf} + \frac{a^2}{\dots} \\
&= \frac{(b^2 + 8ac + 2bcx) \sqrt{a + bx + cx^2}}{8cd} - \frac{(8c^2d + b^2f + 8acf + 2bcfx) \sqrt{a + bx + cx^2}}{8cdf} - \frac{(2}{\dots} \\
&= \frac{(b^2 + 8ac + 2bcx) \sqrt{a + bx + cx^2}}{8cd} - \frac{(8c^2d + b^2f + 8acf + 2bcfx) \sqrt{a + bx + cx^2}}{8cdf} - \frac{a^3}{\dots} \\
&= \frac{(b^2 + 8ac + 2bcx) \sqrt{a + bx + cx^2}}{8cd} - \frac{(8c^2d + b^2f + 8acf + 2bcfx) \sqrt{a + bx + cx^2}}{8cdf} - \frac{a^3}{\dots}
\end{aligned}$$

Mathematica [A] time = 0.50, size = 755, normalized size = 1.61

$$2a^{3/2} f^{3/2} \tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}} \right) + 2cd\sqrt{f}\sqrt{a+x(b+cx)} - af\sqrt{af+b(-\sqrt{d})}\sqrt{f} + cd \tanh^{-1} \left(\frac{2a\sqrt{f}-b\sqrt{d}}{2\sqrt{a+x(b+cx)}\sqrt{a}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + c*x^2)^(3/2)/(x*(d - f*x^2)), x]

[Out] -1/2*(2*c*d*Sqrt[f]*Sqrt[a + x*(b + c*x)] + 2*a^(3/2)*f^(3/2)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])] + 3*b*Sqrt[c]*d*Sqrt[f]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] - c*d*Sqrt[c*d - b*Sqrt[d]*Sq

$$\begin{aligned} & \text{rt}[f] + a*f] * \text{ArcTanh} [(-b*\text{Sqrt}[d]) + 2*a*\text{Sqrt}[f] - 2*c*\text{Sqrt}[d]*x + b*\text{Sqrt}[f] \\ &]*x)/(2*\text{Sqrt}[c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{Sqrt}[a + x*(b + c*x)]) + b*\text{Sqr} \\ & \text{t}[d]*\text{Sqrt}[f]*\text{Sqrt}[c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{ArcTanh} [(-b*\text{Sqrt}[d]) + 2* \\ & a*\text{Sqrt}[f] - 2*c*\text{Sqrt}[d]*x + b*\text{Sqrt}[f]*x)/(2*\text{Sqrt}[c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + \\ & a*f]*\text{Sqrt}[a + x*(b + c*x)]) - a*f*\text{Sqrt}[c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{ArcT} \\ & \text{anh} [(-b*\text{Sqrt}[d]) + 2*a*\text{Sqrt}[f] - 2*c*\text{Sqrt}[d]*x + b*\text{Sqrt}[f]*x)/(2*\text{Sqrt}[c*d \\ & - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{Sqrt}[a + x*(b + c*x)]) + c*d*\text{Sqrt}[c*d + b*\text{Sqrt}[\\ & d]*\text{Sqrt}[f] + a*f]*\text{ArcTanh} [(-2*(a*\text{Sqrt}[f] + c*\text{Sqrt}[d]*x) - b*(\text{Sqrt}[d] + \text{Sqrt} \\ & [f]*x))/(2*\text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{Sqrt}[a + x*(b + c*x)]) + b* \\ & \text{Sqrt}[d]*\text{Sqrt}[f]*\text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{ArcTanh} [(-2*(a*\text{Sqrt}[f] \\ & + c*\text{Sqrt}[d]*x) - b*(\text{Sqrt}[d] + \text{Sqrt}[f]*x))/(2*\text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + \\ & a*f]*\text{Sqrt}[a + x*(b + c*x)]) + a*f*\text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{Arc} \\ & \text{Tanh} [(-2*(a*\text{Sqrt}[f] + c*\text{Sqrt}[d]*x) - b*(\text{Sqrt}[d] + \text{Sqrt}[f]*x))/(2*\text{Sqrt}[c*d + \\ & b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{Sqrt}[a + x*(b + c*x)])]/(d*f^(3/2)) \end{aligned}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(3/2)/x/(-f*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(3/2)/x/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:index.cc index_m operator + Error: Bad Argum
ent Value

maple [B] time = 0.02, size = 4765, normalized size = 10.16

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x+a)^(3/2)/x/(-f*x^2+d),x)

[Out] $\frac{1}{3}d*(c*x^2+b*x+a)^{3/2}-\frac{1}{6}d*((x-(d*f)^{1/2}/f)^2*c+(b*f+2*(d*f)^{1/2}*c)* (x-(d*f)^{1/2}/f)/f+(a*f+c*d+(d*f)^{1/2}*b)/f)^{3/2}-\frac{1}{6}d*((x+(d*f)^{1/2}$

$$\begin{aligned}
&)/f)^2*c+(b*f-2*(d*f)^{(1/2)*c}*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b})/ \\
& f)^{(3/2)+1/4/d*b*(c*x^2+b*x+a)^{(1/2)*x+1/8/d/c*(c*x^2+b*x+a)^{(1/2)*b^2-1/16} \\
& /d/c^{(3/2)*\ln((c*x+1/2*b)/c^{(1/2)+(c*x^2+b*x+a)^{(1/2)*b^2-1/8/d*((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c}*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)*x*b-1/16/d/c*((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c}*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)*b^2-3/4/f*\ln(((x-(d*f)^{(1/2)/f})^2*c+1/2*(b*f+2*(d*f)^{(1/2)*c})/f)/c^{(1/2)+((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c}*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2))*c^{(1/2)*b-1/2/f^2*\ln(((x-(d*f)^{(1/2)/f})^2*c+1/2*(b*f+2*(d*f)^{(1/2)*c})/f)/c^{(1/2)+((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c}*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2))*c^{(3/2)*(d*f)^{(1/2)+1/2/f/((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)*\ln((2*(a*f+c*d+(d*f)^{(1/2)*b})/f+(b*f+2*(d*f)^{(1/2)*c}*(x-(d*f)^{(1/2)/f})/f)+2*((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2))*((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c}*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)))/(x-(d*f)^{(1/2)/f))*b^2+1/2/d/((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)*\ln((2*(a*f+c*d+(d*f)^{(1/2)*b})/f+(b*f+2*(d*f)^{(1/2)*c}*(x-(d*f)^{(1/2)/f})/f)+2*((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2))*((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c}*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)))/(x-(d*f)^{(1/2)/f))*a^2+1/2/d/((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)*\ln((2*(a*f+c*d-(d*f)^{(1/2)*b})/f+(b*f-2*(d*f)^{(1/2)*c}*(x+(d*f)^{(1/2)/f})/f)+2*((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2))*((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c}*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)))/(x+(d*f)^{(1/2)/f))*a^2-1/8/d*((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c}*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)*x*b-1/16/d/c*((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c}*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)*b^2+1/32/d/c^{(3/2)*\ln(((x+(d*f)^{(1/2)/f})^2*c+1/2*(b*f-2*(d*f)^{(1/2)*c})/f)/c^{(1/2)+((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c}*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2))*b^3-3/4/f*\ln(((x+(d*f)^{(1/2)/f})^2*c+1/2*(b*f-2*(d*f)^{(1/2)*c})/f)/c^{(1/2)+((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c}*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2))*c^{(1/2)*b+1/2/f^2*\ln(((x+(d*f)^{(1/2)/f})^2*c+1/2*(b*f-2*(d*f)^{(1/2)*c})/f)/c^{(1/2)+((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c}*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2))*c^{(3/2)*(d*f)^{(1/2)+1/2/f/((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)*\ln((2*(a*f+c*d-(d*f)^{(1/2)*b})/f+(b*f-2*(d*f)^{(1/2)*c}*(x+(d*f)^{(1/2)/f})/f)+2*((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2))*((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c}*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)))/(x+(d*f)^{(1/2)/f))*b^2+1/32/d/c^{(3/2)*\ln(((x-(d*f)^{(1/2)/f})^2*c+1/2*(b*f+2*(d*f)^{(1/2)*c})/f)/c^{(1/2)+((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c}*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2))*b^3-1/4/d/f*((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c}*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)*x*(d*f)^{(1/2)*c-3/4/d/f*\ln(((x-(d*f)^{(1/2)/f})^2*c+1/2*(b*f+2*(d*f)^{(1/2)*c})/f)/c^{(1/2)+((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c}*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2))*c^{(1/2)*(d*f)^{(1/2)*a-3/16/d/f*\ln(((x-(d*f)^{(1/2)/f})^2*c+1/2*(b*f+2*(d*f)^{(1/2)*c})/f)/c^{(1/2)+((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c}*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2))*c^{(1/2)*b^2*(d*f)^{(1/2)+1/f^2/((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)*\ln((2*(a*
\end{aligned}$$

$$\begin{aligned}
& f+c*d+(d*f)^{(1/2)*b}/f+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f})/f+2*((a*f+c*d+(d*f)^{(1/2)*b}/f)^{(1/2)*((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b}/f)^{(1/2)})/(x-(d*f)^{(1/2)/f}))*((d*f)^{(1/2)*b*c+3/16/d/f*\ln(((x+(d*f)^{(1/2)/f})^c+1/2*(b*f-2*(d*f)^{(1/2)*c})/f)/c^{(1/2)+((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b}/f)^{(1/2)})/c^{(1/2)*b^2*(d*f)^{(1/2)}-1/f^2/((a*f+c*d-(d*f)^{(1/2)*b}/f)^{(1/2)*\ln((2*(a*f+c*d-(d*f)^{(1/2)*b}/f+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+2*((a*f+c*d-(d*f)^{(1/2)*b}/f)^{(1/2)*((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b}/f)^{(1/2)})/(x+(d*f)^{(1/2)/f}))*((d*f)^{(1/2)*b*c+1/4/d/f*((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b}/f)^{(1/2)*x*(d*f)^{(1/2)*c+3/4/d/f*\ln(((x+(d*f)^{(1/2)/f})^c+1/2*(b*f-2*(d*f)^{(1/2)*c})/f)/c^{(1/2)+((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b}/f)^{(1/2))*\ln((2*(a*f+c*d+(d*f)^{(1/2)*b}/f+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f})/f+2*((a*f+c*d+(d*f)^{(1/2)*b}/f)^{(1/2)*((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b}/f)^{(1/2)})/(x-(d*f)^{(1/2)/f}))*a*c+1/2*d/f^2/((a*f+c*d+(d*f)^{(1/2)*b}/f)^{(1/2)*\ln((2*(a*f+c*d+(d*f)^{(1/2)*b}/f+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f})/f+2*((a*f+c*d+(d*f)^{(1/2)*b}/f)^{(1/2)*((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b}/f)^{(1/2))*((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b}/f)^{(1/2)})/(x-(d*f)^{(1/2)/f}))*c^2+3/4/d*b/c^{(1/2)*\ln((c*x+1/2*b)/c^{(1/2)+((c*x^2+b*x+a)^{(1/2))*a+1/f/((a*f+c*d-(d*f)^{(1/2)*b}/f)^{(1/2)*\ln((2*(a*f+c*d-(d*f)^{(1/2)*b}/f+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+2*((a*f+c*d-(d*f)^{(1/2)*b}/f)^{(1/2)*((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b}/f)^{(1/2)})/(x+(d*f)^{(1/2)/f}))*a*c+1/2*d/f^2/((a*f+c*d-(d*f)^{(1/2)*b}/f)^{(1/2)*\ln((2*(a*f+c*d-(d*f)^{(1/2)*b}/f+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+2*((a*f+c*d-(d*f)^{(1/2)*b}/f)^{(1/2)*((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b}/f)^{(1/2))*((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b}/f)^{(1/2)})/(x+(d*f)^{(1/2)/f}))*c^2+5/8/d/f*((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b}/f)^{(1/2)*b*(d*f)^{(1/2)}-3/8/d/c^{(1/2)*\ln(((x+(d*f)^{(1/2)/f})^c+1/2*(b*f-2*(d*f)^{(1/2)*c})/f)/c^{(1/2)+((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b}/f)^{(1/2))*a*b+1/d/f/((a*f+c*d+(d*f)^{(1/2)*b}/f)^{(1/2)*\ln((2*(a*f+c*d+(d*f)^{(1/2)*b}/f+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f})/f+2*((a*f+c*d+(d*f)^{(1/2)*b}/f)^{(1/2)*((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b}/f)^{(1/2)})/(x-(d*f)^{(1/2)/f}))*((d*f)^{(1/2)*b*a-1/d/f/((a*f+c*d-(d*f)^{(1/2)*b}/f)^{(1/2)*\ln((2*(a*f+c*d-(d*f)^{(1/2)*b}/f+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+2*((a*f+c*d-(d*f)^{(1/2)*b}/f)^{(1/2)*((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b}/f)^{(1/2)})/(x+(d*f)^{(1/2)/f}))*((d*f)^{(1/2)*b*a-1/2/d*((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b}/f)^{(1/2)*a-1/2/f*((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b}/f)^{(1/2)*c+1/d*a*((c*x^2+b*x+a)^{(1/2)}-1/d*a^{(3/2)*\ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)*a^{(1/2)})/x)-1/2/d*((x-(d*f)^{(1/2)/f})^2*c+(b*f+2
\end{aligned}$$

$(d*f)^{(1/2)*c}*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b}/f)^{(1/2)*a-1/2}/$
 $f*((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c}*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d$
 $+(d*f)^{(1/2)*b}/f)^{(1/2)*c-5/8}/d*f*((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)$
 $)*c)*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b}/f)^{(1/2)*b*(d*f)^{(1/2)-3/8}$
 $/d/c^{(1/2)*ln(((x-(d*f)^{(1/2)/f})*c+1/2*(b*f+2*(d*f)^{(1/2)*c})/f)/c^{(1/2)+((x$
 $-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c}*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f$
 $)^{(1/2)*b}/f)^{(1/2))*a*b$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(cx^2 + bx + a)^{\frac{3}{2}}}{(fx^2 - d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(3/2)/x/(-f*x^2+d),x, algorithm="maxima")

[Out] -integrate((c*x^2 + b*x + a)^(3/2)/((f*x^2 - d)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^2 + bx + a)^{3/2}}{x(d - fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x + c*x^2)^(3/2)/(x*(d - f*x^2)),x)

[Out] int((a + b*x + c*x^2)^(3/2)/(x*(d - f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a\sqrt{a + bx + cx^2}}{-dx + fx^3} dx - \int \frac{bx\sqrt{a + bx + cx^2}}{-dx + fx^3} dx - \int \frac{cx^2\sqrt{a + bx + cx^2}}{-dx + fx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x+a)**(3/2)/x/(-f*x**2+d),x)

[Out] -Integral(a*sqrt(a + b*x + c*x**2)/(-d*x + f*x**3), x) - Integral(b*x*sqrt(a + b*x + c*x**2)/(-d*x + f*x**3), x) - Integral(c*x**2*sqrt(a + b*x + c*x**2)/(-d*x + f*x**3), x)

$$3.89 \quad \int \frac{(a+bx+cx^2)^{3/2}}{x^2(d-fx^2)} dx$$

Optimal. Leaf size=463

$$\frac{(12acf + 3b^2f + 8c^2d) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}df} + \frac{3(4ac + b^2) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}d} + \frac{(af + b(-\sqrt{d})\sqrt{f} + c\sqrt{d})}{8\sqrt{c}d}$$

[Out] $-(c*x^2+b*x+a)^{(3/2)}/d/x-3/2*b*\operatorname{arctanh}(1/2*(b*x+2*a)/a^{(1/2)})/(c*x^2+b*x+a)^{(1/2)}*a^{(1/2)}/d+3/8*(4*a*c+b^2)*\operatorname{arctanh}(1/2*(2*c*x+b)/c^{(1/2)})/(c*x^2+b*x+a)^{(1/2)}/d/c^{(1/2)}-1/8*(12*a*c*f+3*b^2*f+8*c^2*d)*\operatorname{arctanh}(1/2*(2*c*x+b)/c^{(1/2)})/(c*x^2+b*x+a)^{(1/2)}/d/f/c^{(1/2)}+1/2*\operatorname{arctanh}(1/2*(b*d^{(1/2)}-2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}-b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)}/(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(1/2)}*(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(3/2)}/d^{(3/2)}/f+1/2*\operatorname{arctanh}(1/2*(b*d^{(1/2)}+2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}+b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)}/(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(1/2)}*(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(3/2)}/d^{(3/2)}/f+3/4*(2*c*x+3*b)*(c*x^2+b*x+a)^{(1/2)}/d-1/4*(2*c*x+5*b)*(c*x^2+b*x+a)^{(1/2)}/d$

Rubi [A] time = 1.20, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6725, 732, 814, 843, 621, 206, 724, 978, 1078, 1033}

$$\frac{(12acf + 3b^2f + 8c^2d) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}df} + \frac{3(4ac + b^2) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}d} + \frac{(af + b(-\sqrt{d})\sqrt{f} + c\sqrt{d})}{8\sqrt{c}d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + c*x^2)^(3/2)/(x^2*(d - f*x^2)), x]

[Out] $(3*(3*b + 2*c*x)*\operatorname{Sqrt}[a + b*x + c*x^2])/(4*d) - ((5*b + 2*c*x)*\operatorname{Sqrt}[a + b*x + c*x^2])/(4*d) - (a + b*x + c*x^2)^{(3/2)}/(d*x) - (3*\operatorname{Sqrt}[a]*b*\operatorname{ArcTanh}[(2*a + b*x)/(2*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*d) + (3*(b^2 + 4*a*c)*\operatorname{ArcTanh}[(b + 2*c*x)/(2*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(8*\operatorname{Sqrt}[c]*d) - ((8*c^2*d + 3*b^2*f + 12*a*c*f)*\operatorname{ArcTanh}[(b + 2*c*x)/(2*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(8*\operatorname{Sqrt}[c]*d*f) + ((c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)^{(3/2)}*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] - 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] - b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*d^{(3/2)}*f) + ((c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)^{(3/2)}*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] + 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] + b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*d^{(3/2)}*f)$

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/
Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Sym
bol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_S
ymbol] := Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
|| LtQ[m, -1]) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c
_)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p +
2) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1)))*x, x], x]
/; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843


```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 978

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((b*(3*p + 2*q) + 2*c*(p + q)*x)*(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^(q + 1))/(2*f*(p + q)*(2*p + 2*q + 1)), x] - Dist[1/(2*f*(p + q)*(2*p + 2*q + 1)), Int[(a + b*x + c*x^2)^(p - 2)*(d + f*x^2)^q*Simp[b^2*d*(p - 1)*(2*p + q) - (p + q)*(b^2*d*(1 - p) - 2*a*(c*d - a*f*(2*p + 2*q + 1)) - (2*b*(c*d - a*f)*(1 - p)*(2*p + q) - 2*(p + q)*b*(2*c*d*(2*p + q) - (c*d + a*f)*(2*p + 2*q + 1)))*x + (b^2*f*p*(1 - p) + 2*c*(p + q)*(c*d*(2*p - 1) - a*f*(4*p + 2*q - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*q + 1, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2], x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2], x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx+cx^2)^{3/2}}{x^2(d-fx^2)} dx &= \int \left(\frac{(a+bx+cx^2)^{3/2}}{dx^2} + \frac{f(a+bx+cx^2)^{3/2}}{d(d-fx^2)} \right) dx \\
&= \frac{\int \frac{(a+bx+cx^2)^{3/2}}{x^2} dx}{d} + \frac{f \int \frac{(a+bx+cx^2)^{3/2}}{d-fx^2} dx}{d} \\
&= -\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4d} - \frac{(a+bx+cx^2)^{3/2}}{dx} + \frac{\int \frac{\frac{1}{4}(5b^2d+4a(cd+2af))+4b(cd+af)x+\frac{1}{4}(8c^2d+3b^2)}{\sqrt{a+bx+cx^2}(d-fx^2)} dx}{2d} \\
&= \frac{3(3b+2cx)\sqrt{a+bx+cx^2}}{4d} - \frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4d} - \frac{(a+bx+cx^2)^{3/2}}{dx} - \frac{3 \int \frac{-4abc}{x\sqrt{a+bx+cx^2}} dx}{4d} \\
&= \frac{3(3b+2cx)\sqrt{a+bx+cx^2}}{4d} - \frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4d} - \frac{(a+bx+cx^2)^{3/2}}{dx} + \frac{(3ab) \int \frac{1}{x} dx}{4d} \\
&= \frac{3(3b+2cx)\sqrt{a+bx+cx^2}}{4d} - \frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4d} - \frac{(a+bx+cx^2)^{3/2}}{dx} - \frac{(8c^2d+3b^2) \sqrt{a+bx+cx^2}}{4d} \\
&= \frac{3(3b+2cx)\sqrt{a+bx+cx^2}}{4d} - \frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4d} - \frac{(a+bx+cx^2)^{3/2}}{dx} - \frac{3\sqrt{a}b \operatorname{arctanh}\left(\frac{b+cx}{\sqrt{a+bx+cx^2}}\right)}{4d}
\end{aligned}$$

Mathematica [A] time = 0.64, size = 765, normalized size = 1.65

$$2c^{3/2}d^{3/2}x \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right) + 2a\sqrt{d}f\sqrt{a+x(b+cx)} + cdx\sqrt{af+b(-\sqrt{d})}\sqrt{f+cd} \tanh^{-1}\left(\frac{2a\sqrt{f}-bv}{2\sqrt{a+x(b+cx)}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + c*x^2)^(3/2)/(x^2*(d - f*x^2)), x]

[Out] $-1/2*(2*a*\sqrt{d}*f*\sqrt{a+x*(b+c*x)} + 3*\sqrt{a}*b*\sqrt{d}*f*x*\operatorname{ArcTanh}[(2*a+b*x)/(2*\sqrt{a}*\sqrt{a+x*(b+c*x)})] + 2*c^(3/2)*d^(3/2)*x*\operatorname{ArcTanh}[(b+2*c*x)/(2*\sqrt{c}*\sqrt{a+x*(b+c*x)})] + c*d*\sqrt{c*d-b}*\sqrt{d}*\sqrt{f+a*f}*\operatorname{ArcTanh}[(-b*\sqrt{d})+2*a*\sqrt{f}-2*c*\sqrt{d}*x+b*\sqrt{f}*x)/(2*\sqrt{c*d-b}*\sqrt{d}*\sqrt{f+a*f}*\sqrt{a+x*(b+c*x)})] - b*\sqrt{d}*\sqrt{f}*\sqrt{c*d-b}*\sqrt{d}*\sqrt{f+a*f}*\operatorname{ArcTanh}[(-b*\sqrt{d})+2*a*\sqrt{f}-2*c*\sqrt{d}*x+b*\sqrt{f}*x)/(2*\sqrt{c*d-b}*\sqrt{d}*\sqrt{f+a*f}*\sqrt{a+x*(b+c*x)})]$

$$d]) + 2*a*\text{Sqrt}[f] - 2*c*\text{Sqrt}[d]*x + b*\text{Sqrt}[f]*x)/(2*\text{Sqrt}[c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{Sqrt}[a + x*(b + c*x)]) + a*f*\text{Sqrt}[c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*x*\text{ArcTanh}[(-b*\text{Sqrt}[d]) + 2*a*\text{Sqrt}[f] - 2*c*\text{Sqrt}[d]*x + b*\text{Sqrt}[f]*x)/(2*\text{Sqrt}[c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{Sqrt}[a + x*(b + c*x)]) + c*d*\text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*x*\text{ArcTanh}[(-2*(a*\text{Sqrt}[f] + c*\text{Sqrt}[d]*x) - b*(\text{Sqrt}[d] + \text{Sqrt}[f]*x))/(2*\text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{Sqrt}[a + x*(b + c*x)]) + b*\text{Sqrt}[d]*\text{Sqrt}[f]*\text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*x*\text{ArcTanh}[(-2*(a*\text{Sqrt}[f] + c*\text{Sqrt}[d]*x) - b*(\text{Sqrt}[d] + \text{Sqrt}[f]*x))/(2*\text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{Sqrt}[a + x*(b + c*x)]) + a*f*\text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*x*\text{ArcTanh}[(-2*(a*\text{Sqrt}[f] + c*\text{Sqrt}[d]*x) - b*(\text{Sqrt}[d] + \text{Sqrt}[f]*x))/(2*\text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{Sqrt}[a + x*(b + c*x)])]/(d^(3/2)*f*x)$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(3/2)/x^2/(-f*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(3/2)/x^2/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);;OUTPUT:Error: Bad Argument Type

maple [B] time = 0.02, size = 4799, normalized size = 10.37

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x+a)^(3/2)/x^2/(-f*x^2+d),x)

[Out]
$$-1/6*f/d/(d*f)^{(1/2)}*((x-(d*f)^{(1/2)}/f)^{2*c+(b*f+2*(d*f)^{(1/2)}*c)}*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)}*b)/f)^{(3/2)}-1/4/d*((x-(d*f)^{(1/2)}/f)^{2*c+(b*f+2*(d*f)^{(1/2)}*c)}*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)}*b)/f)^{(1/2)}*x*c-3/4/d*\ln(((x-(d*f)^{(1/2)}/f)*c+1/2*(b*f+2*(d*f)^{(1/2)}*c)/f)/c^{(1/2)}+(x-(d*f)^{(1/2)}/f)^{2*c+(b*f+2*(d*f)^{(1/2)}*c)}*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)}*b)/f)$$

$$\begin{aligned}
& x+(d*f)^{(1/2)}/f)/f+2*((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*((x+(d*f)^{(1/2)}/f)^2 \\
& *c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)} \\
&)/(x+(d*f)^{(1/2)}/f))*a^2-1/2/f*d/(d*f)^{(1/2)}/((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)} \\
& *ln((2*(a*f+c*d-(d*f)^{(1/2)*b})/f+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/ \\
& f)/f+2*((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f) \\
&)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)})/(x+(d*f)^{(\\
& 1/2)}/f))*c^2+1/d/((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*ln((2*(a*f+c*d+(d*f)^{(1/ \\
& 2)*b)/f+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+2*((a*f+c*d+(d*f)^{(1/2)*b} \\
&)/f)^{(1/2)}*((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f \\
& +(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)})/(x-(d*f)^{(1/2)}/f))*b*a+1/f/((a*f+c*d+(d*f) \\
&)^{(1/2)*b})/f)^{(1/2)}*ln((2*(a*f+c*d+(d*f)^{(1/2)*b})/f+(b*f+2*(d*f)^{(1/2)*c})* \\
& (x-(d*f)^{(1/2)}/f)/f+2*((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*((x-(d*f)^{(1/2)}/f)^ \\
& 2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1 \\
& /2)})/(x-(d*f)^{(1/2)}/f))*b*c+1/2*f/d/(d*f)^{(1/2)}*((x+(d*f)^{(1/2)}/f)^2*c+(b*f \\
& -2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*a+1/ \\
& d/((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*ln((2*(a*f+c*d-(d*f)^{(1/2)*b})/f+(b*f-2* \\
& (d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+2*((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*((x+ \\
& (d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f) \\
&)^{(1/2)*b})/f)^{(1/2)})/(x+(d*f)^{(1/2)}/f))*b*a+1/f/((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(\\
& 1/2)}*ln((2*(a*f+c*d-(d*f)^{(1/2)*b})/f+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/ \\
& f)/f+2*((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f) \\
&)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)})/(x+(d*f)^{(\\
& 1/2)}/f))*b*c-1/(d*f)^{(1/2)}/((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*ln((2*(a*f+c*d \\
& -(d*f)^{(1/2)*b})/f+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+2*((a*f+c*d-(d* \\
& f)^{(1/2)*b})/f)^{(1/2)}*((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(\\
& 1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)})/(x+(d*f)^{(1/2)}/f))*a*c+1/d*c/a \\
& *(c*x^2+b*x+a)^{(3/2)}*x-5/8/d*((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)*c})*(\\
& x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*b-1/2/(d*f)^{(1/2)}*((x-(\\
& d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(\\
& 1/2)*b})/f)^{(1/2)}*c-1/2/f*ln(((x-(d*f)^{(1/2)}/f)*c+1/2*(b*f+2*(d*f)^{(1/2)*c} \\
&)/f)/c^{(1/2)}+((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/ \\
& f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)})*c^{(3/2)}+9/4/d*b*(c*x^2+b*x+a)^{(1/2)}-5/8 \\
& /d*((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+(a*f+c* \\
& d-(d*f)^{(1/2)*b})/f)^{(1/2)}*b+1/2/(d*f)^{(1/2)}*((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(\\
& d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*c-1/2/f* \\
& ln(((x+(d*f)^{(1/2)}/f)*c+1/2*(b*f-2*(d*f)^{(1/2)*c})/f)/c^{(1/2)}+((x+(d*f)^{(1/2) \\
&)/f)^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b})/ \\
& f)^{(1/2)})*c^{(3/2)}+3/2/d*c*(c*x^2+b*x+a)^{(1/2)}*x+3/2/d*c^{(1/2)}*a*ln((c*x+1/2 \\
& *b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})-1/d/a/x*(c*x^2+b*x+a)^{(5/2)}+3/8*f/d/(d*f)^{(\\
& 1/2)}/c^{(1/2)}*ln(((x+(d*f)^{(1/2)}/f)*c+1/2*(b*f-2*(d*f)^{(1/2)*c})/f)/c^{(1/2)}+ \\
& ((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(\\
& d*f)^{(1/2)*b})/f)^{(1/2)})*a*b-3/8*f/d/(d*f)^{(1/2)}/c^{(1/2)}*ln(((x-(d*f)^{(1/2)}/ \\
& f)*c+1/2*(b*f+2*(d*f)^{(1/2)*c})/f)/c^{(1/2)}+((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d* \\
& f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)})*a*b+1/(d* \\
& f)^{(1/2)}/((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*ln((2*(a*f+c*d+(d*f)^{(1/2)*b})/f+
\end{aligned}$$

$$(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+2*((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)*((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2))}/(x-(d*f)^{(1/2)}/f))*a*c-1/2*f/d/(d*f)^{(1/2)*((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)*a-3/2/d*b*a^{(1/2)*\ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)*a^{(1/2)})/x)}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(cx^2 + bx + a)^{\frac{3}{2}}}{(fx^2 - d)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(3/2)/x^2/(-f*x^2+d),x, algorithm="maxima")

[Out] -integrate((c*x^2 + b*x + a)^(3/2)/((f*x^2 - d)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^2 + bx + a)^{3/2}}{x^2 (d - fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x + c*x^2)^(3/2)/(x^2*(d - f*x^2)),x)

[Out] int((a + b*x + c*x^2)^(3/2)/(x^2*(d - f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a\sqrt{a+bx+cx^2}}{-dx^2+fx^4} dx - \int \frac{bx\sqrt{a+bx+cx^2}}{-dx^2+fx^4} dx - \int \frac{cx^2\sqrt{a+bx+cx^2}}{-dx^2+fx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x+a)**(3/2)/x**2/(-f*x**2+d),x)

[Out] -Integral(a*sqrt(a + b*x + c*x**2)/(-d*x**2 + f*x**4), x) - Integral(b*x*sqrt(a + b*x + c*x**2)/(-d*x**2 + f*x**4), x) - Integral(c*x**2*sqrt(a + b*x + c*x**2)/(-d*x**2 + f*x**4), x)

$$3.90 \quad \int \frac{(a+bx+cx^2)^{3/2}}{x^3(d-fx^2)} dx$$

Optimal. Leaf size=614

$$\frac{a^{3/2} f \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2} - \frac{bf(b^2-12ac) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d^2} - \frac{\sqrt{a+bx+cx^2}(8acf+b^2f+2bcfx+8c^2d)}{8cd^2}$$

[Out] $-1/2*(c*x^2+b*x+a)^{(3/2)}/d/x^2-a^{(3/2)}*f*\operatorname{arctanh}(1/2*(b*x+2*a)/a^{(1/2)})/(c*x^2+b*x+a)^{(1/2)}/d^2-1/16*b*(-12*a*c+b^2)*f*\operatorname{arctanh}(1/2*(2*c*x+b)/c^{(1/2)})/(c*x^2+b*x+a)^{(1/2)}/c^{(3/2)}/d^2-1/16*b*(12*a*c*f-b^2*f+24*c^2*d)*\operatorname{arctanh}(1/2*(2*c*x+b)/c^{(1/2)})/(c*x^2+b*x+a)^{(1/2)}/c^{(3/2)}/d^2-3/8*(4*a*c+b^2)*\operatorname{arctanh}(1/2*(b*x+2*a)/a^{(1/2)})/(c*x^2+b*x+a)^{(1/2)}/d/a^{(1/2)}+3/2*b*\operatorname{arctanh}(1/2*(2*c*x+b)/c^{(1/2)})/(c*x^2+b*x+a)^{(1/2)}*c^{(1/2)}/d-1/2*\operatorname{arctanh}(1/2*(b*d^{(1/2)}-2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}-b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)}/(c*d+a*f-b*d^{(1/2)})*f^{(1/2)}^{(1/2)}*(c*d+a*f-b*d^{(1/2)})*f^{(1/2)}^{(3/2)}/d^2/f^{(1/2)}+1/2*\operatorname{arctanh}(1/2*(b*d^{(1/2)}+2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}+b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)}/(c*d+a*f+b*d^{(1/2)})*f^{(1/2)}^{(1/2)}*(c*d+a*f+b*d^{(1/2)})*f^{(1/2)}^{(3/2)}/d^2/f^{(1/2)}-3/4*(-2*c*x+b)*(c*x^2+b*x+a)^{(1/2)}/d/x+1/8*f*(2*b*c*x+8*a*c+b^2)*(c*x^2+b*x+a)^{(1/2)}/c/d^2-1/8*(2*b*c*f*x+8*a*c*f+b^2*f+8*c^2*d)*(c*x^2+b*x+a)^{(1/2)}/c/d^2$

Rubi [A] time = 1.44, antiderivative size = 614, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {6725, 732, 812, 843, 621, 206, 724, 734, 814, 1021, 1070, 1078, 1033}

$$\frac{a^{3/2} f \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2} - \frac{\sqrt{a+bx+cx^2}(8acf+b^2f+2bcfx+8c^2d)}{8cd^2} - \frac{bf(b^2-12ac) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + c*x^2)^(3/2)/(x^3*(d - f*x^2)), x]

[Out] $(-3*(b-2*c*x)*\operatorname{Sqrt}[a+b*x+c*x^2])/(4*d*x) + (f*(b^2+8*a*c+2*b*c*x)*\operatorname{Sqrt}[a+b*x+c*x^2])/(8*c*d^2) - ((8*c^2*d+b^2*f+8*a*c*f+2*b*c*f*x)*\operatorname{Sqrt}[a+b*x+c*x^2])/(8*c*d^2) - (a+b*x+c*x^2)^{(3/2)}/(2*d*x^2) - (3*(b^2+4*a*c)*\operatorname{ArcTanh}[(2*a+b*x)/(2*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[a+b*x+c*x^2])])/(8*\operatorname{Sqrt}[a]*d) - (a^{(3/2)}*f*\operatorname{ArcTanh}[(2*a+b*x)/(2*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[a+b*x+c*x^2])])/d^2 + (3*b*\operatorname{Sqrt}[c]*\operatorname{ArcTanh}[(b+2*c*x)/(2*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a+b*x+c*x^2])])$

$$\frac{1}{(2*d)} - \frac{(b*(b^2 - 12*a*c)*f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])}{(16*c^(3/2)*d^2)} - \frac{(b*(24*c^2*d - b^2*f + 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])}{(16*c^(3/2)*d^2)} - \frac{((c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x]/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + b*x + c*x^2]])}{(2*d^2*Sqrt[f])} + \frac{((c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x]/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + b*x + c*x^2]])}{(2*d^2*Sqrt[f])}$$

Rule 206

$$\text{Int}[(a_) + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$$

Rule 621

$$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] \rightarrow \text{Dist}[2, \text{Subst}[\text{Int}[1/(4*c - x^2), x], x, (b + 2*c*x)/\text{Sqrt}[a + b*x + c*x^2]], x] /; \text{FreeQ}\{a, b, c\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$$

Rule 724

$$\text{Int}[1/(((d_) + (e_)*(x_))*\text{Sqrt}[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/\text{Sqrt}[a + b*x + c*x^2]], x] /; \text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0]$$

Rule 732

$$\text{Int}(((d_) + (e_)*(x_))^{(m_)}*((a_) + (b_)*(x_) + (c_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}(((d + e*x)^{(m+1)}*(a + b*x + c*x^2)^p)/(e*(m+1)), x] - \text{Dist}[p/(e*(m+1)), \text{Int}[(d + e*x)^{(m+1)}*(b + 2*c*x)*(a + b*x + c*x^2)^{(p-1)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, m\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ (\text{IntegerQ}[p] \ || \ \text{LtQ}[m, -1]) \ \&\& \ \text{NeQ}[m, -1] \ \&\& \ !\text{LtQ}[m + 2*p + 1, 0] \ \&\& \ \text{IntQuadraticQ}[a, b, c, d, e, m, p, x]$$

Rule 734

$$\text{Int}(((d_) + (e_)*(x_))^{(m_)}*((a_) + (b_)*(x_) + (c_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}(((d + e*x)^{(m+1)}*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x] - \text{Dist}[p/(e*(m + 2*p + 1)), \text{Int}[(d + e*x)^m*\text{Simp}[b*d - 2*a*e + (2*c*d - b*e)*x, x]*(a + b*x + c*x^2)^{(p-1)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, m\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{NeQ}[m + 2*p + 1, 0] \ \&\& \ (!\text{RationalQ}[m] \ || \ \text{LtQ}[m, 1]) \ \&$$

& !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]

Rule 812

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m + 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !RationalQ[m])) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 814

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1)))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 843

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

Rule 1021

Int[((g_.) + (h_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q + 1))

1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]

Rule 1033

Int[((g_.) + (h_.)*(x_))/(((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]

Rule 1070

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_.) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((B*c*f*(2*p + 2*q + 3) + C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*(C*(-(b*f)))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3)))] + (2*p*(c*d - a*f)*(C*(-(b*f)))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(-(b*c*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3))))*x + (p*(-(b*f))*(C*(-(b*f)))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]

Rule 1078

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f, 0]

Rule 6725

Int[(u_)/((a_.) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx+cx^2)^{3/2}}{x^3(d-fx^2)} dx &= \int \left(\frac{(a+bx+cx^2)^{3/2}}{dx^3} + \frac{f(a+bx+cx^2)^{3/2}}{d^2x} + \frac{f^2x(a+bx+cx^2)^{3/2}}{d^2(d-fx^2)} \right) dx \\
&= \frac{\int \frac{(a+bx+cx^2)^{3/2}}{x^3} dx}{d} + \frac{f \int \frac{(a+bx+cx^2)^{3/2}}{x} dx}{d^2} + \frac{f^2 \int \frac{x(a+bx+cx^2)^{3/2}}{d-fx^2} dx}{d^2} \\
&= -\frac{(a+bx+cx^2)^{3/2}}{2dx^2} + \frac{3 \int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x^2} dx}{4d} + \frac{f \int \frac{\sqrt{a+bx+cx^2} \left(\frac{3bd}{2} + 3(cd+af)x + \frac{3}{2}bfx^2 \right)}{d-fx^2} dx}{3d^2} \\
&= -\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx} + \frac{f(b^2+8ac+2bcx)\sqrt{a+bx+cx^2}}{8cd^2} - \frac{(8c^2d+b^2f+8a)}{8cd^2} \\
&= -\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx} + \frac{f(b^2+8ac+2bcx)\sqrt{a+bx+cx^2}}{8cd^2} - \frac{(8c^2d+b^2f+8a)}{8cd^2} \\
&= -\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx} + \frac{f(b^2+8ac+2bcx)\sqrt{a+bx+cx^2}}{8cd^2} - \frac{(8c^2d+b^2f+8a)}{8cd^2} \\
&= -\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx} + \frac{f(b^2+8ac+2bcx)\sqrt{a+bx+cx^2}}{8cd^2} - \frac{(8c^2d+b^2f+8a)}{8cd^2} \\
&= -\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx} + \frac{f(b^2+8ac+2bcx)\sqrt{a+bx+cx^2}}{8cd^2} - \frac{(8c^2d+b^2f+8a)}{8cd^2}
\end{aligned}$$

Mathematica [A] time = 0.97, size = 303, normalized size = 0.49

$$\frac{(4a(2af+3cd)+3b^2d) \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{\sqrt{a}} - \frac{4(af+b(-\sqrt{d})\sqrt{f}+cd)^{3/2} \tanh^{-1}\left(\frac{2a\sqrt{f}-b\sqrt{d}+b\sqrt{f}x-2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{\sqrt{f}} + \frac{4(af+b\sqrt{d}\sqrt{f}+cd)^{3/2}}{8d^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + c*x^2)^(3/2)/(x^3*(d - f*x^2)), x]

[Out] -1/8*((2*d*(2*a + 5*b*x)*Sqrt[a + x*(b + c*x)]/x^2 + ((3*b^2*d + 4*a*(3*c*d + 2*a*f))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/Sqrt[a]

$$- (4*(c*d - b*\sqrt{d}*\sqrt{f} + a*f)^{(3/2)}*\text{ArcTanh}[(-b*\sqrt{d}) + 2*a*\sqrt{f} - 2*c*\sqrt{d}*x + b*\sqrt{f}*x)/(2*\sqrt{c*d - b*\sqrt{d}*\sqrt{f} + a*f}*\sqrt{a + x*(b + c*x)}]))/\sqrt{f} + (4*(c*d + b*\sqrt{d}*\sqrt{f} + a*f)^{(3/2)}*\text{ArcTanh}[(-2*(a*\sqrt{f} + c*\sqrt{d}*x) - b*(\sqrt{d} + \sqrt{f}*x))/(2*\sqrt{c*d + b*\sqrt{d}*\sqrt{f} + a*f}*\sqrt{a + x*(b + c*x)}]))/\sqrt{f})/d^2$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(3/2)/x^3/(-f*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage₂

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(3/2)/x^3/(-f*x^2+d),x, algorithm="giac")

[Out] sage2

maple [B] time = 0.02, size = 5056, normalized size = 8.23

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x+a)^(3/2)/x^3/(-f*x^2+d),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{(cx^2 + bx + a)^{\frac{3}{2}}}{(fx^2 - d)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(3/2)/x^3/(-f*x^2+d),x, algorithm="maxima")

[Out] -integrate((c*x^2 + b*x + a)^(3/2)/((f*x^2 - d)*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^2 + bx + a)^{3/2}}{x^3 (d - fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x + c*x^2)^(3/2)/(x^3*(d - f*x^2)), x)

[Out] int((a + b*x + c*x^2)^(3/2)/(x^3*(d - f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a\sqrt{a + bx + cx^2}}{-dx^3 + fx^5} dx - \int \frac{bx\sqrt{a + bx + cx^2}}{-dx^3 + fx^5} dx - \int \frac{cx^2\sqrt{a + bx + cx^2}}{-dx^3 + fx^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x+a)**(3/2)/x**3/(-f*x**2+d), x)

[Out] -Integral(a*sqrt(a + b*x + c*x**2)/(-d*x**3 + f*x**5), x) - Integral(b*x*sqrt(a + b*x + c*x**2)/(-d*x**3 + f*x**5), x) - Integral(c*x**2*sqrt(a + b*x + c*x**2)/(-d*x**3 + f*x**5), x)

$$3.91 \quad \int \frac{(a+bx+cx^2)^{3/2}}{1-x^2} dx$$

Optimal. Leaf size=189

$$-\frac{(12ac + 3b^2 + 8c^2) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}} - \frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2} - \frac{1}{2}(a-b+c)^{3/2} \tanh^{-1}\left(\frac{2a+x(b-2c)}{2\sqrt{a-b+c}\sqrt{a+bx+cx^2}}\right)$$

[Out] $-1/2*(a-b+c)^{(3/2)}*\operatorname{arctanh}(1/2*(2*a-b+(b-2*c)*x)/(a-b+c)^{(1/2)/(c*x^2+b*x+a)^{(1/2)})+1/2*(a+b+c)^{(3/2)}*\operatorname{arctanh}(1/2*(2*a+b+(b+2*c)*x)/(a+b+c)^{(1/2)/(c*x^2+b*x+a)^{(1/2)})-1/8*(12*a*c+3*b^2+8*c^2)*\operatorname{arctanh}(1/2*(2*c*x+b)/c^{(1/2)/(c*x^2+b*x+a)^{(1/2)})/c^{(1/2)}-1/4*(2*c*x+5*b)*(c*x^2+b*x+a)^{(1/2)}$

Rubi [A] time = 0.28, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {978, 1078, 621, 206, 1033, 724}

$$-\frac{(12ac + 3b^2 + 8c^2) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}} - \frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2} - \frac{1}{2}(a-b+c)^{3/2} \tanh^{-1}\left(\frac{2a+x(b-2c)}{2\sqrt{a-b+c}\sqrt{a+bx+cx^2}}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + c*x^2)^(3/2)/(1 - x^2), x]

[Out] $-((5*b + 2*c*x)*\operatorname{Sqrt}[a + b*x + c*x^2])/4 - ((a - b + c)^{(3/2)}*\operatorname{ArcTanh}[(2*a - b + (b - 2*c)*x)/(2*\operatorname{Sqrt}[a - b + c]*\operatorname{Sqrt}[a + b*x + c*x^2]])/2 - ((3*b^2 + 12*a*c + 8*c^2)*\operatorname{ArcTanh}[(b + 2*c*x)/(2*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(8*\operatorname{Sqrt}[c]) + ((a + b + c)^{(3/2)}*\operatorname{ArcTanh}[(2*a + b + (b + 2*c)*x)/(2*\operatorname{Sqrt}[a + b + c]*\operatorname{Sqrt}[a + b*x + c*x^2])])/2$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol]
:> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 978

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x_Symbol]
:> Simp[((b*(3*p + 2*q) + 2*c*(p + q)*x)*(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^(q + 1))/(2*f*(p + q)*(2*p + 2*q + 1)), x] - Dist[1/(2*f*(p + q)*(2*p + 2*q + 1)), Int[(a + b*x + c*x^2)^(p - 2)*(d + f*x^2)^q*Simp[b^2*d*(p - 1)*(2*p + q) - (p + q)*(b^2*d*(1 - p) - 2*a*(c*d - a*f*(2*p + 2*q + 1)) - (2*b*(c*d - a*f)*(1 - p)*(2*p + q) - 2*(p + q)*b*(2*c*d*(2*p + q) - (c*d + a*f)*(2*p + 2*q + 1)))*x + (b^2*f*p*(1 - p) + 2*c*(p + q)*(c*d*(2*p - 1) - a*f*(4*p + 2*q - 1)))*x^2, x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*q + 1, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol]
:> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol]
:> Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx + cx^2)^{3/2}}{1 - x^2} dx &= -\frac{1}{4}(5b + 2cx)\sqrt{a + bx + cx^2} + \frac{1}{2} \int \frac{\frac{1}{4}(8a^2 + 5b^2 + 4ac) + 4b(a + c)x + \frac{1}{4}(3b^2 + 12ac - 8c^2)}{(1 - x^2)\sqrt{a + bx + cx^2}} dx \\
&= -\frac{1}{4}(5b + 2cx)\sqrt{a + bx + cx^2} - \frac{1}{2} \int \frac{\frac{1}{4}(-8a^2 - 5b^2 - 4ac) + \frac{1}{4}(-3b^2 - 12ac - 8c^2) - 4b(a + c)x}{(1 - x^2)\sqrt{a + bx + cx^2}} dx \\
&= -\frac{1}{4}(5b + 2cx)\sqrt{a + bx + cx^2} - \frac{1}{2}(a - b + c)^2 \int \frac{1}{(-1 - x)\sqrt{a + bx + cx^2}} dx + \frac{1}{2}(a + b + c) \int \frac{1}{(-1 + x)\sqrt{a + bx + cx^2}} dx \\
&= -\frac{1}{4}(5b + 2cx)\sqrt{a + bx + cx^2} - \frac{(3b^2 + 12ac + 8c^2) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}} + (a - b + c) \int \frac{1}{(-1 + x)\sqrt{a + bx + cx^2}} dx \\
&= -\frac{1}{4}(5b + 2cx)\sqrt{a + bx + cx^2} - \frac{1}{2}(a - b + c)^{3/2} \tanh^{-1}\left(\frac{2a - b + (b - 2c)x}{2\sqrt{a - b + c}\sqrt{a + bx + cx^2}}\right) - \frac{1}{2}(a + b + c) \int \frac{1}{(-1 + x)\sqrt{a + bx + cx^2}} dx
\end{aligned}$$

Mathematica [A] time = 0.58, size = 181, normalized size = 0.96

$$\frac{1}{8} \left(\frac{(4c(3a + 2c) + 3b^2) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}} - 2(5b + 2cx)\sqrt{a + x(b + cx)} - 4(a - b + c)^{3/2} \tanh^{-1}\left(\frac{2a + (b - 2c)x}{2\sqrt{a - b + c}\sqrt{a + bx + cx^2}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + c*x^2)^(3/2)/(1 - x^2), x]

[Out] (-2*(5*b + 2*c*x)*Sqrt[a + x*(b + c*x)] - 4*(a - b + c)^(3/2)*ArcTanh[(2*a + b*(-1 + x) - 2*c*x)/(2*Sqrt[a - b + c]*Sqrt[a + x*(b + c*x)]] - ((3*b^2 + 4*c*(3*a + 2*c))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + 4*(a + b + c)^(3/2)*ArcTanh[(2*a + b + b*x + 2*c*x)/(2*Sqrt[a + b + c]*Sqrt[a + x*(b + c*x)])])/8

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(3/2)/(-x^2+1), x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(3/2)/(-x^2+1),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT>Error: Bad Argument Type

maple [B] time = 0.02, size = 1346, normalized size = 7.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x+a)^(3/2)/(-x^2+1),x)

[Out]
$$-1/2*c^{3/2}*ln((1/2*b-c+c*(x+1))/c^{1/2}+((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^{(1/2)})-1/2*c^{3/2}*ln((1/2*b+c+c*(x-1))/c^{1/2}+((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^{(1/2)})+1/2*a*((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^{(1/2)}+1/2*c*((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^{(1/2)}-5/8*((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^{(1/2)}*b-1/2*a*((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^{(1/2)}-1/2*c*((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^{(1/2)}-5/8*((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^{(1/2)}*b+3/8*b/c^{1/2}*ln((1/2*b-c+c*(x+1))/c^{1/2}+((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^{(1/2)})*a-3/8*b/c^{1/2}*ln((1/2*b+c+c*(x-1))/c^{1/2}+((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^{(1/2)})*a-1/2*a*(a-b+c)^{(1/2)}*ln((2*a-2*b+2*c+(b-2*c)*(x+1)+2*(a-b+c)^{(1/2)}*((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^{(1/2)})/(x+1))+3/4*b*ln((1/2*b-c+c*(x+1))/c^{1/2}+((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^{(1/2)})*c^{1/2}+1/2*b*(a-b+c)^{(1/2)}*ln((2*a-2*b+2*c+(b-2*c)*(x+1)+2*(a-b+c)^{(1/2)}*((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^{(1/2)})/(x+1))-1/2*c*(a-b+c)^{(1/2)}*ln((2*a-2*b+2*c+(b-2*c)*(x+1)+2*(a-b+c)^{(1/2)}*((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^{(1/2)})/(x+1))+1/8*b*((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^{(1/2)}*x+1/16/c*((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^{(1/2)}*b^2-1/32/c^{3/2}*ln((1/2*b-c+c*(x+1))/c^{1/2}+((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^{(1/2)})*b^3-1/4*c*((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^{(1/2)}*x+1/2*a*(a+b+c)^{(1/2)}*ln((2*a+2*b+2*c+(b+2*c)*(x-1)+2*(a+b+c)^{(1/2)}*((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^{(1/2)})/(x-1))-3/4*b*ln((1/2*b+c+c*(x-1))/c^{1/2}+((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^{(1/2)})*c^{1/2}+1/2*b*(a+b+c)^{(1/2)}*ln((2*a+2*b+2*c+(b+2*c)*(x-1)+2*(a+b+c)^{(1/2)}*((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^{(1/2)})/(x-1))+1/2*c*(a+b+c)^{(1/2)}*ln((2*a+2*b+2*c+(b+2*c)*(x-1)+2*(a+b+c)^{(1/2)}*((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^{(1/2)})/(x-1))-1/8*b*((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^{(1/2)}*x-1/16/c*((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^{(1/2)}*b^2+1/32/c^{3/2}*ln((1/2*b+c+c*(x-1))/c^{1/2}+((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^{(1/2)})*b^3-1/4*c*((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^{(1/2)}*x-3/16/c^{1/2}*ln((1/2*b-c+c*(x+1))/c^{1/2}+((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^{(1/2)})*b^2-3/4*c^{1/2}*ln((1/2*b-c+c*(x+1))/c^{1/2}+((x+1)^2*c+(b-2*c)$$

```

*(x+1)+a-b+c)^(1/2))*a-3/4*c^(1/2)*ln((1/2*b+c+c*(x-1))/c^(1/2)+((x-1)^2*c+
(b+2*c)*(x-1)+a+b+c)^(1/2))*a-3/16/c^(1/2)*ln((1/2*b+c+c*(x-1))/c^(1/2)+((x
-1)^2*c+(b+2*c)*(x-1)+a+b+c)^(1/2))*b^2+1/6*((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)
^(3/2)-1/6*((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^(3/2)

```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(-x^2+1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{(cx^2 + bx + a)^{3/2}}{x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*x + c*x^2)^(3/2)/(x^2 - 1),x)
```

```
[Out] -int((a + b*x + c*x^2)^(3/2)/(x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a\sqrt{a + bx + cx^2}}{x^2 - 1} dx - \int \frac{bx\sqrt{a + bx + cx^2}}{x^2 - 1} dx - \int \frac{cx^2\sqrt{a + bx + cx^2}}{x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(-x**2+1),x)
```

```
[Out] -Integral(a*sqrt(a + b*x + c*x**2)/(x**2 - 1), x) - Integral(b*x*sqrt(a + b
*x + c*x**2)/(x**2 - 1), x) - Integral(c*x**2*sqrt(a + b*x + c*x**2)/(x**2
- 1), x)
```

$$3.92 \quad \int \frac{\sqrt{-1-x+x^2}}{1-x^2} dx$$

Optimal. Leaf size=75

$$-\frac{1}{2} \tan^{-1}\left(\frac{3-x}{2\sqrt{x^2-x-1}}\right) + \tanh^{-1}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right) + \frac{1}{2} \tanh^{-1}\left(\frac{3x+1}{2\sqrt{x^2-x-1}}\right)$$

[Out] $-1/2*\arctan(1/2*(3-x)/(x^2-x-1)^{(1/2)})+\operatorname{arctanh}(1/2*(1-2*x)/(x^2-x-1)^{(1/2)})+1/2*\operatorname{arctanh}(1/2*(1+3*x)/(x^2-x-1)^{(1/2)})$

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {990, 621, 206, 1033, 724, 204}

$$-\frac{1}{2} \tan^{-1}\left(\frac{3-x}{2\sqrt{x^2-x-1}}\right) + \tanh^{-1}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right) + \frac{1}{2} \tanh^{-1}\left(\frac{3x+1}{2\sqrt{x^2-x-1}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-1 - x + x^2]/(1 - x^2), x]

[Out] $-\operatorname{ArcTan}[(3-x)/(2*\operatorname{Sqrt}[-1-x+x^2])]/2 + \operatorname{ArcTanh}[(1-2*x)/(2*\operatorname{Sqrt}[-1-x+x^2])] + \operatorname{ArcTanh}[(1+3*x)/(2*\operatorname{Sqrt}[-1-x+x^2])]/2$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol]
:> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 990

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (f_.)*(x_)^2), x_Symbol]
:> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*
f - b*f*x)/(Sqrt[a + b*x + c*x^2]*(d + f*x^2)), x], x] /; FreeQ[{a, b, c, d
, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{-1-x+x^2}}{1-x^2} dx &= -\int \frac{1}{\sqrt{-1-x+x^2}} dx - \int \frac{x}{(1-x^2)\sqrt{-1-x+x^2}} dx \\ &= -\left(\frac{1}{2} \int \frac{1}{(-1-x)\sqrt{-1-x+x^2}} dx\right) - \frac{1}{2} \int \frac{1}{(1-x)\sqrt{-1-x+x^2}} dx - 2 \operatorname{Subst}\left(\int \frac{1}{4-x^2} dx, x, \frac{3-x}{\sqrt{-1-x+x^2}}\right) \\ &= \tanh^{-1}\left(\frac{1-2x}{2\sqrt{-1-x+x^2}}\right) + \operatorname{Subst}\left(\int \frac{1}{-4-x^2} dx, x, \frac{3-x}{\sqrt{-1-x+x^2}}\right) + \operatorname{Subst}\left(\int \frac{1}{4-x^2} dx, x, \frac{3-x}{\sqrt{-1-x+x^2}}\right) \\ &= -\frac{1}{2} \tan^{-1}\left(\frac{3-x}{2\sqrt{-1-x+x^2}}\right) + \tanh^{-1}\left(\frac{1-2x}{2\sqrt{-1-x+x^2}}\right) + \frac{1}{2} \tanh^{-1}\left(\frac{1+3x}{2\sqrt{-1-x+x^2}}\right) \end{aligned}$$

Mathematica [A] time = 0.04, size = 75, normalized size = 1.00

$$-\frac{1}{2} \tan^{-1}\left(\frac{3-x}{2\sqrt{x^2-x-1}}\right) + \tanh^{-1}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right) + \frac{1}{2} \tanh^{-1}\left(\frac{3x+1}{2\sqrt{x^2-x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 - x + x^2]/(1 - x^2), x]
```

[Out] $-1/2*\text{ArcTan}[(3-x)/(2*\text{Sqrt}[-1-x+x^2])] + \text{ArcTanh}[(1-2*x)/(2*\text{Sqrt}[-1-x+x^2])] + \text{ArcTanh}[(1+3*x)/(2*\text{Sqrt}[-1-x+x^2])]/2$

fricas [A] time = 1.39, size = 70, normalized size = 0.93

$$\arctan\left(-x + \sqrt{x^2 - x - 1} + 1\right) - \frac{1}{2} \log\left(-x + \sqrt{x^2 - x - 1}\right) + \frac{1}{2} \log\left(-x + \sqrt{x^2 - x - 1} - 2\right) + \log\left(-2x + 2\sqrt{x^2 - x - 1} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2-x-1)^(1/2)/(-x^2+1),x, algorithm="fricas")`

[Out] $\arctan(-x + \text{sqrt}(x^2 - x - 1) + 1) - 1/2*\log(-x + \text{sqrt}(x^2 - x - 1)) + 1/2*\log(-x + \text{sqrt}(x^2 - x - 1) - 2) + \log(-2*x + 2*\text{sqrt}(x^2 - x - 1) + 1)$

giac [A] time = 0.20, size = 73, normalized size = 0.97

$$\arctan\left(-x + \sqrt{x^2 - x - 1} + 1\right) - \frac{1}{2} \log\left(\left|-x + \sqrt{x^2 - x - 1}\right|\right) + \frac{1}{2} \log\left(\left|-x + \sqrt{x^2 - x - 1} - 2\right|\right) + \log\left(\left|-2x + 2\sqrt{x^2 - x - 1} + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2-x-1)^(1/2)/(-x^2+1),x, algorithm="giac")`

[Out] $\arctan(-x + \text{sqrt}(x^2 - x - 1) + 1) - 1/2*\log(\text{abs}(-x + \text{sqrt}(x^2 - x - 1))) + 1/2*\log(\text{abs}(-x + \text{sqrt}(x^2 - x - 1) - 2)) + \log(\text{abs}(-2*x + 2*\text{sqrt}(x^2 - x - 1) + 1))$

maple [A] time = 0.02, size = 102, normalized size = 1.36

$$\frac{\arctanh\left(\frac{-3x-1}{2\sqrt{-3x+(x+1)^2-2}}\right)}{2} + \frac{\arctan\left(\frac{x-3}{2\sqrt{x+(x-1)^2-2}}\right)}{2} - \frac{3 \ln\left(x - \frac{1}{2} + \sqrt{-3x + (x+1)^2 - 2}\right)}{4} - \frac{\ln\left(x - \frac{1}{2} + \sqrt{x + (x-1)^2 - 2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2-x-1)^(1/2)/(-x^2+1),x)`

[Out] $-1/2*((x-1)^2+x-2)^(1/2)-1/4*\ln(-1/2+x+((x-1)^2+x-2)^(1/2))+1/2*\arctan(1/2*(-3+x)/((x-1)^2+x-2)^(1/2))+1/2*((x+1)^2-3*x-2)^(1/2)-3/4*\ln(-1/2+x+((x+1)^2-3*x-2)^(1/2))-1/2*\arctanh(1/2*(-1-3*x)/((x+1)^2-3*x-2)^(1/2))$

maxima [A] time = 0.97, size = 83, normalized size = 1.11

$$\frac{1}{2} \arcsin\left(\frac{2\sqrt{5}x}{5|2x-2|} - \frac{6\sqrt{5}}{5|2x-2|}\right) - \log\left(x + \sqrt{x^2 - x - 1} - \frac{1}{2}\right) - \frac{1}{2} \log\left(\frac{2\sqrt{x^2 - x - 1}}{|2x+2|} + \frac{2}{|2x+2|} - \frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-x-1)^(1/2)/(-x^2+1),x, algorithm="maxima")

[Out] 1/2*arcsin(2/5*sqrt(5)*x/abs(2*x - 2) - 6/5*sqrt(5)/abs(2*x - 2)) - log(x + sqrt(x^2 - x - 1) - 1/2) - 1/2*log(2*sqrt(x^2 - x - 1)/abs(2*x + 2) + 2/abs(2*x + 2) - 3/2)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{x^2 - x - 1}}{x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^2 - x - 1)^(1/2)/(x^2 - 1),x)

[Out] -int((x^2 - x - 1)^(1/2)/(x^2 - 1), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{x^2 - x - 1}}{x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2-x-1)**(1/2)/(-x**2+1),x)

[Out] -Integral(sqrt(x**2 - x - 1)/(x**2 - 1), x)

$$3.93 \quad \int \frac{(x+x^2)^{3/2}}{1+x^2} dx$$

Optimal. Leaf size=130

$$\frac{1}{4} \sqrt{x^2+x} (2x+5) + \sqrt{1+\sqrt{2}} \tan^{-1} \left(\frac{-x+\sqrt{2}+1}{\sqrt{2}(1+\sqrt{2})\sqrt{x^2+x}} \right) - \sqrt{\sqrt{2}-1} \tanh^{-1} \left(\frac{-x-\sqrt{2}+1}{\sqrt{2}(\sqrt{2}-1)\sqrt{x^2+x}} \right) - \frac{5}{4} \tan^{-1} \left(\frac{x}{\sqrt{x^2+x}} \right)$$

[Out] $-5/4*\arctanh(x/(x^2+x)^{(1/2)})+1/4*(5+2*x)*(x^2+x)^{(1/2)}-\arctanh((1-x-2^{(1/2)})/(x^2+x)^{(1/2)/(-2+2*2^{(1/2)})^{(1/2)})*(2^{(1/2)}-1)^{(1/2)}+\arctan((1-x+2^{(1/2)})/(x^2+x)^{(1/2)/(2+2*2^{(1/2)})^{(1/2)})*(1+2^{(1/2)})^{(1/2)})$

Rubi [A] time = 0.16, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {978, 1078, 620, 206, 12, 1036, 1030, 207, 203}

$$\frac{1}{4} \sqrt{x^2+x} (2x+5) + \sqrt{1+\sqrt{2}} \tan^{-1} \left(\frac{-x+\sqrt{2}+1}{\sqrt{2}(1+\sqrt{2})\sqrt{x^2+x}} \right) - \sqrt{\sqrt{2}-1} \tanh^{-1} \left(\frac{-x-\sqrt{2}+1}{\sqrt{2}(\sqrt{2}-1)\sqrt{x^2+x}} \right) - \frac{5}{4} \tan^{-1} \left(\frac{x}{\sqrt{x^2+x}} \right)$$

Antiderivative was successfully verified.

[In] Int[(x + x^2)^(3/2)/(1 + x^2), x]

[Out] $((5 + 2*x)*\text{Sqrt}[x + x^2])/4 + \text{Sqrt}[1 + \text{Sqrt}[2]]*\text{ArcTan}[(1 + \text{Sqrt}[2] - x)/(\text{Sqrt}[2*(1 + \text{Sqrt}[2]])*\text{Sqrt}[x + x^2])] - \text{Sqrt}[-1 + \text{Sqrt}[2]]*\text{ArcTanh}[(1 - \text{Sqrt}[2] - x)/(\text{Sqrt}[2*(-1 + \text{Sqrt}[2]])*\text{Sqrt}[x + x^2])] - (5*\text{ArcTanh}[x/\text{Sqrt}[x + x^2]])/4$

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

$Q[a, 0] \parallel LtQ[b, 0]$

Rule 207

$Int[((a_) + (b_)*(x_)^2)^{-1}, x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[\{a, b\}, x] \&\& NegQ[a/b] \&\& (LtQ[a, 0] \parallel GtQ[b, 0])$

Rule 620

$Int[1/Sqrt[(b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[\{b, c\}, x]$

Rule 978

$Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^{(p_)}*((d_) + (f_)*(x_)^2)^{(q_)}, x_Symbol] := Simp[((b*(3*p + 2*q) + 2*c*(p + q)*x)*(a + b*x + c*x^2)^{(p - 1)}*(d + f*x^2)^{(q + 1)})/(2*f*(p + q)*(2*p + 2*q + 1)), x] - Dist[1/(2*f*(p + q)*(2*p + 2*q + 1)), Int[(a + b*x + c*x^2)^{(p - 2)}*(d + f*x^2)^q*Simp[b^2*d*(p - 1)*(2*p + q) - (p + q)*(b^2*d*(1 - p) - 2*a*(c*d - a*f*(2*p + 2*q + 1))) - (2*b*(c*d - a*f)*(1 - p)*(2*p + q) - 2*(p + q)*b*(2*c*d*(2*p + q) - (c*d + a*f)*(2*p + 2*q + 1)))*x + (b^2*f*p*(1 - p) + 2*c*(p + q)*(c*d*(2*p - 1) - a*f*(4*p + 2*q - 1)))*x^2, x], x] /; FreeQ[\{a, b, c, d, f, q\}, x] \&\& NeQ[b^2 - 4*a*c, 0] \&\& GtQ[p, 1] \&\& NeQ[p + q, 0] \&\& NeQ[2*p + 2*q + 1, 0] \&\& !IGtQ[p, 0] \&\& !IGtQ[q, 0]$

Rule 1030

$Int[((g_) + (h_)*(x_))/(((a_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[\{a, c, d, e, f, g, h\}, x] \&\& EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]$

Rule 1036

$Int[((g_) + (h_)*(x_))/(((a_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := With[\{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]\}, Dist[1/(2*q), Int[Simp[-(a*h*e) - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[-(a*h*e) - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[\{a, c, d, e, f, g, h\}, x] \&\& NeQ[e^2 - 4*d*f, 0] \&\& NegQ[-(a*c)]$

Rule 1078


```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
+ f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(x+x^2)^{3/2}}{1+x^2} dx &= \frac{1}{4}(5+2x)\sqrt{x+x^2} - \frac{1}{2} \int \frac{\frac{5}{4} + 4x + \frac{5x^2}{4}}{(1+x^2)\sqrt{x+x^2}} dx \\
&= \frac{1}{4}(5+2x)\sqrt{x+x^2} - \frac{1}{2} \int \frac{4x}{(1+x^2)\sqrt{x+x^2}} dx - \frac{5}{8} \int \frac{1}{\sqrt{x+x^2}} dx \\
&= \frac{1}{4}(5+2x)\sqrt{x+x^2} - \frac{5}{4} \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \frac{x}{\sqrt{x+x^2}}\right) - 2 \int \frac{x}{(1+x^2)\sqrt{x+x^2}} dx \\
&= \frac{1}{4}(5+2x)\sqrt{x+x^2} - \frac{5}{4} \tanh^{-1}\left(\frac{x}{\sqrt{x+x^2}}\right) + \frac{\int \frac{-1+(-1-\sqrt{2})x}{(1+x^2)\sqrt{x+x^2}} dx}{\sqrt{2}} - \frac{\int \frac{-1+(-1+\sqrt{2})x}{(1+x^2)\sqrt{x+x^2}} dx}{\sqrt{2}} \\
&= \frac{1}{4}(5+2x)\sqrt{x+x^2} - \frac{5}{4} \tanh^{-1}\left(\frac{x}{\sqrt{x+x^2}}\right) + (-2+\sqrt{2}) \operatorname{Subst}\left(\int \frac{1}{2(1-\sqrt{2})+x^2} dx, x, \frac{x}{\sqrt{x+x^2}}\right) \\
&= \frac{1}{4}(5+2x)\sqrt{x+x^2} + \sqrt{1+\sqrt{2}} \tan^{-1}\left(\frac{1+\sqrt{2}-x}{\sqrt{2}(1+\sqrt{2})\sqrt{x+x^2}}\right) - \sqrt{-1+\sqrt{2}} \tanh^{-1}\left(\frac{x}{\sqrt{2}(1+\sqrt{2})\sqrt{x+x^2}}\right)
\end{aligned}$$

Mathematica [C] time = 0.16, size = 120, normalized size = 0.92

$$\frac{\sqrt{x}\sqrt{x+1}\left(2\sqrt{x+1}x^{3/2}+5\sqrt{x+1}\sqrt{x}+4(-1+i)^{3/2}\tan^{-1}\left(\sqrt{-1+i}\sqrt{\frac{x}{x+1}}\right)-5\sinh^{-1}(\sqrt{x})+4(1+i)^{3/2}\tanh^{-1}\left(\frac{x}{\sqrt{2}(1+\sqrt{2})\sqrt{x+x^2}}\right)\right)}{4\sqrt{x(x+1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(x + x^2)^(3/2)/(1 + x^2), x]

[Out] (Sqrt[x]*Sqrt[1 + x]*(5*Sqrt[x]*Sqrt[1 + x] + 2*x^(3/2)*Sqrt[1 + x] - 5*ArcSinh[Sqrt[x]] + 4*(-1 + I)^(3/2)*ArcTan[Sqrt[-1 + I]*Sqrt[x/(1 + x)]] + 4*(1 + I)^(3/2)*ArcTanh[Sqrt[1 + I]*Sqrt[x/(1 + x)]])/(4*Sqrt[x*(1 + x)])

fricas [B] time = 1.28, size = 777, normalized size = 5.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)^(3/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/8*8^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)*log(8*x^2 - 8*sqrt(x^2 + x)*
x + 2*(8^(1/4)*sqrt(x^2 + x)*(sqrt(2) - 1) - 8^(1/4)*(sqrt(2)*x - x - 1))*s
qrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4) + 1/8*8^(1/4)*sqrt(2*sqrt(2) + 4)
*(sqrt(2) - 2)*log(8*x^2 - 8*sqrt(x^2 + x)*x - 2*(8^(1/4)*sqrt(x^2 + x)*(sq
rt(2) - 1) - 8^(1/4)*(sqrt(2)*x - x - 1))*sqrt(2*sqrt(2) + 4) + 4*x + 4*sq
rt(2) + 4) + 1/2*8^(1/4)*sqrt(2)*sqrt(2*sqrt(2) + 4)*arctan(1/7*sqrt(2)*(sq
rt(2)*(5*x + 1) + 6*x + 4) + 1/112*sqrt(8*x^2 - 8*sqrt(x^2 + x)*x - 2*(8^(1/
4)*sqrt(x^2 + x)*(sqrt(2) - 1) - 8^(1/4)*(sqrt(2)*x - x - 1))*sqrt(2*sqrt(2
) + 4) + 4*x + 4*sqrt(2) + 4)*(8*sqrt(2)*(5*sqrt(2) + 6) + (8^(3/4)*(5*sqrt
(2) + 6) + 8*8^(1/4)*(2*sqrt(2) + 1))*sqrt(2*sqrt(2) + 4) + 64*sqrt(2) + 32
) - 1/7*sqrt(x^2 + x)*(sqrt(2)*(5*sqrt(2) + 6) + 8*sqrt(2) + 4) + 1/7*sqrt(
2)*(8*x + 3) + 1/56*(8^(3/4)*(sqrt(2)*(5*x + 1) + 6*x + 4) - sqrt(x^2 + x)*
(8^(3/4)*(5*sqrt(2) + 6) + 8*8^(1/4)*(2*sqrt(2) + 1))) + 8*8^(1/4)*(sqrt(2)*
(2*x - 1) + x + 3))*sqrt(2*sqrt(2) + 4) + 4/7*x + 5/7) + 1/2*8^(1/4)*sqrt(2
)*sqrt(2*sqrt(2) + 4)*arctan(-1/7*sqrt(2)*(sqrt(2)*(5*x + 1) + 6*x + 4) - 1
/112*sqrt(8*x^2 - 8*sqrt(x^2 + x)*x + 2*(8^(1/4)*sqrt(x^2 + x)*(sqrt(2) - 1
) - 8^(1/4)*(sqrt(2)*x - x - 1))*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4)
*(8*sqrt(2)*(5*sqrt(2) + 6) - (8^(3/4)*(5*sqrt(2) + 6) + 8*8^(1/4)*(2*sqrt(
2) + 1))*sqrt(2*sqrt(2) + 4) + 64*sqrt(2) + 32) + 1/7*sqrt(x^2 + x)*(sqrt(2
)*(5*sqrt(2) + 6) + 8*sqrt(2) + 4) - 1/7*sqrt(2)*(8*x + 3) + 1/56*(8^(3/4)*
(sqrt(2)*(5*x + 1) + 6*x + 4) - sqrt(x^2 + x)*(8^(3/4)*(5*sqrt(2) + 6) + 8*
8^(1/4)*(2*sqrt(2) + 1))) + 8*8^(1/4)*(sqrt(2)*(2*x - 1) + x + 3))*sqrt(2*sq
rt(2) + 4) - 4/7*x - 5/7) + 1/4*sqrt(x^2 + x)*(2*x + 5) + 5/8*log(-2*x + 2*
sqrt(x^2 + x) - 1)
```

```
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)^(3/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Unable to divide, perhaps due to rounding er
ror%%{poly1[29378258633931653019799718485334848549113596143372369654301
4566614744867108514065050877682658325195312500,-181983623107889624631149356
157406719337484171310120143823570373289503707930369037595766180522699542236
3281250,-457037569568074726029024220476354238878560215202443954711967761282
5585897607153309996504109115565063476562500,5657088028436513895060368238114
045087528706221186372034279204932374221645044901876954909719232468137744140
6250000]:[1,0,-31106,-99112,-65203241]%%}, [6]%%}+%%{poly1[563214846686
```

808121989333072018490142234944998851250120714869543444162112888787695442685
25719225646972656250, -69671463056492042794032844569028505467609156013361581
33975389261008891579718157298328760901400260223388671875, -18501639301691189
171876668611027555220095054439393285921280605772669698963742122921653330793
61994541992187500000, 211543560165990204169085849967411852854582551443377700
410708031605115841400857697288402456445542251729797363281250] : [1, 0, -31106, -
99112, -65203241]%%}, [5]%%}%+%%{%poly1 [2497527834892881694485372868168218
93432606364903446504002320498492700674161652642483438265732405822753906250,
102899415255489529312024941498712410711762889441826371792182609371037315453
9467963565591544030347656250000000, -829533110278394819601873642487583961356
9744462926182727988533904129604785618275766584587664596969303680419921875, -
621314982897707824179256504166985600883550135798254936235154038672746633126
77694266518451768241345782073974609375] : [1, 0, -31106, -99112, -65203241]%%}, [4
]%%}%+%%{%poly1 [13406167943869060069832903395837439547234692769525032921
5956874109101434753056577258503485208281677246093750, 1448223635470015710411
105400521262660890267467474254168325112744678537043998787547620879624916147
4060058593750, -450743946228316845834503722080596923970970618377104928698371
4907113445787303583904941659518660699478698730468750, -501259088879939665231
933655971301952587189466470689047006275328975844492780545187141383181367765
715754302978515625] : [1, 0, -31106, -99112, -65203241]%%}, [3]%%}%+%%{%poly1 [-
381381386714161407348746082633738466949446345681605193953880120440269250489
670513355644056050692291259765625, -9280904840184584672813286675892310136426
54084462078716361857082560926680726081331041009281187046966552734375, 125754
762806389252665809919975988058534449424198247881435471309616603663322695610
14277652901709172789611816406250, 163580374369001064797328406125711787375406
38352696122276658871622820423281266441048684041183868245192871093750000] : [1
, 0, -31106, -99112, -65203241]%%}, [2]%%}%+%%{%poly1 [-2829165046514258991518
864770923875171856122508474820224931553878066663155128213074011486795035416
25976562500, -11047522024183640405812419383946846813110461930170928876192826
646859750070859837003681504660562347747802734375, 92622410573620422001390283
280266603413021660935063007466600699398333831523463749530142976259339105900
26855468750, 379944404413115369887516401121805461986384147356195938788669206
085547964391733575642579455121057385054046630859375] : [1, 0, -31106, -99112, -65
203241]%%}, [1]%%}%+%%{%poly1 [1457442570281580782727937412636382766061047
29476650851683626206635230909249444607603805733373105438232421875, 451515992
466773368261871056920647955432794275385491983650499399141514582936757420365
8363698710825103759765625, -494995282225192497001838548818146861751126606116
9463951678445919411830447649751834787235187730954876190185546875, -118464124
267461539474328024609355477174113903628048164370055623880000124338618560214
721618505583680160430908203125] : [1, 0, -31106, -99112, -65203241]%%}, [0]%%}% /
%%{%poly1 [-44900316662769750, 278135011567527216375, 698514226322709000750
, 1028664866672275019250] : [1, 0, -31106, -99112, -65203241]%%}, [1]%%}%+%%{%[2
929989585103994875, 368557731538442832750, 907666859350191995250, -45807860415
265619064625] : [1, 0, -31106, -99112, -65203241]%%}, [0]%%}% Error: Bad Argument
Value

maple [B] time = 0.14, size = 789, normalized size = 6.07

$$\frac{\sqrt{x^2+x} x}{2} - \frac{5 \ln\left(x + \frac{1}{2} + \sqrt{x^2+x}\right)}{8} + \frac{5\sqrt{x^2+x}}{4} + \frac{\sqrt{\frac{4(x-\sqrt{2}-1)^2}{(-x+1-\sqrt{2})^2} - \frac{3\sqrt{2}(x-\sqrt{2}-1)^2}{(-x+1-\sqrt{2})^2} + 4 + 3\sqrt{2}}}{\sqrt{2}} \left(-4\sqrt{2} \operatorname{arctanh} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2+x)^(3/2)/(x^2+1),x)`

[Out] $\frac{1}{2}x(x^2+x)^{1/2} + \frac{5}{4}(x^2+x)^{1/2} - \frac{5}{8}\ln\left(\frac{1}{2}+x+(x^2+x)^{1/2}\right) + \frac{1}{2}\left(4(-2^{1/2}-1+x)^2/(1-x-2^{1/2})^2 - 3*2^{1/2}*(-2^{1/2}-1+x)^2/(1-x-2^{1/2})^2 + 4 + 3*2^{1/2}\right)^{1/2} * 2^{1/2} * ((-2+2*2^{1/2})^{1/2} * \arctan(1/2*(-2+2*2^{1/2})^{1/2}) * ((3*2^{1/2}-4)*(-2^{1/2}-1+x)^2/(1-x-2^{1/2})^2 + 12*2^{1/2}+17))^{1/2} * (24*(-2^{1/2}-1+x)^2/(1-x-2^{1/2})^2 + 17*2^{1/2}*(-2^{1/2}-1+x)^2/(1-x-2^{1/2})^2 - 2^{1/2}) * (-2^{1/2}-1+x)/(1-x-2^{1/2}) * (3*2^{1/2}-4)/((-2^{1/2}-1+x)^4/(1-x-2^{1/2})^4 - 34*(-2^{1/2}-1+x)^2/(1-x-2^{1/2})^2 + 1) * (1+2^{1/2})^{1/2} * 2^{1/2} - 2*(-2+2*2^{1/2})^{1/2} * \arctan(1/2*(-2+2*2^{1/2})^{1/2}) * ((3*2^{1/2}-4)*(-2^{1/2}-1+x)^2/(1-x-2^{1/2})^2 + 12*2^{1/2}+17))^{1/2} * (24*(-2^{1/2}-1+x)^2/(1-x-2^{1/2})^2 + 17*2^{1/2}*(-2^{1/2}-1+x)^2/(1-x-2^{1/2})^2 - 2^{1/2}) * (-2^{1/2}-1+x)/(1-x-2^{1/2}) * (3*2^{1/2}-4)/((-2^{1/2}-1+x)^4/(1-x-2^{1/2})^4 - 34*(-2^{1/2}-1+x)^2/(1-x-2^{1/2})^2 + 1) * (1+2^{1/2})^{1/2} - 4 * \operatorname{arctanh}\left(\frac{1}{2} * (4*(-2^{1/2}-1+x)^2/(1-x-2^{1/2})^2 - 3*2^{1/2}*(-2^{1/2}-1+x)^2/(1-x-2^{1/2})^2 + 4 + 3*2^{1/2})^{1/2} / (1+2^{1/2})^{1/2}\right) * 2^{1/2} + 6 * \operatorname{arctanh}\left(\frac{1}{2} * (4*(-2^{1/2}-1+x)^2/(1-x-2^{1/2})^2 - 3*2^{1/2}*(-2^{1/2}-1+x)^2/(1-x-2^{1/2})^2 + 4 + 3*2^{1/2})^{1/2} / (1+2^{1/2})^{1/2}\right) / (-3*2^{1/2}*(-2^{1/2}-1+x)^2/(1-x-2^{1/2})^2 - 4*(-2^{1/2}-1+x)^2/(1-x-2^{1/2})^2 + 3*2^{1/2}-4) / ((-2^{1/2}-1+x)/(1-x-2^{1/2})+1)^2)^{1/2} / ((-2^{1/2}-1+x)/(1-x-2^{1/2})+1) / (3*2^{1/2}-4) / (1+2^{1/2})^{1/2})^{1/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2+x)^{3/2}}{x^2+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+x)^(3/2)/(x^2+1),x, algorithm="maxima")`

[Out] integrate((x² + x)^(3/2)/(x² + 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x^2 + x)^{3/2}}{x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + x²)^(3/2)/(x² + 1), x)

[Out] int((x + x²)^(3/2)/(x² + 1), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x(x + 1))^{3/2}}{x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+x)**(3/2)/(x**2+1), x)

[Out] Integral((x*(x + 1))**(3/2)/(x**2 + 1), x)

$$3.94 \quad \int \frac{x^4}{\sqrt{a+bx+cx^2}(d-fx^2)} dx$$

Optimal. Leaf size=369

$$-\frac{(3b^2 - 4ac) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}f} + \frac{3b\sqrt{a+bx+cx^2}}{4c^2f} + \frac{d^{3/2} \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2f^2\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} + \frac{d^{3/2} \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2f^2}$$

[Out] $-1/8*(-4*a*c+3*b^2)*\operatorname{arctanh}(1/2*(2*c*x+b)/c^{(1/2)/(c*x^2+b*x+a)^{(1/2)})}/c^{(5/2)}/f-d*\operatorname{arctanh}(1/2*(2*c*x+b)/c^{(1/2)/(c*x^2+b*x+a)^{(1/2)})}/f^2/c^{(1/2)}+3/4*b*(c*x^2+b*x+a)^{(1/2)}/c^2/f-1/2*x*(c*x^2+b*x+a)^{(1/2)}/c/f+1/2*d^{(3/2)}*\operatorname{arctanh}(1/2*(b*d^{(1/2)}-2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}-b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)})/(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(1/2)}/f^2/(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(1/2)}+1/2*d^{(3/2)}*\operatorname{arctanh}(1/2*(b*d^{(1/2)}+2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}+b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)}/(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(1/2)}/f^2/(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(1/2)}$

Rubi [A] time = 0.81, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6725, 621, 206, 742, 640, 984, 724}

$$-\frac{(3b^2 - 4ac) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}f} + \frac{3b\sqrt{a+bx+cx^2}}{4c^2f} + \frac{d^{3/2} \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2f^2\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} + \frac{d^{3/2} \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2f^2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^4/(\operatorname{Sqrt}[a + b*x + c*x^2]*(d - f*x^2)), x]$

[Out] $(3*b*\operatorname{Sqrt}[a + b*x + c*x^2])/(4*c^2*f) - (x*\operatorname{Sqrt}[a + b*x + c*x^2])/(2*c*f) - (d*\operatorname{ArcTanh}[(b + 2*c*x)/(2*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(\operatorname{Sqrt}[c]*f^2) - ((3*b^2 - 4*a*c)*\operatorname{ArcTanh}[(b + 2*c*x)/(2*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(8*c^{(5/2)}*f) + (d^{(3/2)}*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] - 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] - b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*f^2*\operatorname{Sqrt}[c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]) + (d^{(3/2)}*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] + 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] + b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*f^2*\operatorname{Sqrt}[c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f])$

Rule 206

$\operatorname{Int}[(a_0 + (b_0*x)^2)^{-1}, x_Symbol] := \operatorname{Simp}[(1*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*x]/\operatorname{Rt}[a, 2])]/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& \operatorname{Gt}$

$Q[a, 0] \parallel LtQ[b, 0]$

Rule 621

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] \rightarrow \text{Dist}[2, \text{Subst}[\text{Int}[1/(4*c - x^2), x], x, (b + 2*c*x)/\text{Sqrt}[a + b*x + c*x^2]], x] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 640

$\text{Int}[(d_) + (e_)*(x_)]*((a_) + (b_)*(x_) + (c_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(e*(a + b*x + c*x^2)^{(p + 1)})/(2*c*(p + 1)), x] + \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[(a + b*x + c*x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, p\}, x] \&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{NeQ}[p, -1]$

Rule 724

$\text{Int}[1/(((d_) + (e_)*(x_))*\text{Sqrt}[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/\text{Sqrt}[a + b*x + c*x^2]], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[2*c*d - b*e, 0]$

Rule 742

$\text{Int}[(d_) + (e_)*(x_)]^{(m_)}*((a_) + (b_)*(x_) + (c_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(e*(d + e*x)^{(m - 1)}*(a + b*x + c*x^2)^{(p + 1)})/(c*(m + 2*p + 1)), x] + \text{Dist}[1/(c*(m + 2*p + 1)), \text{Int}[(d + e*x)^{(m - 2)}*\text{Simp}[c*d^{2*(m + 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x], x]*(a + b*x + c*x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, m, p\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{If}[\text{RationalQ}[m], \text{GtQ}[m, 1], \text{SumSimplerQ}[m, -2]] \&\& \text{NeQ}[m + 2*p + 1, 0] \&\& \text{IntQuadraticQ}[a, b, c, d, e, m, p, x]$

Rule 984

$\text{Int}[1/(((a_) + (c_)*(x_)^2)*\text{Sqrt}[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] \rightarrow \text{Dist}[1/2, \text{Int}[1/((a - \text{Rt}[-(a*c), 2]*x)*\text{Sqrt}[d + e*x + f*x^2]), x], x] + \text{Dist}[1/2, \text{Int}[1/((a + \text{Rt}[-(a*c), 2]*x)*\text{Sqrt}[d + e*x + f*x^2]), x], x] /; \text{FreeQ}\{a, c, d, e, f\}, x] \&\& \text{NeQ}[e^2 - 4*d*f, 0] \&\& \text{PosQ}[-(a*c)]$

Rule 6725

$\text{Int}[(u_)/((a_) + (b_)*(x_)^{(n_)}), x_Symbol] \rightarrow \text{With}\{v = \text{RationalFunctionExpand}[u/(a + b*x^n), x]\}, \text{Int}[v, x] /; \text{SumQ}[v] /; \text{FreeQ}\{a, b\}, x] \&\& \text{IGtQ}$

[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt{a+bx+cx^2}(d-fx^2)} dx &= \int \left(-\frac{d}{f^2\sqrt{a+bx+cx^2}} - \frac{x^2}{f\sqrt{a+bx+cx^2}} + \frac{d^2}{f^2\sqrt{a+bx+cx^2}(d-fx^2)} \right) dx \\
&= -\frac{d \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{f^2} + \frac{d^2 \int \frac{1}{\sqrt{a+bx+cx^2}(d-fx^2)} dx}{f^2} - \frac{\int \frac{x^2}{\sqrt{a+bx+cx^2}} dx}{f} \\
&= -\frac{x\sqrt{a+bx+cx^2}}{2cf} - \frac{(2d) \operatorname{Subst}\left(\int \frac{1}{4c-x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f^2} + \frac{d^2 \int \frac{1}{(d-\sqrt{d}\sqrt{fx})\sqrt{a+bx+cx^2}} dx}{2f^2} \\
&= \frac{3b\sqrt{a+bx+cx^2}}{4c^2f} - \frac{x\sqrt{a+bx+cx^2}}{2cf} - \frac{d \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2} - \frac{d^2 \operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{2f^2} \\
&= \frac{3b\sqrt{a+bx+cx^2}}{4c^2f} - \frac{x\sqrt{a+bx+cx^2}}{2cf} - \frac{d \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2} + \frac{d^{3/2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2f^2} \\
&= \frac{3b\sqrt{a+bx+cx^2}}{4c^2f} - \frac{x\sqrt{a+bx+cx^2}}{2cf} - \frac{d \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2} - \frac{(3b^2-4ac)}{8f^2}
\end{aligned}$$

Mathematica [A] time = 1.71, size = 300, normalized size = 0.81

$$\frac{(-4acf+3b^2f+8c^2d) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right) - \frac{2f(2cx-3b)\sqrt{a+x(b+cx)}}{c^2} + \frac{4d^{3/2} \tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} + \frac{4d^{3/2} \tanh^{-1}\left(\frac{-2a\sqrt{f}}{2\sqrt{a+x(b+cx)}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}}{8f^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)), x]

[Out] $\frac{((-2*f*(-3*b + 2*c*x))*\operatorname{Sqrt}[a + x*(b + c*x)])/c^2 - ((8*c^2*d + 3*b^2*f - 4*a*c*f)*\operatorname{ArcTanh}[(b + 2*c*x)/(2*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a + x*(b + c*x)])])/c^{5/2} + (4*d^{3/2}*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] + 2*a*\operatorname{Sqrt}[f] + 2*c*\operatorname{Sqrt}[d]*x + b*\operatorname{Sqrt}[f]*x)/(2$

$\frac{\sqrt{c*d + b*\sqrt{d}*\sqrt{f} + a*f}*\sqrt{a + x*(b + c*x)}}{\sqrt{c*d + b*\sqrt{d}*\sqrt{f} + a*f} + (4*d^{(3/2)}*ArcTanh[(-2*a*\sqrt{f} + 2*c*\sqrt{d})*x + b*(\sqrt{d} - \sqrt{f}*x)]/(2*\sqrt{c*d - b*\sqrt{d}*\sqrt{f} + a*f}*\sqrt{a + x*(b + c*x)})]/\sqrt{c*d - b*\sqrt{d}*\sqrt{f} + a*f})/(8*f^2)}$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);;OUTPUT:Error: Bad Argument Type

maple [A] time = 0.03, size = 516, normalized size = 1.40

$$\frac{d^2 \ln \left(\frac{\frac{2af+2cd-2\sqrt{df}b}{f} + \frac{(bf-2\sqrt{df}c)\left(x+\frac{\sqrt{df}}{f}\right)}{f} + 2\sqrt{\frac{af+cd-\sqrt{df}b}{f}} \sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2 c + \frac{(bf-2\sqrt{df}c)\left(x+\frac{\sqrt{df}}{f}\right) + af+cd-\sqrt{df}b}{f}}}{x+\frac{\sqrt{df}}{f}} \right)}{2\sqrt{df} \sqrt{\frac{af+cd-\sqrt{df}b}{f}} f^2} + \frac{d^2 \ln \left(\frac{2af+2cd+2\sqrt{df}b}{f} \right)}{2\sqrt{df} \sqrt{\frac{af+cd-\sqrt{df}b}{f}} f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)

[Out]
$$-1/2*x*(c*x^2+b*x+a)^{(1/2)}/c/f+3/4*b*(c*x^2+b*x+a)^{(1/2)}/c^2/f-3/8/f*b^2/c^{(5/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})+1/2/f*a/c^{(3/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})-1/f^2*d*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})/c^{(1/2)}-1/2/f^2*d^2/(d*f)^{(1/2)}/((a*f+c*d-(d*f)^{(1/2)}*b)/f)^{(1/2)}*\ln((2*(a*f+c*d-(d*f)^{(1/2)}*b)/f+(b*f-2*(d*f)^{(1/2)}*c)*(x+(d*f)^{(1/2)}/f)/f+2*((a*f+c*d-(d*f)^{(1/2)}*b)/f)^{(1/2)}*((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)}*c)*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)}*b)/f)^{(1/2)})/(x+(d*f)^{(1/2)}/f)$$

) / f)) + 1/2 / f^2 * d^2 / (d * f)^(1/2) / ((a * f + c * d + (d * f)^(1/2) * b) / f)^(1/2) * ln((2 * (a * f + c * d + (d * f)^(1/2) * b) / f + (b * f + 2 * (d * f)^(1/2) * c) * (x - (d * f)^(1/2) / f) / f + 2 * ((a * f + c * d + (d * f)^(1/2) * b) / f)^(1/2) * ((x - (d * f)^(1/2) / f)^2 * c + (b * f + 2 * (d * f)^(1/2) * c) * (x - (d * f)^(1/2) / f) / f + (a * f + c * d + (d * f)^(1/2) * b) / f)^(1/2)) / (x - (d * f)^(1/2) / f))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see 'assume?' for more details) Is ((c*sqrt(4*d*f))/(2*f^2) + b/(2*f)) ^2 - (c*((b*sqrt(4*d*f)) / (2*f) + (c*d)/f+a)) / f^2 zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{(d - f x^2) \sqrt{c x^2 + b x + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)

[Out] int(x^4/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4}{-d\sqrt{a + bx + cx^2} + fx^2\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)

[Out] -Integral(x**4/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)), x)

$$3.95 \quad \int \frac{x^3}{\sqrt{a+bx+cx^2}(d-fx^2)} dx$$

Optimal. Leaf size=287

$$\frac{b \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f} - \frac{d \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2f^{3/2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} + \frac{d \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{3/2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{\sqrt{a+bx+cx^2}}{2c^{3/2}f}$$

[Out] $\frac{1}{2}b \operatorname{arctanh}\left(\frac{1}{2} \frac{(2cx+b)/c}{(cx^2+bx+a)^{1/2}}\right) / c^{3/2} / f - (cx^2+bx+a)^{1/2} / c / f - \frac{1}{2}d \operatorname{arctanh}\left(\frac{1}{2} \frac{(b\sqrt{d}-2af+2cd-b\sqrt{f})}{(cx^2+bx+a)^{1/2}(cd+af-b\sqrt{d}\sqrt{f})^{1/2}}\right) / f^{3/2} / (cd+af-b\sqrt{d}\sqrt{f})^{1/2} + \frac{1}{2}d \operatorname{arctanh}\left(\frac{1}{2} \frac{(b\sqrt{d}+2af+2cd+b\sqrt{f})}{(cx^2+bx+a)^{1/2}(cd+af+b\sqrt{d}\sqrt{f})^{1/2}}\right) / f^{3/2} / (cd+af+b\sqrt{d}\sqrt{f})^{1/2}$

Rubi [A] time = 0.63, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {6725, 640, 621, 206, 1033, 724}

$$\frac{b \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f} - \frac{d \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2f^{3/2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} + \frac{d \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{3/2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{\sqrt{a+bx+cx^2}}{2c^{3/2}f}$$

Antiderivative was successfully verified.

[In] $\int \frac{x^3}{(\sqrt{a+bx+cx^2})(d-fx^2)} dx$

[Out] $-\frac{(\sqrt{a+bx+cx^2}/(cf)) + (b \operatorname{ArcTanh}[(b+2cx)/(2\sqrt{c}\sqrt{a+bx+cx^2}]))/(2c^{3/2}f) - (d \operatorname{ArcTanh}[(b\sqrt{d}-2a\sqrt{f}+2cd-b\sqrt{f})/(2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}]) + (d \operatorname{ArcTanh}[(b\sqrt{d}+2a\sqrt{f}+2cd+b\sqrt{f})/(2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}]))/(2f^{3/2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}) + (d \operatorname{ArcTanh}[(b\sqrt{d}+2a\sqrt{f}+2cd+b\sqrt{f})/(2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}]))/(2f^{3/2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}) - \sqrt{a+bx+cx^2}/(2c^{3/2}f)}$

Rule 206

$\operatorname{Int}[(a_+ + (b_-)(x_-)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1 \operatorname{ArcTanh}[(Rt[-b, 2]x)/Rt[a, 2]])/(Rt[a, 2]Rt[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 621

```
Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 724

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1033

```
Int[((g_) + (h_)*(x_))/((a_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2], x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_)*(x_)^n), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{a+bx+cx^2}(d-fx^2)} dx &= \int \left(-\frac{x}{f\sqrt{a+bx+cx^2}} + \frac{dx}{f\sqrt{a+bx+cx^2}(d-fx^2)} \right) dx \\
&= -\frac{\int \frac{x}{\sqrt{a+bx+cx^2}} dx}{f} + \frac{d \int \frac{x}{\sqrt{a+bx+cx^2}(d-fx^2)} dx}{f} \\
&= -\frac{\sqrt{a+bx+cx^2}}{cf} + \frac{b \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{2cf} + \frac{d \int \frac{1}{(-\sqrt{d}\sqrt{f}-fx)\sqrt{a+bx+cx^2}} dx}{2f} + \frac{d \int \frac{1}{(\sqrt{d}\sqrt{f}-fx)\sqrt{a+bx+cx^2}} dx}{2f} \\
&= -\frac{\sqrt{a+bx+cx^2}}{cf} + \frac{b \operatorname{Subst}\left(\int \frac{1}{4c-x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{cf} - \frac{d \operatorname{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}\sqrt{f}-fx^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{cf} \\
&= -\frac{\sqrt{a+bx+cx^2}}{cf} + \frac{b \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f} - \frac{d \tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+(2c\sqrt{d}-b)\sqrt{a+bx+cx^2}}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^{3/2}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}}
\end{aligned}$$

Mathematica [A] time = 1.16, size = 325, normalized size = 1.13

$$\frac{b\sqrt{f} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{3/2}} + \frac{d \tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{d \tanh^{-1}\left(\frac{-2a\sqrt{f}+b(\sqrt{d}-\sqrt{f}x)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} - \frac{2\sqrt{f}x^2}{\sqrt{a+x(b+cx)}} - \frac{2b}{c\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)), x]

[Out] ((-2*a*Sqrt[f])/(c*Sqrt[a + x*(b + c*x)]) - (2*b*Sqrt[f]*x)/(c*Sqrt[a + x*(b + c*x)])) - (2*Sqrt[f]*x^2)/Sqrt[a + x*(b + c*x)] + (b*Sqrt[f]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/c^(3/2) + (d*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d] + b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + x*(b + c*x)])]/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f] - (d*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + x*(b + c*x)])]/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f])/(2*f^(3/2))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:Error: Bad Argument Type

maple [A] time = 0.02, size = 410, normalized size = 1.43

$$d \ln \left(\frac{\frac{2af+2cd-2\sqrt{df}b}{f} + \frac{(bf-2\sqrt{df}c)\left(x+\frac{\sqrt{df}}{f}\right)}{f} + 2\sqrt{\frac{af+cd-\sqrt{df}b}{f}} \sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2 c + \frac{(bf-2\sqrt{df}c)\left(x+\frac{\sqrt{df}}{f}\right) + af+cd-\sqrt{df}b}{f}}}{x+\frac{\sqrt{df}}{f}} \right) + d \ln \left(\frac{\frac{2af+2cd+2\sqrt{df}b}{f} + \frac{(bf+2\sqrt{df}c)\left(x+\frac{\sqrt{df}}{f}\right)}{f} + 2\sqrt{\frac{af+cd+\sqrt{df}b}{f}} \sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2 c + \frac{(bf+2\sqrt{df}c)\left(x+\frac{\sqrt{df}}{f}\right) + af+cd+\sqrt{df}b}{f}}}{x+\frac{\sqrt{df}}{f}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)

[Out] $-(c*x^2+b*x+a)^{(1/2)}/c/f+1/2/f*b/c^{(3/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})+1/2/f^2*d/((a*f+c*d-(d*f)^{(1/2)}*b)/f)^{(1/2)}*\ln((2*(a*f+c*d-(d*f)^{(1/2)}*b)/f+(b*f-2*(d*f)^{(1/2)}*c)*(x+(d*f)^{(1/2)}/f)/f+2*((a*f+c*d-(d*f)^{(1/2)}*b)/f)^{(1/2)}*((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)}*c)*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)}*b)/f)^{(1/2)})/(x+(d*f)^{(1/2)}/f))+1/2/f^2*d/((a*f+c*d+(d*f)^{(1/2)}*b)/f)^{(1/2)}*\ln((2*(a*f+c*d+(d*f)^{(1/2)}*b)/f+(b*f+2*(d*f)^{(1/2)}*c)*(x-(d*f)^{(1/2)}/f)/f+2*((a*f+c*d+(d*f)^{(1/2)}*b)/f)^{(1/2)}*((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)}*c)*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)}*b)/f)^{(1/2)})/(x-(d*f)^{(1/2)}/f))$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see 'assume?' for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2 - (c*((b*sqrt(4*d*f)) / (2*f) + (c*d)/f+a)) /f^2 zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{(d - f x^2) \sqrt{c x^2 + b x + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)

[Out] int(x^3/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{x^3}{-d\sqrt{a + bx + cx^2} + fx^2\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)

[Out] -Integral(x**3/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)), x)

$$3.96 \quad \int \frac{x^2}{\sqrt{a+bx+cx^2} (d-fx^2)} dx$$

Optimal. Leaf size=266

$$\frac{\sqrt{d} \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2f\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} + \frac{\sqrt{d} \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}$$

[Out] $-\operatorname{arctanh}\left(\frac{1/2*(2*c*x+b)/c^{1/2}/(c*x^2+b*x+a)^{1/2}}{f/c^{1/2}+1/2*\operatorname{arctanh}\left(\frac{1/2*(b*d^{1/2}-2*a*f^{1/2}+x*(2*c*d^{1/2}-b*f^{1/2}))}{(c*x^2+b*x+a)^{1/2}/(c*d+a*f-b*d^{1/2}*f^{1/2})^{1/2}}\right)*d^{1/2}/f/(c*d+a*f-b*d^{1/2}*f^{1/2})^{1/2}\right)+1/2*\operatorname{arctanh}\left(\frac{1/2*(b*d^{1/2}+2*a*f^{1/2}+x*(2*c*d^{1/2}+b*f^{1/2}))}{(c*x^2+b*x+a)^{1/2}/(c*d+a*f+b*d^{1/2}*f^{1/2})^{1/2}}\right)*d^{1/2}/f/(c*d+a*f+b*d^{1/2}*f^{1/2})^{1/2}\right)$

Rubi [A] time = 0.22, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1079, 621, 206, 984, 724}

$$\frac{\sqrt{d} \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2f\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} + \frac{\sqrt{d} \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^2/(\operatorname{Sqrt}[a + b*x + c*x^2]*(d - f*x^2)), x]$

[Out] $-(\operatorname{ArcTanh}[(b + 2*c*x)/(2*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a + b*x + c*x^2])]/(\operatorname{Sqrt}[c]*f)) + (\operatorname{Sqrt}[d]*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] - 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] - b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)*\operatorname{Sqrt}[a + b*x + c*x^2])]/(2*f*\operatorname{Sqrt}[c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]) + (\operatorname{Sqrt}[d]*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] + 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] + b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)*\operatorname{Sqrt}[a + b*x + c*x^2])]/(2*f*\operatorname{Sqrt}[c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f))$

Rule 206

$\operatorname{Int}[(a_) + (b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{Gt}Q[a, 0] \ || \ \operatorname{Lt}Q[b, 0])$

Rule 621


```
Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 984

```
Int[1/(((a_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[1/2, Int[1/((a - Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[1/2, Int[1/((a + Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1079

```
Int[((A_) + (C_)*(x_)^2)/(((a_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[(A*c - a*C)/c, Int[1/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, C}, x] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x^2}{\sqrt{a+bx+cx^2} (d-fx^2)} dx &= -\frac{\int \frac{1}{\sqrt{a+bx+cx^2}} dx}{f} + \frac{d \int \frac{1}{\sqrt{a+bx+cx^2} (d-fx^2)} dx}{f} \\ &= -\frac{2 \operatorname{Subst}\left(\int \frac{1}{4c-x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f} + \frac{d \int \frac{1}{(d-\sqrt{d}\sqrt{f}x)\sqrt{a+bx+cx^2}} dx}{2f} + \frac{d \int \frac{1}{(d+\sqrt{d}\sqrt{f}x)\sqrt{a+bx+cx^2}} dx}{2f} \\ &= -\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f} - \frac{d \operatorname{Subst}\left(\int \frac{1}{4cd^2-4bd^3/2\sqrt{f}+4adf-x^2} dx, x, \frac{-bd+2a\sqrt{d}\sqrt{f}}{\sqrt{a+bx+cx^2}}\right)}{f} \\ &= -\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f} + \frac{\sqrt{d} \tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+(2c\sqrt{d}-b\sqrt{f})x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} + \frac{\sqrt{d} \tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{f}+(2c\sqrt{d}+b\sqrt{f})x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f\sqrt{cd+b\sqrt{d}\sqrt{f}+af}} \end{aligned}$$

Mathematica [A] time = 0.43, size = 250, normalized size = 0.94

$$\frac{\sqrt{d} \left(\frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\sqrt{d}-b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} + \frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} \right) - \frac{2 \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}}}{2f}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]

[Out] ((-2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + Sqrt[d]*(ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + 2*c*Sqrt[d]*x - b*Sqrt[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f] + ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f))/(2*f)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:Error: Bad Argument Type

maple [A] time = 0.02, size = 399, normalized size = 1.50

$$\frac{d \ln \left(\frac{\frac{2af+2cd-2\sqrt{df}b}{f} + \frac{(bf-2\sqrt{df}c)\left(x+\frac{\sqrt{df}}{f}\right)}{f} + 2\sqrt{\frac{af+cd-\sqrt{df}b}{f}} \sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2 + \frac{(bf-2\sqrt{df}c)\left(x+\frac{\sqrt{df}}{f}\right) + af+cd-\sqrt{df}b}{f}}}{x+\frac{\sqrt{df}}{f}} \right)}{2\sqrt{df} \sqrt{\frac{af+cd-\sqrt{df}b}{f}} f} + \frac{d \ln \left(\frac{2af+2cd+2\sqrt{df}b}{f} + \frac{(bf-2\sqrt{df}c)\left(x+\frac{\sqrt{df}}{f}\right)}{f} + 2\sqrt{\frac{af+cd+\sqrt{df}b}{f}} \sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2 + \frac{(bf-2\sqrt{df}c)\left(x+\frac{\sqrt{df}}{f}\right) + af+cd+\sqrt{df}b}{f}}}{x+\frac{\sqrt{df}}{f}} \right)}{2\sqrt{df} \sqrt{\frac{af+cd+\sqrt{df}b}{f}} f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)`

[Out]
$$-1/f*\ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-1/2*d/(d*f)^(1/2)/f/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*\ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))+1/2*d/(d*f)^(1/2)/f/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*\ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see 'assume?' for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2 -c*((b*sqrt(4*d*f)) /((2*f) + (c*d)/f+a)) /f^2 zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{(d - f x^2) \sqrt{c x^2 + b x + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)`

[Out] `int(x^2/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{-d\sqrt{a + bx + cx^2} + fx^2\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x**2/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)),  
x)
```

$$3.97 \quad \int \frac{x}{\sqrt{a+bx+cx^2}(d-fx^2)} dx$$

Optimal. Leaf size=220

$$\frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}$$

[Out] $-1/2*\operatorname{arctanh}(1/2*(b*d^{(1/2)}-2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}-b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)/(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(1/2)}/f^{(1/2)/(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(1/2)}+1/2*\operatorname{arctanh}(1/2*(b*d^{(1/2)}+2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}+b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)/(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(1/2)}/f^{(1/2)/(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(1/2)})$

Rubi [A] time = 0.13, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1033, 724, 206}

$$\frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]

[Out] $-\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] - 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] - b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]*\operatorname{Sqrt}[a + b*x + c*x^2])]/(2*\operatorname{Sqrt}[f]*\operatorname{Sqrt}[c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]) + \operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] + 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] + b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]*\operatorname{Sqrt}[a + b*x + c*x^2])]/(2*\operatorname{Sqrt}[f]*\operatorname{Sqrt}[c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f])$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x], (2

`*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]`

Rule 1033

`Int[((g_.) + (h_.)*(x_))/(((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]`

Rubi steps

$$\begin{aligned} \int \frac{x}{\sqrt{a+bx+cx^2} (d-fx^2)} dx &= \frac{1}{2} \int \frac{1}{(-\sqrt{d}\sqrt{f}-fx)\sqrt{a+bx+cx^2}} dx + \frac{1}{2} \int \frac{1}{(\sqrt{d}\sqrt{f}-fx)\sqrt{a+bx+cx^2}} dx \\ &= -\text{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2} dx, x, \frac{b\sqrt{d}\sqrt{f}-2af-(-2c\sqrt{d}\sqrt{f})}{\sqrt{a+bx+cx^2}}\right) \\ &= -\frac{\tanh^{-1}\left(\frac{b\sqrt{d}\sqrt{f}-2af-(-2c\sqrt{d}\sqrt{f}+bf)x}{2\sqrt{f}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{f}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} - \frac{\tanh^{-1}\left(\frac{-b\sqrt{d}\sqrt{f}-2af-(2c\sqrt{d}\sqrt{f}+bf)x}{2\sqrt{f}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{f}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}} \end{aligned}$$

Mathematica [A] time = 0.17, size = 211, normalized size = 0.96

$$\frac{\frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+b(\sqrt{d}-\sqrt{f}x)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} + \frac{\tanh^{-1}\left(\frac{-2(a\sqrt{f}+c\sqrt{d}x)-b(\sqrt{d}+\sqrt{f}x)}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}}{2\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]

[Out]
$$-1/2*(\text{ArcTanh}[(-2*a*\text{Sqrt}[f] + 2*c*\text{Sqrt}[d]*x + b*(\text{Sqrt}[d] - \text{Sqrt}[f]*x))/(2*\text{Sqrt}[c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f)*\text{Sqrt}[a + x*(b + c*x)]])/\text{Sqrt}[c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f] + \text{ArcTanh}[(-2*(a*\text{Sqrt}[f] + c*\text{Sqrt}[d]*x) - b*(\text{Sqrt}[d] + \text{Sqrt}[f]*x))/(2*\text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f)*\text{Sqrt}[a + x*(b + c*x)]])/\text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f])/ \text{Sqrt}[f]$$

fricas [B] time = 2.40, size = 2753, normalized size = 12.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")

[Out]
$$\frac{1}{4} \sqrt{(c*d + a*f + (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*\sqrt{b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)}}/(c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)) * \log((2*b*c*d*x + b^2*d + 2*(b^2*d*f - (c^3*d^3*f + a^3*f^4 - (b^2*c - 3*a*c^2)*d^2*f^2 - (a*b^2 - 3*a^2*c)*d*f^3)*\sqrt{b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)}}*\sqrt{c*x^2 + b*x + a}*\sqrt{(c*d + a*f + (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*\sqrt{b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)}})/(c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)) - (2*a*c^2*d^2*f + 2*a^3*f^3 - 2*(a*b^2 - 2*a^2*c)*d*f^2 + (b*c^2*d^2*f + a^2*b*f^3 - (b^3 - 2*a*b*c)*d*f^2)*x)*\sqrt{b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)}}/x) - 1/4*\sqrt{(c*d + a*f + (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*\sqrt{b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)}})/(c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)) * \log((2*b*c*d*x + b^2*d - 2*(b^2*d*f - (c^3*d^3*f + a^3*f^4 - (b^2*c - 3*a*c^2)*d^2*f^2 - (a*b^2 - 3*a^2*c)*d*f^3)*\sqrt{b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)}}*\sqrt{c*x^2 + b*x + a}*\sqrt{(c*d + a*f + (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*\sqrt{b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)}})/(c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)) - (2*a*c^2*d^2*f + 2*a^3*f^3 - 2*(a*b^2 - 2*a^2*c)*d*f^2 + (b*c^2*d^2*f + a^2*b*f^3 - (b^3 - 2*a*b*c)*d*f^2)*x)*\sqrt{b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)}}/x) + 1/4*\sqrt{(c*d + a*f - (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*\sqrt{b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)}})/(c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)) * \log((2*b*c*d*x + b^2*d + 2*(b^2*d*f + (c^3*d^3*f + a^3*f^4 - (b^2*c - 3*a*c^2)*d^2*f^2 - (a*b^2 - 3*a^2*c)*d*f^3)*\sqrt{b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)}}*\sqrt{c*x^2 + b*x + a}*\sqrt{(c*d + a*f - (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*\sqrt{b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)}})/(c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)) * \log((2*b*c*d*x + b^2*d + 2*(b^2*d*f + (c^3*d^3*f + a^3*f^4 - (b^2*c - 3*a*c^2)*d^2*f^2 - (a*b^2 - 3*a^2*c)*d*f^3)*\sqrt{b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)}}*\sqrt{c*x^2 + b*x + a}*\sqrt{(c*d + a*f - (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*\sqrt{b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)}})/(c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2))$$

$$f^3 - (b^2 - 2ac)df^2) + (2ac^2d^2f + 2a^3f^3 - 2(ab^2 - 2a^2c)df^2 + (bc^2d^2f + a^2bf^3 - (b^3 - 2abc)df^2)x)\sqrt{b^2d/(c^4d^4f + a^4f^5 - 2(b^2c^2 - 2ac^3)d^3f^2 + (b^4 - 4ab^2c + 6a^2c^2)d^2f^3 - 2(a^2b^2 - 2a^3c)df^4)))/x) - 1/4\sqrt{(cd + af - (c^2d^2f + a^2f^3 - (b^2 - 2ac)df^2)\sqrt{b^2d/(c^4d^4f + a^4f^5 - 2(b^2c^2 - 2ac^3)d^3f^2 + (b^4 - 4ab^2c + 6a^2c^2)d^2f^3 - 2(a^2b^2 - 2a^3c)df^4)))/(c^2d^2f + a^2f^3 - (b^2 - 2ac)df^2))} \log((2b^2cdx + b^2d - 2(b^2df + (c^3d^3f + a^3f^4 - (b^2c - 3ac^2)d^2f^2 - (ab^2 - 3a^2c)df^3)\sqrt{b^2d/(c^4d^4f + a^4f^5 - 2(b^2c^2 - 2ac^3)d^3f^2 + (b^4 - 4ab^2c + 6a^2c^2)d^2f^3 - 2(a^2b^2 - 2a^3c)df^4))}\sqrt{cx^2 + bx + a})\sqrt{(cd + af - (c^2d^2f + a^2f^3 - (b^2 - 2ac)df^2)\sqrt{b^2d/(c^4d^4f + a^4f^5 - 2(b^2c^2 - 2ac^3)d^3f^2 + (b^4 - 4ab^2c + 6a^2c^2)d^2f^3 - 2(a^2b^2 - 2a^3c)df^4)))/(c^2d^2f + a^2f^3 - (b^2 - 2ac)df^2))} + (2ac^2d^2f + 2a^3f^3 - 2(ab^2 - 2a^2c)df^2 + (bc^2d^2f + a^2bf^3 - (b^3 - 2abc)df^2)x)\sqrt{b^2d/(c^4d^4f + a^4f^5 - 2(b^2c^2 - 2ac^3)d^3f^2 + (b^4 - 4ab^2c + 6a^2c^2)d^2f^3 - 2(a^2b^2 - 2a^3c)df^4)))/x)$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 0.52sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [B] time = 0.02, size = 354, normalized size = 1.61

$$\frac{\ln\left(\frac{\frac{2af+2cd-2\sqrt{df}b}{f} + \frac{(bf-2\sqrt{df}c)\left(x+\frac{\sqrt{df}}{f}\right)}{f} + 2\sqrt{\frac{af+cd-\sqrt{df}b}{f}} \sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2 c + \frac{(bf-2\sqrt{df}c)\left(x+\frac{\sqrt{df}}{f}\right)}{f} + \frac{af+cd-\sqrt{df}b}{f}}}{x+\frac{\sqrt{df}}{f}}\right)}{2\sqrt{\frac{af+cd-\sqrt{df}b}{f}} f} + \ln\left(\frac{\frac{2af+2cd+2\sqrt{df}b}{f} + \frac{(bf+2\sqrt{df}c)\left(x+\frac{\sqrt{df}}{f}\right)}{f}}{\dots}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)


```
[Out] 1/2/f/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))+1/2/f/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see 'assume?' for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2 -c*((b*sqrt(4*d*f))/(2*f) + (c*d)/f+a) /f^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{(d - f x^2) \sqrt{c x^2 + b x + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int(x/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-d\sqrt{a + bx + cx^2} + fx^2\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)), x)
```

$$3.98 \quad \int \frac{1}{\sqrt{a+bx+cx^2} (d-fx^2)} dx$$

Optimal. Leaf size=220

$$\frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2\sqrt{d}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} + \frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}$$

[Out] $1/2*\operatorname{arctanh}(1/2*(b*d^{(1/2)}-2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}-b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)/(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(1/2)}/d^{(1/2)/(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(1/2)}+1/2*\operatorname{arctanh}(1/2*(b*d^{(1/2)}+2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}+b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)/(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(1/2)}/d^{(1/2)/(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(1/2)}}$

Rubi [A] time = 0.12, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {984, 724, 206}

$$\frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2\sqrt{d}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} + \frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]

[Out] ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]/(2*Sqrt[d]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f)) + ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]/(2*Sqrt[d]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x], (2

*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 984

Int[1/(((a_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] :> Dist[1/2, Int[1/((a - Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[1/2, Int[1/((a + Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{a+bx+cx^2} (d-fx^2)} dx &= \frac{1}{2} \int \frac{1}{(d-\sqrt{d}\sqrt{f}x)\sqrt{a+bx+cx^2}} dx + \frac{1}{2} \int \frac{1}{(d+\sqrt{d}\sqrt{f}x)\sqrt{a+bx+cx^2}} dx \\ &= -\text{Subst}\left(\int \frac{1}{4cd^2-4bd^{3/2}\sqrt{f}+4adf-x^2} dx, x, \frac{-bd+2a\sqrt{d}\sqrt{f}-(2cd-b\sqrt{d}\sqrt{f})x}{\sqrt{a+bx+cx^2}}\right) \\ &= -\frac{\tanh^{-1}\left(\frac{-bd+2a\sqrt{d}\sqrt{f}-(2cd-b\sqrt{d}\sqrt{f})x}{2\sqrt{d}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} - \frac{\tanh^{-1}\left(\frac{-bd-2a\sqrt{d}\sqrt{f}-(2cd+b\sqrt{d}\sqrt{f})x}{2\sqrt{d}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}} \end{aligned}$$

Mathematica [A] time = 0.10, size = 209, normalized size = 0.95

$$\frac{\frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} + \frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+b(\sqrt{d}-\sqrt{f}x)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}}{2\sqrt{d}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]

[Out] (ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f] + ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f])/(2*Sqrt[d])

fricas [B] time = 2.15, size = 2641, normalized size = 12.00

result too large to display


```
*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3))/(c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f))*log((2*b*c*x + b^2 - 2*(b*c*d + a*b*f + (b*c^2*d^3 + a^2*b*d*f^2 - (b^3 - 2*a*b*c)*d^2*f))*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d + a*f - (c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f))*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)) + (2*a*c^2*d^2 + 2*a^3*f^2 - 2*(a*b^2 - 2*a^2*c)*d*f + (b*c^2*d^2 + a^2*b*f^2 - (b^3 - 2*a*b*c)*d*f)*x)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 0.58sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [B] time = 0.02, size = 358, normalized size = 1.63

$$\frac{\ln \left(\frac{\frac{2af+2cd-2\sqrt{df}b}{f} + \frac{(bf-2\sqrt{df}c)\left(x+\frac{\sqrt{df}}{f}\right)}{f}}{x+\frac{\sqrt{df}}{f}} + 2\sqrt{\frac{af+cd-\sqrt{df}b}{f}} \sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2 + \frac{(bf-2\sqrt{df}c)\left(x+\frac{\sqrt{df}}{f}\right) + af+cd-\sqrt{df}b}{f}}}{2\sqrt{df} \sqrt{\frac{af+cd-\sqrt{df}b}{f}}} \right) + \ln \left(\frac{\frac{2af+2cd+2\sqrt{df}b}{f} + \frac{(bf+2\sqrt{df}c)\left(x+\frac{\sqrt{df}}{f}\right)}{f}}{x+\frac{\sqrt{df}}{f}} + 2\sqrt{\frac{af+cd+\sqrt{df}b}{f}} \sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2 + \frac{(bf+2\sqrt{df}c)\left(x+\frac{\sqrt{df}}{f}\right) + af+cd+\sqrt{df}b}{f}}}{2\sqrt{df} \sqrt{\frac{af+cd+\sqrt{df}b}{f}}} \right)}{2\sqrt{df} \sqrt{\frac{af+cd-\sqrt{df}b}{f}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)

[Out]
$$-1/2/(d*f)^{(1/2)}/((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*\ln((2*(a*f+c*d-(d*f)^{(1/2)*b})/f+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+2*((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*((x+(d*f)^{(1/2)/f})^2+c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)})/(x+(d*f)^{(1/2)/f})+1/2/(d*f)^{(1/2)}/((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*\ln((2*(a*f+c*d+(d*f)^{(1/2)*b})/f+(b*f+2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+2*((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*((x+(d*f)^{(1/2)/f})^2+c+(b*f+2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)})/(x+(d*f)^{(1/2)/f})$$

$$\frac{1}{2}c) * (x - (d*f)^{(1/2)}/f) / f + 2 * ((a*f + c*d + (d*f)^{(1/2)} * b) / f)^{(1/2)} * ((x - (d*f)^{(1/2)}/f)^2 * c + (b*f + 2 * (d*f)^{(1/2)} * c) * (x - (d*f)^{(1/2)}/f) / f + (a*f + c*d + (d*f)^{(1/2)} * b) / f)^{(1/2)} / (x - (d*f)^{(1/2)}/f)$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see 'assume?' for more details)Is ((c*sqrt(4*d*f))/(2*f^2) + b/(2*f)) ^2 - (c*((b*sqrt(4*d*f)) / (2*f) + (c*d)/f+a)) / f^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(d - f x^2) \sqrt{c x^2 + b x + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)

[Out] int(1/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{1}{-d\sqrt{a + bx + cx^2} + fx^2\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)

[Out] -Integral(1/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)), x)

$$3.99 \quad \int \frac{1}{x \sqrt{a+bx+cx^2} (d-fx^2)} dx$$

Optimal. Leaf size=267

$$\frac{\sqrt{f} \tanh^{-1} \left(\frac{-2a\sqrt{f} + x(2c\sqrt{d} - b\sqrt{f}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2} \sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} \right)}{2d\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} + \frac{\sqrt{f} \tanh^{-1} \left(\frac{2a\sqrt{f} + x(b\sqrt{f} + 2c\sqrt{d}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2} \sqrt{af+b\sqrt{d}\sqrt{f}+cd}} \right)}{2d\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{\tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}} \right)}{\sqrt{a}d}$$

[Out] $-\operatorname{arctanh}\left(\frac{1}{2} \cdot \frac{(b \cdot x + 2 \cdot a) / a^{1/2}}{(c \cdot x^2 + b \cdot x + a)^{1/2}}\right) / d / a^{1/2} - \frac{1}{2} \cdot \operatorname{arctanh}\left(\frac{1}{2} \cdot \frac{(b \cdot d^{1/2} - 2 \cdot a \cdot f^{1/2} + x \cdot (2 \cdot c \cdot d^{1/2} - b \cdot f^{1/2}))}{(c \cdot x^2 + b \cdot x + a)^{1/2}}\right) / (c \cdot d + a \cdot f - b \cdot d^{1/2} \cdot f^{1/2})^{1/2} \cdot f^{1/2} / d / (c \cdot d + a \cdot f - b \cdot d^{1/2} \cdot f^{1/2})^{1/2} + \frac{1}{2} \cdot \operatorname{arctanh}\left(\frac{1}{2} \cdot \frac{(b \cdot d^{1/2} + 2 \cdot a \cdot f^{1/2} + x \cdot (2 \cdot c \cdot d^{1/2} + b \cdot f^{1/2}))}{(c \cdot x^2 + b \cdot x + a)^{1/2}}\right) / (c \cdot d + a \cdot f + b \cdot d^{1/2} \cdot f^{1/2})^{1/2} \cdot f^{1/2} / d / (c \cdot d + a \cdot f + b \cdot d^{1/2} \cdot f^{1/2})^{1/2}$

Rubi [A] time = 0.66, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6725, 724, 206, 1033}

$$\frac{\sqrt{f} \tanh^{-1} \left(\frac{-2a\sqrt{f} + x(2c\sqrt{d} - b\sqrt{f}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2} \sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} \right)}{2d\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} + \frac{\sqrt{f} \tanh^{-1} \left(\frac{2a\sqrt{f} + x(b\sqrt{f} + 2c\sqrt{d}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2} \sqrt{af+b\sqrt{d}\sqrt{f}+cd}} \right)}{2d\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{\tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}} \right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}\left[\frac{1}{x \sqrt{a + b \cdot x + c \cdot x^2}} (d - f \cdot x^2)\right], x$

[Out] $-\frac{\operatorname{ArcTanh}\left[\frac{2 \cdot a + b \cdot x}{2 \cdot \sqrt{a} \cdot \sqrt{a + b \cdot x + c \cdot x^2}}\right]}{\sqrt{a} \cdot d} - \frac{\sqrt{f} \cdot \operatorname{ArcTanh}\left[\frac{(b \cdot \sqrt{d} - 2 \cdot a \cdot \sqrt{f} + (2 \cdot c \cdot \sqrt{d} - b \cdot \sqrt{f}) \cdot x)}{2 \cdot \sqrt{c \cdot d - b \cdot \sqrt{d} \cdot \sqrt{f} + a \cdot f}}\right]}{2 \cdot d \cdot \sqrt{c \cdot d - b \cdot \sqrt{d} \cdot \sqrt{f} + a \cdot f}} + \frac{\sqrt{f} \cdot \operatorname{ArcTanh}\left[\frac{(b \cdot \sqrt{d} + 2 \cdot a \cdot \sqrt{f} + (2 \cdot c \cdot \sqrt{d} + b \cdot \sqrt{f}) \cdot x)}{2 \cdot \sqrt{c \cdot d + b \cdot \sqrt{d} \cdot \sqrt{f} + a \cdot f}}\right]}{2 \cdot d \cdot \sqrt{c \cdot d + b \cdot \sqrt{d} \cdot \sqrt{f} + a \cdot f}}$

Rule 206

$\operatorname{Int}\left[\frac{(a + (b \cdot x)^2)^{-1}}{x}, x\right] \rightarrow \operatorname{Simp}\left[\frac{\operatorname{ArcTanh}\left[\frac{Rt[-b, 2] \cdot x}{Rt[a, 2]}\right]}{Rt[a, 2] \cdot Rt[-b, 2]}, x\right] /; \operatorname{FreeQ}\{a, b, x\} \ \&\& \operatorname{NegQ}[a/b] \ \&\& \operatorname{GtQ}[a, 0] \ \|\| \operatorname{LtQ}[b, 0]$

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1033

Int[((g_.) + (h_.)*(x_))/(((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]

Rule 6725

Int[(u_)/((a_) + (b_.)*(x_)^n), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x\sqrt{a+bx+cx^2}(d-fx^2)} dx &= \int \left(\frac{1}{dx\sqrt{a+bx+cx^2}} - \frac{fx}{d\sqrt{a+bx+cx^2}(d-fx^2)} \right) dx \\
 &= \frac{\int \frac{1}{x\sqrt{a+bx+cx^2}} dx}{d} - \frac{f \int \frac{x}{\sqrt{a+bx+cx^2}(d-fx^2)} dx}{d} \\
 &= -\frac{2 \operatorname{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d} - \frac{f \int \frac{1}{(-\sqrt{d}\sqrt{f}+fx)\sqrt{a+bx+cx^2}} dx}{2d} - \frac{f \int \frac{1}{(\sqrt{d}\sqrt{f}+fx)\sqrt{a+bx+cx^2}} dx}{2d} \\
 &= -\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d} + \frac{f \operatorname{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2} dx, x, \frac{-b\sqrt{d}\sqrt{f}+2a}{\sqrt{a+bx+cx^2}}\right)}{d} - \frac{f \operatorname{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2} dx, x, \frac{b\sqrt{d}\sqrt{f}+2a}{\sqrt{a+bx+cx^2}}\right)}{d} \\
 &= -\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d} - \frac{\sqrt{f} \tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+(2c\sqrt{d}-b\sqrt{f})x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} + \frac{\sqrt{f} \tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{f}+(2c\sqrt{d}+b\sqrt{f})x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d\sqrt{cd-b\sqrt{d}\sqrt{f}+af}}
 \end{aligned}$$

Mathematica [A] time = 0.44, size = 252, normalized size = 0.94

$$\frac{\sqrt{f} \left(\frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}}\sqrt{f+cd}}\right)}{\sqrt{af+b\sqrt{d}}\sqrt{f+cd}} - \frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\sqrt{d}-b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b(-\sqrt{d})}\sqrt{f+cd}}\right)}{\sqrt{af+b(-\sqrt{d})}\sqrt{f+cd}} \right) - \frac{2 \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{\sqrt{a}}}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]

[Out] $\left(\frac{-2 \operatorname{ArcTanh}\left[\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right]}{\sqrt{a}} + \frac{\operatorname{ArcTanh}\left[\frac{b\sqrt{d}-2a\sqrt{f}+2c\sqrt{d}x-b\sqrt{f}x}{2\sqrt{c*d-b\sqrt{d}}\sqrt{f}+a\sqrt{f}}\right]}{\sqrt{c*d-b\sqrt{d}}\sqrt{f}+a\sqrt{f}} - \frac{\operatorname{ArcTanh}\left[\frac{b\sqrt{d}+2a\sqrt{f}+2c\sqrt{d}x+b\sqrt{f}x}{2\sqrt{c*d+b\sqrt{d}}\sqrt{f}+a\sqrt{f}}\right]}{\sqrt{c*d+b\sqrt{d}}\sqrt{f}+a\sqrt{f}} \right) / (2*d)$

fricas [B] time = 149.60, size = 5995, normalized size = 22.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")

[Out] $\frac{1}{4} \left(\frac{a d \sqrt{(c d f + a f^2 + (c^2 d^4 + a^2 d^2 f^2 - (b^2 - 2 a c) d^3 f) \sqrt{b^2 f^3 / (c^4 d^7 + a^4 d^3 f^4 - 2 (b^2 c^2 - 2 a c^3) d^6 f + (b^4 - 4 a b^2 c + 6 a^2 c^2) d^5 f^2 - 2 (a^2 b^2 - 2 a^3 c) d^4 f^3)}}}{(c^2 d^4 + a^2 d^2 f^2 - (b^2 - 2 a c) d^3 f)} \log\left(\frac{2 b c f^2 x + b^2 f^2 + 2 (b^2 d f^2 - (c^3 d^5 + a^3 d^2 f^3 - (b^2 c - 3 a c^2) d^4 f - (a b^2 - 3 a^2 c) d^3 f^2) \sqrt{b^2 f^3 / (c^4 d^7 + a^4 d^3 f^4 - 2 (b^2 c^2 - 2 a c^3) d^6 f + (b^4 - 4 a b^2 c + 6 a^2 c^2) d^5 f^2 - 2 (a^2 b^2 - 2 a^3 c) d^4 f^3}}}{(c^2 d^4 + a^2 d^2 f^2 - (b^2 - 2 a c) d^3 f)} - \frac{2 a c^2 d^3 f + 2 a^3 d f^3 - 2 (a b^2 - 2 a^2 c) d^2 f^2 + (b c^2 d^3 f + a^2 b d f^3 - (b^3 - 2 a b c) d^2 f^2) x}{(c^2 d^4 + a^2 d^2 f^2 - (b^2 - 2 a c) d^3 f)} \sqrt{b^2 f^3 / (c^4 d^7 + a^4 d^3 f^4 - 2 (b^2 c^2 - 2 a c^3) d^6 f + (b^4 - 4 a b^2 c + 6 a^2 c^2) d^5 f^2 - 2 (a^2 b^2 - 2 a^3 c) d^4 f^3)} \right) / x - a d \sqrt{(c d f + a f^2 + (c^2 d^4 + a^2 d^2 f^2 - (b^2 - 2 a c) d^3 f) \sqrt{b^2 f^3 / (c^4 d^7 + a^4 d^3 f^4 - 2 (b^2 c^2 - 2 a c^3) d^6 f + (b^4 - 4 a b^2 c + 6 a^2 c^2) d^5 f^2 - 2 (a^2 b^2 - 2 a^3 c) d^4 f^3)}}}{(c^2 d^4 + a^2 d^2 f^2 - (b^2 - 2 a c) d^3 f)} \log\left(\frac{2 b c f^2 x + b^2 f^2 - 2 (b^2 d f^2 - (c^3 d^5 + a^3 d^2 f^3 - (b^2 c - 3 a c^2) d^4 f}{(c^2 d^4 + a^2 d^2 f^2 - (b^2 - 2 a c) d^3 f)} \right) \right)$

$$\begin{aligned} & \sqrt{2f^2 - (b^2 - 2ac)d^3f} \sqrt{(b^2f^3/(c^4d^7 + a^4d^3f^4 - 2(b^2c^2 - 2ac^3)d^6f + (b^4 - 4ab^2c + 6a^2c^2)d^5f^2 - 2(a^2b^2 - 2a^3c)d^4f^3)))/(c^2d^4 + a^2d^2f^2 - (b^2 - 2ac)d^3f)} + (2ac^2d^3f + 2a^3d^2f^3 - 2(ab^2 - 2a^2c)d^2f^2 + (bc^2d^3f + a^2b^2d^2f^3 - (b^3 - 2abc)d^2f^2)x) \sqrt{(b^2f^3/(c^4d^7 + a^4d^3f^4 - 2(b^2c^2 - 2ac^3)d^6f + (b^4 - 4ab^2c + 6a^2c^2)d^5f^2 - 2(a^2b^2 - 2a^3c)d^4f^3)))/x} + 4\sqrt{-a} \arctan(1/2\sqrt{cx^2 + bx + a}(bx + 2a)\sqrt{-a}/(acx^2 + abx + a^2))/(ad) \end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage2

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(cx^2+bx+a)^(1/2)/(-fx^2+d),x, algorithm="giac")

[Out] sage2

maple [A] time = 0.02, size = 391, normalized size = 1.46

$$\frac{\ln\left(\frac{\frac{2af+2cd-2\sqrt{df}b}{f} + \frac{(bf-2\sqrt{df}c)\left(x+\frac{\sqrt{df}}{f}\right)}{f} + 2\sqrt{\frac{af+cd-\sqrt{df}b}{f}} \sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2 c + \frac{(bf-2\sqrt{df}c)\left(x+\frac{\sqrt{df}}{f}\right) + af+cd-\sqrt{df}b}{f}}}{x+\frac{\sqrt{df}}{f}}\right)}{2\sqrt{\frac{af+cd-\sqrt{df}b}{f}} d} + \frac{\ln\left(\frac{2af+2cd+2\sqrt{df}b}{f} + \frac{(bf+2\sqrt{df}c)\left(x+\frac{\sqrt{df}}{f}\right)}{f}\right)}{2\sqrt{\frac{af+cd+\sqrt{df}b}{f}} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(cx^2+bx+a)^(1/2)/(-fx^2+d),x)

[Out] 1/2/d/((af+cd-(df)^(1/2)*b)/f)^(1/2)*ln((2*(af+cd-(df)^(1/2)*b)/f+(bf-f-2*(df)^(1/2)*c)*(x+(df)^(1/2)/f)/f+2*((af+cd-(df)^(1/2)*b)/f)^(1/2)*((x+(df)^(1/2)/f)^2*c+(bf-2*(df)^(1/2)*c)*(x+(df)^(1/2)/f)/f+(af+cd-(df)^(1/2)*b)/f)^(1/2))/(x+(df)^(1/2)/f))+1/2/d/((af+cd+(df)^(1/2)*b)/f)^(1/2)*ln((2*(af+cd+(df)^(1/2)*b)/f+(bf+2*(df)^(1/2)*c)*(x-(df)^(1/2)/f)/f+2*((af+cd+(df)^(1/2)*b)/f)^(1/2)*((x-(df)^(1/2)/f)^2*c+(bf+2*(df)^(1/2)*c)*(x-(df)^(1/2)/f)/f+(af+cd+(df)^(1/2)*b)/f)^(1/2))/(x-(df)^(1/2)/f))-1/d/a^(1/2)*ln((bx+2a+2*(cx^2+bx+a)^(1/2)*a^(1/2))/x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{cx^2 + bx + a}(fx^2 - d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")`

[Out] `-integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)*x), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x (d - f x^2) \sqrt{c x^2 + b x + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)`

[Out] `int(1/(x*(d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-dx\sqrt{a + bx + cx^2} + fx^3\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)`

[Out] `-Integral(1/(-d*x*sqrt(a + b*x + c*x**2) + f*x**3*sqrt(a + b*x + c*x**2)), x)`

$$3.100 \quad \int \frac{1}{x^2 \sqrt{a+bx+cx^2} (d-fx^2)} dx$$

Optimal. Leaf size=291

$$\frac{b \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d} + \frac{f \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2d^{3/2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} + \frac{f \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{3/2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{\sqrt{a+bx+cx^2}}{d}$$

[Out] $1/2*b*\operatorname{arctanh}(1/2*(b*x+2*a)/a^{(1/2)/(c*x^2+b*x+a)^{(1/2)})/a^{(3/2)}/d-(c*x^2+b*x+a)^{(1/2)}/a/d/x+1/2*f*\operatorname{arctanh}(1/2*(b*d^{(1/2)}-2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}-b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)/(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(1/2)})/d^{(3/2)/(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(1/2)}+1/2*f*\operatorname{arctanh}(1/2*(b*d^{(1/2)}+2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}+b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)/(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(1/2)})/d^{(3/2)/(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(1/2)}}$

Rubi [A] time = 0.66, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {6725, 730, 724, 206, 984}

$$\frac{b \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d} + \frac{f \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2d^{3/2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} + \frac{f \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{3/2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{\sqrt{a+bx+cx^2}}{d}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]

[Out] $-(\operatorname{sqrt}[a + b*x + c*x^2]/(a*d*x)) + (b*\operatorname{ArcTanh}[(2*a + b*x)/(2*\operatorname{sqrt}[a]*\operatorname{sqrt}[a + b*x + c*x^2])])/(2*a^{(3/2)*d} + (f*\operatorname{ArcTanh}[(b*\operatorname{sqrt}[d] - 2*a*\operatorname{sqrt}[f] + (2*c*\operatorname{sqrt}[d] - b*\operatorname{sqrt}[f])*x)/(2*\operatorname{sqrt}[c*d - b*\operatorname{sqrt}[d]*\operatorname{sqrt}[f] + a*f]*\operatorname{sqrt}[a + b*x + c*x^2])])/(2*d^{(3/2)*\operatorname{sqrt}[c*d - b*\operatorname{sqrt}[d]*\operatorname{sqrt}[f] + a*f]} + (f*\operatorname{ArcTanh}[(b*\operatorname{sqrt}[d] + 2*a*\operatorname{sqrt}[f] + (2*c*\operatorname{sqrt}[d] + b*\operatorname{sqrt}[f])*x)/(2*\operatorname{sqrt}[c*d + b*\operatorname{sqrt}[d]*\operatorname{sqrt}[f] + a*f]*\operatorname{sqrt}[a + b*x + c*x^2])])/(2*d^{(3/2)*\operatorname{sqrt}[c*d + b*\operatorname{sqrt}[d]*\operatorname{sqrt}[f] + a*f]})$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol]
:> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x]
+ Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
&& EqQ[m + 2*p + 3, 0]
```

Rule 984

```
Int[1/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol]
:> Dist[1/2, Int[1/((a - Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x], x]
+ Dist[1/2, Int[1/((a + Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f}, x]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol]
:> With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x]
&& IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 \sqrt{a+bx+cx^2} (d-fx^2)} dx &= \int \left(\frac{1}{dx^2 \sqrt{a+bx+cx^2}} + \frac{f}{d \sqrt{a+bx+cx^2} (d-fx^2)} \right) dx \\
&= \frac{\int \frac{1}{x^2 \sqrt{a+bx+cx^2}} dx}{d} + \frac{f \int \frac{1}{\sqrt{a+bx+cx^2} (d-fx^2)} dx}{d} \\
&= -\frac{\sqrt{a+bx+cx^2}}{adx} - \frac{b \int \frac{1}{x \sqrt{a+bx+cx^2}} dx}{2ad} + \frac{f \int \frac{1}{(d-\sqrt{d}\sqrt{f}x) \sqrt{a+bx+cx^2}} dx}{2d} + \frac{f \int \frac{1}{(d+\sqrt{d}\sqrt{f}x) \sqrt{a+bx+cx^2}} dx}{2d} \\
&= -\frac{\sqrt{a+bx+cx^2}}{adx} + \frac{b \operatorname{Subst} \left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}} \right)}{ad} - \frac{f \operatorname{Subst} \left(\int \frac{1}{4cd^2-4bd^3/4} dx, x, \frac{b\sqrt{d}-2a\sqrt{f}x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} \right)}{2d^3/2} + \frac{f \operatorname{Subst} \left(\int \frac{1}{4cd^2-4bd^3/4} dx, x, \frac{b\sqrt{d}+2a\sqrt{f}x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} \right)}{2d^3/2} \\
&= -\frac{\sqrt{a+bx+cx^2}}{adx} + \frac{b \tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}} \right)}{2a^{3/2}d} + \frac{f \tanh^{-1} \left(\frac{b\sqrt{d}-2a\sqrt{f}x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} \right)}{2d^{3/2}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} + \frac{f \tanh^{-1} \left(\frac{b\sqrt{d}+2a\sqrt{f}x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} \right)}{2d^{3/2}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}}
\end{aligned}$$

Mathematica [A] time = 1.03, size = 325, normalized size = 1.12

$$\frac{b\sqrt{d} \tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}} \right)}{a^{3/2}} + \frac{f \tanh^{-1} \left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} \right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} + \frac{f \tanh^{-1} \left(\frac{-2a\sqrt{f}+b(\sqrt{d}-\sqrt{f}x)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} \right)}{\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} - \frac{2b\sqrt{d}}{a\sqrt{a+x(b+cx)}} - \frac{2c\sqrt{d}}{a\sqrt{a+x(b+cx)}}$$

$$2d^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*sqrt[a + b*x + c*x^2]*(d - f*x^2)), x]

[Out] ((-2*b*sqrt[d])/(a*sqrt[a + x*(b + c*x)]) - (2*sqrt[d])/(x*sqrt[a + x*(b + c*x)])) - (2*c*sqrt[d]*x)/(a*sqrt[a + x*(b + c*x)]) + (b*sqrt[d]*ArcTanh[(2*a + b*x)/(2*sqrt[a]*sqrt[a + x*(b + c*x)])])/a^(3/2) + (f*ArcTanh[(b*sqrt[d] + 2*a*sqrt[f] + 2*c*sqrt[d]*x + b*sqrt[f]*x)/(2*sqrt[c*d + b*sqrt[d]*sqrt[f] + a*f]*sqrt[a + x*(b + c*x)])])/sqrt[c*d + b*sqrt[d]*sqrt[f] + a*f] + (f*ArcTanh[(-2*a*sqrt[f] + 2*c*sqrt[d]*x + b*(sqrt[d] - sqrt[f]*x))/(2*sqrt[c*d - b*sqrt[d]*sqrt[f] + a*f]*sqrt[a + x*(b + c*x)])])/sqrt[c*d - b*sqrt[d]*sqrt[f] + a*f]/(2*d^(3/2))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 1.11sym2pol
y/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argum
ent Value

maple [A] time = 0.02, size = 427, normalized size = 1.47

$$\frac{f \ln \left(\frac{\frac{2af+2cd-2\sqrt{df}b}{f} + \frac{(bf-2\sqrt{df}c)\left(x+\frac{\sqrt{df}}{f}\right)}{f} + 2\sqrt{\frac{af+cd-\sqrt{df}b}{f}} \sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2 c + \frac{(bf-2\sqrt{df}c)\left(x+\frac{\sqrt{df}}{f}\right) + \frac{af+cd-\sqrt{df}b}{f}}}{x+\frac{\sqrt{df}}{f}} \right)}{2\sqrt{df} \sqrt{\frac{af+cd-\sqrt{df}b}{f}} d} + \frac{f \ln \left(\frac{2af+2cd+2\sqrt{df}b}{f} \right)}{2\sqrt{df} \sqrt{\frac{af+cd-\sqrt{df}b}{f}} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)

[Out]
$$-1/2*f/d/(d*f)^{(1/2)}/((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*\ln((2*(a*f+c*d-(d*f)^{(1/2)*b})/f+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+2*((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)})/(x+(d*f)^{(1/2)/f})-(c*x^2+b*x+a)^{(1/2)}/a/d/x+1/2/d*b/a^{(3/2)}*\ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)*a}^{(1/2)})/x)+1/2*f/d/(d*f)^{(1/2)}/((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*\ln((2*(a*f+c*d+(d*f)^{(1/2)*b})/f+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f})/f+2*((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)})/(x-(d*f)^{(1/2)/f})$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{cx^2 + bx + a}(fx^2 - d)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] -integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2 (d - f x^2) \sqrt{c x^2 + b x + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)

[Out] int(1/(x^2*(d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{1}{-d x^2 \sqrt{a + b x + c x^2} + f x^4 \sqrt{a + b x + c x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)

[Out] -Integral(1/(-d*x**2*sqrt(a + b*x + c*x**2) + f*x**4*sqrt(a + b*x + c*x**2)), x)

$$3.101 \quad \int \frac{1}{x^3 \sqrt{a+bx+cx^2} (d-fx^2)} dx$$

Optimal. Leaf size=376

$$\frac{(3b^2 - 4ac) \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2dx} - \frac{f^{3/2} \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f+cd}}\right)}{2d^2\sqrt{af+b(-\sqrt{d})}\sqrt{f+cd}} + \frac{f^{3/2} \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f+cd}}\right)}{2d^2\sqrt{af+b(-\sqrt{d})}\sqrt{f+cd}}$$

[Out] $-1/8*(-4*a*c+3*b^2)*\operatorname{arctanh}(1/2*(b*x+2*a)/a^{(1/2)/(c*x^2+b*x+a)^{(1/2)})}/a^{(5/2)}/d-f*\operatorname{arctanh}(1/2*(b*x+2*a)/a^{(1/2)/(c*x^2+b*x+a)^{(1/2)})}/d^2/a^{(1/2)-1/2*(c*x^2+b*x+a)^{(1/2)}/a/d/x^2+3/4*b*(c*x^2+b*x+a)^{(1/2)}/a^2/d/x-1/2*f^{(3/2)*a}\operatorname{rctanh}(1/2*(b*d^{(1/2)}-2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}-b*f^{(1/2)}))/((c*x^2+b*x+a)^{(1/2)/(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(1/2)})/d^2/(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(1/2)}+1/2*f^{(3/2)*a}\operatorname{arctanh}(1/2*(b*d^{(1/2)}+2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}+b*f^{(1/2)}))/((c*x^2+b*x+a)^{(1/2)/(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(1/2)})/d^2/(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(1/2)}$

Rubi [A] time = 0.73, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {6725, 744, 806, 724, 206, 1033}

$$\frac{(3b^2 - 4ac) \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2dx} - \frac{f^{3/2} \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f+cd}}\right)}{2d^2\sqrt{af+b(-\sqrt{d})}\sqrt{f+cd}} + \frac{f^{3/2} \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f+cd}}\right)}{2d^2\sqrt{af+b(-\sqrt{d})}\sqrt{f+cd}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]

[Out] $-\operatorname{Sqrt}[a + b*x + c*x^2]/(2*a*d*x^2) + (3*b*\operatorname{Sqrt}[a + b*x + c*x^2])/(4*a^2*d*x) - ((3*b^2 - 4*a*c)*\operatorname{ArcTanh}[(2*a + b*x)/(2*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(8*a^{(5/2)*d} - (f*\operatorname{ArcTanh}[(2*a + b*x)/(2*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(\operatorname{Sqrt}[a]*d^2) - (f^{(3/2)}*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] - 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] - b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*d^2*\operatorname{Sqrt}[c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]) + (f^{(3/2)}*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] + 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] + b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*d^2*\operatorname{Sqrt}[c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f))$

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 744

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x, x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])

Rule 806

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]

Rule 1033

Int[((g_.) + (h_.)*(x_))/((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]

Rule 6725

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 \sqrt{a+bx+cx^2} (d-fx^2)} dx &= \int \left(\frac{1}{dx^3 \sqrt{a+bx+cx^2}} + \frac{f}{d^2 x \sqrt{a+bx+cx^2}} + \frac{f^2 x}{d^2 \sqrt{a+bx+cx^2} (d-fx^2)} \right) dx \\
&= \frac{\int \frac{1}{x^3 \sqrt{a+bx+cx^2}} dx}{d} + \frac{f \int \frac{1}{x \sqrt{a+bx+cx^2}} dx}{d^2} + \frac{f^2 \int \frac{x}{\sqrt{a+bx+cx^2} (d-fx^2)} dx}{d^2} \\
&= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} - \frac{\int \frac{\frac{3b}{2}+cx}{x^2 \sqrt{a+bx+cx^2}} dx}{2ad} - \frac{(2f) \text{Subst} \left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}} \right)}{d^2} \\
&= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2 dx} - \frac{f \tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a} \sqrt{a+bx+cx^2}} \right)}{\sqrt{a} d^2} + \frac{(3b^2 - 4ac) \tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a} \sqrt{a+bx+cx^2}} \right)}{8a^{5/2} d} \\
&= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2 dx} - \frac{f \tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a} \sqrt{a+bx+cx^2}} \right)}{\sqrt{a} d^2} - \frac{f^{3/2} \tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a} \sqrt{a+bx+cx^2}} \right)}{8a^{5/2} d} \\
&= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2 dx} - \frac{(3b^2 - 4ac) \tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a} \sqrt{a+bx+cx^2}} \right)}{8a^{5/2} d}
\end{aligned}$$

Mathematica [A] time = 2.01, size = 314, normalized size = 0.84

$$2\sqrt{a} \left(\frac{2a^2 f^{3/2} \tanh^{-1} \left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}}\sqrt{f+cd}} \right)}{\sqrt{af+b\sqrt{d}}\sqrt{f+cd}} - \frac{2a^2 f^{3/2} \tanh^{-1} \left(\frac{-2a\sqrt{f}+b(\sqrt{d}-\sqrt{f}x)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b(-\sqrt{d})}\sqrt{f+cd}} \right)}{\sqrt{af+b(-\sqrt{d})}\sqrt{f+cd}} - \frac{d(2a-3bx)\sqrt{a+x(b+cx)}}{x^2} \right) + (4ac)$$

$$8a^{5/2}d^2$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]

[Out] ((-3*b^2*d + 4*a*(c*d - 2*a*f))*ArcTanh[(2*a + b*x)/(2*sqrt[a]*sqrt[a + x*(b + c*x)])] + 2*sqrt[a]*(-((d*(2*a - 3*b*x)*sqrt[a + x*(b + c*x)])/x^2) + (2*a^2*f^(3/2)*ArcTanh[(b*sqrt[d] + 2*a*sqrt[f] + 2*c*sqrt[d]*x + b*sqrt[f]*x)/(2*sqrt[c*d + b*sqrt[d]*sqrt[f] + a*f]*sqrt[a + x*(b + c*x)])))/sqrt[c*d + b*sqrt[d]*sqrt[f] + a*f] - (2*a^2*f^(3/2)*ArcTanh[(-2*a*sqrt[f] + 2*c*sqrt[d]*x + b*(sqrt[d] - sqrt[f]*x))/(2*sqrt[c*d - b*sqrt[d]*sqrt[f] + a*f]*sqrt[a + x*(b + c*x)])))/sqrt[c*d + b*sqrt[d]*sqrt[f] + a*f]

$\text{qrt}[a + x*(b + c*x)]])/\text{Sqrt}[c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f])/(8*a^{(5/2)}*d^2)$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 1.43sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [A] time = 0.02, size = 519, normalized size = 1.38

$$f \ln \left(\frac{\frac{2af+2cd-2\sqrt{df}b}{f} + \frac{(bf-2\sqrt{df}c)\left(x+\frac{\sqrt{df}}{f}\right)}{f} + 2\sqrt{\frac{af+cd-\sqrt{df}b}{f}} \sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2 c + \frac{(bf-2\sqrt{df}c)\left(x+\frac{\sqrt{df}}{f}\right) + af+cd-\sqrt{df}b}{f}}}{x+\frac{\sqrt{df}}{f}} \right) + f \ln \left(\frac{\frac{2af+2cd+2\sqrt{df}b}{f} + \frac{(bf+2\sqrt{df}c)\left(x+\frac{\sqrt{df}}{f}\right)}{f} + 2\sqrt{\frac{af+cd+\sqrt{df}b}{f}} \sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2 c + \frac{(bf+2\sqrt{df}c)\left(x+\frac{\sqrt{df}}{f}\right) + af+cd+\sqrt{df}b}{f}}}{x+\frac{\sqrt{df}}{f}} \right) + \frac{2\sqrt{\frac{af+cd-\sqrt{df}b}{f}} d^2}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)

[Out] $\frac{1}{2}f/d^2/((a*f+c*d-(d*f)^{(1/2)}*b)/f)^{(1/2)}*\ln((2*(a*f+c*d-(d*f)^{(1/2)}*b)/f+(b*f-2*(d*f)^{(1/2)}*c)*(x+(d*f)^{(1/2)}/f)/f+2*((a*f+c*d-(d*f)^{(1/2)}*b)/f)^{(1/2)}*((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)}*c)*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)}*b)/f)^{(1/2)})/(x+(d*f)^{(1/2)}/f))+1/2*f/d^2/((a*f+c*d+(d*f)^{(1/2)}*b)/f)^{(1/2)}*\ln((2*(a*f+c*d+(d*f)^{(1/2)}*b)/f+(b*f+2*(d*f)^{(1/2)}*c)*(x-(d*f)^{(1/2)}/f)/f+2*((a*f+c*d+(d*f)^{(1/2)}*b)/f)^{(1/2)}*((x-(d*f)^{(1/2)}/f)^2*c+($

$$\frac{b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)}}{(x-(d*f)^{(1/2)/f})^{-1/2*(c*x^2+b*x+a)^{(1/2)}/a/d/x^2+3/4*b*(c*x^2+b*x+a)^{(1/2)}/a^2/d/x-3/8/d*b^2/a^{(5/2)*\ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)*a^{(1/2))}/x)+1/2/d*c/a^{(3/2)*\ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)*a^{(1/2))}/x)-f/d^2/a^{(1/2)*\ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)*a^{(1/2))}/x)$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{cx^2 + bx + a} (fx^2 - d)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] -integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3 (d - f x^2) \sqrt{c x^2 + b x + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)

[Out] int(1/(x^3*(d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-dx^3\sqrt{a + bx + cx^2} + fx^5\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)

[Out] -Integral(1/(-d*x**3*sqrt(a + b*x + c*x**2) + f*x**5*sqrt(a + b*x + c*x**2)), x)

$$3.102 \quad \int \frac{x^4}{(a+bx+cx^2)^{3/2}(d-fx^2)} dx$$

Optimal. Leaf size=466

$$\frac{2d^2(b(b^2f - c(3af + cd)) - cx(2acf + b^2(-f) + 2c^2d))}{f^2(b^2 - 4ac)\sqrt{a + bx + cx^2}(b^2df - (af + cd)^2)} + \frac{2d(b + 2cx)}{f^2(b^2 - 4ac)\sqrt{a + bx + cx^2}} + \frac{2b\sqrt{a + bx + cx^2}}{cf(b^2 - 4ac)} - \frac{f}{f}$$

[Out] $-\operatorname{arctanh}\left(\frac{1}{2}(2cx+b)/c\right)^{1/2}/(cx^2+bx+a)^{1/2}/c^{3/2}/f+1/2d^{3/2}*\operatorname{arctanh}\left(\frac{1}{2}(bd^{1/2}-2*af^{1/2}+x*(2*cd^{1/2}-b*f^{1/2}))\right)/(cx^2+bx+a)^{1/2}/(cd+af-b*d^{1/2}*f^{1/2})^{1/2}/f/(cd+af-b*d^{1/2}*f^{1/2})^{3/2}+1/2d^{3/2}*\operatorname{arctanh}\left(\frac{1}{2}(bd^{1/2}+2*af^{1/2}+x*(2*cd^{1/2}+b*f^{1/2}))\right)/(cx^2+bx+a)^{1/2}/(cd+af+b*d^{1/2}*f^{1/2})^{1/2}/f/(cd+af+b*d^{1/2}*f^{1/2})^{3/2}-2*x*(bx+2*a)/(-4*a*c+b^2)/f/(cx^2+bx+a)^{1/2}+2*d*(2*cx+b)/(-4*a*c+b^2)/f^2/(cx^2+bx+a)^{1/2}-2*d^2*(b*(b^2*f-c*(3*af+cd))-c*(2*a*c*f-b^2*f+2*c^2*d)*x)/(-4*a*c+b^2)/f^2/(b^2*d*f-(a*f+c*d)^2)/(cx^2+bx+a)^{1/2}+2*b*(cx^2+bx+a)^{1/2}/c/(-4*a*c+b^2)/f$

Rubi [A] time = 1.35, antiderivative size = 466, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {6725, 613, 738, 640, 621, 206, 975, 1033, 724}

$$\frac{2d^2(b(b^2f - c(3af + cd)) - cx(2acf + b^2(-f) + 2c^2d))}{f^2(b^2 - 4ac)\sqrt{a + bx + cx^2}(b^2df - (af + cd)^2)} + \frac{2d(b + 2cx)}{f^2(b^2 - 4ac)\sqrt{a + bx + cx^2}} + \frac{2b\sqrt{a + bx + cx^2}}{cf(b^2 - 4ac)} - \frac{f}{f}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^4/((a + b*x + c*x^2)^{3/2}*(d - f*x^2)), x]$

[Out] $(-2*x*(2*a + b*x))/((b^2 - 4*a*c)*f*\operatorname{Sqrt}[a + b*x + c*x^2]) + (2*d*(b + 2*c*x))/((b^2 - 4*a*c)*f^2*\operatorname{Sqrt}[a + b*x + c*x^2]) - (2*d^2*(b*(b^2*f - c*(c*d + 3*a*f)) - c*(2*c^2*d - b^2*f + 2*a*c*f)*x))/((b^2 - 4*a*c)*f^2*(b^2*d*f - (c*d + a*f)^2)*\operatorname{Sqrt}[a + b*x + c*x^2]) + (2*b*\operatorname{Sqrt}[a + b*x + c*x^2])/((c*(b^2 - 4*a*c)*f) - \operatorname{ArcTanh}[(b + 2*c*x)/(2*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(c^{3/2}*f) + (d^{3/2}*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] - 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] - b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)*\operatorname{Sqrt}[a + b*x + c*x^2]))/(2*f*(c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)^{3/2}) + (d^{3/2}*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] + 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] + b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)*\operatorname{Sqrt}[a + b*x + c*x^2]))/(2*f*(c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)^{3/2})$

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 613

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-3/2), x_Symbol] := Simp[(-2*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 621

Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 640

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 724

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 738

Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] && IntQuadraticQ[a, b, c, d, e, m, p, x]

Rule 975

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*

```
f)))*x)*(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1))/((b^2 - 4*a*c)*(b^2*d
*f + (c*d - a*f)^2)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d -
a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^2
*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1)
- c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d +
b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2*
p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] && !( !IntegerQ[
p] && ILtQ[q, -1]) && !IGtQ[q, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{(a+bx+cx^2)^{3/2}(d-fx^2)} dx &= \int \left(-\frac{d}{f^2(a+bx+cx^2)^{3/2}} - \frac{x^2}{f(a+bx+cx^2)^{3/2}} + \frac{d^2}{f^2(a+bx+cx^2)^{3/2}(d-fx^2)} \right) dx \\
&= -\frac{d \int \frac{1}{(a+bx+cx^2)^{3/2}} dx}{f^2} + \frac{d^2 \int \frac{1}{(a+bx+cx^2)^{3/2}(d-fx^2)} dx}{f^2} - \frac{\int \frac{x^2}{(a+bx+cx^2)^{3/2}} dx}{f} \\
&= -\frac{2x(2a+bx)}{(b^2-4ac)f\sqrt{a+bx+cx^2}} + \frac{2d(b+2cx)}{(b^2-4ac)f^2\sqrt{a+bx+cx^2}} - \frac{2d^2(b(b^2f-2ac))}{(b^2-4ac)^2f\sqrt{a+bx+cx^2}} \\
&= -\frac{2x(2a+bx)}{(b^2-4ac)f\sqrt{a+bx+cx^2}} + \frac{2d(b+2cx)}{(b^2-4ac)f^2\sqrt{a+bx+cx^2}} - \frac{2d^2(b(b^2f-2ac))}{(b^2-4ac)^2f\sqrt{a+bx+cx^2}} \\
&= -\frac{2x(2a+bx)}{(b^2-4ac)f\sqrt{a+bx+cx^2}} + \frac{2d(b+2cx)}{(b^2-4ac)f^2\sqrt{a+bx+cx^2}} - \frac{2d^2(b(b^2f-2ac))}{(b^2-4ac)^2f\sqrt{a+bx+cx^2}} \\
&= -\frac{2x(2a+bx)}{(b^2-4ac)f\sqrt{a+bx+cx^2}} + \frac{2d(b+2cx)}{(b^2-4ac)f^2\sqrt{a+bx+cx^2}} - \frac{2d^2(b(b^2f-2ac))}{(b^2-4ac)^2f\sqrt{a+bx+cx^2}}
\end{aligned}$$

Mathematica [A] time = 1.41, size = 562, normalized size = 1.21

$$\frac{f \left(a(b^2-4ac) \tanh^{-1} \left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}} \right) - 2\sqrt{c} \left(a(b-2cx)+bcx^2 \right) \sqrt{a+x(b+cx)} \right)}{ac^{3/2}(4ac-b^2)} + \frac{d^{3/2} f \left(\frac{(b^2-4ac)(af+b\sqrt{d}\sqrt{f}+cd) \tanh^{-1} \left(\frac{-2a\sqrt{f}+b(\sqrt{d}-\sqrt{f}x)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} \right)}{\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} \right)}{2(b^2-4ac)(af+b\sqrt{d}\sqrt{f}+cd)}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)), x]

[Out] ((2*d*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]) - (2*f*x^3*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]) - (2*d^2*(b^3*f - b*c*(c*d + 3*a*f) + b^2*c*f*x - 2*c^2*(c*d + a*f)*x))/((b^2 - 4*a*c)*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + x*(b + c*x)]) + (f*(-2*Sqrt[c]*(b*c*x^2 + a*(b - 2*c*x))*Sqrt[a + x*(b + c*x)] + a*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sq

$$\frac{\text{rt}[c] \cdot \sqrt{a + x(b + cx)}}{\left(\frac{b^2 - 4ac}{c^2} \right)^{3/2} \sqrt{-b^2 + 4ac}} + \frac{d^{3/2} f \left(\frac{b^2 - 4ac}{c^2} \right)^{3/2} \left(\frac{c^2 d + b \sqrt{d} \sqrt{f} + af}{2 \sqrt{c^2 d - b \sqrt{d} \sqrt{f} + af}} \right) \text{ArcTanh} \left[\frac{-2a \sqrt{f} + 2c \sqrt{d} x + b(\sqrt{d} - \sqrt{f} x)}{2 \sqrt{c^2 d - b \sqrt{d} \sqrt{f} + af}} \right] \sqrt{a + x(b + cx)}}{\sqrt{c^2 d - b \sqrt{d} \sqrt{f} + af}} + \frac{(-b^2 + 4ac) \left(\frac{c^2 d - b \sqrt{d} \sqrt{f} + af}{c^2} \right) \text{ArcTanh} \left[\frac{-2(a \sqrt{f} + c \sqrt{d} x) - b(\sqrt{d} + \sqrt{f} x)}{2 \sqrt{c^2 d + b \sqrt{d} \sqrt{f} + af}} \right] \sqrt{a + x(b + cx)}}{\sqrt{c^2 d + b \sqrt{d} \sqrt{f} + af}} \right) / (2(b^2 - 4ac)(-b^2 d f + (c^2 d + af)^2)) / f^2$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);OUTPUT:Error: Bad Argument Type

maple [B] time = 0.02, size = 1648, normalized size = 3.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)

[Out] $\frac{1}{f} \frac{x}{c} \frac{1}{(c^2 x^2 + b^2 x + a)^{1/2}} - \frac{1}{2} \frac{f b}{c^2} \frac{1}{(c^2 x^2 + b^2 x + a)^{1/2}} - \frac{1}{f} \frac{b^2}{c} \frac{1}{(4ac - b^2) (c^2 x^2 + b^2 x + a)^{1/2}} - \frac{1}{f} \frac{1}{c^{3/2}} \ln \left(\frac{c^2 x + 1/2 b}{c^{1/2} + (c^2 x^2 + b^2 x + a)^{1/2}} \right) - \frac{4}{f^2} \frac{d}{(4ac - b^2) (c^2 x^2 + b^2 x + a)^{1/2}} + \frac{c^2 x - 2}{f^2} \frac{d}{(4ac - b^2) (c^2 x^2 + b^2 x + a)^{1/2}} + \frac{b + 1/2 f d^2}{(d f)^{1/2}} \frac{1}{(a f + c d - (d f)^{1/2} b)} \left(\frac{x + (d f)^{1/2}}{f} \right)^2 c + \frac{b f - 2 (d f)^{1/2} c}{(1/2) c} \left(\frac{x + (d f)^{1/2}}{f} \right) / f + \frac{a f + c d - (d f)^{1/2} b}{f} \left(\frac{x + (d f)^{1/2}}{f} \right)^2 + \frac{2}{f^2} \frac{d^2}{(a f + c d - (d f)^{1/2} b)} \frac{1}{(4ac - b^2) (c^2 x^2 + b^2 x + a)^{1/2}} + \frac{(x + (d f)^{1/2}) / f}{f + (a f + c d - (d f)^{1/2} b) / f} \left(\frac{x + (d f)^{1/2}}{f} \right)^2 c - \frac{1}{f} \frac{d^2}{(d f)^{1/2}} \frac{1}{(a f + c d - (d f)^{1/2} b)} \frac{1}{(4ac - b^2) (c^2 x^2 + b^2 x + a)^{1/2}} + \frac{(x + (d f)^{1/2}) / f}{f + (a f + c d - (d f)^{1/2} b) / f} \left(\frac{x + (d f)^{1/2}}{f} \right)^2 c + \frac{b f - 2 (d f)^{1/2} c}{(1/2) c} \left(\frac{x + (d f)^{1/2}}{f} \right) / f + \frac{a f + c d - (d f)^{1/2} b}{f} \left(\frac{x + (d f)^{1/2}}{f} \right)^2 + \frac{2}{f^2} \frac{d^2}{(a f + c d - (d f)^{1/2} b)} \frac{1}{(4ac - b^2) (c^2 x^2 + b^2 x + a)^{1/2}}$

$$f)^{(1/2)*c}*(x+(d*f)^{(1/2)/f}/f+(a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2)*x*b*c+1/f^2*d^2/(a*f+c*d-(d*f)^{(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c}*(x+(d*f)^{(1/2)/f}/f+(a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2)*b*c-1/2/f*d^2/(d*f)^{(1/2)/(a*f+c*d-(d*f)^{(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c}*(x+(d*f)^{(1/2)/f}/f+(a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2)*b^2-1/2/f*d^2/(d*f)^{(1/2)/(a*f+c*d-(d*f)^{(1/2)*b)/((a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2)*ln((2*(a*f+c*d-(d*f)^{(1/2)*b)/f+(b*f-2*(d*f)^{(1/2)*c}*(x+(d*f)^{(1/2)/f}/f+2*((a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2)*((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c}*(x+(d*f)^{(1/2)/f}/f+(a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2)))/(x+(d*f)^{(1/2)/f})))-1/2/f*d^2/(d*f)^{(1/2)/(a*f+c*d+(d*f)^{(1/2)*b)/((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c}*(x-(d*f)^{(1/2)/f}/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)+2/f^2*d^2/(a*f+c*d+(d*f)^{(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c}*(x-(d*f)^{(1/2)/f}/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*x*c^2+1/f*d^2/(d*f)^{(1/2)/(a*f+c*d+(d*f)^{(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c}*(x-(d*f)^{(1/2)/f}/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*x*b*c+1/f^2*d^2/(a*f+c*d+(d*f)^{(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c}*(x-(d*f)^{(1/2)/f}/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*b*c+1/2/f*d^2/(d*f)^{(1/2)/(a*f+c*d+(d*f)^{(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c}*(x-(d*f)^{(1/2)/f}/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*b^2+1/2/f*d^2/(d*f)^{(1/2)/(a*f+c*d+(d*f)^{(1/2)*b)/((a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*ln((2*(a*f+c*d+(d*f)^{(1/2)*b)/f+(b*f+2*(d*f)^{(1/2)*c}*(x-(d*f)^{(1/2)/f}/f+2*((a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c}*(x-(d*f)^{(1/2)/f}/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)))/(x-(d*f)^{(1/2)/f}))$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2 -c*((b*sqrt(4*d*f)) /((2*f) + (c*d)/f+a)) /f^2 zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{(d - f x^2) (c x^2 + b x + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int(x^4/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)
```

```
sympy [F]    time = 0.00, size = 0, normalized size = 0.00
```

$$-\int \frac{x^4}{-ad\sqrt{a+bx+cx^2} + afx^2\sqrt{a+bx+cx^2} - bdx\sqrt{a+bx+cx^2} + bfx^3\sqrt{a+bx+cx^2} - cdx^2\sqrt{a+bx+cx^2} + \dots} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x**4/(-a*d*sqrt(a + b*x + c*x**2) + a*f*x**2*sqrt(a + b*x + c*x**2) - b*d*x*sqrt(a + b*x + c*x**2) + b*f*x**3*sqrt(a + b*x + c*x**2) - c*d*x**2*sqrt(a + b*x + c*x**2) + c*f*x**4*sqrt(a + b*x + c*x**2)), x)
```

$$3.103 \quad \int \frac{x^3}{(a+bx+cx^2)^{3/2}(d-fx^2)} dx$$

Optimal. Leaf size=341

$$\frac{2d(a(2acf + b^2(-f) + 2c^2d) + bcx(cd - af))}{f(b^2 - 4ac)\sqrt{a+bx+cx^2}(b^2df - (af + cd)^2)} - \frac{2(2a + bx)}{f(b^2 - 4ac)\sqrt{a+bx+cx^2}} - \frac{d \tanh^{-1}\left(\frac{-2a\sqrt{f} + x(2c\sqrt{d} - b\sqrt{f})}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f} + c}\right)}{2\sqrt{f}(af + b(-\sqrt{d})\sqrt{f} + c)}$$

[Out] $-1/2*d*\operatorname{arctanh}(1/2*(b*d^{(1/2)}-2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}-b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)}/(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(1/2)}/f^{(1/2)}/(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(3/2)}+1/2*d*\operatorname{arctanh}(1/2*(b*d^{(1/2)}+2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}+b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)}/(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(1/2)}/f^{(1/2)}/(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(3/2)}-2*(b*x+2*a)/(-4*a*c+b^2)/f/(c*x^2+b*x+a)^{(1/2)}-2*d*(a*(2*a*c*f-b^2*f+2*c^2*d)+b*c*(-a*f+c*d)*x)/(-4*a*c+b^2)/f/(b^2*d*f-(a*f+c*d)^2)/(c*x^2+b*x+a)^{(1/2)}$

Rubi [A] time = 1.04, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {6725, 636, 1018, 1033, 724, 206}

$$\frac{2d(a(2acf + b^2(-f) + 2c^2d) + bcx(cd - af))}{f(b^2 - 4ac)\sqrt{a+bx+cx^2}(b^2df - (af + cd)^2)} - \frac{2(2a + bx)}{f(b^2 - 4ac)\sqrt{a+bx+cx^2}} - \frac{d \tanh^{-1}\left(\frac{-2a\sqrt{f} + x(2c\sqrt{d} - b\sqrt{f})}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f} + c}\right)}{2\sqrt{f}(af + b(-\sqrt{d})\sqrt{f} + c)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^3/((a + b*x + c*x^2)^{(3/2)}*(d - f*x^2)), x]$

[Out] $(-2*(2*a + b*x))/((b^2 - 4*a*c)*f*\operatorname{Sqrt}[a + b*x + c*x^2]) - (2*d*(a*(2*c^2*d - b^2*f + 2*a*c*f) + b*c*(c*d - a*f)*x))/((b^2 - 4*a*c)*f*(b^2*d*f - (c*d + a*f)^2)*\operatorname{Sqrt}[a + b*x + c*x^2]) - (d*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] - 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] - b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)*\operatorname{Sqrt}[a + b*x + c*x^2]])/(2*\operatorname{Sqrt}[f]*(c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)^{(3/2)}) + (d*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] + 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] + b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)*\operatorname{Sqrt}[a + b*x + c*x^2]])/(2*\operatorname{Sqrt}[f]*(c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)^{(3/2)})$

Rule 206

$\operatorname{Int}[(a + (b \cdot x)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

Rule 636

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] := Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1018

Int[((g_.) + (h_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1)*((g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f)*x))/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p + q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] && !(IntegerQ[p] && ILtQ[q, -1])

Rule 1033

Int[((g_.) + (h_.)*(x_))/(((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]

Rule 6725

Int[(u_)/((a_.) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ

[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d-fx^2)} dx &= \int \left(\frac{x}{f(a+bx+cx^2)^{3/2}} + \frac{dx}{f(a+bx+cx^2)^{3/2}(d-fx^2)} \right) dx \\
&= -\frac{\int \frac{x}{(a+bx+cx^2)^{3/2}} dx}{f} + \frac{d \int \frac{x}{(a+bx+cx^2)^{3/2}(d-fx^2)} dx}{f} \\
&= -\frac{2(2a+bx)}{(b^2-4ac)f\sqrt{a+bx+cx^2}} - \frac{2d(a(2c^2d-b^2f+2acf)+bc(cd-af)x)}{(b^2-4ac)f(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} \\
&= -\frac{2(2a+bx)}{(b^2-4ac)f\sqrt{a+bx+cx^2}} - \frac{2d(a(2c^2d-b^2f+2acf)+bc(cd-af)x)}{(b^2-4ac)f(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} \\
&= -\frac{2(2a+bx)}{(b^2-4ac)f\sqrt{a+bx+cx^2}} - \frac{2d(a(2c^2d-b^2f+2acf)+bc(cd-af)x)}{(b^2-4ac)f(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} \\
&= -\frac{2(2a+bx)}{(b^2-4ac)f\sqrt{a+bx+cx^2}} - \frac{2d(a(2c^2d-b^2f+2acf)+bc(cd-af)x)}{(b^2-4ac)f(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}}
\end{aligned}$$

Mathematica [A] time = 1.20, size = 414, normalized size = 1.21

$$\frac{1}{2} \left(\frac{8a^3f + 4a^2(bfx + 2cd) - 4abd(b - 3cx) - 4b^3dx}{(b^2 - 4ac)\sqrt{a + x(b + cx)}(f(b^2d - a^2f) - 2acdf - c^2d^2)} - \frac{d \log(\sqrt{d}\sqrt{f} - fx)}{\sqrt{f}(af + b\sqrt{d}\sqrt{f} + cd)^{3/2}} - \frac{d \log(\sqrt{d}\sqrt{f} + fx)}{\sqrt{f}(af + b(-\sqrt{d}\sqrt{f} + cd)^{3/2})} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)), x]

```

[Out] ((8*a^3*f - 4*b^3*d*x - 4*a*b*d*(b - 3*c*x) + 4*a^2*(2*c*d + b*f*x))/((b^2
- 4*a*c)*(-(c^2*d^2) - 2*a*c*d*f + f*(b^2*d - a^2*f))*Sqrt[a + x*(b + c*x)]
) - (d*Log[Sqrt[d]*Sqrt[f] - f*x])/(Sqrt[f]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)

```

$$\begin{aligned} & \left(\frac{d \cdot \log(\sqrt{d} \cdot \sqrt{f} + f \cdot x)}{(\sqrt{f} \cdot (c \cdot d - b \cdot \sqrt{d} \cdot \sqrt{f} + a \cdot f)^{3/2})} + \frac{d \cdot \log(\sqrt{d} \cdot (-b \cdot \sqrt{d}) + 2 \cdot a \cdot \sqrt{f} - 2 \cdot c \cdot \sqrt{d} \cdot x + b \cdot \sqrt{f} \cdot x + 2 \cdot \sqrt{c \cdot d - b \cdot \sqrt{d} \cdot \sqrt{f} + a \cdot f}) \cdot \sqrt{a + x \cdot (b + c \cdot x)}}{(\sqrt{f} \cdot (c \cdot d - b \cdot \sqrt{d} \cdot \sqrt{f} + a \cdot f)^{3/2})} + \frac{d \cdot \log(\sqrt{d} \cdot (b \cdot (\sqrt{d} + \sqrt{f} \cdot x) + 2 \cdot (a \cdot \sqrt{f} + c \cdot \sqrt{d}) \cdot x + \sqrt{c \cdot d + b \cdot \sqrt{d} \cdot \sqrt{f} + a \cdot f}) \cdot \sqrt{a + x \cdot (b + c \cdot x)})}{(\sqrt{f} \cdot (c \cdot d + b \cdot \sqrt{d} \cdot \sqrt{f} + a \cdot f)^{3/2})} \right) / 2 \end{aligned}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
i,const vecteur & l) Error: Bad Argument Valueint() Bad Argument Type

maple [B] time = 0.02, size = 1480, normalized size = 4.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)

[Out]
$$\begin{aligned} & \frac{1}{f} \cdot \frac{c}{(c \cdot x^2 + b \cdot x + a)^{1/2}} + \frac{2}{f} \cdot \frac{b}{(4 \cdot a \cdot c - b^2)^{1/2}} \cdot \frac{1}{(c \cdot x^2 + b \cdot x + a)^{1/2}} \cdot x + \frac{1}{f} \cdot \frac{b^2}{c} \\ & \cdot \frac{1}{(4 \cdot a \cdot c - b^2)^{1/2}} \cdot \frac{1}{(c \cdot x^2 + b \cdot x + a)^{1/2}} - \frac{1}{2} \cdot \frac{f \cdot d}{(a \cdot f + c \cdot d - (d \cdot f)^{1/2} \cdot b)} \cdot \frac{1}{((x + (d \cdot f)^{1/2}) / f)^{2 \cdot c + (b \cdot f - 2 \cdot (d \cdot f)^{1/2} \cdot c) \cdot (x + (d \cdot f)^{1/2}) / f + (a \cdot f + c \cdot d - (d \cdot f)^{1/2} \cdot b) / f)^{1/2}} \\ & - \frac{2}{f} \cdot \frac{d}{(a \cdot f + c \cdot d - (d \cdot f)^{1/2} \cdot b)} \cdot \frac{1}{(4 \cdot a \cdot c - b^2)^{1/2}} \cdot \frac{1}{((x + (d \cdot f)^{1/2}) / f)^{2 \cdot c + (b \cdot f - 2 \cdot (d \cdot f)^{1/2} \cdot c) \cdot (x + (d \cdot f)^{1/2}) / f + (a \cdot f + c \cdot d - (d \cdot f)^{1/2} \cdot b) / f)^{1/2}} \\ & \cdot \frac{1}{(d \cdot f)^{1/2}} \cdot \frac{x \cdot c^2 + 1}{f \cdot d} \cdot \frac{1}{(a \cdot f + c \cdot d - (d \cdot f)^{1/2} \cdot b)} \cdot \frac{1}{(4 \cdot a \cdot c - b^2)^{1/2}} \cdot \frac{1}{((x + (d \cdot f)^{1/2}) / f)^{2 \cdot c + (b \cdot f - 2 \cdot (d \cdot f)^{1/2} \cdot c) \cdot (x + (d \cdot f)^{1/2}) / f + (a \cdot f + c \cdot d - (d \cdot f)^{1/2} \cdot b) / f)^{1/2}} \\ & \cdot \frac{x \cdot b \cdot c - 1}{f} \cdot \frac{d}{(a \cdot f + c \cdot d - (d \cdot f)^{1/2} \cdot b)} \cdot \frac{1}{(4 \cdot a \cdot c - b^2)^{1/2}} \cdot \frac{1}{((x + (d \cdot f)^{1/2}) / f)^{2 \cdot c + (b \cdot f - 2 \cdot (d \cdot f)^{1/2} \cdot c) \cdot (x + (d \cdot f)^{1/2}) / f + (a \cdot f + c \cdot d - (d \cdot f)^{1/2} \cdot b) / f)^{1/2}} \end{aligned}$$

```

)/f)^(1/2)*(d*f)^(1/2)*b*c+1/2/f*d/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+
(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)
^(1/2)*b)/f)^(1/2)*b^2+1/2/f*d/(a*f+c*d-(d*f)^(1/2)*b)/((a*f+c*d-(d*f)^(1/2)
)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)
)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*
f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x
+(d*f)^(1/2)/f))-1/2/f*d/(a*f+c*d+(d*f)^(1/2)*b)/((x-(d*f)^(1/2)/f)^2*c+(b*
f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)+2/f
^2*d/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)
)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*(d*f)^(1/2)
*x*c^2+1/f*d/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(b*
f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*x*b
*c+1/f^2*d/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(b*f+
2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*(d*f)
^(1/2)*b*c+1/2/f*d/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2
*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/
2)*b^2+1/2/f*d/(a*f+c*d+(d*f)^(1/2)*b)/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln
((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*(
(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*
c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))

```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2 -c*((b*sqrt(4*d*f)) /((2*f) +(c*d)/f+a)) /f^2 zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{(d - f x^2) (c x^2 + b x + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)

[Out] int(x^3/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3}{-ad\sqrt{a+bx+cx^2} + afx^2\sqrt{a+bx+cx^2} - bdx\sqrt{a+bx+cx^2} + bfx^3\sqrt{a+bx+cx^2} - cdx^2\sqrt{a+bx+cx^2} + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d), x)

[Out] -Integral(x**3/(-a*d*sqrt(a + b*x + c*x**2) + a*f*x**2*sqrt(a + b*x + c*x**2) - b*d*x*sqrt(a + b*x + c*x**2) + b*f*x**3*sqrt(a + b*x + c*x**2) - c*d*x**2*sqrt(a + b*x + c*x**2) + c*f*x**4*sqrt(a + b*x + c*x**2)), x)

$$3.104 \quad \int \frac{x^2}{(a+bx+cx^2)^{3/2}(d-fx^2)} dx$$

Optimal. Leaf size=297

$$\frac{2(cx(b^2d - 2a(af + cd)) + ab(cd - af))}{(b^2 - 4ac)\sqrt{a + bx + cx^2}(b^2df - (af + cd)^2)} + \frac{\sqrt{d} \tanh^{-1}\left(\frac{-2a\sqrt{f} + x(2c\sqrt{d} - b\sqrt{f}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2(af + b(-\sqrt{d})\sqrt{f} + cd)^{3/2}} + \frac{\sqrt{d} \tanh^{-1}\left(\frac{2a\sqrt{f} + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2(af + b\sqrt{d}\sqrt{f} + cd)^{3/2}}$$

[Out] $1/2*\operatorname{arctanh}(1/2*(b*d^{(1/2)}-2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}-b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)}/(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(1/2)}*d^{(1/2)}/(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(3/2)}+1/2*\operatorname{arctanh}(1/2*(b*d^{(1/2)}+2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}+b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)}/(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(1/2)}*d^{(1/2)}/(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(3/2)}+2*(a*b*(-a*f+c*d)+c*(b^2*d-2*a*(a*f+c*d))*x)/(-4*a*c+b^2)/(b^2*d*f-(a*f+c*d)^2)/(c*x^2+b*x+a)^{(1/2)}$

Rubi [A] time = 0.45, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1065, 1033, 724, 206}

$$\frac{2(cx(b^2d - 2a(af + cd)) + ab(cd - af))}{(b^2 - 4ac)\sqrt{a + bx + cx^2}(b^2df - (af + cd)^2)} + \frac{\sqrt{d} \tanh^{-1}\left(\frac{-2a\sqrt{f} + x(2c\sqrt{d} - b\sqrt{f}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2(af + b(-\sqrt{d})\sqrt{f} + cd)^{3/2}} + \frac{\sqrt{d} \tanh^{-1}\left(\frac{2a\sqrt{f} + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2(af + b\sqrt{d}\sqrt{f} + cd)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^2/((a + b*x + c*x^2)^{(3/2)}*(d - f*x^2)), x]$

[Out] $(2*(a*b*(c*d - a*f) + c*(b^2*d - 2*a*(c*d + a*f))*x)/((b^2 - 4*a*c)*(b^2*d*f - (c*d + a*f)^2)*\operatorname{Sqrt}[a + b*x + c*x^2]) + (\operatorname{Sqrt}[d]*\operatorname{ArcTanH}[(b*\operatorname{Sqrt}[d] - 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] - b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)*\operatorname{Sqrt}[a + b*x + c*x^2]])/(2*(c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)^{(3/2)}) + (\operatorname{Sqrt}[d]*\operatorname{ArcTanH}[(b*\operatorname{Sqrt}[d] + 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] + b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)*\operatorname{Sqrt}[a + b*x + c*x^2]])/(2*(c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)^{(3/2)})$

Rule 206

$\operatorname{Int}(((a_) + (b_)*(x_)^2)^{-1}, x_Symbol] := \operatorname{Simp}[(1*\operatorname{ArcTanH}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol]
:> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol]
:> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1065

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_.) + (f_.)*(x_)^2)^(q_), x_Symbol]
:> Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1)*((A*c - a*C)*(-(b*(c*d + a*f))) + (A*b)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(2*a*f)) + C*(b^2*d - 2*a*(c*d - a*f)))x) / ((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[(-2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*((A*c - a*C)*(-(b*(c*d + a*f))) + (A*b)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(b*f*(p + 1)))*x - c*f*(b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, A, C, q}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(a+bx+cx^2)^{3/2}(d-fx^2)} dx &= \frac{2(ab(cd-af)+c(b^2d-2a(cd+af))x)}{(b^2-4ac)(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} + \frac{2\int \frac{-\frac{1}{2}(b^2-4ac)d(cd+af)+\frac{1}{2}b(b^2-4ac)}{\sqrt{a+bx+cx^2}(d-fx^2)} dx}{(b^2-4ac)(b^2df-(cd+af)^2)} \\
&= \frac{2(ab(cd-af)+c(b^2d-2a(cd+af))x)}{(b^2-4ac)(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} - \frac{(\sqrt{d}\sqrt{f})\int \frac{1}{(-\sqrt{d}\sqrt{f}-fx)\sqrt{a+bx+cx^2}} dx}{2(cd-b\sqrt{d}\sqrt{f}+a)} \\
&= \frac{2(ab(cd-af)+c(b^2d-2a(cd+af))x)}{(b^2-4ac)(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} + \frac{(\sqrt{d}\sqrt{f})\text{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}\sqrt{f}-fx} dx\right)}{2(cd-b\sqrt{d}\sqrt{f}+a)} \\
&= \frac{2(ab(cd-af)+c(b^2d-2a(cd+af))x)}{(b^2-4ac)(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} + \frac{\sqrt{d}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+(2\sqrt{cd-b\sqrt{d}\sqrt{f}+a})x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+a}}\right)}{2(cd-b\sqrt{d}\sqrt{f}+a)}
\end{aligned}$$

Mathematica [A] time = 0.40, size = 352, normalized size = 1.19

$$\frac{2\left(\frac{a^2f(b+2cx)+acd(2cx-b)-b^2cdx}{\sqrt{a+x(b+cx)}} + \frac{\sqrt{d}(b^2-4ac)(af+b\sqrt{d}\sqrt{f}+cd)\tanh^{-1}\left(\frac{-2a\sqrt{f}+b(\sqrt{d}-\sqrt{f}x)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{4\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} + \frac{\sqrt{d}(4ac-b^2)(af+b(-\sqrt{d})\sqrt{f}+cd)}{4\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{(b^2-4ac)((af+cd)^2-b^2df)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)), x]

[Out] (2*((-(b^2*c*d*x) + a*c*d*(-b + 2*c*x) + a^2*f*(b + 2*c*x))/Sqrt[a + x*(b + c*x)] + ((b^2 - 4*a*c)*Sqrt[d]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f])*Sqrt[a + x*(b + c*x)]))/(4*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + ((-b^2 + 4*a*c)*Sqrt[d]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])*Sqrt[a + x*(b + c*x)]))/(4*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]))/((b^2 - 4*a*c)*(-(b^2*d*f) + (c*d + a*f)^2))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

```
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
i,const vecteur & l) Error: Bad Argument Valueint() Bad Argument Type
```

```
maple [B] time = 0.02, size = 1427, normalized size = 4.80
```

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] -2/f*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+1/2*d/(d*f)^(1/2)/(a*f+c*d-(
d*f)^(1/2)*b)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f
)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)+2*d/f/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b
^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c
*d-(d*f)^(1/2)*b)/f)^(1/2)*x*c^2-d/(d*f)^(1/2)/(a*f+c*d-(d*f)^(1/2)*b)/(4*a
*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a
*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*b*c+d/f/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2
)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d
-(d*f)^(1/2)*b)/f)^(1/2)*b*c-1/2*d/(d*f)^(1/2)/(a*f+c*d-(d*f)^(1/2)*b)/(4*a
*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a
*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b^2-1/2*d/(d*f)^(1/2)/(a*f+c*d-(d*f)^(1/2)*b
)/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*
(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+
(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)
^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))-1/2*d/(d*f)^(1/2)/(a*f+c*d+(d*f)^(1/
2)*b)/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f
+c*d+(d*f)^(1/2)*b)/f)^(1/2)+2*d/f/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-
(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)
^(1/2)*b)/f)^(1/2)*x*c^2+d/(d*f)^(1/2)/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/
((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(
d*f)^(1/2)*b)/f)^(1/2)*x*b*c+d/f/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d
*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(
1/2)*b)/f)^(1/2)*b*c+1/2*d/(d*f)^(1/2)/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/
```


$$\left(\frac{x-(d*f)^{1/2}}{f}\right)^2*c+(b*f+2*(d*f)^{1/2}*c)*\left(\frac{x-(d*f)^{1/2}}{f}\right)/f+(a*f+c*d+(d*f)^{1/2}*b)/f^{1/2}*b^2+1/2*d/(d*f)^{1/2}/(a*f+c*d+(d*f)^{1/2}*b)/((a*f+c*d+(d*f)^{1/2}*b)/f)^{1/2}*\ln\left(\frac{2*(a*f+c*d+(d*f)^{1/2}*b)/f+(b*f+2*(d*f)^{1/2}*c)*\left(\frac{x-(d*f)^{1/2}}{f}\right)/f+2*(a*f+c*d+(d*f)^{1/2}*b)/f^{1/2}}{2*(a*f+c*d+(d*f)^{1/2}*b)/f+(b*f+2*(d*f)^{1/2}*c)*\left(\frac{x-(d*f)^{1/2}}{f}\right)/f+(a*f+c*d+(d*f)^{1/2}*b)/f^{1/2}}\right)/\left(\frac{x-(d*f)^{1/2}}{f}\right)$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation may help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see 'assume?' for more details)Is $\frac{(c*\sqrt{4*d*f})/(2*f^2) + b/(2*f)}{-c*((b*\sqrt{4*d*f})/(2*f) + (c*d)/f+a)}$ zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{(d - f x^2) (c x^2 + b x + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)

[Out] int(x^2/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{-ad\sqrt{a+bx+cx^2} + afx^2\sqrt{a+bx+cx^2} - bdx\sqrt{a+bx+cx^2} + bfx^3\sqrt{a+bx+cx^2} - cdx^2\sqrt{a+bx+cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)

[Out] -Integral(x**2/(-a*d*sqrt(a + b*x + c*x**2) + a*f*x**2*sqrt(a + b*x + c*x**2) - b*d*x*sqrt(a + b*x + c*x**2) + b*f*x**3*sqrt(a + b*x + c*x**2) - c*d*x**2*sqrt(a + b*x + c*x**2) + c*f*x**4*sqrt(a + b*x + c*x**2)), x)

$$3.105 \quad \int \frac{x}{(a+bx+cx^2)^{3/2}(d-fx^2)} dx$$

Optimal. Leaf size=299

$$\frac{2(a(2acf + b^2(-f) + 2c^2d) + bcx(cd - af))}{(b^2 - 4ac)\sqrt{a + bx + cx^2}(b^2df - (af + cd)^2)} - \frac{\sqrt{f} \tanh^{-1}\left(\frac{-2a\sqrt{f} + x(2c\sqrt{d} - b\sqrt{f}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f+cd}}\right)}{2(af + b(-\sqrt{d})\sqrt{f} + cd)^{3/2}} + \frac{\sqrt{f} \tanh^{-1}\left(\frac{2a\sqrt{f} + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f+cd}}\right)}{2(af + b\sqrt{d})\sqrt{f+cd}}$$

[Out] $-1/2*\operatorname{arctanh}(1/2*(b*d^{(1/2)}-2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}-b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)}/(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(1/2)}*f^{(1/2)}/(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(3/2)}+1/2*\operatorname{arctanh}(1/2*(b*d^{(1/2)}+2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}+b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)}/(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(1/2)}*f^{(1/2)}/(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(3/2)}-2*(a*(2*a*c*f-b^2*f+2*c^2*d)+b*c*(-a*f+c*d)*x)/(-4*a*c+b^2)/(b^2*d*f-(a*f+c*d)^2)/(c*x^2+b*x+a)^{(1/2)}$

Rubi [A] time = 0.40, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1018, 1033, 724, 206}

$$\frac{2(a(2acf + b^2(-f) + 2c^2d) + bcx(cd - af))}{(b^2 - 4ac)\sqrt{a + bx + cx^2}(b^2df - (af + cd)^2)} - \frac{\sqrt{f} \tanh^{-1}\left(\frac{-2a\sqrt{f} + x(2c\sqrt{d} - b\sqrt{f}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f+cd}}\right)}{2(af + b(-\sqrt{d})\sqrt{f} + cd)^{3/2}} + \frac{\sqrt{f} \tanh^{-1}\left(\frac{2a\sqrt{f} + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f+cd}}\right)}{2(af + b\sqrt{d})\sqrt{f+cd}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x/((a + b*x + c*x^2)^{(3/2)}*(d - f*x^2)), x]$

[Out] $(-2*(a*(2*c^2*d - b^2*f + 2*a*c*f) + b*c*(c*d - a*f)*x))/((b^2 - 4*a*c)*(b^2*d*f - (c*d + a*f)^2)*\operatorname{Sqrt}[a + b*x + c*x^2]) - (\operatorname{Sqrt}[f]*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] - 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] - b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)*\operatorname{Sqrt}[a + b*x + c*x^2]])/(2*(c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)^{(3/2)}) + (\operatorname{Sqrt}[f]*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] + 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] + b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)*\operatorname{Sqrt}[a + b*x + c*x^2]])/(2*(c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)^{(3/2)})$

Rule 206

$\operatorname{Int}[(a + b*x)(x^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b, x\} \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_.))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol]
:= Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1018

```
Int[((g_.) + (h_.)*(x_.))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)*((d_.) + (f_.)*(x_)^2)^(q_.), x_Symbol]
:= Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1)*((g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p + q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] && !(IntegerQ[p] && ILtQ[q, -1])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_.))/(((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol]
:= With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(a+bx+cx^2)^{3/2}(d-fx^2)} dx &= -\frac{2(a(2c^2d-b^2f+2acf)+bc(cd-af)x)}{(b^2-4ac)(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} + \frac{2\int \frac{\frac{1}{2}b(b^2-4ac)df-\frac{1}{2}(b^2-4ac)f(c)}{\sqrt{a+bx+cx^2}(d-fx^2)}}{(b^2-4ac)(b^2df-(cd+af)^2)} \\
&= -\frac{2(a(2c^2d-b^2f+2acf)+bc(cd-af)x)}{(b^2-4ac)(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} + \frac{f\int \frac{1}{(-\sqrt{d}\sqrt{f}-fx)\sqrt{a+bx+cx^2}}}{2(cd-b\sqrt{d}\sqrt{f}+af)} \\
&= -\frac{2(a(2c^2d-b^2f+2acf)+bc(cd-af)x)}{(b^2-4ac)(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} - \frac{f\text{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4cd}\right)}{cd} \\
&= -\frac{2(a(2c^2d-b^2f+2acf)+bc(cd-af)x)}{(b^2-4ac)(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} - \frac{\sqrt{f}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+(2\sqrt{cd-b\sqrt{d}\sqrt{f}+af})}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}}\right)}{2(cd-b\sqrt{d}\sqrt{f}+af)}
\end{aligned}$$

Mathematica [A] time = 0.35, size = 356, normalized size = 1.19

$$\frac{2\left(\frac{2a^2cf+a(b^2(-f)-bcfx+2c^2d)+bc^2dx}{\sqrt{a+x(b+cx)}} - \frac{\sqrt{f}(b^2-4ac)(af+b\sqrt{d}\sqrt{f}+cd)\tanh^{-1}\left(\frac{-2a\sqrt{f}+b(\sqrt{d}-\sqrt{f}x)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{4\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right) + \frac{\sqrt{f}(4ac-b^2)(af+b(-\sqrt{d})\sqrt{f}+cd)}{4\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}}{(b^2-4ac)((af+cd)^2-b^2df)}$$

Antiderivative was successfully verified.

[In] Integrate[x/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)), x]

[Out] (2*((2*a^2*c*f + b*c^2*d*x + a*(2*c^2*d - b^2*f - b*c*f*x))/Sqrt[a + x*(b + c*x)] - ((b^2 - 4*a*c)*Sqrt[f]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])))/(4*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f)) + ((-b^2 + 4*a*c)*Sqrt[f]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])))/(4*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f)))/((b^2 - 4*a*c)*(-(b^2*d*f) + (c*d + a*f)^2))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
i,const vecteur & l) Error: Bad Argument Valueint() Bad Argument Type

maple [B] time = 0.02, size = 1360, normalized size = 4.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)

[Out]
$$-1/2/(a*f+c*d-(d*f)^{(1/2)*b})/((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}-2/f/(a*f+c*d-(d*f)^{(1/2)*b})/(4*a*c-b^2)/((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*(d*f)^{(1/2)*x*c^2+1/(a*f+c*d-(d*f)^{(1/2)*b})/(4*a*c-b^2)/((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)*x*b*c-1/f/(a*f+c*d-(d*f)^{(1/2)*b})/(4*a*c-b^2)/((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*(d*f)^{(1/2)*b*c+1/2/(a*f+c*d-(d*f)^{(1/2)*b})/(4*a*c-b^2)/((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+2*((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)})/(x+(d*f)^{(1/2)}/f))-1/2/(a*f+c*d+(d*f)^{(1/2)*b})/((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}+2/f/(a*f+c*d+(d*f)^{(1/2)*b})/(4*a*c-b^2)/((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*(d*f)^{(1/2)*x*c^2+1/(a*f+c*d+(d*f)^{(1/2)*b})/(4*a*c-b^2)/((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)*x*b*c+1/f/(a*f+c*d+(d*f)^{(1/2)*b})/(4*a*c-b^2)/((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*(d*f)^{(1/2)*b*c+1/2/(a*f+c*d+(d*f)^{(1/2)*b})/(4*a*c-b^2)/((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}$$

```
) * b^2 + 1/2 / (a * f + c * d + (d * f)^(1/2) * b) / ((a * f + c * d + (d * f)^(1/2) * b) / f)^(1/2) * ln((2 * (a * f + c * d + (d * f)^(1/2) * b) / f + (b * f + 2 * (d * f)^(1/2) * c) * (x - (d * f)^(1/2) / f) / f + 2 * ((a * f + c * d + (d * f)^(1/2) * b) / f)^(1/2) * ((x - (d * f)^(1/2) / f)^2 * c + (b * f + 2 * (d * f)^(1/2) * c) * (x - (d * f)^(1/2) / f) / f + (a * f + c * d + (d * f)^(1/2) * b) / f)^(1/2)) / (x - (d * f)^(1/2) / f))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2) + b/(2*f)) ^2 - (c*((b*sqrt(4*d*f)) / (2*f) + (c*d)/f+a)) / f^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{(d - f x^2) (c x^2 + b x + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int(x/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-ad\sqrt{a+bx+cx^2} + afx^2\sqrt{a+bx+cx^2} - bdx\sqrt{a+bx+cx^2} + bfx^3\sqrt{a+bx+cx^2} - cdx^2\sqrt{a+bx+cx^2} + \dots} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x/(-a*d*sqrt(a + b*x + c*x**2) + a*f*x**2*sqrt(a + b*x + c*x**2) - b*d*x*sqrt(a + b*x + c*x**2) + b*f*x**3*sqrt(a + b*x + c*x**2) - c*d*x**2*sqrt(a + b*x + c*x**2) + c*f*x**4*sqrt(a + b*x + c*x**2)), x)
```

$$3.106 \quad \int \frac{1}{(a+bx+cx^2)^{3/2}(d-fx^2)} dx$$

Optimal. Leaf size=310

$$\frac{2(b(b^2f - c(3af + cd)) - cx(2acf + b^2(-f) + 2c^2d))}{(b^2 - 4ac)\sqrt{a+bx+cx^2}(b^2df - (af + cd)^2)} + \frac{f \tanh^{-1}\left(\frac{-2a\sqrt{f} + x(2c\sqrt{d} - b\sqrt{f}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2\sqrt{d}(af + b(-\sqrt{d})\sqrt{f} + cd)^{3/2}} + \frac{f \tanh^{-1}\left(\frac{2a\sqrt{f} + x(2c\sqrt{d} + b\sqrt{f}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(\sqrt{d})\sqrt{f}+cd}}\right)}{2\sqrt{d}(af + b(\sqrt{d})\sqrt{f} + cd)^{3/2}}$$

[Out] $1/2*f*\operatorname{arctanh}(1/2*(b*d^{(1/2)}-2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}-b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)}/(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(1/2)}/d^{(1/2)}/(c*d+a*f-b*d^{(1/2)}*f^{(1/2)})^{(3/2)}+1/2*f*\operatorname{arctanh}(1/2*(b*d^{(1/2)}+2*a*f^{(1/2)}+x*(2*c*d^{(1/2)}+b*f^{(1/2)}))/(c*x^2+b*x+a)^{(1/2)}/(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(1/2)}/d^{(1/2)}/(c*d+a*f+b*d^{(1/2)}*f^{(1/2)})^{(3/2)}-2*(b*(b^2*f-c*(3*a*f+c*d))-c*(2*a*c*f-b^2*f+2*c^2*d)*x)/(-4*a*c+b^2)/(b^2*d*f-(a*f+c*d)^2)/(c*x^2+b*x+a)^{(1/2)}$

Rubi [A] time = 0.41, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {975, 1033, 724, 206}

$$\frac{2(b(b^2f - c(3af + cd)) - cx(2acf + b^2(-f) + 2c^2d))}{(b^2 - 4ac)\sqrt{a+bx+cx^2}(b^2df - (af + cd)^2)} + \frac{f \tanh^{-1}\left(\frac{-2a\sqrt{f} + x(2c\sqrt{d} - b\sqrt{f}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2\sqrt{d}(af + b(-\sqrt{d})\sqrt{f} + cd)^{3/2}} + \frac{f \tanh^{-1}\left(\frac{2a\sqrt{f} + x(2c\sqrt{d} + b\sqrt{f}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(\sqrt{d})\sqrt{f}+cd}}\right)}{2\sqrt{d}(af + b(\sqrt{d})\sqrt{f} + cd)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)), x]

[Out] $(-2*(b*(b^2*f - c*(c*d + 3*a*f)) - c*(2*c^2*d - b^2*f + 2*a*c*f)*x))/((b^2 - 4*a*c)*(b^2*d*f - (c*d + a*f)^2)*\operatorname{Sqrt}[a + b*x + c*x^2]) + (f*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] - 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] - b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]*\operatorname{Sqrt}[a + b*x + c*x^2])))/(2*\operatorname{Sqrt}[d]*(c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)^{(3/2)}) + (f*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] + 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] + b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f]*\operatorname{Sqrt}[a + b*x + c*x^2])))/(2*\operatorname{Sqrt}[d]*(c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)^{(3/2)})$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol]
:> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 975

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x_Symbol]
:> Simp[((b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*f)))*x*(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1))/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^2*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f)))*(p + q + 2) - (2*c^2*d + b^2*f - c*(2*a*f))*(b*f*(p + 1))]*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol]
:> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx+cx^2)^{3/2}(d-fx^2)} dx &= -\frac{2(b(b^2f-c(cd+3af))-c(2c^2d-b^2f+2acf)x)}{(b^2-4ac)(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} - \frac{2\int \frac{\frac{1}{2}(b^2-4ac)f(cd+af)}{\sqrt{a+bx+cx^2}}}{(b^2-4ac)(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} \\
&= -\frac{2(b(b^2f-c(cd+3af))-c(2c^2d-b^2f+2acf)x)}{(b^2-4ac)(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} - \frac{f^{3/2}\int \frac{1}{(-\sqrt{d}\sqrt{f}-fx)\sqrt{a+bx+cx^2}}}{2\sqrt{d}(cd-b\sqrt{d}\sqrt{f})\sqrt{a+bx+cx^2}} \\
&= -\frac{2(b(b^2f-c(cd+3af))-c(2c^2d-b^2f+2acf)x)}{(b^2-4ac)(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} + \frac{f^{3/2}\text{Subst}\left(\int \frac{1}{4cdf-}\right)}{2\sqrt{d}(cd-b\sqrt{d}\sqrt{f})\sqrt{a+bx+cx^2}} \\
&= -\frac{2(b(b^2f-c(cd+3af))-c(2c^2d-b^2f+2acf)x)}{(b^2-4ac)(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} + \frac{f \tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}}{2\sqrt{cd-b\sqrt{d}\sqrt{f}}}\right)}{2\sqrt{d}(cd-b\sqrt{d}\sqrt{f})\sqrt{a+bx+cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.40, size = 360, normalized size = 1.16

$$2 \frac{\frac{f(b^2-4ac)(af+b\sqrt{d}\sqrt{f}+cd) \tanh^{-1}\left(\frac{-2a\sqrt{f}+b(\sqrt{d}-\sqrt{f}x)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{4\sqrt{d}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} + \frac{f(4ac-b^2)(af+b(-\sqrt{d})\sqrt{f}+cd) \tanh^{-1}\left(\frac{-2(a\sqrt{f}+c\sqrt{d}x)-b(\sqrt{d}+\sqrt{f}x)}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{4\sqrt{d}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}}{(b^2-4ac)((af+cd)^2-b^2df)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]

[Out] (2*((b^3*f - b*c*(c*d + 3*a*f) + b^2*c*f*x - 2*c^2*(c*d + a*f)*x)/Sqrt[a + x*(b + c*x)] + ((b^2 - 4*a*c)*f*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + x*(b + c*x)])]/(4*Sqrt[d]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f) + ((-b^2 + 4*a*c)*f*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + x*(b + c*x)])]/(4*Sqrt[d]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f)))/((b^2 - 4*a*c)*(-b^2*d*f) + (c*d + a*f)^2)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

```
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
i,const vecteur & l) Error: Bad Argument Valueint() Bad Argument Type
```

```
maple [B] time = 0.02, size = 1376, normalized size = 4.44
```

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] 1/2/(d*f)^(1/2)/(a*f+c*d-(d*f)^(1/2)*b)*f/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*
f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)+2/(a*f+c*d
-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x
+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*c^2-1/(d*f)^(1/2)/(a*f
+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c
)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*b*c*f+1/(a*f+c*d-(
d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(
d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b*c-1/2/(d*f)^(1/2)/(a*f+c
*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*
(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b^2*f-1/2/(d*f)^(1/2)/
(a*f+c*d-(d*f)^(1/2)*b)*f/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-
(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*(a*f+c*d-(d*f
)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(
1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))-1/2/(d*f)^(1
/2)/(a*f+c*d+(d*f)^(1/2)*b)*f/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*
(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)+2/(a*f+c*d+(d*f)^(1/2)
*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)
/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*x*c^2+1/(d*f)^(1/2)/(a*f+c*d+(d*f)^(
1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(
1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*x*b*c*f+1/(a*f+c*d+(d*f)^(1/2)*b
)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f
)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*b*c+1/2/(d*f)^(1/2)/(a*f+c*d+(d*f)^(1
/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/
```

2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*b^2*f+1/2/(d*f)^(1/2)/(a*f+c*d+(d*f)^(1/2)*b)*f/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see 'assume?' for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2 -c*((b*sqrt(4*d*f)) / (2*f) + (c*d)/f+a) /f^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(d - f x^2) (c x^2 + b x + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)

[Out] int(1/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-ad\sqrt{a+bx+cx^2} + afx^2\sqrt{a+bx+cx^2} - bdx\sqrt{a+bx+cx^2} + bfx^3\sqrt{a+bx+cx^2} - cdx^2\sqrt{a+bx+cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)

[Out] -Integral(1/(-a*d*sqrt(a + b*x + c*x**2) + a*f*x**2*sqrt(a + b*x + c*x**2) - b*d*x*sqrt(a + b*x + c*x**2) + b*f*x**3*sqrt(a + b*x + c*x**2) - c*d*x**2*sqrt(a + b*x + c*x**2) + c*f*x**4*sqrt(a + b*x + c*x**2)), x)

$$3.107 \quad \int \frac{1}{x(a+bx+cx^2)^{3/2}(d-fx^2)} dx$$

Optimal. Leaf size=394

$$\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{a^{3/2}d} - \frac{2f\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{d\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)} + \frac{2\left(-2ac+b^2+bcx\right)}{ad\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} - \frac{f^{3/2}\operatorname{arctanh}\left(\frac{b\sqrt{d-fx^2}}{a+bx+cx^2}\right)}{2\sqrt{a}\sqrt{a+bx+cx^2}}$$

[Out] $-\operatorname{arctanh}\left(\frac{1}{2}\frac{(b*x+2*a)/a^{1/2}}{(c*x^2+b*x+a)^{1/2}}\right)/a^{3/2}/d-1/2*f^{3/2}*a$
 $\operatorname{rctanh}\left(\frac{1}{2}\frac{(b*d^{1/2}-2*a*f^{1/2})+x*(2*c*d^{1/2}-b*f^{1/2})}{(c*x^2+b*x+a)^{1/2}}\right)/((c*d+a*f-b*d^{1/2})f^{1/2})^{1/2}/d/((c*d+a*f-b*d^{1/2})f^{1/2})^{3/2}$
 $+1/2*f^{3/2}*\operatorname{arctanh}\left(\frac{1}{2}\frac{(b*d^{1/2}+2*a*f^{1/2})+x*(2*c*d^{1/2}+b*f^{1/2})}{(c*x^2+b*x+a)^{1/2}}\right)/((c*d+a*f+b*d^{1/2})f^{1/2})^{1/2}/d/((c*d+a*f+b*d^{1/2})f^{1/2})^{3/2}$
 $+2*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/d/((c*x^2+b*x+a)^{1/2})-2$
 $*f*(a*(2*a*c*f-b^2*f+2*c^2*d)+b*c*(-a*f+c*d)*x)/(-4*a*c+b^2)/d/(b^2*d*f-(a*f+c*d)^2)/((c*x^2+b*x+a)^{1/2})$

Rubi [A] time = 1.18, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6725, 740, 12, 724, 206, 1018, 1033}

$$\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{a^{3/2}d} - \frac{2f\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{d\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)} + \frac{2\left(-2ac+b^2+bcx\right)}{ad\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} - \frac{f^{3/2}\operatorname{arctanh}\left(\frac{b\sqrt{d-fx^2}}{a+bx+cx^2}\right)}{2\sqrt{a}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}\left[1/(x*(a + b*x + c*x^2)^{(3/2)}*(d - f*x^2)), x\right]$

[Out] $(2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*d*\operatorname{Sqrt}[a + b*x + c*x^2]) - (2*f*(a*(2*c^2*d - b^2*f + 2*a*c*f) + b*c*(c*d - a*f)*x))/((b^2 - 4*a*c)*d*(b^2*d*f - (c*d + a*f)^2)*\operatorname{Sqrt}[a + b*x + c*x^2]) - \operatorname{ArcTanh}[(2*a + b*x)/(2*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[a + b*x + c*x^2])]/(a^{3/2}*d) - (f^{3/2}*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] - 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] - b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)*\operatorname{Sqrt}[a + b*x + c*x^2])]/(2*d*(c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)^{3/2}) + (f^{3/2}*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] + 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] + b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)*\operatorname{Sqrt}[a + b*x + c*x^2])]/(2*d*(c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)^{3/2}))$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 740

Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]

Rule 1018

Int[((g_) + (h_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1)*((g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f)*x))/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p + q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] && !(IntegerQ[p] && ILtQ[q, -1])

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(a+bx+cx^2)^{3/2}(d-fx^2)} dx &= \int \left(\frac{1}{dx(a+bx+cx^2)^{3/2}} - \frac{fx}{d(a+bx+cx^2)^{3/2}(-d+fx^2)} \right) dx \\
&= \frac{\int \frac{1}{x(a+bx+cx^2)^{3/2}} dx}{d} - \frac{f \int \frac{x}{(a+bx+cx^2)^{3/2}(-d+fx^2)} dx}{d} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)d\sqrt{a+bx+cx^2}} - \frac{2f(a(2c^2d - b^2f + 2acf) + bc(cd - af)x)}{(b^2 - 4ac)d(b^2df - (cd + af)^2)\sqrt{a+bx+cx^2}} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)d\sqrt{a+bx+cx^2}} - \frac{2f(a(2c^2d - b^2f + 2acf) + bc(cd - af)x)}{(b^2 - 4ac)d(b^2df - (cd + af)^2)\sqrt{a+bx+cx^2}} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)d\sqrt{a+bx+cx^2}} - \frac{2f(a(2c^2d - b^2f + 2acf) + bc(cd - af)x)}{(b^2 - 4ac)d(b^2df - (cd + af)^2)\sqrt{a+bx+cx^2}} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)d\sqrt{a+bx+cx^2}} - \frac{2f(a(2c^2d - b^2f + 2acf) + bc(cd - af)x)}{(b^2 - 4ac)d(b^2df - (cd + af)^2)\sqrt{a+bx+cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.87, size = 436, normalized size = 1.11

$$\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{a^{3/2}} - \frac{2f(a(2acf+b^2(-f)+2c^2d)+bcx(cd-af))}{(b^2-4ac)\sqrt{a+x(b+cx)}(b^2df-(af+cd)^2)} + \frac{f^{3/2}\left((af+b(-\sqrt{d})\sqrt{f+cd})^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}}\sqrt{f+cd}}\right)\right)+af}{2\sqrt{af+b(-\sqrt{d})\sqrt{f+cd}}\sqrt{af+b\sqrt{d}}\sqrt{f+cd}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]

[Out]
$$\left(\frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)\sqrt{a + x(b + cx)}} - (2f(a(2c^2d - b^2f + 2ac^2f) + bc(c^2d - af)x) / ((b^2 - 4ac)(b^2df - (c^2d + af)^2)\sqrt{a + x(b + cx)}) - \text{ArcTanh}[(2a + bx)/(2\sqrt{a}\sqrt{a + x(b + cx)})]) / a^{3/2} + (f^{3/2}((c^2d + b\sqrt{d}\sqrt{f} + af)^{3/2}\text{ArcTanh}[-(b\sqrt{d}) + 2a\sqrt{f} - 2c\sqrt{d}x + b\sqrt{f}x] / (2\sqrt{c^2d - b\sqrt{d}\sqrt{f} + af})\sqrt{a + x(b + cx)}) + (c^2d - b\sqrt{d}\sqrt{f} + af)^{3/2}\text{ArcTanh}[(b\sqrt{d} + 2a\sqrt{f} + 2c\sqrt{d}x + b\sqrt{f}x) / (2\sqrt{c^2d + b\sqrt{d}\sqrt{f} + af})\sqrt{a + x(b + cx)})]) / (2\sqrt{c^2d - b\sqrt{d}\sqrt{f} + af})\sqrt{c^2d + b\sqrt{d}\sqrt{f} + af} * (-b^2df + (c^2d + af)^2)) / d$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage2

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")

[Out] sage2

maple [B] time = 0.02, size = 1518, normalized size = 3.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)

[Out]
$$-1/2/d/(a*f+c*d-(d*f)^{(1/2)*b})*f/((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}-2/d/(a*f+c*d-(d*f)^{(1/2)*b})/(4*a*c-b^2)/((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*(d*f)^{(1/2)*x*c^2+1/d/(a*f+c*d-(d*f)^{(1/2)*b})/(4*a*c-b^2)/((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*x*b*c*f-1/d/(a*f+c*d-(d*f)^{(1/2)*b})/(4*a*c-b^2)/((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*(d*f)^{(1/2)*b*c+1/2/d/(a*f+c*d-(d*f)^{(1/2)*b})/(4*a*c-b^2)/((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*b^2*f+1/2/d/(a*f+c*d-(d*f)^{(1/2)*b})*f/((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*\ln((2*(a*f+c*d-(d*f)^{(1/2)*b})/f+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+2*((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)})/(x+(d*f)^{(1/2)/f))-1/2/d/(a*f+c*d+(d*f)^{(1/2)*b})*f/((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}+2/d/(a*f+c*d+(d*f)^{(1/2)*b})/(4*a*c-b^2)/((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*x*b*c*f+1/d/(a*f+c*d+(d*f)^{(1/2)*b})/(4*a*c-b^2)/((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*(d*f)^{(1/2)*b*c+1/2/d/(a*f+c*d+(d*f)^{(1/2)*b})/(4*a*c-b^2)/((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*\ln((2*(a*f+c*d+(d*f)^{(1/2)*b})/f+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f})/f+2*((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)})/(x-(d*f)^{(1/2)/f))+1/d/a/(c*x^2+b*x+a)^(1/2)-2/d*b/a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-1/d*b^2/a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-1/d/a^(3/2)*\ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{(cx^2 + bx + a)^{\frac{3}{2}}(fx^2 - d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] -integrate(1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x (d - f x^2) (c x^2 + b x + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)

[Out] int(1/(x*(d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-adx\sqrt{a+bx+cx^2} + afx^3\sqrt{a+bx+cx^2} - bdx^2\sqrt{a+bx+cx^2} + bfx^4\sqrt{a+bx+cx^2} - cdx^3\sqrt{a+bx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)

[Out] -Integral(1/(-a*d*x*sqrt(a + b*x + c*x**2) + a*f*x**3*sqrt(a + b*x + c*x**2) - b*d*x**2*sqrt(a + b*x + c*x**2) + b*f*x**4*sqrt(a + b*x + c*x**2) - c*d*x**3*sqrt(a + b*x + c*x**2) + c*f*x**5*sqrt(a + b*x + c*x**2)), x)

$$3.108 \quad \int \frac{1}{x^2(a+bx+cx^2)^{3/2}(d-fx^2)} dx$$

Optimal. Leaf size=454

$$\frac{3b \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{5/2}d} - \frac{(3b^2 - 8ac)\sqrt{a+bx+cx^2}}{a^2dx(b^2 - 4ac)} - \frac{2f(b(b^2f - c(3af + cd)) - cx(2acf + b^2(-f) + 2c^2d))}{d(b^2 - 4ac)\sqrt{a+bx+cx^2}(b^2df - (af + cd)^2)}$$

[Out] $\frac{3/2*b*\operatorname{arctanh}(1/2*(b*x+2*a)/a^{1/2}/(c*x^2+b*x+a)^{1/2})/a^{5/2}/d+1/2*f^2*\operatorname{arctanh}(1/2*(b*d^{1/2}-2*a*f^{1/2}+x*(2*c*d^{1/2}-b*f^{1/2}))/((c*x^2+b*x+a)^{1/2}/(c*d+a*f-b*d^{1/2})*f^{1/2})^{1/2})/d^{3/2}/(c*d+a*f-b*d^{1/2})*f^{1/2})^{3/2}+1/2*f^2*\operatorname{arctanh}(1/2*(b*d^{1/2}+2*a*f^{1/2}+x*(2*c*d^{1/2}+b*f^{1/2}))/((c*x^2+b*x+a)^{1/2}/(c*d+a*f+b*d^{1/2})*f^{1/2})^{1/2})/d^{3/2}/(c*d+a*f+b*d^{1/2})*f^{1/2})^{3/2}+2*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/d/x/(c*x^2+b*x+a)^{1/2}-2*f*(b*(b^2*f-c*(3*a*f+c*d))-c*(2*a*c*f-b^2*f+2*c^2*d)*x)/(-4*a*c+b^2)/d/(b^2*d*f-(a*f+c*d)^2)/(c*x^2+b*x+a)^{1/2}-(-8*a*c+3*b^2)*(c*x^2+b*x+a)^{1/2}/a^2/(-4*a*c+b^2)/d/x$

Rubi [A] time = 1.19, antiderivative size = 454, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6725, 740, 806, 724, 206, 975, 1033}

$$-\frac{(3b^2 - 8ac)\sqrt{a+bx+cx^2}}{a^2dx(b^2 - 4ac)} + \frac{3b \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{5/2}d} - \frac{2f(b(b^2f - c(3af + cd)) - cx(2acf + b^2(-f) + 2c^2d))}{d(b^2 - 4ac)\sqrt{a+bx+cx^2}(b^2df - (af + cd)^2)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[1/(x^2*(a + b*x + c*x^2)^{3/2}*(d - f*x^2)), x]$

[Out] $\frac{2*(b^2 - 2*a*c + b*c*x)}{(a*(b^2 - 4*a*c)*d*x*\operatorname{Sqrt}[a + b*x + c*x^2])} - (2*f*(b*(b^2*f - c*(c*d + 3*a*f)) - c*(2*c^2*d - b^2*f + 2*a*c*f)*x))/((b^2 - 4*a*c)*d*(b^2*d*f - (c*d + a*f)^2)*\operatorname{Sqrt}[a + b*x + c*x^2]) - ((3*b^2 - 8*a*c)*\operatorname{Sqrt}[a + b*x + c*x^2])/(a^2*(b^2 - 4*a*c)*d*x) + (3*b*\operatorname{ArcTanh}[(2*a + b*x)/(2*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*a^{5/2}*d) + (f^2*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] - 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] - b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*d^{3/2}*(c*d - b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)^{3/2}) + (f^2*\operatorname{ArcTanh}[(b*\operatorname{Sqrt}[d] + 2*a*\operatorname{Sqrt}[f] + (2*c*\operatorname{Sqrt}[d] + b*\operatorname{Sqrt}[f])*x)/(2*\operatorname{Sqrt}[c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*d^{3/2}*(c*d + b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[f] + a*f)^{3/2})$

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Sym
bol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_S
ymbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c
_)*(x_)^2)^(p_), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
+ d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 975

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_) + (f_)*(x_)^2)^(q_), x
_Symbol] := Simp[((b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*
f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1))/((b^2 - 4*a*c)*(b^2*d
*f + (c*d - a*f)^2)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d -
a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^2
*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1)
- c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d +
b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] && !( !IntegerQ[
```

p] && ILtQ[q, -1]) && !IGtQ[q, 0]

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^2 (a + bx + cx^2)^{3/2} (d - fx^2)} dx &= \int \left(\frac{1}{dx^2 (a + bx + cx^2)^{3/2}} + \frac{f}{d (a + bx + cx^2)^{3/2} (d - fx^2)} \right) dx \\
 &= \frac{\int \frac{1}{x^2 (a + bx + cx^2)^{3/2}} dx}{d} + \frac{f \int \frac{1}{(a + bx + cx^2)^{3/2} (d - fx^2)} dx}{d} \\
 &= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac) dx \sqrt{a + bx + cx^2}} - \frac{2f(b(b^2 f - c(cd + 3af)) - c(2c^2 d - b^2 f))}{(b^2 - 4ac) d (b^2 d f - (cd + af)^2) \sqrt{a + bx + cx^2}} \\
 &= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac) dx \sqrt{a + bx + cx^2}} - \frac{2f(b(b^2 f - c(cd + 3af)) - c(2c^2 d - b^2 f))}{(b^2 - 4ac) d (b^2 d f - (cd + af)^2) \sqrt{a + bx + cx^2}} \\
 &= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac) dx \sqrt{a + bx + cx^2}} - \frac{2f(b(b^2 f - c(cd + 3af)) - c(2c^2 d - b^2 f))}{(b^2 - 4ac) d (b^2 d f - (cd + af)^2) \sqrt{a + bx + cx^2}} \\
 &= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac) dx \sqrt{a + bx + cx^2}} - \frac{2f(b(b^2 f - c(cd + 3af)) - c(2c^2 d - b^2 f))}{(b^2 - 4ac) d (b^2 d f - (cd + af)^2) \sqrt{a + bx + cx^2}}
 \end{aligned}$$

Mathematica [A] time = 1.19, size = 488, normalized size = 1.07

$$\frac{3b(b^2-4ac) \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{a^{5/2}} + \frac{2(8ac-3b^2)\sqrt{a+x(b+cx)}}{a^2x} - \frac{f^2 \left(\frac{(b^2-4ac)(af+b\sqrt{d}\sqrt{f+cd}) \tanh^{-1}\left(\frac{2a\sqrt{f}-b\sqrt{d}+b\sqrt{f}x-2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b(-\sqrt{d})\sqrt{f+cd}}}\right)}{\sqrt{af+b(-\sqrt{d})\sqrt{f+cd}}} \right) + \frac{(4ac-b^2)(a\sqrt{d}((af+cd)^2-b^2df))}{2d(b^2-4ac)}}{\sqrt{d}((af+cd)^2-b^2df)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]

[Out]
$$\left(\frac{4(b^2 - 2ac + bcx)}{ax\sqrt{a + x(b + cx)}} - \frac{4f(b^3f - b^2c^2d + 3acf) + b^2c^2fx - 2c^2(cd + af)x}{(b^2df - (cd + af)^2)\sqrt{a + x(b + cx)}} + \frac{2(-3b^2 + 8ac)\sqrt{a + x(b + cx)}}{a^2x} + \frac{3b(b^2 - 4ac)\operatorname{ArcTanh}\left(\frac{2a + bx}{2\sqrt{a}\sqrt{a + x(b + cx)}}\right)}{a^{5/2}} - \frac{f^2((b^2 - 4ac)(cd + b\sqrt{d}\sqrt{f} + af)\operatorname{ArcTanh}\left(\frac{-(b\sqrt{d}) + 2a\sqrt{f} - 2c\sqrt{d}x + b\sqrt{f}x}{2\sqrt{cd - b\sqrt{d}\sqrt{f} + af}\sqrt{a + x(b + cx)}}\right) + ((-b^2 + 4ac)(cd - b\sqrt{d}\sqrt{f} + af)\operatorname{ArcTanh}\left(\frac{b\sqrt{d} + 2a\sqrt{f} + 2c\sqrt{d}x + b\sqrt{f}x}{2\sqrt{cd + b\sqrt{d}\sqrt{f} + af}\sqrt{a + x(b + cx)}}\right))}{\sqrt{cd - b\sqrt{d}\sqrt{f} + af}\sqrt{cd + b\sqrt{d}\sqrt{f} + af}} \right) / ((\sqrt{d}(-(b^2df) + (cd + af)^2)) / (2(b^2 - 4ac)d))$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 1.84int() Bad Argument Type

maple [B] time = 0.02, size = 1656, normalized size = 3.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/x^2/(c*x^2+b*x+a)^{3/2}/(-f*x^2+d), x)$

[Out] $\frac{1}{2} \frac{f^2}{d} \frac{1}{(df)^{1/2}} \frac{1}{(af+cd-(df)^{1/2}b)} \frac{1}{((x+(df)^{1/2}/f)^2c+(bf-2*(df)^{1/2}c)*(x+(df)^{1/2}/f)/f+(af+cd-(df)^{1/2}b)/f)^{1/2}} + 2 \frac{f}{d} \frac{1}{(af+cd-(df)^{1/2}b)} \frac{1}{(4a^2c-b^2)} \frac{1}{((x+(df)^{1/2}/f)^2c+(bf-2*(df)^{1/2}c)*(x+(df)^{1/2}/f)/f+(af+cd-(df)^{1/2}b)/f)^{1/2}} + x \frac{c^2-f^2}{d} \frac{1}{(df)^{1/2}} \frac{1}{(af+cd-(df)^{1/2}b)} \frac{1}{(4a^2c-b^2)} \frac{1}{((x+(df)^{1/2}/f)^2c+(bf-2*(df)^{1/2}c)*(x+(df)^{1/2}/f)/f+(af+cd-(df)^{1/2}b)/f)^{1/2}} + x \frac{b^2c+f}{d} \frac{1}{(af+cd-(df)^{1/2}b)} \frac{1}{(4a^2c-b^2)} \frac{1}{((x+(df)^{1/2}/f)^2c+(bf-2*(df)^{1/2}c)*(x+(df)^{1/2}/f)/f+(af+cd-(df)^{1/2}b)/f)^{1/2}} + x \frac{b^2c-1}{2} \frac{f^2}{d} \frac{1}{(df)^{1/2}} \frac{1}{(af+cd-(df)^{1/2}b)} \frac{1}{(4a^2c-b^2)} \frac{1}{((x+(df)^{1/2}/f)^2c+(bf-2*(df)^{1/2}c)*(x+(df)^{1/2}/f)/f+(af+cd-(df)^{1/2}b)/f)^{1/2}} + \ln\left(\frac{2*(af+cd-(df)^{1/2}b)/f+(bf-2*(df)^{1/2}c)*(x+(df)^{1/2}/f)/f+2*((af+cd-(df)^{1/2}b)/f)^{1/2}*((x+(df)^{1/2}/f)^2c+(bf-2*(df)^{1/2}c)*(x+(df)^{1/2}/f)/f+(af+cd-(df)^{1/2}b)/f)^{1/2}}{(x+(df)^{1/2}/f)-1/d/a/x/(c*x^2+b*x+a)^{1/2}-3/2/d*b/a^2/(c*x^2+b*x+a)^{1/2}+3/d*b^2/a^2/(4*a^2c-b^2)/(c*x^2+b*x+a)^{1/2}*c*x+3/2/d*b^3/a^2/(4*a^2c-b^2)/(c*x^2+b*x+a)^{1/2}+3/2/d*b/a^{5/2}*ln((b*x+2*a+2*(c*x^2+b*x+a)^{1/2})*a^{1/2})/x-8/d*c^2/a/(4*a^2c-b^2)/(c*x^2+b*x+a)^{1/2}*x-4/d*c/a/(4*a^2c-b^2)/(c*x^2+b*x+a)^{1/2}*b-1}{2} \frac{f^2}{d} \frac{1}{(df)^{1/2}} \frac{1}{(af+cd+(df)^{1/2}b)} \frac{1}{((x-(df)^{1/2}/f)^2c+(bf+2*(df)^{1/2}c)*(x-(df)^{1/2}/f)/f+(af+cd+(df)^{1/2}b)/f)^{1/2}} + 2 \frac{f}{d} \frac{1}{(af+cd+(df)^{1/2}b)} \frac{1}{(4a^2c-b^2)} \frac{1}{((x-(df)^{1/2}/f)^2c+(bf+2*(df)^{1/2}c)*(x-(df)^{1/2}/f)/f+(af+cd+(df)^{1/2}b)/f)^{1/2}} + x \frac{c^2+f^2}{d} \frac{1}{(df)^{1/2}} \frac{1}{(af+cd+(df)^{1/2}b)} \frac{1}{(4a^2c-b^2)} \frac{1}{((x-(df)^{1/2}/f)^2c+(bf+2*(df)^{1/2}c)*(x-(df)^{1/2}/f)/f+(af+cd+(df)^{1/2}b)/f)^{1/2}} + x \frac{b^2c+f}{d} \frac{1}{(af+cd+(df)^{1/2}b)} \frac{1}{(4a^2c-b^2)} \frac{1}{((x-(df)^{1/2}/f)^2c+(bf+2*(df)^{1/2}c)*(x-(df)^{1/2}/f)/f+(af+cd+(df)^{1/2}b)/f)^{1/2}} + x \frac{b^2c+1}{2} \frac{f^2}{d} \frac{1}{(df)^{1/2}} \frac{1}{(af+cd+(df)^{1/2}b)} \frac{1}{(4a^2c-b^2)} \frac{1}{((x-(df)^{1/2}/f)^2c+(bf+2*(df)^{1/2}c)*(x-(df)^{1/2}/f)/f+(af+cd+(df)^{1/2}b)/f)^{1/2}} + \ln\left(\frac{2*(af+cd+(df)^{1/2}b)/f+(bf+2*(df)^{1/2}c)*(x-(df)^{1/2}/f)/f+2*((af+cd+(df)^{1/2}b)/f)^{1/2}*((x-(df)^{1/2}/f)^2c+(bf+2*(df)^{1/2}c)*(x-(df)^{1/2}/f)/f+(af+cd+(df)^{1/2}b)/f)^{1/2}}{(x-(df)^{1/2}/f)}\right)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{(cx^2 + bx + a)^{\frac{3}{2}}(fx^2 - d)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")`

[Out] `-integrate(1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)*x^2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2 (d - f x^2) (c x^2 + b x + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2*(d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)`

[Out] `int(1/(x^2*(d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-adx^2\sqrt{a+bx+cx^2} + afx^4\sqrt{a+bx+cx^2} - bdx^3\sqrt{a+bx+cx^2} + bfx^5\sqrt{a+bx+cx^2} - cdx^4\sqrt{a+bx+cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)`

[Out] `-Integral(1/(-a*d*x**2*sqrt(a + b*x + c*x**2) + a*f*x**4*sqrt(a + b*x + c*x**2) - b*d*x**3*sqrt(a + b*x + c*x**2) + b*f*x**5*sqrt(a + b*x + c*x**2) - c*d*x**4*sqrt(a + b*x + c*x**2)), x)`

$$3.109 \quad \int \frac{x^2 \sqrt{a+bx+cx^2}}{d+ex+fx^2} dx$$

Optimal. Leaf size=761

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4cf(be-af)+b^2f^2-8c^2(e^2-df)\right)\left(f\left(af\left(-e\sqrt{e^2-4df}-2df+e^2\right)-b\left(-e^2\sqrt{e^2-4df}\right)\right)\right)}{8c^{3/2}f^3}$$

[Out] $-1/8*(b^2*f^2+4*c*f*(-a*f+b*e)-8*c^2*(-d*f+e^2))*\operatorname{arctanh}(1/2*(2*c*x+b)/c^{(1/2)})/(c*x^2+b*x+a)^{(1/2)}/c^{(3/2)}/f^3-1/4*(-2*c*f*x-b*f+4*c*e)*(c*x^2+b*x+a)^{(1/2)}/c/f^2-1/2*\operatorname{arctanh}(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^{(1/2)}))-b*(e-(-4*d*f+e^2)^{(1/2)}))*2^{(1/2)})/(c*x^2+b*x+a)^{(1/2)}/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)})*(c*(e^4-4*d*e^2*f+2*d^2*f^2-e^3*(-4*d*f+e^2)^{(1/2)}+2*d*e*f*(-4*d*f+e^2)^{(1/2)})+f*(a*f*(-(-4*d*f+e^2)^{(1/2)})*e-2*d*f+e^2)-b*(e^3-3*d*e*f-e^2*(-4*d*f+e^2)^{(1/2)}+d*f*(-4*d*f+e^2)^{(1/2)})))/f^3*2^{(1/2)}/(-4*d*f+e^2)^{(1/2)}/(f*(2*a*f-b*(e-(-4*d*f+e^2)^{(1/2)}))+(-(-4*d*f+e^2)^{(1/2)})*e-2*d*f+e^2)*c)^{(1/2)}+1/2*\operatorname{arctanh}(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^{(1/2)}))+2*x*(b*f-c*(e+(-4*d*f+e^2)^{(1/2)}))) * 2^{(1/2)})/(c*x^2+b*x+a)^{(1/2)}/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)})*(c*(e^4-4*d*e^2*f+2*d^2*f^2+e^3*(-4*d*f+e^2)^{(1/2)}-2*d*e*f*(-4*d*f+e^2)^{(1/2)})+f*(a*f*(-(-4*d*f+e^2)^{(1/2)})*e-2*d*f+e^2)-b*(e^3-3*d*e*f+e^2*(-4*d*f+e^2)^{(1/2)}-d*f*(-4*d*f+e^2)^{(1/2)})))/f^3*2^{(1/2)}/(-4*d*f+e^2)^{(1/2)}/(((4*d*f+e^2)^{(1/2)})*e-2*d*f+e^2)*c+f*(2*a*f-b*(e+(-4*d*f+e^2)^{(1/2)})))^{(1/2)}$

Rubi [A] time = 3.14, antiderivative size = 761, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1067, 1076, 621, 206, 1032, 724}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4cf(be-af)+b^2f^2-8c^2(e^2-df)\right)\left(f\left(af\left(-e\sqrt{e^2-4df}-2df+e^2\right)-b\left(-e^2\sqrt{e^2-4df}\right)\right)\right)}{8c^{3/2}f^3}$$

Antiderivative was successfully verified.

[In] Int[(x^2*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2), x]

[Out] $-((4*c*e - b*f - 2*c*f*x)*\operatorname{Sqrt}[a + b*x + c*x^2])/(4*c*f^2) - ((b^2*f^2 + 4*c*f*(b*e - a*f) - 8*c^2*(e^2 - d*f))*\operatorname{ArcTanh}[(b + 2*c*x)/(2*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(8*c^{(3/2)}*f^3) - ((c*(e^4 - 4*d*e^2*f + 2*d^2*f^2 - e^3*\operatorname{Sqrt}[e^2 - 4*d*f] + 2*d*e*f*\operatorname{Sqrt}[e^2 - 4*d*f]) + f*(a*f*(e^2 - 2*d*f - e*\operatorname{Sqrt}[e^2 - 4*d*f]) - b*(e^3 - 3*d*e*f - e^2*\operatorname{Sqrt}[e^2 - 4*d*f] + d*f*\operatorname{Sqrt}[e^2 - 4*d*f])))/f^3$

$$\begin{aligned}
& - 4*d*f)))*\text{ArcTanh}[(4*a*f - b*(e - \text{Sqrt}[e^2 - 4*d*f]) + 2*(b*f - c*(e - \text{Sqrt}[e^2 - 4*d*f]))*x)/(2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*\text{Sqrt}[e^2 - 4*d*f]]*\text{Sqrt}[a + b*x + c*x^2]])/(\text{Sqrt}[2]*f^3*\text{Sqrt}[e^2 - 4*d*f]*\text{Sqrt}[c*(e^2 - 2*d*f - e*\text{Sqrt}[e^2 - 4*d*f]) + f*(2*a*f - b*(e - \text{Sqrt}[e^2 - 4*d*f]))]) + ((c*(e^4 - 4*d*e^2*f + 2*d^2*f^2 + e^3*\text{Sqrt}[e^2 - 4*d*f]) - 2*d*e*f*\text{Sqrt}[e^2 - 4*d*f]) + f*(a*f*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]) - b*(e^3 - 3*d*e*f + e^2*\text{Sqrt}[e^2 - 4*d*f] - d*f*\text{Sqrt}[e^2 - 4*d*f])))*\text{ArcTanh}[(4*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]) + 2*(b*f - c*(e + \text{Sqrt}[e^2 - 4*d*f]))*x)/(2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*\text{Sqrt}[e^2 - 4*d*f]]*\text{Sqrt}[a + b*x + c*x^2]])/(\text{Sqrt}[2]*f^3*\text{Sqrt}[e^2 - 4*d*f]*\text{Sqrt}[c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]) + f*(2*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]))])])
\end{aligned}$$

Rule 206

$$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] \text{ /; FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}\{a/b\} \ \&\& \ (\text{GtQ}\{a, 0\} \ || \ \text{LtQ}\{b, 0\})$$

Rule 621

$$\text{Int}[1/\text{Sqrt}[(a_ + (b_)*(x_) + (c_)*(x_)^2)], x_Symbol] \rightarrow \text{Dist}[2, \text{Subst}[\text{Int}[1/(4*c - x^2), x], x, (b + 2*c*x)/\text{Sqrt}[a + b*x + c*x^2]], x] \text{ /; FreeQ}\{a, b, c\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$$

Rule 724

$$\text{Int}[1/(((d_ + (e_)*(x_))*\text{Sqrt}[(a_ + (b_)*(x_) + (c_)*(x_)^2)]), x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/\text{Sqrt}[a + b*x + c*x^2]], x] \text{ /; FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0]$$

Rule 1032

$$\text{Int}[(g_ + (h_)*(x_))/(((a_ + (b_)*(x_) + (c_)*(x_)^2)*\text{Sqrt}[(d_ + (e_)*(x_) + (f_)*(x_)^2])], x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(2*c*g - h*(b - q))/q, \text{Int}[1/((b - q + 2*c*x)*\text{Sqrt}[d + e*x + f*x^2]), x], x] - \text{Dist}[(2*c*g - h*(b + q))/q, \text{Int}[1/((b + q + 2*c*x)*\text{Sqrt}[d + e*x + f*x^2]), x], x] \text{ /; FreeQ}\{a, b, c, d, e, f, g, h\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[e^2 - 4*d*f, 0] \ \&\& \ \text{PosQ}[b^2 - 4*a*c]$$

Rule 1067

$$\text{Int}[(a_ + (b_)*(x_) + (c_)*(x_)^2)^{(p_)}*((A_ + (C_)*(x_)^2)*((d_ + (e_)*(x_) + (f_)*(x_)^2)^{(q_)}), x_Symbol] \rightarrow \text{Simp}[(C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + e*x + f*x^2)^{(q + 1)}$$

1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) - c*(C*e)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))]*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))]*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]

Rule 1076

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^2 \sqrt{a + bx + cx^2}}{d + ex + fx^2} dx &= -\frac{(4ce - bf - 2cfx)\sqrt{a + bx + cx^2}}{4cf^2} - \frac{\int \frac{-\frac{1}{4}d(4bce - b^2f - 4acf) - \frac{1}{4}(8c^2de - b^2ef - 4acef + 4bc(e^2 - 2df))x +}{\sqrt{a + bx + cx^2}(d + ex + fx^2)}}{2cf^2} \\
 &= -\frac{(4ce - bf - 2cfx)\sqrt{a + bx + cx^2}}{4cf^2} - \frac{\int \frac{-\frac{1}{4}df(4bce - b^2f - 4acf) - \frac{1}{4}d(b^2f^2 + 4cf(be - af) - 8c^2(e^2 - df)) +}{\sqrt{a + bx + cx^2}}}{4cf^2} \\
 &= -\frac{(4ce - bf - 2cfx)\sqrt{a + bx + cx^2}}{4cf^2} - \frac{(b^2f^2 + 4cf(be - af) - 8c^2(e^2 - df)) \operatorname{Subst}\left(\int \frac{1}{4c\sqrt{u}} du\right)}{4cf^3} \\
 &= -\frac{(4ce - bf - 2cfx)\sqrt{a + bx + cx^2}}{4cf^2} - \frac{(b^2f^2 + 4cf(be - af) - 8c^2(e^2 - df)) \tanh^{-1}\left(\frac{2\sqrt{a + bx + cx^2}}{2\sqrt{a + bx + cx^2}}\right)}{8c^{3/2}f^3} \\
 &= -\frac{(4ce - bf - 2cfx)\sqrt{a + bx + cx^2}}{4cf^2} - \frac{(b^2f^2 + 4cf(be - af) - 8c^2(e^2 - df)) \tanh^{-1}\left(\frac{2\sqrt{a + bx + cx^2}}{2\sqrt{a + bx + cx^2}}\right)}{8c^{3/2}f^3}
 \end{aligned}$$

Mathematica [A] time = 2.31, size = 552, normalized size = 0.73

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(4cf(af-be)-b^2f^2+8c^2(e^2-df)\right)}{8c^{3/2}f^3} + \frac{f\sqrt{e^2-4df}\sqrt{a+x(b+cx)}(bf-4ce+2cfx)}{8c^{3/2}f^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]

[Out]
$$\left(\frac{-(b^2f^2) + 4c*f*(-(b*e) + a*f) + 8c^2*(e^2 - d*f)*\text{ArcTanh}\left[\frac{b + 2c*x}{2*\text{Sqrt}[c]*\text{Sqrt}[a + x*(b + c*x)]}\right]}{(8c^{3/2})f^3} + \frac{f*\text{Sqrt}[e^2 - 4*d*f]*(-4*c*e + b*f + 2*c*f*x)*\text{Sqrt}[a + x*(b + c*x)] + \text{Sqrt}[2]*c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f])*\text{Sqrt}[c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]) + f*(2*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]))]}{(2*\text{Sqrt}[2])*\text{Sqrt}[c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]) + f*(2*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]))]}*\text{Sqrt}[a + x*(b + c*x)] + \text{Sqrt}[2]*c*(-e^2 + 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f])*\text{Sqrt}[c*(e^2 - 2*d*f - e*\text{Sqrt}[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + \text{Sqrt}[e^2 - 4*d*f]))]}*\text{ArcTanh}\left[\frac{(4*a*f + 2*c*(-e + \text{Sqrt}[e^2 - 4*d*f]))*x + b*(-e + \text{Sqrt}[e^2 - 4*d*f] + 2*f*x)}{(2*\text{Sqrt}[2])*\text{Sqrt}[c*(e^2 - 2*d*f - e*\text{Sqrt}[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + \text{Sqrt}[e^2 - 4*d*f]))]}*\text{Sqrt}[a + x*(b + c*x)]}\right)}{(4*c*f^3*\text{Sqrt}[e^2 - 4*d*f])}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);;OUTPUT:Error: Bad Argument Type

maple [B] time = 0.02, size = 14815, normalized size = 19.47

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)`

[Out] result too large to display

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see 'assume?' for more details)Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \sqrt{cx^2 + bx + a}}{fx^2 + ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2*(a + b*x + c*x^2)^(1/2))/(d + e*x + f*x^2),x)`

[Out] `int((x^2*(a + b*x + c*x^2)^(1/2))/(d + e*x + f*x^2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sqrt{a + bx + cx^2}}{d + ex + fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)`

[Out] `Integral(x**2*sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)`

$$3.110 \quad \int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2} dx$$

Optimal. Leaf size=549

$$\frac{\left((e - \sqrt{e^2 - 4df})\right) \left(f(be - af) - c(e^2 - df)\right) + 2df(ce - bf) \tanh^{-1} \left(\frac{4af + 2x(bf - c(e - \sqrt{e^2 - 4df})) - b(e - \sqrt{e^2 - 4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf}} \right)}{\sqrt{2}f^2\sqrt{e^2 - 4df}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf} + ce^2}$$

[Out] $-1/2*(-b*f+2*c*e)*\operatorname{arctanh}(1/2*(2*c*x+b)/c^{(1/2)}/(c*x^2+b*x+a)^{(1/2)})/f^2/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)}/f-1/2*\operatorname{arctanh}(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^{(1/2)})))-b*(e-(-4*d*f+e^2)^{(1/2)}))*2^{(1/2)}/(c*x^2+b*x+a)^{(1/2)}/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}*(2*d*f*(-b*f+c*e)+(f*(-a*f+b*e)-c*(-d*f+e^2))*(e-(-4*d*f+e^2)^{(1/2)}))/f^2*2^{(1/2)}/(-4*d*f+e^2)^{(1/2)}/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}+1/2*\operatorname{arctanh}(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^{(1/2)}))+2*x*(b*f-c*(e+(-4*d*f+e^2)^{(1/2)})))*2^{(1/2)}/(c*x^2+b*x+a)^{(1/2)}/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}*(2*d*f*(-b*f+c*e)+(f*(-a*f+b*e)-c*(-d*f+e^2))*(e+(-4*d*f+e^2)^{(1/2)}))/f^2*2^{(1/2)}/(-4*d*f+e^2)^{(1/2)}/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}$

Rubi [A] time = 7.03, antiderivative size = 549, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1019, 1076, 621, 206, 1032, 724}

$$\frac{\left((e - \sqrt{e^2 - 4df})\right) \left(f(be - af) - c(e^2 - df)\right) + 2df(ce - bf) \tanh^{-1} \left(\frac{4af + 2x(bf - c(e - \sqrt{e^2 - 4df})) - b(e - \sqrt{e^2 - 4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf}} \right)}{\sqrt{2}f^2\sqrt{e^2 - 4df}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf} + ce^2}$$

Antiderivative was successfully verified.

[In] Int[(x*sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2), x]

[Out] $\operatorname{Sqrt}[a + b*x + c*x^2]/f - ((2*c*e - b*f)*\operatorname{ArcTanh}[(b + 2*c*x)/(2*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*\operatorname{Sqrt}[c]*f^2) - ((2*d*f*(c*e - b*f) + (e - \operatorname{Sqrt}[e^2 - 4*d*f])*(f*(b*e - a*f) - c*(e^2 - d*f)))*\operatorname{ArcTanh}[(4*a*f - b*(e - \operatorname{Sqrt}[e^2 - 4*d*f]) + 2*(b*f - c*(e - \operatorname{Sqrt}[e^2 - 4*d*f]))*x)/(2*\operatorname{Sqrt}[2]*\operatorname{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*\operatorname{Sqrt}[e^2 - 4*d*f]]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(\operatorname{Sqrt}[2]*f^2*\operatorname{Sqrt}[e^2 - 4*d*f]*\operatorname{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*\operatorname{Sqrt}[e^2 - 4*d*f]]) + ((2*d*f*(c*e - b*f) + (e + \operatorname{Sqrt}[e^2 - 4*d*f])*(f*(b*e - a*f) - c*(e^2 - d*f)))*\operatorname{ArcTanh}[(4*a*f - b*(e + \operatorname{Sqrt}[e^2 - 4*d*f]) + 2*(b*f - c*(e + \operatorname{Sqrt}[e^2 - 4*d*f]))*x)/(2*\operatorname{Sqrt}[2]*\operatorname{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*\operatorname{Sqrt}[e^2 - 4*d*f]])]$

$$\frac{2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*\sqrt{e^2 - 4*d*f}*\sqrt{a + b*x + c*x^2}}{(\sqrt{2}*f^2*\sqrt{e^2 - 4*d*f}*\sqrt{c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*\sqrt{e^2 - 4*d*f}})}$$

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1019

```
Int[((g_) + (h_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[(h*(a + b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[h*p*(b*d - a*e) + a*(h*e - 2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(h*e - 2*g*f)*(p + q + 1))*x + (h*p*(c*e - b*f) + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1032

```
Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2} dx &= \frac{\sqrt{a+bx+cx^2}}{f} - \frac{\int \frac{\frac{bd}{2} + \frac{1}{2}(2cd+be-2af)x + \frac{1}{2}(2ce-bf)x^2}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx}{f} \\ &= \frac{\sqrt{a+bx+cx^2}}{f} - \frac{\int \frac{\frac{bdf}{2} - \frac{1}{2}d(2ce-bf) + \left(\frac{1}{2}f(2cd+be-2af) - \frac{1}{2}e(2ce-bf)\right)x}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx}{f^2} - \frac{(2ce-bf) \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{2f^2} \\ &= \frac{\sqrt{a+bx+cx^2}}{f} - \frac{(2ce-bf) \operatorname{Subst}\left(\int \frac{1}{4c-x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f^2} - \frac{\left(2f\left(\frac{bdf}{2} - \frac{1}{2}d(2ce-bf)\right) - (2ce-bf)\right) \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{2f^2} \\ &= \frac{\sqrt{a+bx+cx^2}}{f} - \frac{(2ce-bf) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2} + \frac{\left(2\left(2f\left(\frac{bdf}{2} - \frac{1}{2}d(2ce-bf)\right) - (2ce-bf)\right) \int \frac{1}{\sqrt{a+bx+cx^2}} dx\right)}{\sqrt{2}f^2} \\ &= \frac{\sqrt{a+bx+cx^2}}{f} - \frac{(2ce-bf) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2} - \frac{(2f(cde-bdf) + (e - \sqrt{e^2-4df})) \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{\sqrt{2}f^2} \end{aligned}$$

Mathematica [A] time = 1.78, size = 496, normalized size = 0.90

$$4f\sqrt{e^2-4df}\sqrt{a+x(b+cx)} - \sqrt{2}\left(\sqrt{e^2-4df}+e\right)\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}$$

Antiderivative was successfully verified.

[In] Integrate[(x*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2), x]

[Out] -1/2*((2*c*e - b*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]) / (Sqrt[c]*f^2) + (4*f*Sqrt[e^2 - 4*d*f]*Sqrt[a + x*(b + c*x)] - Sqrt[2]*(e + Sqrt[e^2 - 4*d*f])*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f

$$-b*(e + \sqrt{e^2 - 4*d*f}))]*\text{ArcTanh}[(4*a*f - 2*c*(e + \sqrt{e^2 - 4*d*f}))*x - b*(e + \sqrt{e^2 - 4*d*f} - 2*f*x)]/(2*\sqrt{2}*\sqrt{c*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f}) + f*(2*a*f - b*(e + \sqrt{e^2 - 4*d*f}))})*\sqrt{a + x*(b + c*x)}}] - \sqrt{2}*(-e + \sqrt{e^2 - 4*d*f})*\sqrt{f*(-(b*e) + 2*a*f + b*\sqrt{e^2 - 4*d*f}) + c*(e^2 - 2*d*f - e*\sqrt{e^2 - 4*d*f})}]*\text{ArcTanh}[(4*a*f + 2*c*(-e + \sqrt{e^2 - 4*d*f}))*x + b*(-e + \sqrt{e^2 - 4*d*f} + 2*f*x)]/(2*\sqrt{2}*\sqrt{f*(-(b*e) + 2*a*f + b*\sqrt{e^2 - 4*d*f}) + c*(e^2 - 2*d*f - e*\sqrt{e^2 - 4*d*f})})*\sqrt{a + x*(b + c*x)}})]/(4*f^2*\sqrt{e^2 - 4*d*f})$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [B] time = 0.02, size = 10138, normalized size = 18.47

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)

[Out] result too large to display

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see 'assume?' for more details) Is $4*d*f-e^2$ positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x \sqrt{c x^2 + b x + a}}{f x^2 + e x + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(a + b*x + c*x^2)^(1/2))/(d + e*x + f*x^2), x)

[Out] int((x*(a + b*x + c*x^2)^(1/2))/(d + e*x + f*x^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sqrt{a + b x + c x^2}}{d + e x + f x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d), x)

[Out] Integral(x*sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)

$$3.111 \quad \int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2} dx$$

Optimal. Leaf size=431

$$\frac{\sqrt{f(2af - b(e - \sqrt{e^2 - 4df})) + c(-e\sqrt{e^2 - 4df} - 2df + e^2)} \tanh^{-1} \left(\frac{4af + 2x(bf - c(e - \sqrt{e^2 - 4df})) - b(e - \sqrt{e^2 - 4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf}} \right)}{\sqrt{2}f\sqrt{e^2 - 4df}}$$

[Out] arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*c^(1/2)/f-1/2*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2))))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(f*(2*a*f-b*(e-(-4*d*f+e^2)^(1/2)))+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)/f*2^(1/2)/(-4*d*f+e^2)^(1/2)+1/2*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c+f*(2*a*f-b*(e+(-4*d*f+e^2)^(1/2))))^(1/2)/f*2^(1/2)/(-4*d*f+e^2)^(1/2)

Rubi [A] time = 0.65, antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {989, 621, 206, 1032, 724}

$$\frac{\sqrt{f(2af - b(e - \sqrt{e^2 - 4df})) + c(-e\sqrt{e^2 - 4df} - 2df + e^2)} \tanh^{-1} \left(\frac{4af + 2x(bf - c(e - \sqrt{e^2 - 4df})) - b(e - \sqrt{e^2 - 4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf}} \right)}{\sqrt{2}f\sqrt{e^2 - 4df}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2), x]

[Out] (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f - (Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])]/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]) + (Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])]/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]))

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 989

Int[Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]/((d_) + (e_)*(x_) + (f_)*(x_)^2), x_Symbol] := Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*f + (c*e - b*f)*x)/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]

Rule 1032

Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2} dx &= -\frac{\int \frac{cd-af+(ce-bf)x}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx}{f} + \frac{c \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{f} \\
&= \frac{(2c) \text{Subst}\left(\int \frac{1}{4c-x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f} - \frac{(2f(cd-af) - (ce-bf)(e - \sqrt{e^2-4df})) \int \frac{1}{(e-\sqrt{e^2-4df})}}{f\sqrt{e^2-4df}} \\
&= \frac{\sqrt{c} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f} + \frac{(2(2f(cd-af) - (ce-bf)(e - \sqrt{e^2-4df}))) \text{Subst}\left(\int \frac{1}{16a}\right)}{f\sqrt{e^2-4df}} \\
&= \frac{\sqrt{c} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f} - \frac{\sqrt{c}(e^2 - 2df - e\sqrt{e^2-4df}) + f(2af - b(e - \sqrt{e^2-4df}))}{\sqrt{2}f\sqrt{e^2-4df}}
\end{aligned}$$

Mathematica [A] time = 0.74, size = 417, normalized size = 0.97

$$\frac{\sqrt{f(2af - b(\sqrt{e^2-4df} + e)) + c(e\sqrt{e^2-4df} - 2df + e^2)} \tanh^{-1}\left(\frac{4af - b(\sqrt{e^2-4df} + e) - 2cx(\sqrt{e^2-4df} + e)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f(2af - b(\sqrt{e^2-4df} + e)) + c(e\sqrt{e^2-4df} - 2df + e^2)}}\right)}{\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f(2af - b(\sqrt{e^2-4df} + e)) + c(e\sqrt{e^2-4df} - 2df + e^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2), x]

[Out] (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/f + (Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)]] - Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + x*(b + c*x)]])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f])

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
i,const vecteur & l) Error: Bad Argument Value

maple [B] time = 0.02, size = 6019, normalized size = 13.97

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)

[Out] result too large to display

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{fx^2 + ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x + c*x^2)^(1/2)/(d + e*x + f*x^2),x)

```
[Out] int((a + b*x + c*x^2)^(1/2)/(d + e*x + f*x^2), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{\sqrt{a + bx + cx^2}}{d + ex + fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d), x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)
```

$$3.112 \quad \int \frac{\sqrt{a+bx+cx^2}}{x(d+ex+fx^2)} dx$$

Optimal. Leaf size=523

$$\frac{(cd(e - \sqrt{e^2 - 4df}) - f(2bd - a(\sqrt{e^2 - 4df} + e))) \tanh^{-1} \left(\frac{4af + 2x(bf - c(e - \sqrt{e^2 - 4df})) - b(e - \sqrt{e^2 - 4df})}{2\sqrt{2} \sqrt{a+bx+cx^2} \sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}} \right)}{\sqrt{2} d \sqrt{e^2 - 4df} \sqrt{f(2af - b(e - \sqrt{e^2 - 4df})) + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}} \quad (c$$

[Out] $-\operatorname{arctanh}(1/2*(b*x+2*a)/a^{(1/2)}/(c*x^2+b*x+a)^{(1/2)})*a^{(1/2)}/d+1/2*\operatorname{arctanh}(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^{(1/2)}))-b*(e-(-4*d*f+e^2)^{(1/2)}))*2^{(1/2)}/(c*x^2+b*x+a)^{(1/2)}/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)})*(c*d*(e-(-4*d*f+e^2)^{(1/2)})-f*(2*b*d-a*(e+(-4*d*f+e^2)^{(1/2)})))/d*2^{(1/2)}/(-4*d*f+e^2)^{(1/2)}/(f*(2*a*f-b*(e-(-4*d*f+e^2)^{(1/2)}))+(-4*d*f+e^2)^{(1/2)}*e-2*d*f+e^2)*c)^{(1/2)}-1/2*\operatorname{arctanh}(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^{(1/2)}))+2*x*(b*f-c*(e+(-4*d*f+e^2)^{(1/2)}))) * 2^{(1/2)}/(c*x^2+b*x+a)^{(1/2)}/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)})*(-f*(2*b*d-a*(e-(-4*d*f+e^2)^{(1/2)}))+c*d*(e+(-4*d*f+e^2)^{(1/2)}))/d*2^{(1/2)}/(-4*d*f+e^2)^{(1/2)}/(((-4*d*f+e^2)^{(1/2)}*e-2*d*f+e^2)*c+f*(2*a*f-b*(e+(-4*d*f+e^2)^{(1/2)})))^{(1/2)}$

Rubi [A] time = 3.70, antiderivative size = 521, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6728, 734, 843, 621, 206, 724, 1019, 1076, 1032}

$$\frac{(-af(\sqrt{e^2 - 4df} + e) + 2bdf - cd(e - \sqrt{e^2 - 4df})) \tanh^{-1} \left(\frac{4af + 2x(bf - c(e - \sqrt{e^2 - 4df})) - b(e - \sqrt{e^2 - 4df})}{2\sqrt{2} \sqrt{a+bx+cx^2} \sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}} \right)}{\sqrt{2} d \sqrt{e^2 - 4df} \sqrt{f(2af - b(e - \sqrt{e^2 - 4df})) + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}} \quad +$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sqrt}[a + b*x + c*x^2]/(x*(d + e*x + f*x^2)), x]$

[Out] $-\left(\frac{\operatorname{Sqrt}[a]*\operatorname{ArcTanh}[(2*a + b*x)/(2*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[a + b*x + c*x^2])]}{d}\right) - \left(\frac{(2*b*d*f - c*d*(e - \operatorname{Sqrt}[e^2 - 4*d*f]) - a*f*(e + \operatorname{Sqrt}[e^2 - 4*d*f]))*\operatorname{ArcTanh}[(4*a*f - b*(e - \operatorname{Sqrt}[e^2 - 4*d*f]) + 2*(b*f - c*(e - \operatorname{Sqrt}[e^2 - 4*d*f]))*x]/(2*\operatorname{Sqrt}[2]*\operatorname{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*\operatorname{Sqrt}[e^2 - 4*d*f]]*\operatorname{Sqrt}[a + b*x + c*x^2])]}{(\operatorname{Sqrt}[2]*d*\operatorname{Sqrt}[e^2 - 4*d*f]*\operatorname{Sqrt}[c*(e^2 - 2*d*f - e*\operatorname{Sqrt}[e^2 - 4*d*f]) + f*(2*a*f - b*(e - \operatorname{Sqrt}[e^2 - 4*d*f]))])} + \left(\frac{(2*b*d*f - a*f*(e - \operatorname{Sqrt}[e^2 - 4*d*f]) - c*d*(e + \operatorname{Sqrt}[e^2 - 4*d*f]))*\operatorname{ArcTanh}[(4*a*f - b*(e + \operatorname{Sqrt}[e^2 - 4*d*f]) + 2*(b*f - c*(e + \operatorname{Sqrt}[e^2 - 4*d*f]))*x]/(2*\operatorname{Sqrt}[2]*\operatorname{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*\operatorname{Sqrt}[e^2 - 4*d*f]]*\operatorname{Sqrt}[a + b*x + c*x^2])]}{(\operatorname{Sqrt}[2]*d*\operatorname{Sqrt}[e^2 - 4*d*f]*\operatorname{Sqrt}[c*(e^2 - 2*d*f - e*\operatorname{Sqrt}[e^2 - 4*d*f]) + f*(2*a*f - b*(e + \operatorname{Sqrt}[e^2 - 4*d*f]))])}\right)$

```
e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]))/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[c*
(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))
])
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int
[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Sym
bol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_S
ymbol] := Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && (!RationalQ[m] || LtQ[m, 1]) &
& !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c
_)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1019

```
Int[((g_) + (h_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_) + (e
_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[(h*(a + b*x + c*x^2)^p*(d
+ e*x + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), I
```



```

nt[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[h*p*(b*d - a*e) + a*(
h*e - 2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(h*e - 2*g*f)*(p + q + 1)
)*x + (h*p*(c*e - b*f) + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x] /; Fr
eeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d
*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]

```

Rule 1032

```

Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]

```

Rule 1076

```

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]

```

Rule 6728

```

Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx+cx^2}}{x(d+ex+fx^2)} dx &= \int \left(\frac{\sqrt{a+bx+cx^2}}{dx} + \frac{(-e-fx)\sqrt{a+bx+cx^2}}{d(d+ex+fx^2)} \right) dx \\
&= \frac{\int \frac{\sqrt{a+bx+cx^2}}{x} dx}{d} + \frac{\int \frac{(-e-fx)\sqrt{a+bx+cx^2}}{d+ex+fx^2} dx}{d} \\
&= -\frac{\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}} dx}{2d} - \frac{\int \frac{-\frac{1}{2}(bd-2ae)f - \frac{1}{2}f(2cd-be-2af)x + \frac{1}{2}bf^2x^2}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx}{df} \\
&= \frac{a \int \frac{1}{x\sqrt{a+bx+cx^2}} dx}{d} - \frac{\int \frac{-\frac{1}{2}bdf^2 - \frac{1}{2}(bd-2ae)f^2 + \left(-\frac{1}{2}bef^2 - \frac{1}{2}f^2(2cd-be-2af)\right)x}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx}{df^2} \\
&= -\frac{(2a) \text{Subst} \left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}} \right)}{d} - \frac{(2bdf - af(e - \sqrt{e^2 - 4df}) - cd(e + \sqrt{e^2 - 4df}))}{d\sqrt{e^2 - 4df}} \\
&= -\frac{\sqrt{a} \tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}} \right)}{d} + \frac{(2(2bdf - af(e - \sqrt{e^2 - 4df}) - cd(e + \sqrt{e^2 - 4df})))}{d\sqrt{e^2 - 4df}} \\
&= -\frac{\sqrt{a} \tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}} \right)}{d} - \frac{(2bdf - cd(e - \sqrt{e^2 - 4df}) - af(e + \sqrt{e^2 - 4df})) \tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}} \right)}{\sqrt{2}d\sqrt{e^2 - 4df}\sqrt{ce^2 - 2cdf - bef}}
\end{aligned}$$

Mathematica [A] time = 1.31, size = 454, normalized size = 0.87

$$\frac{(\sqrt{e^2 - 4df} - e) \sqrt{f(2af - b(\sqrt{e^2 - 4df} + e)) + c(e\sqrt{e^2 - 4df} - 2df + e^2)} \tanh^{-1} \left(\frac{4af - b(\sqrt{e^2 - 4df} + e - 2f)}{2\sqrt{2}\sqrt{a+x(b+cx)} \sqrt{f(2af - b(\sqrt{e^2 - 4df} + e)) + c(e\sqrt{e^2 - 4df} - 2df + e^2)}} \right)}{\sqrt{2}d\sqrt{e^2 - 4df}\sqrt{ce^2 - 2cdf - bef}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*x + c*x^2]/(x*(d + e*x + f*x^2)),x]

[Out] -((Sqrt[a]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/d) + ((-e + Sqrt[e^2 - 4*d*f])*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])]*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b

$$+ c*x)))] + (e + \sqrt{e^2 - 4*d*f})*\sqrt{f*(-(b*e) + 2*a*f + b*\sqrt{e^2 - 4*d*f})} + c*(e^2 - 2*d*f - e*\sqrt{e^2 - 4*d*f}))*\text{ArcTanh}[(4*a*f + 2*c*(-e + \sqrt{e^2 - 4*d*f}))*x + b*(-e + \sqrt{e^2 - 4*d*f} + 2*f*x)]/(2*\sqrt{2}*\sqrt{f*(-(b*e) + 2*a*f + b*\sqrt{e^2 - 4*d*f})} + c*(e^2 - 2*d*f - e*\sqrt{e^2 - 4*d*f}))*\sqrt{a + x*(b + c*x)}})/(2*\sqrt{2}*d*f*\sqrt{e^2 - 4*d*f})$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Evaluation time: 7.77sym2poly/r2sym(const ge
n & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [B] time = 0.02, size = 6460, normalized size = 12.35

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x+a)^(1/2)/x/(f*x^2+e*x+d),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{(fx^2 + ex + d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 + e*x + d)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{x(fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x + c*x^2)^(1/2)/(x*(d + e*x + f*x^2)), x)

[Out] int((a + b*x + c*x^2)^(1/2)/(x*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx + cx^2}}{x(d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x+a)**(1/2)/x/(f*x**2+e*x+d), x)

[Out] Integral(sqrt(a + b*x + c*x**2)/(x*(d + e*x + f*x**2)), x)

$$3.113 \quad \int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex+fx^2)} dx$$

Optimal. Leaf size=736

$$\frac{f\left(a\left(e\sqrt{e^2-4df}-2df+e^2\right)-bd\left(\sqrt{e^2-4df}+e\right)+2cd^2\right)\tanh^{-1}\left(\frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

[Out] $-1/2*b*\operatorname{arctanh}(1/2*(b*x+2*a)/a^{(1/2)}/(c*x^2+b*x+a)^{(1/2)})/d/a^{(1/2)}+e*\operatorname{arctanh}(1/2*(b*x+2*a)/a^{(1/2)}/(c*x^2+b*x+a)^{(1/2)})*a^{(1/2)}/d^2-1/2*b*e*\operatorname{arctanh}(1/2*(2*c*x+b)/c^{(1/2)}/(c*x^2+b*x+a)^{(1/2)})/d^2/c^{(1/2)}-1/2*(-b*e+2*c*d)*\operatorname{arctanh}(1/2*(2*c*x+b)/c^{(1/2)}/(c*x^2+b*x+a)^{(1/2)})/d^2/c^{(1/2)}+\operatorname{arctanh}(1/2*(2*c*x+b)/c^{(1/2)}/(c*x^2+b*x+a)^{(1/2)})*c^{(1/2)}/d-(c*x^2+b*x+a)^{(1/2)}/d/x-1/2*f*\operatorname{arctanh}(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^{(1/2)}))-b*(e-(-4*d*f+e^2)^{(1/2)}))*2^{(1/2)}/(c*x^2+b*x+a)^{(1/2)}/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)})*(2*c*d^2-b*d*(e+(-4*d*f+e^2)^{(1/2)}))+a*((-4*d*f+e^2)^{(1/2)}*e-2*d*f+e^2))/d^2*2^{(1/2)}/(-4*d*f+e^2)^{(1/2)}/(f*(2*a*f-b*(e-(-4*d*f+e^2)^{(1/2)}))+(-(-4*d*f+e^2)^{(1/2)}*e-2*d*f+e^2)*c)^{(1/2)}+1/2*f*\operatorname{arctanh}(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^{(1/2)}))+2*x*(b*f-c*(e+(-4*d*f+e^2)^{(1/2)}))) * 2^{(1/2)}/(c*x^2+b*x+a)^{(1/2)}/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)})*(2*c*d^2-b*d*(e-(-4*d*f+e^2)^{(1/2)}))+a*(-(-4*d*f+e^2)^{(1/2)}*e-2*d*f+e^2))/d^2*2^{(1/2)}/(-4*d*f+e^2)^{(1/2)}/(((-4*d*f+e^2)^{(1/2)}*e-2*d*f+e^2)*c+f*(2*a*f-b*(e+(-4*d*f+e^2)^{(1/2)})))^{(1/2)}$

Rubi [A] time = 3.48, antiderivative size = 736, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6728, 732, 843, 621, 206, 724, 734, 1019, 1076, 1032}

$$\frac{f\left(a\left(e\sqrt{e^2-4df}-2df+e^2\right)-bd\left(\sqrt{e^2-4df}+e\right)+2cd^2\right)\tanh^{-1}\left(\frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*x + c*x^2]/(x^2*(d + e*x + f*x^2)), x]

[Out] $-(\operatorname{Sqrt}[a + b*x + c*x^2]/(d*x)) - (b*\operatorname{ArcTanh}[(2*a + b*x)/(2*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[a + b*x + c*x^2]))/(2*\operatorname{Sqrt}[a]*d) + (\operatorname{Sqrt}[a]*e*\operatorname{ArcTanh}[(2*a + b*x)/(2*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[a + b*x + c*x^2]))/d^2 + (\operatorname{Sqrt}[c]*\operatorname{ArcTanh}[(b + 2*c*x)/(2*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a + b*x + c*x^2]))/d - (b*e*\operatorname{ArcTanh}[(b + 2*c*x)/(2*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a + b*x + c*x^2]))/(2*\operatorname{Sqrt}[c]*d^2) - ((2*c*d - b*e)*\operatorname{ArcTanh}[(b + 2*c*x)/(2*\operatorname{Sqrt}[c]*$

$$\frac{\sqrt{a + bx + cx^2}}{(2\sqrt{c}d^2) - (f(2cd^2 - b(d + \sqrt{e^2 - 4df})) + a(e^2 - 2df + e\sqrt{e^2 - 4df}))\operatorname{ArcTanh}[(4af - b(e - \sqrt{e^2 - 4df})) + 2(bf - c(e - \sqrt{e^2 - 4df}))x]/(2\sqrt{2}\sqrt{c^2e^2 - 2c^2d^2 - b^2ef + 2a^2f^2 - (ce - bf)\sqrt{e^2 - 4df}})\sqrt{a + bx + cx^2})} / (\sqrt{2}d^2\sqrt{e^2 - 4df}\sqrt{c(e^2 - 2df - e\sqrt{e^2 - 4df}) + f(2af - b(e - \sqrt{e^2 - 4df}))}) + (f(2cd^2 - b(d + \sqrt{e^2 - 4df})) + a(e^2 - 2df - e\sqrt{e^2 - 4df}))\operatorname{ArcTanh}[(4af - b(e + \sqrt{e^2 - 4df})) + 2(bf - c(e + \sqrt{e^2 - 4df}))x]/(2\sqrt{2}\sqrt{c^2e^2 - 2c^2d^2 - b^2ef + 2a^2f^2 + (ce - bf)\sqrt{e^2 - 4df}})\sqrt{a + bx + cx^2})} / (\sqrt{2}d^2\sqrt{e^2 - 4df}\sqrt{c(e^2 - 2df + e\sqrt{e^2 - 4df}) + f(2af - b(e + \sqrt{e^2 - 4df}))})$$

Rule 206

$$\operatorname{Int}[(a_.) + (b_.)x^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]x]/\operatorname{Rt}[a, 2])]/(\operatorname{Rt}[a, 2]\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$$

Rule 621

$$\operatorname{Int}[1/\sqrt{(a_.) + (b_.)x + (c_.)x^2}], x_Symbol] \rightarrow \operatorname{Dist}[2, \operatorname{Subst}[\operatorname{Int}[1/(4c - x^2), x], x, (b + 2cx)/\sqrt{a + bx + cx^2}], x] /; \operatorname{FreeQ}\{a, b, c\}, x \ \&\& \operatorname{NeQ}[b^2 - 4ac, 0]$$

Rule 724

$$\operatorname{Int}[1/(((d_.) + (e_.)x)\sqrt{(a_.) + (b_.)x + (c_.)x^2})], x_Symbol] \rightarrow \operatorname{Dist}[-2, \operatorname{Subst}[\operatorname{Int}[1/(4cd^2 - 4bde + 4ae^2 - x^2), x], x, (2ae - bd - (2cd - be)x)/\sqrt{a + bx + cx^2}], x] /; \operatorname{FreeQ}\{a, b, c, d, e\}, x \ \&\& \operatorname{NeQ}[b^2 - 4ac, 0] \ \&\& \operatorname{NeQ}[2cd - be, 0]$$

Rule 732

$$\operatorname{Int}(((d_.) + (e_.)x)^{m_})((a_.) + (b_.)x + (c_.)x^2)^{p_}), x_Symbol] \rightarrow \operatorname{Simp}(((d + ex)^{m+1}(a + bx + cx^2)^p)/(e(m+1)), x) - \operatorname{Dist}[p/(e(m+1)), \operatorname{Int}[(d + ex)^{m+1}(b + 2cx)(a + bx + cx^2)^{p-1}], x] /; \operatorname{FreeQ}\{a, b, c, d, e, m\}, x \ \&\& \operatorname{NeQ}[b^2 - 4ac, 0] \ \&\& \operatorname{NeQ}[cd^2 - bde + ae^2, 0] \ \&\& \operatorname{NeQ}[2cd - be, 0] \ \&\& \operatorname{GtQ}[p, 0] \ \&\& (\operatorname{IntegerQ}[p] \ || \ \operatorname{LtQ}[m, -1]) \ \&\& \operatorname{NeQ}[m, -1] \ \&\& \operatorname{!ILtQ}[m + 2p + 1, 0] \ \&\& \operatorname{IntQuadraticQ}[a, b, c, d, e, m, p, x]$$

Rule 734

$$\operatorname{Int}(((d_.) + (e_.)x)^{m_})((a_.) + (b_.)x + (c_.)x^2)^{p_}), x_Symbol] \rightarrow \operatorname{Simp}(((d + ex)^{m+1}(a + bx + cx^2)^p)/(e(m + 2p + 1)), x$$

] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && (!RationalQ[m] || LtQ[m, 1]) && !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]

Rule 843

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

Rule 1019

Int[((g_.) + (h_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[(h*(a + b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1)/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[h*p*(b*d - a*e) + a*(h*e - 2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(h*e - 2*g*f)*(p + q + 1))*x + (h*p*(c*e - b*f) + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]

Rule 1032

Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]

Rule 1076

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]

Rule 6728

Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[

{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex+fx^2)} dx &= \int \left(\frac{\sqrt{a+bx+cx^2}}{dx^2} - \frac{e\sqrt{a+bx+cx^2}}{d^2x} + \frac{(e^2-df+efx)\sqrt{a+bx+cx^2}}{d^2(d+ex+fx^2)} \right) dx \\
 &= \frac{\int \frac{(e^2-df+efx)\sqrt{a+bx+cx^2}}{d+ex+fx^2} dx}{d^2} + \frac{\int \frac{\sqrt{a+bx+cx^2}}{x^2} dx}{d} - \frac{e \int \frac{\sqrt{a+bx+cx^2}}{x} dx}{d^2} \\
 &= -\frac{\sqrt{a+bx+cx^2}}{dx} + \frac{\int \frac{b+2cx}{x\sqrt{a+bx+cx^2}} dx}{2d} + \frac{e \int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}} dx}{2d^2} - \frac{\int \frac{\frac{1}{2}f(bde-2ae^2+2adf)+\frac{1}{2}f(2cde-b^2)}{\sqrt{a+bx+cx^2}} dx}{d^2} \\
 &= -\frac{\sqrt{a+bx+cx^2}}{dx} + \frac{b \int \frac{1}{x\sqrt{a+bx+cx^2}} dx}{2d} + \frac{c \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{d} - \frac{(ae) \int \frac{1}{x\sqrt{a+bx+cx^2}} dx}{d^2} - \frac{(be) \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{d^2} \\
 &= -\frac{\sqrt{a+bx+cx^2}}{dx} - \frac{b \operatorname{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d} + \frac{(2c) \operatorname{Subst}\left(\int \frac{1}{4c-x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{d} \\
 &= -\frac{\sqrt{a+bx+cx^2}}{dx} - \frac{b \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}d} + \frac{\sqrt{a}e \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2} + \frac{\sqrt{c} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{d} \\
 &= -\frac{\sqrt{a+bx+cx^2}}{dx} - \frac{b \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}d} + \frac{\sqrt{a}e \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2} + \frac{\sqrt{c} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{d}
 \end{aligned}$$

Mathematica [A] time = 1.68, size = 520, normalized size = 0.71

$$\frac{(2ae - bd) \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right) + 4df\sqrt{e^2 - 4df}\sqrt{a+x(b+cx)} + \sqrt{2}x(e\sqrt{e^2 - 4df} + 2df - e^2)\sqrt{f(2af - b^2)}}{2\sqrt{a}d^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*x + c*x^2]/(x^2*(d + e*x + f*x^2)), x]


```
[Out] ((-(b*d) + 2*a*e)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/(
2*Sqrt[a]*d^2) - (4*d*f*Sqrt[e^2 - 4*d*f]*Sqrt[a + x*(b + c*x)] + Sqrt[2]*(
-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*
f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*x)*ArcTanh[(4*a*f - 2*c*(e + Sqr
t[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(
e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])
*Sqrt[a + x*(b + c*x)])] + Sqrt[2]*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*Sqrt
[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*
f]))]*x)*ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2
- 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) +
f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))])*Sqrt[a + x*(b + c*x)])])/(4*d^2*f*S
qrt[e^2 - 4*d*f]*x)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.02, size = 6765, normalized size = 9.19

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/x^2/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{(fx^2 + ex + d)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 + e*x + d)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c x^2 + b x + a}}{x^2 (f x^2 + e x + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x + c*x^2)^(1/2)/(x^2*(d + e*x + f*x^2)),x)

[Out] int((a + b*x + c*x^2)^(1/2)/(x^2*(d + e*x + f*x^2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x+a)**(1/2)/x**2/(f*x**2+e*x+d),x)

[Out] Timed out

$$3.114 \quad \int \frac{x^3}{\sqrt{a+bx+cx^2} (d+ex+fx^2)} dx$$

Optimal. Leaf size=545

$$\frac{b \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (2def - (e^2 - df)(e - \sqrt{e^2 - 4df})) \tanh^{-1}\left(\frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{2c^{3/2}f \sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

[Out] $-1/2*b*\operatorname{arctanh}(1/2*(2*c*x+b)/c^{(1/2)}/(c*x^2+b*x+a)^{(1/2)})/c^{(3/2)}/f-e*\operatorname{arctanh}(1/2*(2*c*x+b)/c^{(1/2)}/(c*x^2+b*x+a)^{(1/2)})/f^2/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)}/c/f-1/2*\operatorname{arctanh}(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^{(1/2)}))-b*(e-(-4*d*f+e^2)^{(1/2)}))*2^{(1/2)}/(c*x^2+b*x+a)^{(1/2)}/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}*(2*d*e*f-(-d*f+e^2)*(e-(-4*d*f+e^2)^{(1/2)}))/f^2*2^{(1/2)}/(-4*d*f+e^2)^{(1/2)}/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}+1/2*\operatorname{arctanh}(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^{(1/2)}))+2*x*(b*f-c*(e+(-4*d*f+e^2)^{(1/2)}))*2^{(1/2)}/(c*x^2+b*x+a)^{(1/2)}/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}*(2*d*e*f-(-d*f+e^2)*(e+(-4*d*f+e^2)^{(1/2)}))/f^2*2^{(1/2)}/(-4*d*f+e^2)^{(1/2)}/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}$

Rubi [A] time = 3.72, antiderivative size = 545, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6728, 621, 206, 640, 1032, 724}

$$\frac{b \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (2def - (e^2 - df)(e - \sqrt{e^2 - 4df})) \tanh^{-1}\left(\frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{2c^{3/2}f \sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]

[Out] $\operatorname{Sqrt}[a + b*x + c*x^2]/(c*f) - (e*\operatorname{ArcTanh}[(b + 2*c*x)/(2*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(\operatorname{Sqrt}[c]*f^2) - (b*\operatorname{ArcTanh}[(b + 2*c*x)/(2*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*c^{(3/2)}*f) - ((2*d*e*f - (e^2 - d*f)*(e - \operatorname{Sqrt}[e^2 - 4*d*f]))*\operatorname{ArcTanh}[(4*a*f - b*(e - \operatorname{Sqrt}[e^2 - 4*d*f]) + 2*(b*f - c*(e - \operatorname{Sqrt}[e^2 - 4*d*f]))*x)/(2*\operatorname{Sqrt}[2]*\operatorname{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*\operatorname{Sqrt}[e^2 - 4*d*f]]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(\operatorname{Sqrt}[2]*f^2*\operatorname{Sqrt}[e^2 - 4*d*f]*\operatorname{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*\operatorname{Sqrt}[e^2 - 4*d*f]]) + ((2*d*e*f - (e^2 - d*f)*(e + \operatorname{Sqrt}[e^2 - 4*d*f]))*\operatorname{ArcTanh}[(4*a*f - b*(e + \operatorname{Sqrt}[e^2 - 4*d*f]) + 2*(b*f - c*(e + \operatorname{Sqrt}[e^2 - 4*d*f]))*x)/(2*\operatorname{Sqrt}[2]*\operatorname{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*\operatorname{Sqrt}[e^2 - 4*d*f]]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(\operatorname{Sqrt}[2]*f^2*\operatorname{Sqrt}[e^2 - 4*d*f]*\operatorname{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*\operatorname{Sqrt}[e^2 - 4*d*f]])$

$$\frac{[c^2e^2 - 2cd^2f - b^2ef + 2a^2f^2 + (ce - bf)\sqrt{e^2 - 4d^2f}]\sqrt{a + bx + cx^2}}{(\sqrt{2}f^2\sqrt{e^2 - 4d^2f}\sqrt{c^2e^2 - 2cd^2f - b^2ef + 2a^2f^2 + (ce - bf)\sqrt{e^2 - 4d^2f}})}$$

Rule 206

$$\text{Int}[(a_ + (b_)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] \text{ /; FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$$

Rule 621

$$\text{Int}[1/\sqrt{(a_ + (b_)(x_ + (c_)(x_)^2)}, x_Symbol] \rightarrow \text{Dist}[2, \text{Subst}[\text{Int}[1/(4c - x^2), x], x, (b + 2cx)/\sqrt{a + bx + cx^2}], x] \text{ /; FreeQ}\{a, b, c\}, x \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$$

Rule 640

$$\text{Int}[(d_ + (e_)(x_))*((a_ + (b_)(x_ + (c_)(x_)^2)^{p_}), x_Symbol] \rightarrow \text{Simp}[(e*(a + bx + cx^2)^{p+1})/(2c*(p+1)), x] + \text{Dist}[(2cd - b^2e)/(2c), \text{Int}[(a + bx + cx^2)^p, x], x] \text{ /; FreeQ}\{a, b, c, d, e, p\}, x \ \&\& \ \text{NeQ}[2cd - b^2e, 0] \ \&\& \ \text{NeQ}[p, -1]$$

Rule 724

$$\text{Int}[1/(((d_ + (e_)(x_))*\sqrt{(a_ + (b_)(x_ + (c_)(x_)^2)}), x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/(4cd^2 - 4b^2de + 4ae^2 - x^2), x], x, (2ae - b^2d - (2cd - b^2e)x)/\sqrt{a + bx + cx^2}], x] \text{ /; FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{NeQ}[2cd - b^2e, 0]$$

Rule 1032

$$\text{Int}[(g_ + (h_)(x_))/((a_ + (b_)(x_ + (c_)(x_)^2)*\sqrt{(d_ + (e_)(x_ + (f_)(x_)^2)}), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[b^2 - 4ac, 2]\}, \text{Dist}[(2cg - h(b - q))/q, \text{Int}[1/((b - q + 2cx)*\sqrt{d + ex + fx^2}), x], x] - \text{Dist}[(2cg - h(b + q))/q, \text{Int}[1/((b + q + 2cx)*\sqrt{d + ex + fx^2}), x], x] \text{ /; FreeQ}\{a, b, c, d, e, f, g, h\}, x \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{NeQ}[e^2 - 4df, 0] \ \&\& \ \text{PosQ}[b^2 - 4ac]$$

Rule 6728

$$\text{Int}(u_)/((a_ + (b_)(x_)^{n_} + (c_)(x_)^{n2_}), x_Symbol] \rightarrow \text{With}\{v = \text{RationalFunctionExpand}[u/(a + bx^n + cx^{2n}), x]\}, \text{Int}[v, x] \text{ /; SumQ}[v] \text{ /; FreeQ}\{a, b, c\}, x \ \&\& \ \text{EqQ}[n2, 2n] \ \&\& \ \text{IGtQ}[n, 0]$$

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx &= \int \left(-\frac{e}{f^2\sqrt{a+bx+cx^2}} + \frac{x}{f\sqrt{a+bx+cx^2}} + \frac{de+(e^2-df)x}{f^2\sqrt{a+bx+cx^2}(d+ex+fx^2)} \right) dx \\
&= \frac{\int \frac{de+(e^2-df)x}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx}{f^2} - \frac{e \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{f^2} + \frac{\int \frac{x}{\sqrt{a+bx+cx^2}} dx}{f} \\
&= \frac{\sqrt{a+bx+cx^2}}{cf} - \frac{(2e) \operatorname{Subst}\left(\int \frac{1}{4c-x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f^2} - \frac{b \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{2cf} \\
&= \frac{\sqrt{a+bx+cx^2}}{cf} - \frac{e \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2} - \frac{b \operatorname{Subst}\left(\int \frac{1}{4c-x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{cf} \\
&= \frac{\sqrt{a+bx+cx^2}}{cf} - \frac{e \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2} - \frac{b \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f}
\end{aligned}$$

Mathematica [A] time = 2.20, size = 550, normalized size = 1.01

$$\frac{bf \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{3/2}} + \frac{\sqrt{2}\left(-\frac{e(e^2-3df)}{\sqrt{e^2-4df}}-df+e^2\right) \tanh^{-1}\left(\frac{4af+b\left(\sqrt{e^2-4df}-e+2fx\right)+2cx\left(\sqrt{e^2-4df}-e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{\sqrt{2}\left(e^2\sqrt{e^2-4df}\right)}{2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)), x]

[Out] $-1/2*((-2*f*\text{Sqrt}[a + x*(b + c*x)])/c + (2*e*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + x*(b + c*x)]))/\text{Sqrt}[c] + (b*f*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + x*(b + c*x)]))/c^{3/2} + (\text{Sqrt}[2]*(e^3 - 3*d*e*f + e^2*\text{Sqrt}[e^2 - 4*d*f] - d*f*\text{Sqrt}[e^2 - 4*d*f])*\text{ArcTanh}[(4*a*f - 2*c*(e + \text{Sqrt}[e^2 - 4*d*f])]*x - b*(e + \text{Sqrt}[e^2 - 4*d*f] - 2*f*x))/(2*\text{Sqrt}[2]*\text{Sqrt}[c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]) + f*(2*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f])])]*\text{Sqrt}[a + x*(b + c*x)]))/(\text{Sqrt}[e^2 - 4*d*f]*\text{Sqrt}[c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]) + f*(2*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f])])]) + (\text{Sqrt}[2]*(e^2 - d*f - (e*(e^2 - 3*d*$

$$\frac{f)}{\sqrt{e^2 - 4df}} \operatorname{ArcTanh}\left[\frac{(4af + 2c(-e + \sqrt{e^2 - 4df}))x + b(-e + \sqrt{e^2 - 4df} + 2fx)}{(2\sqrt{2}\sqrt{c(e^2 - 2df - e\sqrt{e^2 - 4df})} + f(2af + b(-e + \sqrt{e^2 - 4df})))}\sqrt{a + x(b + cx)}\right]}{\sqrt{c(e^2 - 2df - e\sqrt{e^2 - 4df}) + f(2af + b(-e + \sqrt{e^2 - 4df}))}}\right) \frac{1}{f^2}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type

maple [B] time = 0.03, size = 3131, normalized size = 5.74

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)

[Out]
$$\begin{aligned} & (cx^2+bx+a)^{1/2}/c/f-1/2*b/c^{3/2}/f*\ln((cx+1/2*b)/c^{1/2}+(cx^2+bx+a)^{1/2}) \\ & -1/f^2*e*\ln((cx+1/2*b)/c^{1/2}+(cx^2+bx+a)^{1/2})/c^{1/2}+1/2/f^2*2^{1/2}/((-4df+e^2)^{1/2}*bf+(-4df+e^2)^{1/2}*ce+2af^2-bef-2c*d*f+ce^2)/f^2)^{1/2} \\ & *\ln(((-4df+e^2)^{1/2}*bf+(-4df+e^2)^{1/2}*ce+2af^2-bef-2c*d*f+ce^2)/f^2+1/f*(-c*(-4df+e^2)^{1/2}+bf-ce)*(x+1/2*(e+(-4df+e^2)^{1/2}))/f)+1/2*2^{1/2}*((-4df+e^2)^{1/2}*bf+(-4df+e^2)^{1/2}*ce+2af^2-bef-2c*d*f+ce^2)/f^2)^{1/2} \\ & *(4*(x+1/2*(e+(-4df+e^2)^{1/2}))/f)^2*c+4/f*(-c*(-4df+e^2)^{1/2}+bf-ce)*(x+1/2*(e+(-4df+e^2)^{1/2}))/f)+2*((-4df+e^2)^{1/2}*bf+(-4df+e^2)^{1/2}*ce+2af^2-bef-2c*d*f+ce^2)/f^2)^{1/2} \\ & /((x+1/2*(e+(-4df+e^2)^{1/2}))/f)*d-1/2/f^3*2^{1/2}/((-4df+e^2)^{1/2}*bf+(-4df+e^2)^{1/2}*ce+2af^2-bef-2c*d*f+ce^2)/f^2)^{1/2} \\ & *\ln(((-4df+e^2)^{1/2}*bf+(-4df+e^2)^{1/2}*ce+2af^2-bef-2c*d*f+ce^2)/f^2)^{1/2} \\ & *\ln(((-4df+e^2)^{1/2}*bf+(-4df+e^2)^{1/2}*ce+2af^2-bef-2c*d*f+ce^2)/f^2)^{1/2} \end{aligned}$$

$$+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2)})/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f))*d*e+1/2/f^3/(-4*d*f+e^2)^{(1/2)}*2^{(1/2)}/(((-4*d*f+e^2)^{(1/2)})*b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2)}*\ln((((-4*d*f+e^2)^{(1/2)})*b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^{(1/2)}+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)+1/2*2^{(1/2)}*(((-4*d*f+e^2)^{(1/2)})*b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c+4*(c*(-4*d*f+e^2)^{(1/2)}+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)+2*((-4*d*f+e^2)^{(1/2)})*b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2)})/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f))*e^3$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details)Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{cx^2+bx+a} (fx^2+ex+d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)

[Out] int(x^3/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a+bx+cx^2} (d+ex+fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)

[Out] Integral(x**3/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)

$$3.115 \quad \int \frac{x^2}{\sqrt{a+bx+cx^2} (d+ex+fx^2)} dx$$

Optimal. Leaf size=463

$$\frac{(-e\sqrt{e^2-4df}-2df+e^2) \tanh^{-1}\left(\frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right) (2df-e(\sqrt{e^2-4df}+e))}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} - \frac{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

[Out] arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/f/c^(1/2)-1/2*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)/f*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)-1/2*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(2*d*f-e*(e+(-4*d*f+e^2)^(1/2)))/f*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)

Rubi [A] time = 3.44, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1077, 621, 206, 1032, 724}

$$\frac{(-e\sqrt{e^2-4df}-2df+e^2) \tanh^{-1}\left(\frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right) (2df-e(\sqrt{e^2-4df}+e))}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} - \frac{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]

[Out] ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]/(Sqrt[c]*f) - ((e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x]/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]))/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - ((2*d*f - e*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x]/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]))/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Sym
bol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (
e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1077

```
Int[((A_) + (C_)*(x_)^2)/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) +
(e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C - b*C*x)/((a + b*x + c*x^2)*Sqrt[d
+ e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{a+bx+cx^2} (d+ex+fx^2)} dx &= \frac{\int \frac{1}{\sqrt{a+bx+cx^2}} dx}{f} + \frac{\int \frac{-d-ex}{\sqrt{a+bx+cx^2} (d+ex+fx^2)} dx}{f} \\
&= \frac{2 \operatorname{Subst}\left(\int \frac{1}{4c-x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f} + \frac{(e^2 - 2df - e\sqrt{e^2 - 4df}) \int \frac{1}{(e - \sqrt{e^2 - 4df})}}{f\sqrt{e^2 - 4df}} \\
&= \frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f} - \frac{(2(e^2 - 2df - e\sqrt{e^2 - 4df})) \operatorname{Subst}\left(\int \frac{1}{16af^2 - 8}\right)}{\sqrt{c}f} \\
&= \frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f} - \frac{(e^2 - 2df - e\sqrt{e^2 - 4df}) \tanh^{-1}\left(\frac{4af - b(e - \sqrt{e^2 - 4df})}{2\sqrt{2}\sqrt{ce^2 - 2cdf - bef + 2d^2}}\right)}{\sqrt{2}f\sqrt{e^2 - 4df}\sqrt{ce^2 - 2cdf - bef + 2d^2}}
\end{aligned}$$

Mathematica [A] time = 0.95, size = 468, normalized size = 1.01

$$\frac{\sqrt{2}(e\sqrt{e^2-4df}-2df+e^2) \tanh^{-1}\left(\frac{4af-b(\sqrt{e^2-4df}+e-2fx)-2cx(\sqrt{e^2-4df}+e)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f(2af-b(\sqrt{e^2-4df}+e))+c(e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{e^2-4df}\sqrt{f(2af-b(\sqrt{e^2-4df}+e))+c(e\sqrt{e^2-4df}-2df+e^2)}} + \frac{\sqrt{2}\left(\frac{2df-e^2}{\sqrt{e^2-4df}}+e\right) \tanh^{-1}\left(\frac{4af+b(\sqrt{e^2-4df}-e)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f(2af-b(\sqrt{e^2-4df}-e))+c(-e\sqrt{e^2-4df}+2df-e^2)}}\right)}{2f}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]

[Out] ((2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + (Sqrt[2]*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)]])/((Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]) + (Sqrt[2]*(e + (-e^2 + 2*d*f)/Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)]])/Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))])/(2*f))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

```
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
i,const vecteur & l) Error: Bad Argument Value
```

```
maple [B] time = 0.02, size = 2321, normalized size = 5.01
```

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] 1/c^(1/2)/f*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/2/f^2*2^(1/2)/((-
(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/
f^2)^(1/2)*ln(((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/2/f^2*2^(1/2)/((-
(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/
f^2)+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+
1/2*2^(1/2)*((-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*
(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+
2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))
*e-1/(-4*d*f+e^2)^(1/2)/f*2^(1/2)/((-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*
ln(((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/2/f^2*2^(1/2)/((-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/
f^2)+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*
(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))
*d+1/2/(-4*d*f+e^2)^(1/2)/f^2*2^(1/2)/((-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/2/f^2*2^(1/2)/((-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*
```

$$b\sqrt{-4df+e^2} + (-4df+e^2)^{1/2} c\sqrt{e+2af^2-b^2ef-2cdf+ce^2} / f^2)^{1/2} * (4(x+1/2 * (e+(-4df+e^2)^{1/2})/f)^2 c + 4/f * (-c(-4df+e^2)^{1/2} + b\sqrt{-4df+e^2} - ce) * (x+1/2 * (e+(-4df+e^2)^{1/2})/f) + 2 * (-(-4df+e^2)^{1/2} * b\sqrt{-4df+e^2} + (-4df+e^2)^{1/2} * ce + 2af^2 - b^2ef - 2cdf + ce^2) / f^2)^{1/2}) / (x+1/2 * (e+(-4df+e^2)^{1/2})/f)) * e^2 + 1/2 / f^2 * 2^{1/2} / (((-4df+e^2)^{1/2} * b\sqrt{-4df+e^2} - (-4df+e^2)^{1/2} * ce + 2af^2 - b^2ef - 2cdf + ce^2) / f^2)^{1/2} * \ln(((-4df+e^2)^{1/2} * b\sqrt{-4df+e^2} - (-4df+e^2)^{1/2} * ce + 2af^2 - b^2ef - 2cdf + ce^2) / f^2 + (c(-4df+e^2)^{1/2} + b\sqrt{-4df+e^2} - ce) / f * (x-1/2 * (-e+(-4df+e^2)^{1/2})/f) + 1/2 * 2^{1/2} * (((-4df+e^2)^{1/2} * b\sqrt{-4df+e^2} - (-4df+e^2)^{1/2} * ce + 2af^2 - b^2ef - 2cdf + ce^2) / f^2)^{1/2} * (4(x-1/2 * (-e+(-4df+e^2)^{1/2})/f)^2 c + 4 * (c(-4df+e^2)^{1/2} + b\sqrt{-4df+e^2} - ce) / f * (x-1/2 * (-e+(-4df+e^2)^{1/2})/f) + 2 * ((-4df+e^2)^{1/2} * b\sqrt{-4df+e^2} - (-4df+e^2)^{1/2} * ce + 2af^2 - b^2ef - 2cdf + ce^2) / f^2)^{1/2}) / (x-1/2 * (-e+(-4df+e^2)^{1/2})/f)) * e + 1 / (-4df+e^2)^{1/2} / f * 2^{1/2} / (((-4df+e^2)^{1/2} * b\sqrt{-4df+e^2} - (-4df+e^2)^{1/2} * ce + 2af^2 - b^2ef - 2cdf + ce^2) / f^2)^{1/2} * \ln(((-4df+e^2)^{1/2} * b\sqrt{-4df+e^2} - (-4df+e^2)^{1/2} * ce + 2af^2 - b^2ef - 2cdf + ce^2) / f^2 + (c(-4df+e^2)^{1/2} + b\sqrt{-4df+e^2} - ce) / f * (x-1/2 * (-e+(-4df+e^2)^{1/2})/f) + 1/2 * 2^{1/2} * (((-4df+e^2)^{1/2} * b\sqrt{-4df+e^2} - (-4df+e^2)^{1/2} * ce + 2af^2 - b^2ef - 2cdf + ce^2) / f^2)^{1/2} * (4(x-1/2 * (-e+(-4df+e^2)^{1/2})/f)^2 c + 4 * (c(-4df+e^2)^{1/2} + b\sqrt{-4df+e^2} - ce) / f * (x-1/2 * (-e+(-4df+e^2)^{1/2})/f) + 2 * ((-4df+e^2)^{1/2} * b\sqrt{-4df+e^2} - (-4df+e^2)^{1/2} * ce + 2af^2 - b^2ef - 2cdf + ce^2) / f^2)^{1/2}) / (x-1/2 * (-e+(-4df+e^2)^{1/2})/f)) * d - 1/2 / (-4df+e^2)^{1/2} / f * 2^{1/2} / (((-4df+e^2)^{1/2} * b\sqrt{-4df+e^2} - (-4df+e^2)^{1/2} * ce + 2af^2 - b^2ef - 2cdf + ce^2) / f^2)^{1/2} * \ln(((-4df+e^2)^{1/2} * b\sqrt{-4df+e^2} - (-4df+e^2)^{1/2} * ce + 2af^2 - b^2ef - 2cdf + ce^2) / f^2 + (c(-4df+e^2)^{1/2} + b\sqrt{-4df+e^2} - ce) / f * (x-1/2 * (-e+(-4df+e^2)^{1/2})/f) + 1/2 * 2^{1/2} * (((-4df+e^2)^{1/2} * b\sqrt{-4df+e^2} - (-4df+e^2)^{1/2} * ce + 2af^2 - b^2ef - 2cdf + ce^2) / f^2)^{1/2} * (4(x-1/2 * (-e+(-4df+e^2)^{1/2})/f)^2 c + 4 * (c(-4df+e^2)^{1/2} + b\sqrt{-4df+e^2} - ce) / f * (x-1/2 * (-e+(-4df+e^2)^{1/2})/f) + 2 * ((-4df+e^2)^{1/2} * b\sqrt{-4df+e^2} - (-4df+e^2)^{1/2} * ce + 2af^2 - b^2ef - 2cdf + ce^2) / f^2)^{1/2}) / (x-1/2 * (-e+(-4df+e^2)^{1/2})/f)) * e^2$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details)Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{cx^2 + bx + a} (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)`

[Out] `int(x^2/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a + bx + cx^2} (d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)`

[Out] `Integral(x**2/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)`

$$3.116 \quad \int \frac{x}{\sqrt{a+bx+cx^2} (d+ex+fx^2)} dx$$

Optimal. Leaf size=402

$$\frac{(e - \sqrt{e^2 - 4df}) \tanh^{-1} \left(\frac{4af+2x(bf-c(e-\sqrt{e^2-4df})) - b(e-\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} \right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} \frac{(\sqrt{e^2-4df} + e) \tanh^{-1} \left(\frac{4af+2x(bf-c(e+\sqrt{e^2-4df})) - b(e+\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)+bef-2cdf+ce^2}} \right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)+bef-2cdf+ce^2}}$$

[Out] $1/2*\operatorname{arctanh}(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^{(1/2)}))-b*(e-(-4*d*f+e^2)^{(1/2)}))*2^{(1/2)}/(c*x^2+b*x+a)^{(1/2)}/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}*(e-(-4*d*f+e^2)^{(1/2)})*2^{(1/2)}/(-4*d*f+e^2)^{(1/2)}/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}-1/2*\operatorname{arctanh}(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^{(1/2)})+2*x*(b*f-c*(e+(-4*d*f+e^2)^{(1/2)}))))*2^{(1/2)}/(c*x^2+b*x+a)^{(1/2)}/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}*(e+(-4*d*f+e^2)^{(1/2)})*2^{(1/2)}/(-4*d*f+e^2)^{(1/2)}/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}$

Rubi [A] time = 0.96, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1032, 724, 206}

$$\frac{(e - \sqrt{e^2 - 4df}) \tanh^{-1} \left(\frac{4af+2x(bf-c(e-\sqrt{e^2-4df})) - b(e-\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} \right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} \frac{(\sqrt{e^2-4df} + e) \tanh^{-1} \left(\frac{4af+2x(bf-c(e+\sqrt{e^2-4df})) - b(e+\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)+bef-2cdf+ce^2}} \right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)+bef-2cdf+ce^2}}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]

[Out] $((e - \operatorname{Sqrt}[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e - \operatorname{Sqrt}[e^2 - 4*d*f]) + 2*(b*f - c*(e - \operatorname{Sqrt}[e^2 - 4*d*f]))*x]/(2*\operatorname{Sqrt}[2]*\operatorname{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*\operatorname{Sqrt}[e^2 - 4*d*f]]*\operatorname{Sqrt}[a + b*x + c*x^2]))]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[e^2 - 4*d*f]*\operatorname{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*\operatorname{Sqrt}[e^2 - 4*d*f]] - ((e + \operatorname{Sqrt}[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e + \operatorname{Sqrt}[e^2 - 4*d*f]) + 2*(b*f - c*(e + \operatorname{Sqrt}[e^2 - 4*d*f]))*x]/(2*\operatorname{Sqrt}[2]*\operatorname{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*\operatorname{Sqrt}[e^2 - 4*d*f]]*\operatorname{Sqrt}[a + b*x + c*x^2]))]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[e^2 - 4*d*f]*\operatorname{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*\operatorname{Sqrt}[e^2 - 4*d*f]]))$

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :=> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1032

Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :=> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{x}{\sqrt{a+bx+cx^2} (d+ex+fx^2)} dx &= - \left(\left(-1 - \frac{e}{\sqrt{e^2-4df}} \right) \int \frac{1}{(e + \sqrt{e^2-4df} + 2fx) \sqrt{a+bx+cx^2}} dx \right) + \left(1 - \frac{e}{\sqrt{e^2-4df}} \right) \int \frac{1}{(e - \sqrt{e^2-4df} + 2fx) \sqrt{a+bx+cx^2}} dx \\ &= - \left(2 \left(1 - \frac{e}{\sqrt{e^2-4df}} \right) \right) \text{Subst} \left(\int \frac{1}{16af^2 - 8bf(e - \sqrt{e^2-4df}) + 4c(e - \sqrt{e^2-4df})x} dx \right) \\ &= - \frac{\left(1 - \frac{e}{\sqrt{e^2-4df}} \right) \tanh^{-1} \left(\frac{4af - b(e - \sqrt{e^2-4df}) + 2(bf - c(e - \sqrt{e^2-4df}))x}{2\sqrt{2} \sqrt{ce^2 - 2cdf - bef + 2af^2 - (ce - bf)\sqrt{e^2-4df}} \sqrt{a+bx+cx^2}} \right)}{\sqrt{2} \sqrt{ce^2 - 2cdf - bef + 2af^2 - (ce - bf)\sqrt{e^2-4df}}} \end{aligned}$$

Mathematica [A] time = 0.97, size = 407, normalized size = 1.01

$$\frac{(\sqrt{e^2-4df} + e) \tanh^{-1} \left(\frac{4af - b(\sqrt{e^2-4df} + e) - 2cx(\sqrt{e^2-4df} + e)}{2\sqrt{2} \sqrt{a+x(b+cx)} \sqrt{f(2af - b(\sqrt{e^2-4df} + e)) + c(e\sqrt{e^2-4df} - 2df + e^2)}} \right)}{\sqrt{e^2-4df} \sqrt{f(2af - b(\sqrt{e^2-4df} + e)) + c(e\sqrt{e^2-4df} - 2df + e^2)}} - \frac{\left(1 - \frac{e}{\sqrt{e^2-4df}} \right) \tanh^{-1} \left(\frac{4af + b(\sqrt{e^2-4df} - e) + 2cx(\sqrt{e^2-4df} - e)}{2\sqrt{2} \sqrt{a+x(b+cx)} \sqrt{f(2af + b(\sqrt{e^2-4df} - e)) + c(-e\sqrt{e^2-4df} - 2df + e^2)}} \right)}{\sqrt{f(2af + b(\sqrt{e^2-4df} - e)) + c(-e\sqrt{e^2-4df} - 2df + e^2)}}}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]

[Out]
$$\begin{aligned} & -\left(\left(\left(e + \sqrt{e^2 - 4df}\right) \operatorname{ArcTanh}\left[\frac{4af - 2c(e + \sqrt{e^2 - 4df})}{2\sqrt{2}\sqrt{c(e^2 - 2df + e\sqrt{e^2 - 4df})} + f(2af - b(e + \sqrt{e^2 - 4df}))}\right]\right) \sqrt{a + x(b + cx)}\right) \\ & + \left(\left(\left(e + \sqrt{e^2 - 4df}\right) \operatorname{ArcTanh}\left[\frac{4af + 2c(-e + \sqrt{e^2 - 4df})}{2\sqrt{2}\sqrt{c(e^2 - 2df - e\sqrt{e^2 - 4df})} + f(2af + b(-e + \sqrt{e^2 - 4df}))}\right]\right) \sqrt{a + x(b + cx)}\right) \\ & \left. \right) / \left(\sqrt{e^2 - 4df} \sqrt{c(e^2 - 2df + e\sqrt{e^2 - 4df})} + f(2af - b(e + \sqrt{e^2 - 4df})) \right) - \left(\left(1 - \frac{e}{\sqrt{e^2 - 4df}}\right) \operatorname{ArcTanh}\left[\frac{4af + 2c(-e + \sqrt{e^2 - 4df})}{2\sqrt{2}\sqrt{c(e^2 - 2df - e\sqrt{e^2 - 4df})} + f(2af + b(-e + \sqrt{e^2 - 4df}))}\right] \right) \sqrt{a + x(b + cx)} \right) / \sqrt{c(e^2 - 2df - e\sqrt{e^2 - 4df}) + f(2af + b(-e + \sqrt{e^2 - 4df}))} \end{aligned}$$

fricas [B] time = 11.37, size = 11311, normalized size = 28.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out]
$$\begin{aligned} & \frac{1}{4} \sqrt{2} \sqrt{(2cd^2 - bde + ae^2 - 2adef + (c^2d^2e^2 - bcd^2e^3 + ac^2e^4 - 4a^2d^2f^3 + (4abde + a^2e^2 - 4(b^2 - 2ac)d^2) f^2 - (4c^2d^3 - 4b^2cd^2e + ab^2e^3 - (b^2 - 6ac)d^2e^2) f) \sqrt{(b^2d^2 - 2abd^2e + a^2e^2) / (c^4d^4e^2 - 2b^2c^3d^3e^3 - 2ab^2c^2d^2e^5 + a^2c^2e^6 - 4a^4d^2f^5 + (b^2c^2 + 2ac^3)d^2e^4 + (8a^3bde + a^4e^2 - 8(a^2b^2 - 2a^3c)d^2) f^4 - 2(a^3b^2e^3 + 2(b^4 - 4ab^2c + 6a^2c^2)d^3 - 4(ab^3 - a^2bc)d^2e + (a^2b^2 + 6a^3c)d^2e^2) f^3 - (8(b^2c^2 - 2ac^3)d^4 - 8(b^3c - abc^2)d^3e - (b^4 - 20ab^2c + 22a^2c^2)d^2e^2 + 2(ab^3 - 5a^2bc)d^2e^3 - (a^2b^2 + 2a^3c)e^4) f^2 - 2(2c^4d^5 - 4b^2c^3d^4e + a^2b^2c^2e^5 + (b^2c^2 + 6ac^3)d^3e^2 + (b^3c - 5ab^2c^2)d^2e^3 - 2(ab^2c - 2a^2c^2)d^2e^4) f)} \\ & \left. \right) / \left(\sqrt{e^2 - 4df} \sqrt{c(e^2 - 2df + e\sqrt{e^2 - 4df})} + f(2af - b(e + \sqrt{e^2 - 4df})) \right) \log\left(-\frac{2b^2d^3 - 4abd^2e + 2a^2d^2e^2 + \sqrt{2}(b^2d^2e^2 - 2abd^2e^3 + a^2e^4 - 4(b^2d^3 - 2abd^2e + a^2d^2e^2) f - (2c^3d^4e^2 - 3b^2c^2d^3e^3 - 2ab^2cd^2e^5 + a^2c^2e^6 + 8a^3d^2f^4 + (b^2c + 3ac^2)d^2e^4 - 2(2a^2bd^2e + 3a^3d^2e^2 - 4(ab^2 - 3a^2c)d^3) f^3 + (5a^2bd^2e^3 + a^3e^4 - 8(b^2c - 3ac^2)d^4 + 4(b^3 - 2abc)d^3e - 2(5ab^2 - 11a^2c)d^2e^2) f^2 - (8c^3d^5 - 12b^2c^2d^4e + a^2b^2e^5 + 2(b^2c + 9ac^2)d^3e^2 + (b^3 - 10abc)d^2e^3 - 2(ab^2 - 4a^2c)d^2e^4) f) \sqrt{(b^2d^2 - 2abd^2e + a^2e^2) / (c^4d^4e^2 - 2b^2c^3d^3e^3 - 2ab^2c^2d^2e^5 + a^2c^2e^6 - 4a^4d^2f^5 + (b^2c^2 + 2ac^3)d^2e^4 + (8a^3bde + a^4e^2 - 8(a^2b^2 - 2a^3c)d^2) f^4 - 2(a^3b^2e^3 + 2(b^4 - 4ab^2c + 6a^2c^2)d^3 - 4(ab^3 - a^2bc)d^2e + (a^2b^2 + 6a^3c)d^2e^2) f^3 - (8(b^2c^2 - 2ac^3)d^4 - 8(b^3c - abc^2)d^3e - (b^4 - 20ab^2c + 22a^2c^2)d^2e^2 + 2(ab^3 - 5a^2bc)d^2e^3 - (a^2b^2 + 2a^3c)e^4) f^2 - 2(2c^4d^5 - 4b^2c^3d^4e + a^2b^2c^2e^5 + (b^2c^2 + 6ac^3)d^3e^2 + (b^3c - 5ab^2c^2)d^2e^3 - 2(ab^2c - 2a^2c^2)d^2e^4) f} \right) \end{aligned}$$

$$\begin{aligned}
& 2*c^2*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 \\
& - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 \\
& + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*\sqrt{(c \\
& *x^2 + b*x + a)*\sqrt{(2*c*d^2 - b*d*e + a*e^2 - 2*a*d*f + (c^2*d^2*e^2 - b* \\
& c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2 \\
&)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*\sqrt{(\\
& (b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d* \\
& e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d* \\
& e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4* \\
& a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)* \\
& d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 \\
& - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 \\
& + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 \\
& + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2) \\
&)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e \\
& + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 \\
& - (b^2 - 6*a*c)*d*e^2)*f)) + (4*b*c*d^3 + a*b*d*e^2 - (b^2 + 4*a*c)*d^2*e)* \\
& x - (2*a*c^2*d^3*e^2 - 2*a*b*c*d^2*e^3 + 2*a^2*c*d*e^4 - 8*a^3*d^2*f^3 + 2* \\
& (4*a^2*b*d^2*e + a^3*d*e^2 - 4*(a*b^2 - 2*a^2*c)*d^3)*f^2 - 2*(4*a*c^2*d^4 \\
& - 4*a*b*c*d^3*e + a^2*b*d*e^3 - (a*b^2 - 6*a^2*c)*d^2*e^2)*f + (b*c^2*d^3*e \\
& ^2 - b^2*c*d^2*e^3 + a*b*c*d*e^4 - 4*a^2*b*d^2*f^3 + (4*a*b^2*d^2*e + a^2*b \\
& *d*e^2 - 4*(b^3 - 2*a*b*c)*d^3)*f^2 - (4*b*c^2*d^4 - 4*b^2*c*d^3*e + a*b^2* \\
& d*e^3 - (b^3 - 6*a*b*c)*d^2*e^2)*f)*x)*\sqrt{(b^2*d^2 - 2*a*b*d*e + a^2*e^2) \\
& / (c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f \\
& ^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2* \\
& a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a \\
& *b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a* \\
& c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2* \\
& e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4 \\
& *d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - \\
& 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x - 1/4*\sqrt{2)* \\
& \sqrt{(2*c*d^2 - b*d*e + a*e^2 - 2*a*d*f + (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 \\
& - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 \\
& - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*\sqrt{(b^2*d^2 - 2*a*b \\
& *d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2* \\
& e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - \\
& 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2* \\
& c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8* \\
& (b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 2 \\
& 2*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4) \\
& *f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3 \\
& *e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c \\
& ^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(\\
& b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)* \\
& d*e^2)*f))*\log(-(2*b^2*d^3 - 4*a*b*d^2*e + 2*a^2*d*e^2 - \sqrt{2)*(b^2*d^2*e
\end{aligned}$$

$$\begin{aligned}
&^2 - 2*a*b*d*e^3 + a^2*e^4 - 4*(b^2*d^3 - 2*a*b*d^2*e + a^2*d*e^2)*f - (2*c \\
&^3*d^4*e^2 - 3*b*c^2*d^3*e^3 - 2*a*b*c*d*e^5 + a^2*c*e^6 + 8*a^3*d^2*f^4 + \\
&(b^2*c + 3*a*c^2)*d^2*e^4 - 2*(2*a^2*b*d^2*e + 3*a^3*d*e^2 - 4*(a*b^2 - 3*a \\
&^2*c)*d^3)*f^3 + (5*a^2*b*d*e^3 + a^3*e^4 - 8*(b^2*c - 3*a*c^2)*d^4 + 4*(b^ \\
&3 - 2*a*b*c)*d^3*e - 2*(5*a*b^2 - 11*a^2*c)*d^2*e^2)*f^2 - (8*c^3*d^5 - 12* \\
&b*c^2*d^4*e + a^2*b*e^5 + 2*(b^2*c + 9*a*c^2)*d^3*e^2 + (b^3 - 10*a*b*c)*d^ \\
&2*e^3 - 2*(a*b^2 - 4*a^2*c)*d*e^4)*f)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/ \\
&(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^ \\
&5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a \\
&^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a* \\
&b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c \\
&^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e \\
&^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4* \\
&d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - \\
&5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f))*sqrt(c*x^2 + b*x + \\
&a)*sqrt((2*c*d^2 - b*d*e + a*e^2 - 2*a*d*f + (c^2*d^2*e^2 - b*c*d*e^3 + a* \\
&c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4* \\
&c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((b^2*d^2 - 2 \\
&*a*b*d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2* \\
&c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^ \\
&2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6* \\
&a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - \\
&(8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c \\
&+ 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)* \\
&e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3) \\
&)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)) \\
&)/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - \\
&4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a \\
&*c)*d*e^2)*f)) + (4*b*c*d^3 + a*b*d*e^2 - (b^2 + 4*a*c)*d^2*e)*x - (2*a*c^2 \\
&*d^3*e^2 - 2*a*b*c*d^2*e^3 + 2*a^2*c*d*e^4 - 8*a^3*d^2*f^3 + 2*(4*a^2*b*d^2 \\
&*e + a^3*d*e^2 - 4*(a*b^2 - 2*a^2*c)*d^3)*f^2 - 2*(4*a*c^2*d^4 - 4*a*b*c*d^ \\
&3*e + a^2*b*d*e^3 - (a*b^2 - 6*a^2*c)*d^2*e^2)*f + (b*c^2*d^3*e^2 - b^2*c*d \\
&^2*e^3 + a*b*c*d*e^4 - 4*a^2*b*d^2*f^3 + (4*a*b^2*d^2*e + a^2*b*d*e^2 - 4*(\\
&b^3 - 2*a*b*c)*d^3)*f^2 - (4*b*c^2*d^4 - 4*b^2*c*d^3*e + a*b^2*d*e^3 - (b^3 \\
&- 6*a*b*c)*d^2*e^2)*f)*x)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4*e^ \\
&2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^ \\
&2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)* \\
&f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b \\
&*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8 \\
&*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b \\
&^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c \\
&^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)* \\
&d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x) + 1/4*sqrt(2)*sqrt((2*c*d^ \\
&2 - b*d*e + a*e^2 - 2*a*d*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d* \\
&f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*
\end{aligned}$$

$$\begin{aligned}
& d^2e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*\sqrt{(b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*\log(-(2*b^2*d^3 - 4*a*b*d^2*e + 2*a^2*d*e^2 + \sqrt{2}*(b^2*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4 - 4*(b^2*d^3 - 2*a*b*d^2*e + a^2*d*e^2)*f + (2*c^3*d^4*e^2 - 3*b*c^2*d^3*e^3 - 2*a*b*c*d*e^5 + a^2*c*e^6 + 8*a^3*d^2*f^4 + (b^2*c + 3*a*c^2)*d^2*e^4 - 2*(2*a^2*b*d^2*e + 3*a^3*d*e^2 - 4*(a*b^2 - 3*a^2*c)*d^3)*f^3 + (5*a^2*b*d*e^3 + a^3*e^4 - 8*(b^2*c - 3*a*c^2)*d^4 + 4*(b^3 - 2*a*b*c)*d^3*e - 2*(5*a*b^2 - 11*a^2*c)*d^2*e^2)*f^2 - (8*c^3*d^5 - 12*b*c^2*d^4*e + a^2*b*e^5 + 2*(b^2*c + 9*a*c^2)*d^3*e^2 + (b^3 - 10*a*b*c)*d^2*e^3 - 2*(a*b^2 - 4*a^2*c)*d*e^4)*f)*\sqrt{(b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*\sqrt{c*x^2 + b*x + a)*\sqrt{(2*c*d^2 - b*d*e + a*e^2 - 2*a*d*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*\sqrt{(b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)) + (4*b*c*d^3 + a*b*d*e^2 - (b^2 + 4*a*c)*d^2*e)*x + (2*a*c^2*d^3*e^2 - 2*a*b*c*d^2*e^3 + 2*a^2*c*d*e^4 - 8*a^3*d^2*f^3 + 2*(4*a^2*b*d^2*e + a^3*d*e^2 - 4*(a*b^2 - 2*a^2*c)*d^3)*f^2 - 2*(4*a*c^2*d^4 - 4*a*b*c*d^3*e + a^2*b*d*e^3 - (a*b^2 - 6*a^2*c)*d^2*e^2)*f + (b*c^2*d^3*e^2 - b^2*c*d^2*e^3 + a*b*c*d*e^4 - 4*a^2*b*d^2*f^3 + (4*a*b^2*d^2*e + a^2*b*d*e^2 - 4*(b^3 - 2*a*b*
\end{aligned}$$

$$\begin{aligned}
& c) * d^3) * f^2 - (4 * b * c^2 * d^4 - 4 * b^2 * c * d^3 * e + a * b^2 * d * e^3 - (b^3 - 6 * a * b * c) * \\
& d^2 * e^2) * f) * x) * \text{sqrt}((b^2 * d^2 - 2 * a * b * d * e + a^2 * e^2) / (c^4 * d^4 * e^2 - 2 * b * c^3 * \\
& d^3 * e^3 - 2 * a * b * c^2 * d * e^5 + a^2 * c^2 * e^6 - 4 * a^4 * d * f^5 + (b^2 * c^2 + 2 * a * c^3) \\
& * d^2 * e^4 + (8 * a^3 * b * d * e + a^4 * e^2 - 8 * (a^2 * b^2 - 2 * a^3 * c) * d^2) * f^4 - 2 * (a^3 \\
& * b * e^3 + 2 * (b^4 - 4 * a * b^2 * c + 6 * a^2 * c^2) * d^3 - 4 * (a * b^3 - a^2 * b * c) * d^2 * e + \\
& (a^2 * b^2 + 6 * a^3 * c) * d * e^2) * f^3 - (8 * (b^2 * c^2 - 2 * a * c^3) * d^4 - 8 * (b^3 * c - a * \\
& b * c^2) * d^3 * e - (b^4 - 20 * a * b^2 * c + 22 * a^2 * c^2) * d^2 * e^2 + 2 * (a * b^3 - 5 * a^2 * b \\
& * c) * d * e^3 - (a^2 * b^2 + 2 * a^3 * c) * e^4) * f^2 - 2 * (2 * c^4 * d^5 - 4 * b * c^3 * d^4 * e + a \\
& ^2 * b * c * e^5 + (b^2 * c^2 + 6 * a * c^3) * d^3 * e^2 + (b^3 * c - 5 * a * b * c^2) * d^2 * e^3 - 2 * \\
& (a * b^2 * c - 2 * a^2 * c^2) * d * e^4) * f)) / x) - 1/4 * \text{sqrt}(2) * \text{sqrt}((2 * c * d^2 - b * d * e + \\
& a * e^2 - 2 * a * d * f - (c^2 * d^2 * e^2 - b * c * d * e^3 + a * c * e^4 - 4 * a^2 * d * f^3 + (4 * a * b \\
& * d * e + a^2 * e^2 - 4 * (b^2 - 2 * a * c) * d^2) * f^2 - (4 * c^2 * d^3 - 4 * b * c * d^2 * e + a * b * \\
& e^3 - (b^2 - 6 * a * c) * d * e^2) * f) * \text{sqrt}((b^2 * d^2 - 2 * a * b * d * e + a^2 * e^2) / (c^4 * d^4 \\
& * e^2 - 2 * b * c^3 * d^3 * e^3 - 2 * a * b * c^2 * d * e^5 + a^2 * c^2 * e^6 - 4 * a^4 * d * f^5 + (b^2 \\
& * c^2 + 2 * a * c^3) * d^2 * e^4 + (8 * a^3 * b * d * e + a^4 * e^2 - 8 * (a^2 * b^2 - 2 * a^3 * c) * d^2 \\
&) * f^4 - 2 * (a^3 * b * e^3 + 2 * (b^4 - 4 * a * b^2 * c + 6 * a^2 * c^2) * d^3 - 4 * (a * b^3 - a^2 \\
& * b * c) * d^2 * e + (a^2 * b^2 + 6 * a^3 * c) * d * e^2) * f^3 - (8 * (b^2 * c^2 - 2 * a * c^3) * d^4 \\
& - 8 * (b^3 * c - a * b * c^2) * d^3 * e - (b^4 - 20 * a * b^2 * c + 22 * a^2 * c^2) * d^2 * e^2 + 2 * (\\
& a * b^3 - 5 * a^2 * b * c) * d * e^3 - (a^2 * b^2 + 2 * a^3 * c) * e^4) * f^2 - 2 * (2 * c^4 * d^5 - 4 * \\
& b * c^3 * d^4 * e + a^2 * b * c * e^5 + (b^2 * c^2 + 6 * a * c^3) * d^3 * e^2 + (b^3 * c - 5 * a * b * c^2) \\
&) * d^2 * e^3 - 2 * (a * b^2 * c - 2 * a^2 * c^2) * d * e^4) * f)) / (c^2 * d^2 * e^2 - b * c * d * e^3 + \\
& a * c * e^4 - 4 * a^2 * d * f^3 + (4 * a * b * d * e + a^2 * e^2 - 4 * (b^2 - 2 * a * c) * d^2) * f^2 - \\
& (4 * c^2 * d^3 - 4 * b * c * d^2 * e + a * b * e^3 - (b^2 - 6 * a * c) * d * e^2) * f) * \log(-(2 * b^2 * d \\
& ^3 - 4 * a * b * d^2 * e + 2 * a^2 * d * e^2 - \text{sqrt}(2) * (b^2 * d^2 * e^2 - 2 * a * b * d * e^3 + a^2 * e \\
& ^4 - 4 * (b^2 * d^3 - 2 * a * b * d^2 * e + a^2 * d * e^2) * f + (2 * c^3 * d^4 * e^2 - 3 * b * c^2 * d^3 \\
& * e^3 - 2 * a * b * c * d * e^5 + a^2 * c * e^6 + 8 * a^3 * d^2 * f^4 + (b^2 * c + 3 * a * c^2) * d^2 * e^4 \\
& - 2 * (2 * a^2 * b * d^2 * e + 3 * a^3 * d * e^2 - 4 * (a * b^2 - 3 * a^2 * c) * d^3) * f^3 + (5 * a^2 * \\
& b * d * e^3 + a^3 * e^4 - 8 * (b^2 * c - 3 * a * c^2) * d^4 + 4 * (b^3 - 2 * a * b * c) * d^3 * e - 2 * (\\
& 5 * a * b^2 - 11 * a^2 * c) * d^2 * e^2) * f^2 - (8 * c^3 * d^5 - 12 * b * c^2 * d^4 * e + a^2 * b * e^5 \\
& + 2 * (b^2 * c + 9 * a * c^2) * d^3 * e^2 + (b^3 - 10 * a * b * c) * d^2 * e^3 - 2 * (a * b^2 - 4 * a^2 \\
& * c) * d * e^4) * f) * \text{sqrt}((b^2 * d^2 - 2 * a * b * d * e + a^2 * e^2) / (c^4 * d^4 * e^2 - 2 * b * c^3 * d \\
& ^3 * e^3 - 2 * a * b * c^2 * d * e^5 + a^2 * c^2 * e^6 - 4 * a^4 * d * f^5 + (b^2 * c^2 + 2 * a * c^3) * \\
& d^2 * e^4 + (8 * a^3 * b * d * e + a^4 * e^2 - 8 * (a^2 * b^2 - 2 * a^3 * c) * d^2) * f^4 - 2 * (a^3 * \\
& b * e^3 + 2 * (b^4 - 4 * a * b^2 * c + 6 * a^2 * c^2) * d^3 - 4 * (a * b^3 - a^2 * b * c) * d^2 * e + (\\
& a^2 * b^2 + 6 * a^3 * c) * d * e^2) * f^3 - (8 * (b^2 * c^2 - 2 * a * c^3) * d^4 - 8 * (b^3 * c - a * b \\
& * c^2) * d^3 * e - (b^4 - 20 * a * b^2 * c + 22 * a^2 * c^2) * d^2 * e^2 + 2 * (a * b^3 - 5 * a^2 * b * \\
& c) * d * e^3 - (a^2 * b^2 + 2 * a^3 * c) * e^4) * f^2 - 2 * (2 * c^4 * d^5 - 4 * b * c^3 * d^4 * e + a^2 \\
& * b * c * e^5 + (b^2 * c^2 + 6 * a * c^3) * d^3 * e^2 + (b^3 * c - 5 * a * b * c^2) * d^2 * e^3 - 2 * (\\
& a * b^2 * c - 2 * a^2 * c^2) * d * e^4) * f)) * \text{sqrt}(c * x^2 + b * x + a) * \text{sqrt}((2 * c * d^2 - b * d * \\
& e + a * e^2 - 2 * a * d * f - (c^2 * d^2 * e^2 - b * c * d * e^3 + a * c * e^4 - 4 * a^2 * d * f^3 + (4 \\
& * a * b * d * e + a^2 * e^2 - 4 * (b^2 - 2 * a * c) * d^2) * f^2 - (4 * c^2 * d^3 - 4 * b * c * d^2 * e + \\
& a * b * e^3 - (b^2 - 6 * a * c) * d * e^2) * f) * \text{sqrt}((b^2 * d^2 - 2 * a * b * d * e + a^2 * e^2) / (c^4 \\
& * d^4 * e^2 - 2 * b * c^3 * d^3 * e^3 - 2 * a * b * c^2 * d * e^5 + a^2 * c^2 * e^6 - 4 * a^4 * d * f^5 + \\
& (b^2 * c^2 + 2 * a * c^3) * d^2 * e^4 + (8 * a^3 * b * d * e + a^4 * e^2 - 8 * (a^2 * b^2 - 2 * a^3 * c) \\
&) * d^2) * f^4 - 2 * (a^3 * b * e^3 + 2 * (b^4 - 4 * a * b^2 * c + 6 * a^2 * c^2) * d^3 - 4 * (a * b^3
\end{aligned}$$

$$\begin{aligned}
& - a^2 b c) d^2 e + (a^2 b^2 + 6 a^3 c) d e^2) f^3 - (8 (b^2 c^2 - 2 a c^3) d^4 - 8 (b^3 c - a b c^2) d^3 e - (b^4 - 20 a b^2 c + 22 a^2 c^2) d^2 e^2 + \\
& 2 (a b^3 - 5 a^2 b c) d e^3 - (a^2 b^2 + 2 a^3 c) e^4) f^2 - 2 (2 c^4 d^5 - 4 b c^3 d^4 e + a^2 b c e^5 + (b^2 c^2 + 6 a c^3) d^3 e^2 + (b^3 c - 5 a b c^2) d^2 e^3 - \\
& 2 (a b^2 c - 2 a^2 c^2) d e^4) f) / (c^2 d^2 e^2 - b c d e^3 + a c e^4 - 4 a^2 d f^3 + (4 a b d e + a^2 e^2 - 4 (b^2 - 2 a c) d^2) f^2 - \\
& (4 c^2 d^3 - 4 b c d^2 e + a b e^3 - (b^2 - 6 a c) d e^2) f) + (4 b c d^3 + a b d e^2 - (b^2 + 4 a c) d^2 e) x + (2 a c^2 d^3 e^2 - 2 a b c d^2 e^3 + 2 a^2 c d e^4 - \\
& 8 a^3 d^2 f^3 + 2 (4 a^2 b d^2 e + a^3 d e^2 - 4 (a b^2 - 2 a^2 c) d^3) f^2 - 2 (4 a c^2 d^4 - 4 a b c d^3 e + a^2 b d e^3 - (a b^2 - \\
& 6 a^2 c) d^2 e^2) f + (b c^2 d^3 e^2 - b^2 c d^2 e^3 + a b c d e^4 - 4 a^2 b d^2 f^3 + (4 a b^2 d^2 e + a^2 b d e^2 - 4 (b^3 - 2 a b c) d^3) f^2 - \\
& (4 b c^2 d^4 - 4 b^2 c d^3 e + a b^2 d e^3 - (b^3 - 6 a b c) d^2 e^2) f) x) \sqrt{(b^2 d^2 - 2 a b d e + a^2 e^2) / (c^4 d^4 e^2 - 2 b c^3 d^3 e^3 - 2 a b c^2 d e^5 + \\
& a^2 c^2 e^6 - 4 a^4 d f^5 + (b^2 c^2 + 2 a c^3) d^2 e^4 + (8 a^3 b d e + a^4 e^2 - 8 (a^2 b^2 - 2 a^3 c) d^2) f^4 - 2 (a^3 b e^3 + 2 (b^4 - 4 a b^2 c + 6 a^2 c^2) d^3 - 4 (a b^3 - a^2 b c) d^2 e + (a^2 b^2 + 6 a^3 c) d e^2) f^3 - (8 (b^2 c^2 - 2 a c^3) d^4 - 8 (b^3 c - a b c^2) d^3 e - (b^4 - 20 a b^2 c + 22 a^2 c^2) d^2 e^2 + 2 (a b^3 - 5 a^2 b c) d e^3 - (a^2 b^2 + 2 a^3 c) e^4) f^2 - 2 (2 c^4 d^5 - 4 b c^3 d^4 e + a^2 b c e^5 + (b^2 c^2 + 6 a c^3) d^3 e^2 + (b^3 c - 5 a b c^2) d^2 e^3 - 2 (a b^2 c - 2 a^2 c^2) d e^4) f) / x)
\end{aligned}$$

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.01, size = 1516, normalized size = 3.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)

[Out]
$$\begin{aligned}
& -1/2 / (-4 d f + e^2)^{1/2} / f^2^{1/2} / ((2 a f^2 - b e f - 2 c d f + c e^2 - (-4 d f + e^2)^{1/2})^{1/2} * b f + (-4 d f + e^2)^{1/2} * c e) / f^2)^{1/2} * \ln(((b f - c e - (-4 d f + e^2)^{1/2})^{1/2} * c) * (x + 1/2 * (e + (-4 d f + e^2)^{1/2}) / f) / f + (2 a f^2 - b e f - 2 c d f + c e^2 - (-4 d f + e^2)^{1/2}) * b f + (-4 d f + e^2)^{1/2} * c e) / f^2 + 1/2 * 2^{1/2} * ((2 a f^2 - b e f - 2 c d f + c e^2 - (-4 d f + e^2)^{1/2}) * b f + (-4 d f + e^2)^{1/2} * c e) / f^2)^{1/2} * (4 *
\end{aligned}$$

$$\begin{aligned} & (x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)^2*c+4*(b*f-c*e-(-4*d*f+e^2)^{(1/2)}*c)*(x+1/2 \\ & * (e+(-4*d*f+e^2)^{(1/2)})/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)} \\ & *b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}/(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/ \\ & f))*e^{-1/2}/f^2)^{(1/2)}/((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(- \\ & 4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*\ln(((b*f-c*e-(-4*d*f+e^2)^{(1/2)}*c)*(x+1/2* \\ & (e+(-4*d*f+e^2)^{(1/2)})/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)} \\ & *b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2)^{(1/2)}*((2*a*f^2-b*e*f-2*c*d*f+c*e^2- \\ & (-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(x+1/2*(e+(-4* \\ & d*f+e^2)^{(1/2)})/f)^2*c+4*(b*f-c*e-(-4*d*f+e^2)^{(1/2)}*c)*(x+1/2*(e+(-4*d*f+e \\ & ^2)^{(1/2)})/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d \\ & *f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}/(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f))-1/2/f^2^(1 \\ & /2)/((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)} \\ & *c*e)/f^2)^{(1/2)}*\ln(((b*f-c*e+(-4*d*f+e^2)^{(1/2)}*c)*(x-1/2*(-e+(-4*d*f+e^2) \\ & ^{(1/2)})/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^ \\ & 2)^{(1/2)}*c*e)/f^2+1/2*2)^{(1/2)}*((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1 \\ & /2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2) \\ &)/f)^2*c+4*(b*f-c*e+(-4*d*f+e^2)^{(1/2)}*c)*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f) \\ & /f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)} \\ & *c*e)/f^2)^{(1/2)}/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f))+1/2/(-4*d*f+e^2)^{(1/2)} \\ & /f^2)^{(1/2)}/((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2) \\ &)^{(1/2)}*c*e)/f^2)^{(1/2)}*\ln(((b*f-c*e+(-4*d*f+e^2)^{(1/2)}*c)*(x-1/2*(-e+(-4*d \\ & *f+e^2)^{(1/2)})/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4 \\ & *d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2)^{(1/2)}*((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+ \\ & e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2) \\ &)^{(1/2)})/f)^2*c+4*(b*f-c*e+(-4*d*f+e^2)^{(1/2)}*c)*(x-1/2*(-e+(-4*d*f+e^2)^{(1 \\ & /2)})/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2) \\ &)^{(1/2)}*c*e)/f^2)^{(1/2)}/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f))*e \end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details)Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{cx^2 + bx + a} (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)`

[Out] `int(x/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+bx+cx^2} (d+ex+fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)`

[Out] `Integral(x/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)`

Q[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 983

Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c)/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e - b*f, 0] && PosQ[b^2 - 4*a*c]

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2} (d+ex+fx^2)} dx = \frac{(2f) \int \frac{1}{(e-\sqrt{e^2-4df}+2fx)\sqrt{a+bx+cx^2}} dx}{\sqrt{e^2-4df}} - \frac{(2f) \int \frac{1}{(e+\sqrt{e^2-4df}+2fx)\sqrt{a+bx+cx^2}} dx}{\sqrt{e^2-4df}}$$

$$= \frac{(4f) \text{Subst} \left(\int \frac{1}{16af^2-8bf(e-\sqrt{e^2-4df})+4c(e-\sqrt{e^2-4df})^2-x^2} dx, x, \frac{4af-b(e-\sqrt{e^2-4df})}{\sqrt{e^2-4df}} \right)}{\sqrt{e^2-4df}}$$

$$= \frac{\sqrt{2} f \tanh^{-1} \left(\frac{4af-b(e-\sqrt{e^2-4df})+2(bf-c(e-\sqrt{e^2-4df}))x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}} \sqrt{a+bx+cx^2}} \right)}{\sqrt{e^2-4df} \sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}} + \frac{\sqrt{2} f \tanh^{-1} \left(\frac{4af+b(e+\sqrt{e^2-4df})-2cx(\sqrt{e^2-4df}-e)}{2\sqrt{2}\sqrt{a+x(b+cx)} \sqrt{f(2af+b\sqrt{e^2-4df}+b(-e))+c(-e\sqrt{e^2-4df}-2df+e^2)}} \right)}{\sqrt{e^2-4df} \sqrt{f(2af+b\sqrt{e^2-4df}+b(-e))+c(-e\sqrt{e^2-4df}-2df+e^2)}}$$

Mathematica [A] time = 0.79, size = 376, normalized size = 1.01

$$\sqrt{2} f \frac{\left(\frac{\tanh^{-1} \left(\frac{4af-b(\sqrt{e^2-4df}+e-2fx)-2cx(\sqrt{e^2-4df}+e)}{2\sqrt{2}\sqrt{a+x(b+cx)} \sqrt{f(2af-b(\sqrt{e^2-4df}+e))+c(e\sqrt{e^2-4df}-2df+e^2)}} \right)}{\sqrt{f(2af-b(\sqrt{e^2-4df}+e))+c(e\sqrt{e^2-4df}-2df+e^2)}} \right) - \frac{\tanh^{-1} \left(\frac{4af+b(\sqrt{e^2-4df}-e+2fx)+2cx(\sqrt{e^2-4df}-e)}{2\sqrt{2}\sqrt{a+x(b+cx)} \sqrt{f(2af+b\sqrt{e^2-4df}+b(-e))+c(-e\sqrt{e^2-4df}-2df+e^2)}} \right)}{\sqrt{f(2af+b\sqrt{e^2-4df}+b(-e))+c(-e\sqrt{e^2-4df}-2df+e^2)}}}{\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] (Sqrt[2]*f*(ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])*Sqrt[a + x*(b + c*x)])/Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))] - ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])])*Sqrt[a + x*(b + c*x)])/Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]))/Sqrt[e^2 - 4*d*f]
```

fricas [B] time = 10.30, size = 11287, normalized size = 30.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f + (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*log((2*(b^2*d - a*b*e)*f^2 + sqrt(2)*(c^2*d*e^3 - 4*a*b*d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*b*e^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^2 + a*c*e^3)*f - (c^3*d^3*e^3 - b*c^2*d^2*e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^2 - a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3*e^3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*a*b^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^4 - a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*d^3*e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3*a*b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^3*d^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e^4 - a^2*c*e^5 - (3*b^2*c - 5*a*c^2)*d^2*e^3)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a
```

$$\begin{aligned}
& ^2b^2 + 6a^3c) * d * e^2) * f^3 - (8 * (b^2 * c^2 - 2 * a * c^3) * d^4 - 8 * (b^3 * c - a * b * \\
& c^2) * d^3 * e - (b^4 - 20 * a * b^2 * c + 22 * a^2 * c^2) * d^2 * e^2 + 2 * (a * b^3 - 5 * a^2 * b * c \\
&) * d * e^3 - (a^2 * b^2 + 2 * a^3 * c) * e^4) * f^2 - 2 * (2 * c^4 * d^5 - 4 * b * c^3 * d^4 * e + a^2 \\
& * b * c * e^5 + (b^2 * c^2 + 6 * a * c^3) * d^3 * e^2 + (b^3 * c - 5 * a * b * c^2) * d^2 * e^3 - 2 * (a \\
& * b^2 * c - 2 * a^2 * c^2) * d * e^4) * f) * \sqrt{c * x^2 + b * x + a} * \sqrt{(c * e^2 + 2 * a * f^2 \\
& - (2 * c * d + b * e) * f + (c^2 * d^2 * e^2 - b * c * d * e^3 + a * c * e^4 - 4 * a^2 * d * f^3 + (4 * \\
& a * b * d * e + a^2 * e^2 - 4 * (b^2 - 2 * a * c) * d^2) * f^2 - (4 * c^2 * d^3 - 4 * b * c * d^2 * e + a \\
& * b * e^3 - (b^2 - 6 * a * c) * d * e^2) * f) * \sqrt{(c^2 * e^2 - 2 * b * c * e * f + b^2 * f^2) / (c^4 * \\
& d^4 * e^2 - 2 * b * c^3 * d^3 * e^3 - 2 * a * b * c^2 * d * e^5 + a^2 * c^2 * e^6 - 4 * a^4 * d * f^5 + (\\
& b^2 * c^2 + 2 * a * c^3) * d^2 * e^4 + (8 * a^3 * b * d * e + a^4 * e^2 - 8 * (a^2 * b^2 - 2 * a^3 * c) \\
& * d^2) * f^4 - 2 * (a^3 * b * e^3 + 2 * (b^4 - 4 * a * b^2 * c + 6 * a^2 * c^2) * d^3 - 4 * (a * b^3 - \\
& a^2 * b * c) * d^2 * e + (a^2 * b^2 + 6 * a^3 * c) * d * e^2) * f^3 - (8 * (b^2 * c^2 - 2 * a * c^3) * d \\
& ^4 - 8 * (b^3 * c - a * b * c^2) * d^3 * e - (b^4 - 20 * a * b^2 * c + 22 * a^2 * c^2) * d^2 * e^2 + \\
& 2 * (a * b^3 - 5 * a^2 * b * c) * d * e^3 - (a^2 * b^2 + 2 * a^3 * c) * e^4) * f^2 - 2 * (2 * c^4 * d^5 - \\
& 4 * b * c^3 * d^4 * e + a^2 * b * c * e^5 + (b^2 * c^2 + 6 * a * c^3) * d^3 * e^2 + (b^3 * c - 5 * a * b \\
& * c^2) * d^2 * e^3 - 2 * (a * b^2 * c - 2 * a^2 * c^2) * d * e^4) * f) * \sqrt{(c^2 * d^2 * e^2 - b * c * d * e^3 \\
& + a * c * e^4 - 4 * a^2 * d * f^3 + (4 * a * b * d * e + a^2 * e^2 - 4 * (b^2 - 2 * a * c) * d^2) * f^2 \\
& - (4 * c^2 * d^3 - 4 * b * c * d^2 * e + a * b * e^3 - (b^2 - 6 * a * c) * d * e^2) * f) - 2 * (b * c * d \\
& * e - a * c * e^2) * f + ((4 * b * c * d - b^2 * e) * f^2 - (4 * c^2 * d * e - b * c * e^2) * f) * x - (8 * \\
& a^3 * d * f^4 - 2 * (4 * a^2 * b * d * e + a^3 * e^2 - 4 * (a * b^2 - 2 * a^2 * c) * d^2) * f^3 + 2 * (4 * \\
& a * c^2 * d^3 - 4 * a * b * c * d^2 * e + a^2 * b * e^3 - (a * b^2 - 6 * a^2 * c) * d * e^2) * f^2 - 2 * (a \\
& * c^2 * d^2 * e^2 - a * b * c * d * e^3 + a^2 * c * e^4) * f + (4 * a^2 * b * d * f^4 - (4 * a * b^2 * d * e + \\
& a^2 * b * e^2 - 4 * (b^3 - 2 * a * b * c) * d^2) * f^3 + (4 * b * c^2 * d^3 - 4 * b^2 * c * d^2 * e + a * \\
& b^2 * e^3 - (b^3 - 6 * a * b * c) * d * e^2) * f^2 - (b * c^2 * d^2 * e^2 - b^2 * c * d * e^3 + a * b * c \\
& * e^4) * f) * x) * \sqrt{(c^2 * e^2 - 2 * b * c * e * f + b^2 * f^2) / (c^4 * d^4 * e^2 - 2 * b * c^3 * d^3 \\
& * e^3 - 2 * a * b * c^2 * d * e^5 + a^2 * c^2 * e^6 - 4 * a^4 * d * f^5 + (b^2 * c^2 + 2 * a * c^3) * d^2 \\
& * e^4 + (8 * a^3 * b * d * e + a^4 * e^2 - 8 * (a^2 * b^2 - 2 * a^3 * c) * d^2) * f^4 - 2 * (a^3 * b * \\
& e^3 + 2 * (b^4 - 4 * a * b^2 * c + 6 * a^2 * c^2) * d^3 - 4 * (a * b^3 - a^2 * b * c) * d^2 * e + (a^2 * \\
& b^2 + 6 * a^3 * c) * d * e^2) * f^3 - (8 * (b^2 * c^2 - 2 * a * c^3) * d^4 - 8 * (b^3 * c - a * b * c \\
& ^2) * d^3 * e - (b^4 - 20 * a * b^2 * c + 22 * a^2 * c^2) * d^2 * e^2 + 2 * (a * b^3 - 5 * a^2 * b * c) \\
& * d * e^3 - (a^2 * b^2 + 2 * a^3 * c) * e^4) * f^2 - 2 * (2 * c^4 * d^5 - 4 * b * c^3 * d^4 * e + a^2 * \\
& b * c * e^5 + (b^2 * c^2 + 6 * a * c^3) * d^3 * e^2 + (b^3 * c - 5 * a * b * c^2) * d^2 * e^3 - 2 * (a * \\
& b^2 * c - 2 * a^2 * c^2) * d * e^4) * f) * x - 1/4 * \sqrt{2} * \sqrt{(c * e^2 + 2 * a * f^2 - (2 * \\
& c * d + b * e) * f + (c^2 * d^2 * e^2 - b * c * d * e^3 + a * c * e^4 - 4 * a^2 * d * f^3 + (4 * a * b * d * \\
& e + a^2 * e^2 - 4 * (b^2 - 2 * a * c) * d^2) * f^2 - (4 * c^2 * d^3 - 4 * b * c * d^2 * e + a * b * e^3 \\
& - (b^2 - 6 * a * c) * d * e^2) * f) * \sqrt{(c^2 * e^2 - 2 * b * c * e * f + b^2 * f^2) / (c^4 * d^4 * e^2 \\
& - 2 * b * c^3 * d^3 * e^3 - 2 * a * b * c^2 * d * e^5 + a^2 * c^2 * e^6 - 4 * a^4 * d * f^5 + (b^2 * c^2 \\
& + 2 * a * c^3) * d^2 * e^4 + (8 * a^3 * b * d * e + a^4 * e^2 - 8 * (a^2 * b^2 - 2 * a^3 * c) * d^2) * \\
& f^4 - 2 * (a^3 * b * e^3 + 2 * (b^4 - 4 * a * b^2 * c + 6 * a^2 * c^2) * d^3 - 4 * (a * b^3 - a^2 * b * \\
& c) * d^2 * e + (a^2 * b^2 + 6 * a^3 * c) * d * e^2) * f^3 - (8 * (b^2 * c^2 - 2 * a * c^3) * d^4 - 8 \\
& * (b^3 * c - a * b * c^2) * d^3 * e - (b^4 - 20 * a * b^2 * c + 22 * a^2 * c^2) * d^2 * e^2 + 2 * (a * b \\
& ^3 - 5 * a^2 * b * c) * d * e^3 - (a^2 * b^2 + 2 * a^3 * c) * e^4) * f^2 - 2 * (2 * c^4 * d^5 - 4 * b * c \\
& ^3 * d^4 * e + a^2 * b * c * e^5 + (b^2 * c^2 + 6 * a * c^3) * d^3 * e^2 + (b^3 * c - 5 * a * b * c^2) * \\
& d^2 * e^3 - 2 * (a * b^2 * c - 2 * a^2 * c^2) * d * e^4) * f) * \sqrt{(c^2 * d^2 * e^2 - b * c * d * e^3 + a * \\
& c * e^4 - 4 * a^2 * d * f^3 + (4 * a * b * d * e + a^2 * e^2 - 4 * (b^2 - 2 * a * c) * d^2) * f^2 - (4 *
\end{aligned}$$

$$\begin{aligned}
& c^2d^3 - 4b^*c*d^2e + a*b*e^3 - (b^2 - 6a*c)*d*e^2)*f)) * \log((2*(b^2*d - \\
& a*b*e)*f^2 - \sqrt{2}*(c^2*d*e^3 - 4a*b*d*f^3 + (4b^*c*d^2 + 4a*c*d*e + a* \\
& b*e^2)*f^2 - (4c^2*d^2e + b^*c*d*e^2 + a*c*e^3)*f - (c^3*d^3e^3 - b^*c^2*d \\
& ^2e^4 + a*c^2*d*e^5 + 4*(2a^2*b*d^2 - a^3*d*e)*f^4 + (2a^2*b*d*e^2 + a^3 \\
& *e^3 + 8*(b^3 - 2a*b*c)*d^3 - 4*(3a*b^2 - a^2*c)*d^2e)*f^3 + (8b^*c^2*d^ \\
& 4 - a^2*b*e^4 - 4*(3b^2*c - a*c^2)*d^3e - 2*(b^3 - 10a*b*c)*d^2e^2 + (3 \\
& *a*b^2 - 5a^2*c)*d*e^3)*f^2 - (4c^3*d^4e - 2b^*c^2*d^3e^2 + 4a*b^*c*d*e \\
& ^4 - a^2*c*e^5 - (3b^2*c - 5a*c^2)*d^2e^3)*f) * \sqrt{(c^2e^2 - 2b^*c*e*f \\
& + b^2*f^2)/(c^4*d^4e^2 - 2b^*c^3*d^3e^3 - 2a*b^*c^2*d*e^5 + a^2*c^2e^6 - \\
& 4a^4*d*f^5 + (b^2*c^2 + 2a*c^3)*d^2e^4 + (8a^3*b*d*e + a^4e^2 - 8*(a^ \\
& 2*b^2 - 2a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4a*b^2*c + 6a^2*c^2)* \\
& d^3 - 4*(a*b^3 - a^2*b*c)*d^2e + (a^2*b^2 + 6a^3*c)*d*e^2)*f^3 - (8*(b^2*c \\
& ^2 - 2a*c^3)*d^4 - 8*(b^3*c - a*b^*c^2)*d^3e - (b^4 - 20a*b^2*c + 22a^2 \\
& *c^2)*d^2e^2 + 2*(a*b^3 - 5a^2*b*c)*d*e^3 - (a^2*b^2 + 2a^3*c)*e^4)*f^2 \\
& - 2*(2c^4*d^5 - 4b^*c^3*d^4e + a^2*b^*c*e^5 + (b^2*c^2 + 6a*c^3)*d^3e^2 \\
& + (b^3*c - 5a*b^*c^2)*d^2e^3 - 2*(a*b^2*c - 2a^2*c^2)*d*e^4)*f)) * \sqrt{(c \\
& x^2 + b*x + a) * \sqrt{(c*e^2 + 2a*f^2 - (2*c*d + b*e)*f + (c^2*d^2e^2 - b^*c \\
& *d*e^3 + a*c*e^4 - 4a^2*d*f^3 + (4a*b*d*e + a^2e^2 - 4*(b^2 - 2a*c)*d^2 \\
&) * f^2 - (4c^2*d^3 - 4b^*c*d^2e + a*b*e^3 - (b^2 - 6a*c)*d*e^2)*f) * \sqrt{(\\
& c^2e^2 - 2b^*c*e*f + b^2f^2)/(c^4*d^4e^2 - 2b^*c^3*d^3e^3 - 2a*b^*c^2*d \\
& *e^5 + a^2*c^2e^6 - 4a^4*d*f^5 + (b^2*c^2 + 2a*c^3)*d^2e^4 + (8a^3*b*d \\
& *e + a^4e^2 - 8*(a^2*b^2 - 2a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4a \\
& *b^2*c + 6a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2e + (a^2*b^2 + 6a^3*c)*d \\
& *e^2)*f^3 - (8*(b^2*c^2 - 2a*c^3)*d^4 - 8*(b^3*c - a*b^*c^2)*d^3e - (b^4 - \\
& 20a*b^2*c + 22a^2*c^2)*d^2e^2 + 2*(a*b^3 - 5a^2*b*c)*d*e^3 - (a^2*b^2 \\
& + 2a^3*c)*e^4)*f^2 - 2*(2c^4*d^5 - 4b^*c^3*d^4e + a^2*b^*c*e^5 + (b^2*c^2 \\
& + 6a*c^3)*d^3e^2 + (b^3*c - 5a*b^*c^2)*d^2e^3 - 2*(a*b^2*c - 2a^2*c^2) \\
& *d*e^4)*f)))/(c^2*d^2e^2 - b^*c*d*e^3 + a*c*e^4 - 4a^2*d*f^3 + (4a*b*d*e \\
& + a^2e^2 - 4*(b^2 - 2a*c)*d^2)*f^2 - (4c^2*d^3 - 4b^*c*d^2e + a*b*e^3 - \\
& (b^2 - 6a*c)*d*e^2)*f) - 2*(b^*c*d*e - a*c*e^2)*f + ((4b^*c*d - b^2e)*f^ \\
& 2 - (4c^2*d*e - b^*c*e^2)*f)*x - (8a^3*d*f^4 - 2*(4a^2*b*d*e + a^3e^2 - \\
& 4*(a*b^2 - 2a^2*c)*d^2)*f^3 + 2*(4a^*c^2*d^3 - 4a*b^*c*d^2e + a^2*b*e^3 - \\
& (a*b^2 - 6a^2*c)*d*e^2)*f^2 - 2*(a*c^2*d^2e^2 - a*b^*c*d*e^3 + a^2*c*e^4) \\
& *f + (4a^2*b*d*f^4 - (4a*b^2*d*e + a^2*b*e^2 - 4*(b^3 - 2a*b*c)*d^2)*f^3 \\
& + (4b^*c^2*d^3 - 4b^2*c*d^2e + a*b^2e^3 - (b^3 - 6a*b*c)*d*e^2)*f^2 - \\
& (b^*c^2*d^2e^2 - b^2*c*d*e^3 + a*b^*c*e^4)*f)*x) * \sqrt{(c^2e^2 - 2b^*c*e*f + \\
& b^2f^2)/(c^4*d^4e^2 - 2b^*c^3*d^3e^3 - 2a*b^*c^2*d*e^5 + a^2*c^2e^6 - \\
& 4a^4*d*f^5 + (b^2*c^2 + 2a*c^3)*d^2e^4 + (8a^3*b*d*e + a^4e^2 - 8*(a^2 \\
& *b^2 - 2a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4a*b^2*c + 6a^2*c^2)*d \\
& ^3 - 4*(a*b^3 - a^2*b*c)*d^2e + (a^2*b^2 + 6a^3*c)*d*e^2)*f^3 - (8*(b^2*c \\
& ^2 - 2a*c^3)*d^4 - 8*(b^3*c - a*b^*c^2)*d^3e - (b^4 - 20a*b^2*c + 22a^2* \\
& c^2)*d^2e^2 + 2*(a*b^3 - 5a^2*b*c)*d*e^3 - (a^2*b^2 + 2a^3*c)*e^4)*f^2 - \\
& 2*(2c^4*d^5 - 4b^*c^3*d^4e + a^2*b^*c*e^5 + (b^2*c^2 + 6a*c^3)*d^3e^2 + \\
& (b^3*c - 5a*b^*c^2)*d^2e^3 - 2*(a*b^2*c - 2a^2*c^2)*d*e^4)*f)))/x + 1/4 \\
& * \sqrt{2} * \sqrt{(c*e^2 + 2a*f^2 - (2*c*d + b*e)*f - (c^2*d^2e^2 - b^*c*d*e^3
\end{aligned}$$

$$\begin{aligned}
& + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 \\
& - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*\sqrt{(c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f))*\log((2*(b^2*d - a*b*e)*f^2 + \sqrt{2}*(c^2*d*e^3 - 4*a*b*d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*b*e^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^2 + a*c*e^3)*f + (c^3*d^3*e^3 - b*c^2*d^2*e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^2 - a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3*e^3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*a*b^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^4 - a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*d^3*e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3*a*b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^3*d^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e^4 - a^2*c*e^5 - (3*b^2*c - 5*a*c^2)*d^2*e^3)*f)*\sqrt{(c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*\sqrt{c*x^2 + b*x + a}*\sqrt{(c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*\sqrt{(c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d*e - a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^2 - (4*c^2*d*e - b*c*e^2)*f)*x + (8*a^3*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 - 4*(a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*a*c^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - (a*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a*c^2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)*f + (4*a^2*b*d*f^4 - (4*a*b^2*d*e + a^
\end{aligned}$$

$$\begin{aligned}
& 2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3 + (4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*b^2 \\
& *e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 - (b*c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c*e^4) \\
& *f)*x)*\sqrt{(c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d^2*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f))/x) - 1/4*\sqrt{2)*\sqrt{(c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*\sqrt{(c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d^2*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f))/((c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f))*\log((2*(b^2*d - a*b*e)*f^2 - \sqrt{2)*(c^2*d*e^3 - 4*a*b*d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*b*e^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^2 + a*c*e^3)*f + (c^3*d^3*e^3 - b*c^2*d^2*e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^2 - a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3*e^3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*a*b^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^4 - a^2*b*e^4 - 4*(3*b^2*c - 5*a*c^2)*d^3*e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3*a*b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^3*d^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e^4 - a^2*c*e^5 - (3*b^2*c - 5*a*c^2)*d^2*e^3)*f)*\sqrt{(c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d^2*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f))/\sqrt{c*x^2 + b*x + a)*\sqrt{(c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*\sqrt{(c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d^2*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f))/\sqrt{c*x^2 + b*x + a)}
\end{aligned}$$

$$\frac{2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f) - 2*(b*c*d*e - a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^2 - (4*c^2*d*e - b*c*e^2)*f)*x + (8*a^3*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 - 4*(a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*a*c^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - (a*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a*c^2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)*f + (4*a^2*b*d*f^4 - (4*a*b^2*d*e + a^2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3 + (4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*b^2*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 - (b*c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c*e^4)*f)*x)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2))/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x$$

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.01, size = 761, normalized size = 2.03

$$\sqrt{2} \ln \left(\frac{\left(\frac{bf-ce-\sqrt{-4df+e^2}}{f} c \right) \left(x + \frac{e+\sqrt{-4df+e^2}}{2f} \right) + \frac{2af^2-bef-2cdf+ce^2-\sqrt{-4df+e^2}bf+\sqrt{-4df+e^2}ce}{f^2} + \frac{\sqrt{2} \sqrt{\frac{2af^2-bef-2cdf+ce^2-\sqrt{-4df+e^2}bf+\sqrt{-4df+e^2}ce}{f^2}}}{x + \frac{e+\sqrt{-4df+e^2}}{2f}}}{\sqrt{-4df+e^2} \sqrt{\frac{2af^2-bef-2cdf+ce^2-\sqrt{-4df+e^2}bf+\sqrt{-4df+e^2}ce}{f^2}}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)`

[Out]
$$\frac{1/(-4*d*f+e^2)^{(1/2)}*2^{(1/2)} / ((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)} * \ln(((b*f-c*e-(-4*d*f+e^2)^{(1/2)}*c)*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(1/2)}*((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c+4*(b*f-c*e-(-4*d*f+e^2)^{(1/2)}*c)*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}}{(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)}-1/(-4*d*f+e^2)^{(1/2)}*2^{(1/2)} / ((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)} * \ln(((b*f-c*e+(-4*d*f+e^2)^{(1/2)}*c)*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(1/2)}*((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c+4*(b*f-c*e+(-4*d*f+e^2)^{(1/2)}*c)*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}}{(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details)Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a} (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)`

[Out] `int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + bx + cx^2} (d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d), x)

[Out] Integral(1/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)

$$3.118 \quad \int \frac{1}{x \sqrt{a+bx+cx^2} (d+ex+fx^2)} dx$$

Optimal. Leaf size=451

$$\frac{f(\sqrt{e^2-4df}+e) \tanh^{-1}\left(\frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} - \frac{f(e-\sqrt{e^2-4df}) \tanh^{-1}\left(\frac{4af}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)+bef-2cdf+ce^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)+bef-2cdf+ce^2}}$$

[Out] $-\operatorname{arctanh}\left(\frac{1/2*(b*x+2*a)/a^{1/2}/(c*x^2+b*x+a)^{1/2}}{d/a^{1/2}+1/2*f*\operatorname{arctanh}\left(\frac{1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^{1/2}))-b*(e-(-4*d*f+e^2)^{1/2}))}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}\right) + \operatorname{arctanh}\left(\frac{1/4*(4*a*f-b*(e+(-4*d*f+e^2)^{1/2}))+2*x*(b*f-c*(e+(-4*d*f+e^2)^{1/2}))}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)+bef-2cdf+ce^2}}\right) - \operatorname{arctanh}\left(\frac{1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^{1/2}))-b*(e-(-4*d*f+e^2)^{1/2}))}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right) + \operatorname{arctanh}\left(\frac{1/4*(4*a*f-b*(e+(-4*d*f+e^2)^{1/2}))+2*x*(b*f-c*(e+(-4*d*f+e^2)^{1/2}))}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)+bef-2cdf+ce^2}}\right)$

Rubi [A] time = 2.63, antiderivative size = 451, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6728, 724, 206, 1032}

$$\frac{f(\sqrt{e^2-4df}+e) \tanh^{-1}\left(\frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} - \frac{f(e-\sqrt{e^2-4df}) \tanh^{-1}\left(\frac{4af}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)+bef-2cdf+ce^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)+bef-2cdf+ce^2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]

[Out] $-\frac{\operatorname{ArcTanh}\left[\frac{(2*a + b*x)/(2*\sqrt{a}*\sqrt{a + b*x + c*x^2})}{\sqrt{a}*d}\right] + (f*(e + \sqrt{e^2 - 4*d*f})*\operatorname{ArcTanh}\left[\frac{(4*a*f - b*(e - \sqrt{e^2 - 4*d*f}) + 2*(b*f - c*(e - \sqrt{e^2 - 4*d*f}))*x}{2*\sqrt{2}*\sqrt{c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*\sqrt{e^2 - 4*d*f}*\sqrt{a + b*x + c*x^2}}\right])}{\sqrt{2}*d*\sqrt{e^2 - 4*d*f}*\sqrt{c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*\sqrt{e^2 - 4*d*f}*\sqrt{a + b*x + c*x^2}}} - \frac{(f*(e - \sqrt{e^2 - 4*d*f})*\operatorname{ArcTanh}\left[\frac{(4*a*f - b*(e + \sqrt{e^2 - 4*d*f}) + 2*(b*f - c*(e + \sqrt{e^2 - 4*d*f}))*x}{2*\sqrt{2}*\sqrt{c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*\sqrt{e^2 - 4*d*f}*\sqrt{a + b*x + c*x^2}}\right])}{\sqrt{2}*d*\sqrt{e^2 - 4*d*f}*\sqrt{c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*\sqrt{e^2 - 4*d*f}*\sqrt{a + b*x + c*x^2}}}$

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Sym
bol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (
e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x_Symbol] := With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx &= \int \left(\frac{1}{dx\sqrt{a+bx+cx^2}} + \frac{-e-fx}{d\sqrt{a+bx+cx^2}(d+ex+fx^2)} \right) dx \\
&= \frac{\int \frac{1}{x\sqrt{a+bx+cx^2}} dx}{d} + \frac{\int \frac{-e-fx}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx}{d} \\
&= \frac{2 \operatorname{Subst} \left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}} \right)}{d} - \frac{\left(f \left(1 - \frac{e}{\sqrt{e^2-4df}} \right) \right) \int \frac{1}{(e+\sqrt{e^2-4df}+2fx)^2} dx}{d} \\
&= -\frac{\tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}} \right)}{\sqrt{a}d} + \frac{\left(2f \left(1 - \frac{e}{\sqrt{e^2-4df}} \right) \right) \operatorname{Subst} \left(\int \frac{1}{16af^2-8bf(e+\sqrt{e^2-4df}+2fx)^2} dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}} \right)}{\sqrt{a}d} \\
&= -\frac{\tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}} \right)}{\sqrt{a}d} + \frac{f \left(1 + \frac{e}{\sqrt{e^2-4df}} \right) \tanh^{-1} \left(\frac{4af-b(e-\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2}} \right)}{\sqrt{2}d\sqrt{ce^2-2cdf-bef+2af^2}}
\end{aligned}$$

Mathematica [A] time = 2.41, size = 450, normalized size = 1.00

$$\frac{\sqrt{2}f \left(\frac{\left(\sqrt{e^2-4df}-e \right) \tanh^{-1} \left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}} \right)}{\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}} \right) + \frac{\left(\sqrt{e^2-4df}+e \right) \tanh^{-1} \left(\frac{4af+b\left(\sqrt{e^2-4df}-e+2fx\right)+2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af+b\sqrt{e^2-4df}+b(-e)\right)}} \right)}{\sqrt{f\left(2af+b\sqrt{e^2-4df}+b(-e)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}}{\sqrt{e^2-4df}}$$

$2d$

Antiderivative was successfully verified.

[In] Integrate[1/(x*sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]

[Out] ((-2*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/Sqrt[a] + (Sqrt[2]*f*(((-e + Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])]*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)]))/Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))] + ((e + Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])]*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4

$*d*f))]*\text{Sqrt}[a + x*(b + c*x)]])/\text{Sqrt}[f*(-(b*e) + 2*a*f + b*\text{Sqrt}[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*\text{Sqrt}[e^2 - 4*d*f])])]/\text{Sqrt}[e^2 - 4*d*f]/(2*d)$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.02, size = 859, normalized size = 1.90

$$2\sqrt{2} f \ln \left(\frac{\left(bf - ce - \sqrt{-4df + e^2} c \right) \left(x + \frac{e + \sqrt{-4df + e^2}}{2f} \right) + \frac{2af^2 - bef - 2cdf + ce^2 - \sqrt{-4df + e^2} bf + \sqrt{-4df + e^2} ce}{f^2} + \sqrt{2} \sqrt{\frac{2af^2 - bef - 2cdf + ce^2 - \sqrt{-4df + e^2} bf + \sqrt{-4df + e^2} ce}{f^2}}}{x + \frac{e + \sqrt{-4df + e^2}}{2f}} \right)$$

$$\frac{(e + \sqrt{-4df + e^2}) \sqrt{-4df + e^2} \sqrt{2af^2 - bef - 2cdf + ce^2 - \sqrt{-4df + e^2} bf + \sqrt{-4df + e^2} ce}}{(e + \sqrt{-4df + e^2}) \sqrt{-4df + e^2} \sqrt{2af^2 - bef - 2cdf + ce^2 - \sqrt{-4df + e^2} bf + \sqrt{-4df + e^2} ce}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)

[Out] $-2*f/(e+(-4*d*f+e^2)^{(1/2)})/(-4*d*f+e^2)^{(1/2)}*2^{(1/2)}/((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*\ln(((b*f-c*e-(-4*d*f+e^2)^{(1/2)}*c)*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(1/2)}*((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)^2*c+4*(b*f-c*e-(-4*d*f+e^2)^{(1/2)}*c)*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f$

$$+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2))/(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f))-2*f/(-e+(-4*d*f+e^2)^{(1/2)})/(-4*d*f+e^2)^{(1/2)}*2^{(1/2)}/((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*\ln(((b*f-c*e+(-4*d*f+e^2)^{(1/2)}*c)*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(1/2)}*((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f))^2*c+4*(b*f-c*e+(-4*d*f+e^2)^{(1/2)}*c)*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)})/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f))+4*f/(-e+(-4*d*f+e^2)^{(1/2)})/(e+(-4*d*f+e^2)^{(1/2)})/a^{(1/2)}*\ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)}*a^{(1/2)})/x)$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a} (fx^2 + ex + d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x \sqrt{cx^2 + bx + a} (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)

[Out] int(1/(x*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sqrt{a + bx + cx^2} (d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)

[Out] Integral(1/(x*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)

$$3.119 \quad \int \frac{1}{x^2 \sqrt{a+bx+cx^2} (d+ex+fx^2)} dx$$

Optimal. Leaf size=543

$$\frac{b \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right) f(e\sqrt{e^2-4df}-2df+e^2) \tanh^{-1}\left(\frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right) f}{2a^{3/2}d \sqrt{2}d^2\sqrt{e^2-4df}\sqrt{f(2af-b(e-\sqrt{e^2-4df}))+c(-e\sqrt{e^2-4df}-2df+e^2)}} +$$

[Out] $\frac{1}{2}b \operatorname{arctanh}\left(\frac{1}{2}\frac{(b*x+2*a)}{a}\right) / (c*x^2+b*x+a)^{1/2} / a^{3/2} / d + e \operatorname{arctan}h\left(\frac{1}{2}\frac{(b*x+2*a)}{a}\right) / (c*x^2+b*x+a)^{1/2} / d^2 / a^{1/2} - (c*x^2+b*x+a)^{1/2} / a / d / x + \frac{1}{2}f \operatorname{arctanh}\left(\frac{1}{4}\frac{(4*a*f-b*(e+(-4*d*f+e^2)^{1/2}))+2*x*(b*f-c*(e+(-4*d*f+e^2)^{1/2}))}{c*x^2+b*x+a}\right) * 2^{1/2} / (c*x^2+b*x+a)^{1/2} / (c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^{1/2})^{1/2} * (-(-4*d*f+e^2)^{1/2} * e^{-2*d*f+e^2}) / d^2 * 2^{1/2} / (-4*d*f+e^2)^{1/2} / (c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^{1/2})^{1/2} - \frac{1}{2}f \operatorname{arctanh}\left(\frac{1}{4}\frac{(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^{1/2}))-b*(e-(-4*d*f+e^2)^{1/2}))}{c*x^2+b*x+a}\right) * 2^{1/2} / (c*x^2+b*x+a)^{1/2} / (c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^{1/2})^{1/2} * ((-4*d*f+e^2)^{1/2} * e^{-2*d*f+e^2}) / d^2 * 2^{1/2} / (-4*d*f+e^2)^{1/2} / (f*(2*a*f-b*(e-(-4*d*f+e^2)^{1/2}))) + ((-4*d*f+e^2)^{1/2} * e^{-2*d*f+e^2} * c)^{1/2}$

Rubi [A] time = 4.59, antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6728, 730, 724, 206, 1032}

$$\frac{b \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right) f(e\sqrt{e^2-4df}-2df+e^2) \tanh^{-1}\left(\frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right) f}{2a^{3/2}d \sqrt{2}d^2\sqrt{e^2-4df}\sqrt{f(2af-b(e-\sqrt{e^2-4df}))+c(-e\sqrt{e^2-4df}-2df+e^2)}} +$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]

[Out] $-(\sqrt{a+b*x+c*x^2}/(a*d*x)) + (b*\operatorname{ArcTanh}[(2*a+b*x)/(2*\sqrt{a}*\sqrt{a+b*x+c*x^2}])/(2*a^{3/2}*d) + (e*\operatorname{ArcTanh}[(2*a+b*x)/(2*\sqrt{a}*\sqrt{a+b*x+c*x^2}])/(\sqrt{a}*d^2) - (f*(e^2-2*d*f+e*\sqrt{e^2-4*d*f})*\operatorname{ArcTanh}[(4*a*f-b*(e-\sqrt{e^2-4*d*f}))+2*(b*f-c*(e-\sqrt{e^2-4*d*f}))]*x)/(2*\sqrt{2}*\sqrt{c*e^2-2*c*d*f-b*e*f+2*a*f^2-(c*e-b*f)*\sqrt{e^2-4*d*f}}*\sqrt{a+b*x+c*x^2}))/(\sqrt{2}*d^2*\sqrt{e^2-4*d*f}*\sqrt{c*(e^2-2*d*f-e*\sqrt{e^2-4*d*f}))+f*(2*a*f-b*(e-\sqrt{e^2-4*d*f}))}) + (f*(e^2-2*d*f-e*\sqrt{e^2-4*d*f})*\operatorname{ArcTanh}[(4*a*f-b*(e+\sqrt{e^2-4*d*f}))+2*(b*f-c*(e+\sqrt{e^2-4*d*f}))]*x)/(2*\sqrt{2}*\sqrt{c*$

$$\frac{e^2 - 2cd + b^2 + 2af + (c - b)\sqrt{e^2 - 4d}}{\sqrt{2d}\sqrt{e^2 - 4d}\sqrt{c^2 - 2cd + b^2 + 2af + (c - b)\sqrt{e^2 - 4d}}}$$

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 1032

```
Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(2*n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 \sqrt{a+bx+cx^2} (d+ex+fx^2)} dx &= \int \left(\frac{1}{dx^2 \sqrt{a+bx+cx^2}} - \frac{e}{d^2 x \sqrt{a+bx+cx^2}} + \frac{e^2-df+efx}{d^2 \sqrt{a+bx+cx^2} (d+ex+fx^2)} \right) dx \\
&= \frac{\int \frac{e^2-df+efx}{\sqrt{a+bx+cx^2} (d+ex+fx^2)} dx}{d^2} + \frac{\int \frac{1}{x^2 \sqrt{a+bx+cx^2}} dx}{d} - \frac{e \int \frac{1}{x \sqrt{a+bx+cx^2}} dx}{d^2} \\
&= -\frac{\sqrt{a+bx+cx^2}}{adx} - \frac{b \int \frac{1}{x \sqrt{a+bx+cx^2}} dx}{2ad} + \frac{(2e) \text{Subst} \left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}} \right)}{d^2} \\
&= -\frac{\sqrt{a+bx+cx^2}}{adx} + \frac{e \tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a} \sqrt{a+bx+cx^2}} \right)}{\sqrt{a} d^2} + \frac{b \text{Subst} \left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}} \right)}{ad} \\
&= -\frac{\sqrt{a+bx+cx^2}}{adx} + \frac{b \tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a} \sqrt{a+bx+cx^2}} \right)}{2a^{3/2}d} + \frac{e \tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a} \sqrt{a+bx+cx^2}} \right)}{\sqrt{a} d^2}
\end{aligned}$$

Mathematica [A] time = 1.41, size = 533, normalized size = 0.98

$$\frac{bd \tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a} \sqrt{a+bx+cx^2}} \right)}{a^{3/2}} + \frac{\sqrt{2} f (e \sqrt{e^2-4df} + 2df - e^2) \tanh^{-1} \left(\frac{4af-b(\sqrt{e^2-4df}+e-2fx)-2cx(\sqrt{e^2-4df}+e)}{2\sqrt{2} \sqrt{a+x(b+cx)} \sqrt{f(2af-b(\sqrt{e^2-4df}+e))+c(e\sqrt{e^2-4df}-2df+e^2)}} \right)}{\sqrt{e^2-4df} \sqrt{f(2af-b(\sqrt{e^2-4df}+e))+c(e\sqrt{e^2-4df}-2df+e^2)}} + \frac{\sqrt{2} f \left(\frac{e^2-2df}{\sqrt{e^2-4df}} \right)}{2d^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]

[Out]
$$-\frac{1}{2} \left(\frac{(2d \sqrt{a+x(b+cx)})}{(a^3)^{3/2}} - \frac{(b d \text{ArcTanh}[(2a+bx)/(2\sqrt{a} \sqrt{a+x(b+cx)})])}{a^{3/2}} - \frac{(2e \text{ArcTanh}[(2a+bx)/(2\sqrt{a} \sqrt{a+x(b+cx)})])}{\sqrt{a}} + \frac{(\sqrt{2} f (-e^2 + 2d f + e \sqrt{e^2 - 4d f})) \text{ArcTanh}[(4a f - 2c(e + \sqrt{e^2 - 4d f})x - b(e + \sqrt{e^2 - 4d f} - 2f x)) / (2\sqrt{2} \sqrt{c(e^2 - 2d f + e \sqrt{e^2 - 4d f}) + f(2a f - b(e + \sqrt{e^2 - 4d f}))}) \sqrt{a+x(b+cx)}]}{\sqrt{e^2 - 4d f} \sqrt{c(e^2 - 2d f + e \sqrt{e^2 - 4d f}) + f(2a f - b(e + \sqrt{e^2 - 4d f}))}} \right) + \frac{(\sqrt{2} f (e + (e^2 - 2d f) / \sqrt{e^2 - 4d f})) \text{ArcTanh}[(4a f + 2c(-e + \sqrt{e^2 - 4d f})x + b(-e + \sqrt{e^2 - 4d f} + 2f x)) / (2\sqrt{2} \sqrt{c(e^2 - 2d f - e \sqrt{e^2 - 4d f}) + f(2a f + b(-e + \sqrt{e^2 - 4d f}))}) \sqrt{a+x(b+cx)}]}{\sqrt{e^2 - 4d f} \sqrt{c(e^2 - 2d f - e \sqrt{e^2 - 4d f}) + f(2a f + b(-e + \sqrt{e^2 - 4d f}))}} \right) + \frac{\sqrt{2} f \left(\frac{e^2 - 2d f}{\sqrt{e^2 - 4d f}} \right)}{2d^2}$$

+ Sqrt[e^2 - 4*d*f]))*Sqrt[a + x*(b + c*x)))]/Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f])))]/d^2

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.02, size = 983, normalized size = 1.81

$$\frac{16e f^2 \ln\left(\frac{bx+2a+2\sqrt{cx^2+bx+a}\sqrt{a}}{x}\right)}{(-e + \sqrt{-4df + e^2})^2 (e + \sqrt{-4df + e^2})^2 \sqrt{a}} + \frac{4\sqrt{2} f^2 \ln\left(\frac{(bf-ce-\sqrt{-4df+e^2}c)\left(x+\frac{e+\sqrt{-4df+e^2}}{2f}\right)}{f} + \frac{2af^2-bef-2cdf+ce^2-\sqrt{-4df+e^2}bf}{f^2}\right)}{(e + \sqrt{-4df + e^2})^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)

[Out] $4*f^2/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*\ln((b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))^2*c+4*(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c$

$$d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e/f^2)^{(1/2))/(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)+4*f/(-e+(-4*d*f+e^2)^{(1/2)})/(e+(-4*d*f+e^2)^{(1/2)})/a/x*(c*x^2+b*x+a)^{(1/2)}-2*f/(-e+(-4*d*f+e^2)^{(1/2)})/(e+(-4*d*f+e^2)^{(1/2)})*b/a^{(3/2)}*\ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)}*a^{(1/2)})/x)-4*f^2/(-e+(-4*d*f+e^2)^{(1/2)})^2/(-4*d*f+e^2)^{(1/2)}*2^{(1/2)}/((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*\ln(((b*f-c*e+(-4*d*f+e^2)^{(1/2)}*c)*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(1/2)}*((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c+4*(b*f-c*e+(-4*d*f+e^2)^{(1/2)}*c)*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)})/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)+16*f^2*e/(-e+(-4*d*f+e^2)^{(1/2)})^2/(e+(-4*d*f+e^2)^{(1/2)})^2/a^{(1/2)}*\ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)}*a^{(1/2)})/x)$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a} (fx^2 + ex + d)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d), x, algorithm="maxima")

[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2 \sqrt{cx^2 + bx + a} (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)

[Out] int(1/(x^2*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{a + bx + cx^2} (d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d), x)

[Out] Integral(1/(x**2*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)

$$3.120 \quad \int \frac{1}{x^3 \sqrt{a+bx+cx^2} (d+ex+fx^2)} dx$$

Optimal. Leaf size=679

$$\frac{(3b^2 - 4ac) \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d} - \frac{be \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d^2} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2dx} - \frac{(e^2 - df) \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^3}$$

[Out] $-1/8*(-4*a*c+3*b^2)*\operatorname{arctanh}(1/2*(b*x+2*a)/a^{(1/2)/(c*x^2+b*x+a)^{(1/2)})/a^{(5/2)}/d-1/2*b*e*\operatorname{arctanh}(1/2*(b*x+2*a)/a^{(1/2)/(c*x^2+b*x+a)^{(1/2)})/a^{(3/2)}/d^2-(-d*f+e^2)*\operatorname{arctanh}(1/2*(b*x+2*a)/a^{(1/2)/(c*x^2+b*x+a)^{(1/2)})/d^3/a^{(1/2)}-1/2*(c*x^2+b*x+a)^{(1/2)}/a/d/x^2+3/4*b*(c*x^2+b*x+a)^{(1/2)}/a^2/d/x+e*(c*x^2+b*x+a)^{(1/2)}/a/d^2/x+1/2*f*\operatorname{arctanh}(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^{(1/2)}))-b*(e-(-4*d*f+e^2)^{(1/2)}))*2^{(1/2)/(c*x^2+b*x+a)^{(1/2)})/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}*(2*e^3-4*d*e*f-(-d*f+e^2)*(e-(-4*d*f+e^2)^{(1/2)}))/d^3*2^{(1/2)/(-4*d*f+e^2)^{(1/2)})/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}-1/2*f*\operatorname{arctanh}(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^{(1/2)}))+2*x*(b*f-c*(e+(-4*d*f+e^2)^{(1/2)}))) * 2^{(1/2)/(c*x^2+b*x+a)^{(1/2)})/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}*(2*e^3-4*d*e*f-(-d*f+e^2)*(e+(-4*d*f+e^2)^{(1/2)}))/d^3*2^{(1/2)/(-4*d*f+e^2)^{(1/2)})/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}$

Rubi [A] time = 11.23, antiderivative size = 679, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {6728, 744, 806, 724, 206, 730, 1032}

$$\frac{(3b^2 - 4ac) \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d} - \frac{be \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d^2} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2dx} + \frac{f(-e^2 - df)(e - \sqrt{e^2 - 4df})}{\sqrt{2}d^3\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)), x]

[Out] $-\operatorname{Sqrt}[a + b*x + c*x^2]/(2*a*d*x^2) + (3*b*\operatorname{Sqrt}[a + b*x + c*x^2])/(4*a^2*d*x) + (e*\operatorname{Sqrt}[a + b*x + c*x^2])/(a*d^2*x) - ((3*b^2 - 4*a*c)*\operatorname{ArcTanh}[(2*a + b*x)/(2*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(8*a^{(5/2)*d} - (b*e*\operatorname{ArcTanh}[(2*a + b*x)/(2*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(2*a^{(3/2)*d^2} - ((e^2 - d*f)*\operatorname{ArcTanh}[(2*a + b*x)/(2*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(\operatorname{Sqrt}[a]*d^3) + (f*(2*e^3 - 4*d*e*f - (e^2 - d*f)*(e - \operatorname{Sqrt}[e^2 - 4*d*f]))*\operatorname{ArcTanh}[(4*a*f - b*(e$

$$- \sqrt{e^2 - 4df} + 2(bf - c(e - \sqrt{e^2 - 4df}))x / (2\sqrt{2} \sqrt{c^2e^2 - 2cd^2f - b^2e^2f + 2a^2f^2 - (ce - bf)\sqrt{e^2 - 4df}} \sqrt{a + bx + cx^2}) / (\sqrt{2} d^3 \sqrt{e^2 - 4df} \sqrt{c^2e^2 - 2cd^2f - b^2e^2f + 2a^2f^2 - (ce - bf)\sqrt{e^2 - 4df}}) - (f(2e^3 - 4d^2ef - (e^2 - df)(e + \sqrt{e^2 - 4df}))) \operatorname{ArcTanh}[(4af - b(e + \sqrt{e^2 - 4df})) + 2(bf - c(e + \sqrt{e^2 - 4df}))x] / (2\sqrt{2} \sqrt{c^2e^2 - 2cd^2f - b^2e^2f + 2a^2f^2 + (ce - bf)\sqrt{e^2 - 4df}} \sqrt{a + bx + cx^2}) / (\sqrt{2} d^3 \sqrt{e^2 - 4df} \sqrt{c^2e^2 - 2cd^2f - b^2e^2f + 2a^2f^2 + (ce - bf)\sqrt{e^2 - 4df}})$$

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 744

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x, x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c
```

```

_.)*(x_)^2)^(p_.), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
+ d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]

```

Rule 1032

```

Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]

```

Rule 6728

```

Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 \sqrt{a+bx+cx^2} (d+ex+fx^2)} dx &= \int \left(\frac{1}{dx^3 \sqrt{a+bx+cx^2}} - \frac{e}{d^2 x^2 \sqrt{a+bx+cx^2}} + \frac{e^2-df}{d^3 x \sqrt{a+bx+cx^2}} + \frac{1}{d^3 \sqrt{a+bx+cx^2}} \right) dx \\
&= \frac{\int \frac{-e(e^2-2df)-f(e^2-df)x}{\sqrt{a+bx+cx^2} (d+ex+fx^2)} dx}{d^3} + \frac{\int \frac{1}{x^3 \sqrt{a+bx+cx^2}} dx}{d} - \frac{e \int \frac{1}{x^2 \sqrt{a+bx+cx^2}} dx}{d^2} + \frac{\int \frac{1}{\sqrt{a+bx+cx^2}} dx}{d^3} \\
&= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} + \frac{e\sqrt{a+bx+cx^2}}{ad^2x} - \frac{\int \frac{\frac{3b}{2}+cx}{x^2 \sqrt{a+bx+cx^2}} dx}{2ad} + \frac{(be) \int \frac{1}{x \sqrt{a+bx+cx^2}} dx}{2ad^2} \\
&= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2dx} + \frac{e\sqrt{a+bx+cx^2}}{ad^2x} - \frac{(e^2-df) \operatorname{tanh}^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{2a^3} \\
&= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2dx} + \frac{e\sqrt{a+bx+cx^2}}{ad^2x} - \frac{be \operatorname{tanh}^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{2a^3} \\
&= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2dx} + \frac{e\sqrt{a+bx+cx^2}}{ad^2x} - \frac{(3b^2-4ac) \operatorname{tanh}^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{a^{5/2}x}
\end{aligned}$$

Mathematica [A] time = 2.11, size = 669, normalized size = 0.99

$$\frac{d^2 \left((4acx-3b^2x) \operatorname{tanh}^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right) + 6\sqrt{a}b\sqrt{a+x(b+cx)} \right)}{a^{5/2}x} - \frac{4bde \operatorname{tanh}^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{a^{3/2}} - \frac{4d^2\sqrt{a+x(b+cx)}}{ax^2} - \frac{8(e^2-df) \operatorname{tanh}^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]

[Out] ((-4*d^2*sqrt[a + x*(b + c*x)])/(a*x^2) + (8*d*e*sqrt[a + x*(b + c*x)])/(a*x) - (4*b*d*e*ArcTanh[(2*a + b*x)/(2*sqrt[a]*sqrt[a + x*(b + c*x)])])/a^(3/2) - (8*(e^2 - d*f)*ArcTanh[(2*a + b*x)/(2*sqrt[a]*sqrt[a + x*(b + c*x)])])/sqrt[a] + (d^2*(6*sqrt[a]*b*sqrt[a + x*(b + c*x)] + (-3*b^2*x + 4*a*c*x)*ArcTanh[(2*a + b*x)/(2*sqrt[a]*sqrt[a + x*(b + c*x)])]))/a^(5/2)*x) - (4*sqrt[a + x*(b + c*x)]/a^2)


```

rt[2]*f*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f] + d*f*Sqrt[e^2 - 4*d*f])*Arc
Tanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*
f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*
(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])]/(Sqrt[e^2 - 4*d*f]*Sqrt[
c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]
))]) + (4*Sqrt[2]*f*(e^2 - d*f + (e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*f])*ArcTa
nh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*
f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*
(-e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])]/Sqrt[c*(e^2 - 2*d*f - e
*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))])]/(8*d^3)

```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

[Out] Timed out

maple [B] time = 0.02, size = 1296, normalized size = 1.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```

[Out] -8*f^3/(e+(-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-b*e*f-
2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(
((b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2
-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2
*2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(
1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e-(-
4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c
*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/

```

$$2*(e+(-4*d*f+e^2)^{(1/2)})/f)+16*f^2*e/(-e+(-4*d*f+e^2)^{(1/2)})^2/(e+(-4*d*f+e^2)^{(1/2)})^2/a/x*(c*x^2+b*x+a)^{(1/2)}-8*f^2*e/(-e+(-4*d*f+e^2)^{(1/2)})^2/(e+(-4*d*f+e^2)^{(1/2)})^2*b/a^{(3/2)}*\ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)}*a^{(1/2)})/x)+2*f/(-e+(-4*d*f+e^2)^{(1/2)})/(e+(-4*d*f+e^2)^{(1/2)})/a/x^2*(c*x^2+b*x+a)^{(1/2)}-3*f/(-e+(-4*d*f+e^2)^{(1/2)})/(e+(-4*d*f+e^2)^{(1/2)})*b/a^2/x*(c*x^2+b*x+a)^{(1/2)}+3/2*f/(-e+(-4*d*f+e^2)^{(1/2)})/(e+(-4*d*f+e^2)^{(1/2)})*b^2/a^{(5/2)}*\ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)}*a^{(1/2)})/x)-2*f/(-e+(-4*d*f+e^2)^{(1/2)})/(e+(-4*d*f+e^2)^{(1/2)})*c/a^{(3/2)}*\ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)}*a^{(1/2)})/x)-8*f^3/(-e+(-4*d*f+e^2)^{(1/2)})^3/(-4*d*f+e^2)^{(1/2)}*2^{(1/2)}/((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*\ln(((b*f-c*e+(-4*d*f+e^2)^{(1/2)}*c)*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(1/2)}*((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c+4*(b*f-c*e+(-4*d*f+e^2)^{(1/2)}*c)*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)})/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f))-64*f^4/(-e+(-4*d*f+e^2)^{(1/2)})^3/(e+(-4*d*f+e^2)^{(1/2)})^3/a^{(1/2)}*\ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)}*a^{(1/2)})/x)*d+64*f^3/(-e+(-4*d*f+e^2)^{(1/2)})^3/(e+(-4*d*f+e^2)^{(1/2)})^3/a^{(1/2)}*\ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)}*a^{(1/2)})/x)*e^2$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a} (fx^2 + ex + d)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3 \sqrt{cx^2 + bx + a} (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)

[Out] int(1/(x^3*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \sqrt{a + bx + cx^2} (d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(1/(x**3*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

$$3.121 \quad \int \frac{x^3}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)} dx$$

Optimal. Leaf size=779

$$\frac{2 \left(cx \left((e^2 - df)(abf - 2ace + bcd) - de(-c(2af + be) + b^2f + 2c^2d) \right) - (adf - ae^2 + bde) \left(-c(2af + be) + b^2f + \dots \right) \right)}{f^2 (b^2 - 4ac) \sqrt{a + bx + cx^2} \left((cd - af)^2 - (bd - ae)(ce - bf) \right)}$$

[Out] $2*(b*x+2*a)/(-4*a*c+b^2)/f/(c*x^2+b*x+a)^{(1/2)}+2*e*(2*c*x+b)/(-4*a*c+b^2)/f$
 $^2/(c*x^2+b*x+a)^{(1/2)}+2*(c*d*e*(a*b*f-2*a*c*e+b*c*d)-(a*d*f-a*e^2+b*d*e)*$
 $2*c^2*d+b^2*f-c*(2*a*f+b*e))+c*((a*b*f-2*a*c*e+b*c*d)*(-d*f+e^2)-d*e*(2*c^2$
 $*d+b^2*f-c*(2*a*f+b*e)))*x)/(-4*a*c+b^2)/f^2/((-a*f+c*d)^2-(a*e+b*d)*(-b*f$
 $+c*e))/(c*x^2+b*x+a)^{(1/2)}+1/2*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)$
 $)^{(1/2)}))-b*(e-(-4*d*f+e^2)^{(1/2)}))*2^{(1/2)}/(c*x^2+b*x+a)^{(1/2)}/(c*e^2-2*c*$
 $d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}*(2*d*(-a*e+b*d)*f+$
 $c*d^2-b*d*e+a*(-d*f+e^2))*(e-(-4*d*f+e^2)^{(1/2)}))/((-a*f+c*d)^2-(a*e+b*d)*$
 $(-b*f+c*e))*2^{(1/2)}/(-4*d*f+e^2)^{(1/2)}/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c$
 $*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}-1/2*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^{(1/2)}$
 $))+2*x*(b*f-c*(e+(-4*d*f+e^2)^{(1/2)}))*2^{(1/2)}/(c*x^2+b*x+a)^{(1/2)}/(c*e^2-2$
 $*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}*(2*d*(-a*e+b*d)*$
 $f+(c*d^2-b*d*e+a*(-d*f+e^2))*(e+(-4*d*f+e^2)^{(1/2)}))/((-a*f+c*d)^2-(a*e+b$
 $d)*(-b*f+c*e))*2^{(1/2)}/(-4*d*f+e^2)^{(1/2)}/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b$
 $f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}$

Rubi [A] time = 14.17, antiderivative size = 779, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {6728, 613, 636, 1016, 1032, 724, 206}

$$\frac{2 \left(cx \left((e^2 - df)(abf - 2ace + bcd) - de(-c(2af + be) + b^2f + 2c^2d) \right) - (adf - ae^2 + bde) \left(-c(2af + be) + b^2f + \dots \right) \right)}{f^2 (b^2 - 4ac) \sqrt{a + bx + cx^2} \left((cd - af)^2 - (bd - ae)(ce - bf) \right)}$$

Antiderivative was successfully verified.

[In] Int[x^3/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

[Out] $(2*(2*a + b*x))/((b^2 - 4*a*c)*f*\text{Sqrt}[a + b*x + c*x^2]) + (2*e*(b + 2*c*x))$
 $/((b^2 - 4*a*c)*f^2*\text{Sqrt}[a + b*x + c*x^2]) + (2*(c*d*e*(b*c*d - 2*a*c*e + a$
 $*b*f) - (b*d*e - a*e^2 + a*d*f)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*((b$
 $*c*d - 2*a*c*e + a*b*f)*(e^2 - d*f) - d*e*(2*c^2*d + b^2*f - c*(b*e + 2*a*f$
 $)))*x))/((b^2 - 4*a*c)*f^2*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*\text{Sqrt}[a$

$$\begin{aligned}
& + b*x + c*x^2]) + ((2*d*(b*d - a*e)*f + (e - \text{Sqrt}[e^2 - 4*d*f])*(c*d^2 - b \\
& *d*e + a*(e^2 - d*f)))*\text{ArcTanh}[(4*a*f - b*(e - \text{Sqrt}[e^2 - 4*d*f]) + 2*(b*f \\
& - c*(e - \text{Sqrt}[e^2 - 4*d*f]))*x]/(2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2 \\
& *a*f^2 - (c*e - b*f)*\text{Sqrt}[e^2 - 4*d*f]]*\text{Sqrt}[a + b*x + c*x^2])))/(\text{Sqrt}[2]*\text{S} \\
& \text{qrt}[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*\text{Sqrt}[c*e^2 - 2*c \\
& *d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*\text{Sqrt}[e^2 - 4*d*f]]) - ((2*d*(b*d - a*e \\
&)*f + (e + \text{Sqrt}[e^2 - 4*d*f])*(c*d^2 - b*d*e + a*(e^2 - d*f)))*\text{ArcTanh}[(4*a \\
& *f - b*(e + \text{Sqrt}[e^2 - 4*d*f]) + 2*(b*f - c*(e + \text{Sqrt}[e^2 - 4*d*f]))*x]/(2* \\
& \text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*\text{Sqrt}[e^2 - 4*d \\
& *f]]*\text{Sqrt}[a + b*x + c*x^2])))/(\text{Sqrt}[2]*\text{Sqrt}[e^2 - 4*d*f]*((c*d - a*f)^2 - (\\
& b*d - a*e)*(c*e - b*f))*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f \\
&)*\text{Sqrt}[e^2 - 4*d*f]])
\end{aligned}$$

Rule 206

$$\text{Int}[\frac{(a_.) + (b_.)*(x_.)^2}{(x_.)^2}, x_Symbol] \rightarrow \text{Simp}[\frac{1*\text{ArcTanh}[\frac{\text{Rt}[-b, 2]*x}{\text{Rt}[a, 2]}]}{\text{Rt}[a, 2]*\text{Rt}[-b, 2]}, x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{Gt} \\
\text{Q}[a, 0] \ || \ \text{LtQ}[b, 0])$$

Rule 613

$$\text{Int}[\frac{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}{(x_.)^2}, x_Symbol] \rightarrow \text{Simp}[\frac{-2*(b + 2*c*x)}{(b^2 - 4*a*c)*\text{Sqrt}[a + b*x + c*x^2]}, x] /; \text{FreeQ}\{a, b, c\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$$

Rule 636

$$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x_Symbol] \rightarrow \text{Simp}[\frac{-2*(b*d - 2*a*e + (2*c*d - b*e)*x)}{(b^2 - 4*a*c)*\text{Sqrt}[a + b*x + c*x^2]}, x] /; \text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$$

Rule 724

$$\text{Int}[1/((d_.) + (e_.)*(x_.)*\text{Sqrt}[(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2]), x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/\text{Sqrt}[a + b*x + c*x^2]], x] /; \text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0]$$

Rule 1016

$$\text{Int}[\frac{(g_.) + (h_.)*(x_.)*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^p}{(d_.) + (e_.)*(x_.) + (f_.)*(x_.)^2}, x_Symbol] \rightarrow \text{Simp}[\frac{(a + b*x + c*x^2)^{p+1}*(d + e*x + f*x^2)^{q+1}*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*c*d - 2*a*c*e + a*b*f))*x}{(b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d -$$

```

a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
+ (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e
))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4))]*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1])

```

Rule 1032

```

Int[((g_.) + (h_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]

```

Rule 6728

```

Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)} dx &= \int \left(-\frac{e}{f^2(a+bx+cx^2)^{3/2}} + \frac{x}{f(a+bx+cx^2)^{3/2}} + \frac{de+(e^2-df)}{f^2(a+bx+cx^2)^{3/2}} \right) dx \\
&= \frac{\int \frac{de+(e^2-df)x}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)} dx}{f^2} - \frac{e \int \frac{1}{(a+bx+cx^2)^{3/2}} dx}{f^2} + \frac{\int \frac{x}{(a+bx+cx^2)^{3/2}} dx}{f} \\
&= \frac{2(2a+bx)}{(b^2-4ac)f\sqrt{a+bx+cx^2}} + \frac{2e(b+2cx)}{(b^2-4ac)f^2\sqrt{a+bx+cx^2}} + \frac{2(cde(bc-df))}{(b^2-4ac)f^2\sqrt{a+bx+cx^2}} \\
&= \frac{2(2a+bx)}{(b^2-4ac)f\sqrt{a+bx+cx^2}} + \frac{2e(b+2cx)}{(b^2-4ac)f^2\sqrt{a+bx+cx^2}} + \frac{2(cde(bc-df))}{(b^2-4ac)f^2\sqrt{a+bx+cx^2}} \\
&= \frac{2(2a+bx)}{(b^2-4ac)f\sqrt{a+bx+cx^2}} + \frac{2e(b+2cx)}{(b^2-4ac)f^2\sqrt{a+bx+cx^2}} + \frac{2(cde(bc-df))}{(b^2-4ac)f^2\sqrt{a+bx+cx^2}} \\
&= \frac{2(2a+bx)}{(b^2-4ac)f\sqrt{a+bx+cx^2}} + \frac{2e(b+2cx)}{(b^2-4ac)f^2\sqrt{a+bx+cx^2}} + \frac{2(cde(bc-df))}{(b^2-4ac)f^2\sqrt{a+bx+cx^2}}
\end{aligned}$$

Mathematica [A] time = 2.62, size = 1066, normalized size = 1.37

$$\frac{4(2fa^3+(-2cd-be+2cex+bf)x)a^2+b(b(d-ex)-3cdx)a+b^3dx)}{(b^2-4ac)\sqrt{a+bx+cx^2}} + \frac{\sqrt{2}(c(\sqrt{e^2-4df}-e)d^2+b(e^2-\sqrt{e^2-4df}e-2df)d+a(-e^3+\sqrt{e^2-4df}e^2+3dfe-df\sqrt{e^2-4df}-e^2d^2))}{\sqrt{e^2-4df}\sqrt{c(e^2-\sqrt{e^2-4df}e-2df)+f(2af+b(\sqrt{e^2-4df}-e))}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

[Out] ((4*(2*a^3*f + b^3*d*x + a^2*(-2*c*d - b*e + 2*c*e*x + b*f*x) + a*b*(-3*c*d*x + b*(d - e*x))))/((b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]) + (Sqrt[2]*(c*d^2*(-e + Sqrt[e^2 - 4*d*f]) + b*d*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + a*(-e^3 + 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f]))*Log[-e + Sqrt[e^2 - 4*d*f] - 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]) + (Sqrt[2]*(c*d^2*(e + Sqrt[e^2 - 4*d*f]) - b*d*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + a*(e^3 -

$$\begin{aligned} & 3*d*e*f + e^2*\sqrt{e^2 - 4*d*f} - d*f*\sqrt{e^2 - 4*d*f})) * \text{Log}[e + \sqrt{e^2 - 4*d*f} + 2*f*x] / (\sqrt{e^2 - 4*d*f} * \sqrt{c*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f}) + f*(2*a*f - b*(e + \sqrt{e^2 - 4*d*f}))}) - (\sqrt{2}*(c*d^2*(e + \sqrt{e^2 - 4*d*f}) - b*d*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f}) + a*(e^3 - 3*d*e*f + e^2*\sqrt{e^2 - 4*d*f} - d*f*\sqrt{e^2 - 4*d*f})) * \text{Log}[-4*a*f + 2*c*e*x + 2*c*\sqrt{e^2 - 4*d*f}*x + b*(e + \sqrt{e^2 - 4*d*f} - 2*f*x) - 2*\sqrt{2}*\sqrt{c*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f}) + f*(2*a*f - b*(e + \sqrt{e^2 - 4*d*f}))}] * \sqrt{a + x*(b + c*x)}}) / (\sqrt{e^2 - 4*d*f} * \sqrt{c*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f}) + f*(2*a*f - b*(e + \sqrt{e^2 - 4*d*f}))}) - (\sqrt{2}*(c*d^2*(-e + \sqrt{e^2 - 4*d*f}) + b*d*(e^2 - 2*d*f - e*\sqrt{e^2 - 4*d*f}) + a*(-e^3 + 3*d*e*f + e^2*\sqrt{e^2 - 4*d*f} - d*f*\sqrt{e^2 - 4*d*f})) * \text{Log}[b*(-e + \sqrt{e^2 - 4*d*f} + 2*f*x) + 2*(2*a*f - c*e*x + c*\sqrt{e^2 - 4*d*f}*x + \sqrt{2}*\sqrt{f*(-(b*e) + 2*a*f + b*\sqrt{e^2 - 4*d*f}) + c*(e^2 - 2*d*f - e*\sqrt{e^2 - 4*d*f}))}] * \sqrt{a + x*(b + c*x)}}) / (\sqrt{e^2 - 4*d*f} * \sqrt{c*(e^2 - 2*d*f - e*\sqrt{e^2 - 4*d*f}) + f*(2*a*f + b*(-e + \sqrt{e^2 - 4*d*f}))})) / (2*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))) \end{aligned}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.04, size = 14651, normalized size = 18.81

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)

[Out] result too large to display

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x³/(c*x²+b*x+a)^(3/2)/(f*x²+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e²>0)', see `assume?` for more details) Is 4*d*f-e² positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{(cx^2 + bx + a)^{3/2} (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x³/((a + b*x + c*x²)^(3/2)*(d + e*x + f*x²)),x)

[Out] int(x³/((a + b*x + c*x²)^(3/2)*(d + e*x + f*x²)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)

[Out] Timed out

$$3.122 \quad \int \frac{x^2}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)} dx$$

Optimal. Leaf size=609

$$\frac{2\left(cx(-abe - 2a(cd - af) + b^2d) + a(abf - 2ace + bcd)\right)}{(b^2 - 4ac)\sqrt{a + bx + cx^2} \left((cd - af)^2 - (bd - ae)(ce - bf)\right)} \frac{f\left(2d(cd - af) - (e - \sqrt{e^2 - 4df})(bd - ae)\right) \tanh^{-1}\left(\frac{e - \sqrt{e^2 - 4df}}{cd - af}\right)}{\sqrt{2}\sqrt{e^2 - 4df} \left((cd - af)^2 - (bd - ae)(ce - bf)\right) \sqrt{a + bx + cx^2}}$$

[Out]
$$\frac{-2*(a*(a*b*f-2*a*c*e+b*c*d)+c*(b^2*d-a*b*e-2*a*(a*f+c*d))*x)/(-4*a*c+b^2)/((a*f+c*d)^2-(a*e+b*d)*(-b*f+c*e))/(c*x^2+b*x+a)^{(1/2)-1/2*f*\operatorname{arctanh}(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^{(1/2)}))-b*(e-(-4*d*f+e^2)^{(1/2)})))*2^{(1/2)}/(c*x^2+b*x+a)^{(1/2)}/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}*(2*d*(a*f+c*d)-(-a*e+b*d)*(e-(-4*d*f+e^2)^{(1/2)})))/((-a*f+c*d)^2-(a*e+b*d)*(-b*f+c*e))*2^{(1/2)}/(-4*d*f+e^2)^{(1/2)}/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}+1/2*f*\operatorname{arctanh}(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^{(1/2)}))+2*x*(b*f-c*(e+(-4*d*f+e^2)^{(1/2)})))*2^{(1/2)}/(c*x^2+b*x+a)^{(1/2)}/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}*(2*d*(a*f+c*d)-(-a*e+b*d)*(e+(-4*d*f+e^2)^{(1/2)})))/((-a*f+c*d)^2-(a*e+b*d)*(-b*f+c*e))*2^{(1/2)}/(-4*d*f+e^2)^{(1/2)}/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}}$$

Rubi [A] time = 5.84, antiderivative size = 609, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1061, 1032, 724, 206}

$$\frac{2\left(cx(-abe - 2a(cd - af) + b^2d) + a(abf - 2ace + bcd)\right)}{(b^2 - 4ac)\sqrt{a + bx + cx^2} \left((cd - af)^2 - (bd - ae)(ce - bf)\right)} \frac{f\left(2d(cd - af) - (e - \sqrt{e^2 - 4df})(bd - ae)\right) \tanh^{-1}\left(\frac{e - \sqrt{e^2 - 4df}}{cd - af}\right)}{\sqrt{2}\sqrt{e^2 - 4df} \left((cd - af)^2 - (bd - ae)(ce - bf)\right) \sqrt{a + bx + cx^2}}$$

Antiderivative was successfully verified.

[In] Int[x^2/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

[Out]
$$\frac{(-2*(a*(b*c*d - 2*a*c*e + a*b*f) + c*(b^2*d - a*b*e - 2*a*(c*d - a*f))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*\operatorname{Sqrt}[a + b*x + c*x^2]) - (f*(2*d*(c*d - a*f) - (b*d - a*e)*(e - \operatorname{Sqrt}[e^2 - 4*d*f]))*\operatorname{ArcTanh}[(4*a*f - b*(e - \operatorname{Sqrt}[e^2 - 4*d*f]) + 2*(b*f - c*(e - \operatorname{Sqrt}[e^2 - 4*d*f]))*x)/(2*\operatorname{Sqrt}[2]*\operatorname{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*\operatorname{Sqrt}[e^2 - 4*d*f]])*\operatorname{Sqrt}[a + b*x + c*x^2])]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*\operatorname{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*\operatorname{Sqrt}[e^2 - 4*d*f]]) + (f*(2*d*(c*d - a*f) - (b*d - a*e)*(e + \operatorname{Sqrt}[e^2 - 4*d*f]))*\operatorname{ArcTanh}[(4*a*f - b*(e + \operatorname{Sqrt}[e^2 - 4*d*f]) + 2*(b*f - c*(e + \operatorname{Sqrt}[e^2 - 4*d*f]))*x)/(2*\operatorname{Sqrt}[2]*\operatorname{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*\operatorname{Sqrt}[e^2 - 4*d*f]])*\operatorname{Sqrt}[a + b*x + c*x^2])]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*\operatorname{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*\operatorname{Sqrt}[e^2 - 4*d*f]])}$$

$$- 4*d*f)) * \text{ArcTanh}[(4*a*f - b*(e + \sqrt{e^2 - 4*d*f}) + 2*(b*f - c*(e + \sqrt{e^2 - 4*d*f}))*x) / (2*\sqrt{2}*\sqrt{c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*\sqrt{e^2 - 4*d*f}}*\sqrt{a + b*x + c*x^2})] / (\sqrt{2}*\sqrt{e^2 - 4*d*f}) * ((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)) * \sqrt{c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*\sqrt{e^2 - 4*d*f}}$$

Rule 206

$$\text{Int}[(a + (b \cdot x)^{-1}), x_Symbol] \rightarrow \text{Simp}[(1 * \text{ArcTanh}[\text{Rt}[-b, 2] * x] / \text{Rt}[a, 2]) / (\text{Rt}[a, 2] * \text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$$

Rule 724

$$\text{Int}[1 / (((d \cdot x) + (e \cdot x) * \sqrt{(a \cdot x) + (b \cdot x) + (c \cdot x)^2})], x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1 / (4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x) / \sqrt{a + b*x + c*x^2}], x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0]$$

Rule 1032

$$\text{Int}[(g \cdot x) + (h \cdot x) * \sqrt{(a \cdot x) + (b \cdot x) + (c \cdot x)^2}], x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(2*c*g - h*(b - q)) / q, \text{Int}[1 / ((b - q + 2*c*x) * \sqrt{d + e*x + f*x^2}), x], x] - \text{Dist}[(2*c*g - h*(b + q)) / q, \text{Int}[1 / ((b + q + 2*c*x) * \sqrt{d + e*x + f*x^2}), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h\}, x\} \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[e^2 - 4*d*f, 0] \ \&\& \ \text{PosQ}[b^2 - 4*a*c]$$

Rule 1061

$$\text{Int}[(a + (b \cdot x) + (c \cdot x)^2)^{(p)} * ((A \cdot x) + (C \cdot x)^2) * ((d \cdot x) + (e \cdot x) + (f \cdot x)^2)^{(q)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x + c*x^2)^{(p+1)} * (d + e*x + f*x^2)^{(q+1)} * ((A*c - a*C) * (2*a*c*e - b*(c*d + a*f)) + (A*b) * (2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f))) * x) / ((b^2 - 4*a*c) * ((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)) * (p + 1)), x] + \text{Dist}[1 / ((b^2 - 4*a*c) * ((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)) * (p + 1)), \text{Int}[(a + b*x + c*x^2)^{(p+1)} * (d + e*x + f*x^2)^q * \text{Simp}[(-2*A*c - 2*a*C) * ((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)) * (p + 1) + (b^2*(C*d + A*f) - b*((Plus[A])*c*e + a*C*e) + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f))) * (a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C) * (2*a*c*e - b*(c*d + a*f)) + (A*b) * (2*c^2*d + b^2*f - c*(b*e + 2*a*f))) * (p + q + 2) - (2*f*((A*c - a*C) * (2*a*c*e - b*(c*d + a*f)) + (A*b) * (2*c^2*d + b^2*f - c*(b*e + 2*a*f))) * (p + q + 2) - (b^2*(C*d + A*f) - b*(A*c*e + a*C*e) + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f))) * (b*f*(p + 1) - c*e*(2*p + q + 4))] * x - c*f*(b^2*(C*d + A*f) - b*(A*c*e + a*C*e) + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f))) * (2*p + 2*q + 5) * x^2, x], x] /; \text{FreeQ}\{a, b, c, d, e, f$$

, A, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !(!IntegerQ[p] && IntegerQ[q, -1]) && !IGtQ[q, 0]

Rubi steps

$$\int \frac{x^2}{(a + bx + cx^2)^{3/2} (d + ex + fx^2)} dx = -\frac{2(abcd - 2ace + abf) + c(b^2d - abe - 2a(cd - af))x}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a + bx + cx^2}} - \frac{2 \int \frac{1}{\sqrt{a + bx + cx^2}} dx}{(b^2 - 4ac)}$$

$$= -\frac{2(abcd - 2ace + abf) + c(b^2d - abe - 2a(cd - af))x}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a + bx + cx^2}} + \frac{f(2d(c - e) + b^2 - 4ac)}{(b^2 - 4ac)}$$

$$= -\frac{2(abcd - 2ace + abf) + c(b^2d - abe - 2a(cd - af))x}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a + bx + cx^2}} - \frac{2f(2d(c - e) + b^2 - 4ac)}{(b^2 - 4ac)}$$

$$= -\frac{2(abcd - 2ace + abf) + c(b^2d - abe - 2a(cd - af))x}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a + bx + cx^2}} - \frac{f(2d(c - e) + b^2 - 4ac)}{\sqrt{2}\sqrt{e^2 - 4df}}$$

Mathematica [A] time = 6.59, size = 1097, normalized size = 1.80

$$\frac{16\sqrt{2}f\sqrt{ce^2 - bfe - c\sqrt{e^2 - 4df}e + 2af^2 - 2cdf + bf\sqrt{e^2 - 4df}}\left(e + \frac{2df - e^2}{\sqrt{e^2 - 4df}}\right)\tanh^{-1}\left(\frac{4af - b(e - \sqrt{e^2 - 4df})}{2\sqrt{2}\sqrt{ce^2 - bfe - c\sqrt{e^2 - 4df}e + 2af^2 - 2cdf + bf\sqrt{e^2 - 4df}}}\right)}{\left(4af^2 - 2b(e - \sqrt{e^2 - 4df})f + c(e - \sqrt{e^2 - 4df})^2\right)\left(16af^2 - 8b(e - \sqrt{e^2 - 4df})f + 4c(e - \sqrt{e^2 - 4df})^2\right)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

[Out] (-2*(e - (e^2 - 2*d*f)/Sqrt[e^2 - 4*d*f])*(2*b^2*f - 4*a*c*f - b*c*(e - Sqrt[e^2 - 4*d*f]) + 2*c*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)*(a + b*x + c*x^2))/((b^2 - 4*a*c)*f*(4*a*f^2 - 2*b*f*(e - Sqrt[e^2 - 4*d*f]) + c*(e - Sqrt[e^2 - 4*d*f])^2)*(a + x*(b + c*x))^(3/2)) - (2*(e + (e^2 - 2*d*f)/Sqrt[e^2 - 4*d*f])*(2*b^2*f - 4*a*c*f - b*c*(e + Sqrt[e^2 - 4*d*f]) + 2*c*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)*(a + b*x + c*x^2))/((b^2 - 4*a*c)*f*(4*a*f^2 - 2*b*f*(e + Sqrt[e^2 - 4*d*f]) + c*(e + Sqrt[e^2 - 4*d*f])^2)*(a + x*(b + c*x))^(3/2))

$$\begin{aligned} &))^{(3/2)} + (4*(b + 2*c*x)*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^{(3/2)})/ \\ &(c*f*(a + x*(b + c*x))^{(3/2)}*Sqrt[1 - (b + 2*c*x)^2/(b^2 - 4*a*c)]) + (16*S \\ &qrt[2]*f*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b \\ &*f*Sqrt[e^2 - 4*d*f]]*(e + (-e^2 + 2*d*f)/Sqrt[e^2 - 4*d*f])*(a + b*x + c*x \\ &^2)^{(3/2)}*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) - (-2*b*f + 2*c*(e - S \\ &qrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c \\ &*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]))]/((4* \\ &a*f^2 - 2*b*f*(e - Sqrt[e^2 - 4*d*f]) + c*(e - Sqrt[e^2 - 4*d*f])^2)*(16*a* \\ &f^2 - 8*b*f*(e - Sqrt[e^2 - 4*d*f]) + 4*c*(e - Sqrt[e^2 - 4*d*f])^2)*(a + x \\ &*(b + c*x))^{(3/2)}) + (16*Sqrt[2]*f*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + \\ &c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*(e - (-e^2 + 2*d*f)/Sqrt[e^ \\ &2 - 4*d*f])*(a + b*x + c*x^2)^{(3/2)}*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d* \\ &f]) - (-2*b*f + 2*c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c \\ &*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sqr \\ &t[a + b*x + c*x^2]))]/((4*a*f^2 - 2*b*f*(e + Sqrt[e^2 - 4*d*f]) + c*(e + Sq \\ &rt[e^2 - 4*d*f])^2)*(16*a*f^2 - 8*b*f*(e + Sqrt[e^2 - 4*d*f]) + 4*c*(e + Sq \\ &rt[e^2 - 4*d*f])^2)*(a + x*(b + c*x))^{(3/2)}) \end{aligned}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage₂

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] sage2

maple [B] time = 0.02, size = 11341, normalized size = 18.62

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)

[Out] result too large to display

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details)Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{(cx^2 + bx + a)^{3/2} (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)

[Out] int(x^2/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)

[Out] Timed out

$$3.123 \quad \int \frac{x}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)} dx$$

Optimal. Leaf size=609

$$\frac{2(a(-2acf + b^2f - bce + 2c^2d) + cx(abf - 2ace + bcd))}{(b^2 - 4ac)\sqrt{a + bx + cx^2}((cd - af)^2 - (bd - ae)(ce - bf))} + \frac{f(2d(ce - bf) - (e - \sqrt{e^2 - 4df})(cd - af)) \tanh^{-1}(\frac{e - \sqrt{e^2 - 4df}}{\sqrt{2}\sqrt{e^2 - 4df}})}{\sqrt{2}\sqrt{e^2 - 4df}((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a + bx + cx^2}}$$

[Out] $2*(a*(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)+c*(a*b*f-2*a*c*e+b*c*d)*x)/(-4*a*c+b^2)/(((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))/(c*x^2+b*x+a)^(1/2)+1/2*f*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2))))-b*(e-(-4*d*f+e^2)^(1/2))))*(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(2*d*(-b*f+c*e)-(-a*f+c*d)*(e-(-4*d*f+e^2)^(1/2)))/(((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)-1/2*f*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(2*d*(-b*f+c*e)-(-a*f+c*d)*(e+(-4*d*f+e^2)^(1/2)))/(((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))$

Rubi [A] time = 5.64, antiderivative size = 609, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1016, 1032, 724, 206}

$$\frac{2(a(-2acf + b^2f - bce + 2c^2d) + cx(abf - 2ace + bcd))}{(b^2 - 4ac)\sqrt{a + bx + cx^2}((cd - af)^2 - (bd - ae)(ce - bf))} + \frac{f(2d(ce - bf) - (e - \sqrt{e^2 - 4df})(cd - af)) \tanh^{-1}(\frac{e - \sqrt{e^2 - 4df}}{\sqrt{2}\sqrt{e^2 - 4df}})}{\sqrt{2}\sqrt{e^2 - 4df}((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a + bx + cx^2}}$$

Antiderivative was successfully verified.

[In] Int[x/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

[Out] $(2*(a*(2*c^2*d - b*c*e + b^2*f - 2*a*c*f) + c*(b*c*d - 2*a*c*e + a*b*f)*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[a + b*x + c*x^2]) + (f*(2*d*(c*e - b*f) - (c*d - a*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])*Sqrt[a + b*x + c*x^2])]/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - (f*(2*d*(c*e - b*f) - (c*d - a*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])*Sqrt[a + b*x + c*x^2])]/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])$

$$- 4*d*f)) * \text{ArcTanh}[(4*a*f - b*(e + \sqrt{e^2 - 4*d*f})) + 2*(b*f - c*(e + \sqrt{e^2 - 4*d*f})) * x] / (2*\sqrt{2}*\sqrt{c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*\sqrt{e^2 - 4*d*f}}*\sqrt{a + b*x + c*x^2}) / (\sqrt{2}*\sqrt{e^2 - 4*d*f}) * ((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)) * \sqrt{c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*\sqrt{e^2 - 4*d*f}}$$

Rule 206

$$\text{Int}[(a + (b \cdot x)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1 * \text{ArcTanh}[\text{Rt}[-b, 2] * x] / \text{Rt}[a, 2]) / (\text{Rt}[a, 2] * \text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b, x\} \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$$

Rule 724

$$\text{Int}[1 / (((d \cdot x) + (e \cdot x)) * \sqrt{(a \cdot x) + (b \cdot x) + (c \cdot x)^2})], x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1 / (4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x) / \sqrt{a + b*x + c*x^2}], x] /; \text{FreeQ}\{a, b, c, d, e, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[2*c*d - b*e, 0]$$

Rule 1016

$$\text{Int}[(g \cdot x + (h \cdot x)) * ((a \cdot x) + (b \cdot x) + (c \cdot x)^2)^{p} * ((d \cdot x) + (e \cdot x) + (f \cdot x)^2)^{q}, x_Symbol] \rightarrow \text{Simp}[(a + b*x + c*x^2)^{p+1} * (d + e*x + f*x^2)^{q+1} * (g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*c*d - 2*a*c*e + a*b*f)) * x] / ((b^2 - 4*a*c) * ((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)) * (p + 1)), x] + \text{Dist}[1 / ((b^2 - 4*a*c) * ((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)) * (p + 1)), \text{Int}[(a + b*x + c*x^2)^{p+1} * (d + e*x + f*x^2)^q * \text{Simp}[(b*h - 2*g*c) * ((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)) * (p + 1) + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e))) * (a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))) * (p + q + 2) - (2*f*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))) * (p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e))) * (b*f*(p + 1) - c*e*(2*p + q + 4))) * x - c*f*(b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e)) * (2*p + 2*q + 5) * x^2, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h, q, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[e^2 - 4*d*f, 0] \&\& \text{LtQ}[p, -1] \&\& \text{NeQ}[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] \&\& !(IntegerQ[p] \&\& ILtQ[q, -1])$$

Rule 1032

$$\text{Int}[(g \cdot x + (h \cdot x)) / (((a \cdot x) + (b \cdot x) + (c \cdot x)^2) * \sqrt{(d \cdot x) + (e \cdot x) + (f \cdot x)^2}), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(2*c*g - h*(b - q)) / q, \text{Int}[1 / ((b - q + 2*c*x) * \sqrt{d + e*x + f*x^2}), x], x] - \text{Dist}[(2*c*g - h*(b + q)) / q, \text{Int}[1 / ((b + q + 2*c*x) * \sqrt{d + e*x + f*x^2}), x], x]$$

$\wedge 2)), x], x]] /; \text{FreeQ}[\{a, b, c, d, e, f, g, h\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$
 $\&\& \ \text{NeQ}[e^2 - 4*d*f, 0] \ \&\& \ \text{PosQ}[b^2 - 4*a*c]$

Rubi steps

$$\int \frac{x}{(a + bx + cx^2)^{3/2} (d + ex + fx^2)} dx = \frac{2(a(2c^2d - bce + b^2f - 2acf) + c(bcd - 2ace + abf)x)}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a + bx + cx^2}} - \frac{2 \int \frac{1}{2} \left(\frac{2d(c^2d - bce + b^2f - 2acf) + c(bcd - 2ace + abf)}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a + bx + cx^2}} \right) dx}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a + bx + cx^2}} - \frac{f(2d(c^2d - bce + b^2f - 2acf) + c(bcd - 2ace + abf))}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a + bx + cx^2}} + \frac{2f(2d(c^2d - bce + b^2f - 2acf) + c(bcd - 2ace + abf))}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a + bx + cx^2}} + \frac{f(2d(c^2d - bce + b^2f - 2acf) + c(bcd - 2ace + abf))}{\sqrt{2}\sqrt{e^2 - 4df}}$$

Mathematica [A] time = 5.68, size = 770, normalized size = 1.26

$$\frac{2\left(1 - \frac{e}{\sqrt{e^2 - 4df}}\right)\left(2c\left(cx\left(\sqrt{e^2 - 4df} - e\right) - 2af\right) + 2b^2f + bc\left(\sqrt{e^2 - 4df} - e + 2fx\right)\right)}{(b^2 - 4ac)\sqrt{a + x(b + cx)}\left(4af^2 + 2bf\left(\sqrt{e^2 - 4df} - e\right) + c\left(e - \sqrt{e^2 - 4df}\right)^2\right)} + \frac{2\left(\frac{e}{\sqrt{e^2 - 4df}} + 1\right)\left(-2c\left(2a\left(\sqrt{e^2 - 4df} - e\right) + b\left(\sqrt{e^2 - 4df} - e + 2fx\right)\right)\right)}{(b^2 - 4ac)\sqrt{a + x(b + cx)}}$$

Antiderivative was successfully verified.

[In] Integrate[x/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

[Out] $(2*(1 - e/\text{Sqrt}[e^2 - 4*d*f])*(2*b^2*f + b*c*(-e + \text{Sqrt}[e^2 - 4*d*f] + 2*f*x) + 2*c*(-2*a*f + c*(-e + \text{Sqrt}[e^2 - 4*d*f])*x)))/((b^2 - 4*a*c)*(4*a*f^2 + c*(e - \text{Sqrt}[e^2 - 4*d*f])^2 + 2*b*f*(-e + \text{Sqrt}[e^2 - 4*d*f]))*\text{Sqrt}[a + x*(b + c*x)]) + (2*(1 + e/\text{Sqrt}[e^2 - 4*d*f])*(2*b^2*f - b*c*(e + \text{Sqrt}[e^2 - 4*d*f] - 2*f*x) - 2*c*(2*a*f + c*(e + \text{Sqrt}[e^2 - 4*d*f])*x)))/((b^2 - 4*a*c)*(4*a*f^2 - 2*b*f*(e + \text{Sqrt}[e^2 - 4*d*f]) + c*(e + \text{Sqrt}[e^2 - 4*d*f])^2)*\text{Sqrt}[a + x*(b + c*x)]) - (\text{Sqrt}[2]*f^2*(e + \text{Sqrt}[e^2 - 4*d*f])*ArcTanh[(4*a*f - 2*c*(e + \text{Sqrt}[e^2 - 4*d*f])*x - b*(e + \text{Sqrt}[e^2 - 4*d*f] - 2*f*x))/(2*\text{Sqrt}$

$$\frac{[2]*\text{Sqrt}[c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]) + f*(2*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]))]*\text{Sqrt}[a + x*(b + c*x)]]}{(\text{Sqrt}[e^2 - 4*d*f]*(c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]) + f*(2*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f])))^{(3/2)}) - (\text{Sqrt}[2]*f^2*(-e + \text{Sqrt}[e^2 - 4*d*f])*\text{Sqrt}[c*(e^2 - 2*d*f - e*\text{Sqrt}[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + \text{Sqrt}[e^2 - 4*d*f]))]*\text{ArcTanh}[(4*a*f + 2*c*(-e + \text{Sqrt}[e^2 - 4*d*f])*x + b*(-e + \text{Sqrt}[e^2 - 4*d*f] + 2*f*x))/(2*\text{Sqrt}[2]*\text{Sqrt}[c*(e^2 - 2*d*f - e*\text{Sqrt}[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + \text{Sqrt}[e^2 - 4*d*f]))]*\text{Sqrt}[a + x*(b + c*x)])])]/(\text{Sqrt}[e^2 - 4*d*f]*(c*(-e^2 + 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]) + f*(-2*a*f + b*(e - \text{Sqrt}[e^2 - 4*d*f])))^2)}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage₂

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] sage2

maple [B] time = 0.02, size = 7163, normalized size = 11.76

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)

[Out] result too large to display

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details) Is $4*d*f-e^2$ positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{(cx^2 + bx + a)^{3/2} (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)

[Out] int(x/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)

[Out] Timed out

$$3.124 \quad \int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)} dx$$

Optimal. Leaf size=666

$$\frac{2(-cx(-2acf + b^2f - bce + 2c^2d) - bc(cd - 3af) - 2ac^2e + b^3(-f) + b^2ce)}{(b^2 - 4ac)\sqrt{a + bx + cx^2}((cd - af)^2 - (bd - ae)(ce - bf))} \frac{f(f(2af - b(\sqrt{e^2 - 4df} + e)) + c)}{\sqrt{2}\sqrt{e^2 - 4df}((cd - af)^2 - (bd - ae)(ce - bf))}$$

[Out] $2*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f) - c*(2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x)/((-4*a*c + b^2)/((-a*f + c*d)^2 - (-a*e + b*d)*(-b*f + c*e))/(c*x^2 + b*x + a)^{(1/2)} - 1/2*f*arctanh(1/4*(4*a*f + 2*x*(b*f - c*(e - (-4*d*f + e^2)^{(1/2)})) - b*(e - (-4*d*f + e^2)^{(1/2)})))*2^{(1/2)}/(c*x^2 + b*x + a)^{(1/2)}/(c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (-b*f + c*e)*(-4*d*f + e^2)^{(1/2)})^{(1/2)})*(((-4*d*f + e^2)^{(1/2)}*e - 2*d*f + e^2)*c + f*(2*a*f - b*(e + (-4*d*f + e^2)^{(1/2)})))/((-a*f + c*d)^2 - (-a*e + b*d)*(-b*f + c*e))*2^{(1/2)}/((-4*d*f + e^2)^{(1/2)}/(f*(2*a*f - b*(e - (-4*d*f + e^2)^{(1/2)})) + (-(-4*d*f + e^2)^{(1/2)}*e - 2*d*f + e^2)*c)^{(1/2)} + 1/2*f*arctanh(1/4*(4*a*f - b*(e + (-4*d*f + e^2)^{(1/2)})))*2^{(1/2)}/(c*x^2 + b*x + a)^{(1/2)}/(c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (-b*f + c*e)*(-4*d*f + e^2)^{(1/2)})^{(1/2)})*((f*(2*a*f - b*(e - (-4*d*f + e^2)^{(1/2)})) + (-(-4*d*f + e^2)^{(1/2)}*e - 2*d*f + e^2)*c)/((-a*f + c*d)^2 - (-a*e + b*d)*(-b*f + c*e))*2^{(1/2)}/((-4*d*f + e^2)^{(1/2)}/(((-4*d*f + e^2)^{(1/2)}*e - 2*d*f + e^2)*c + f*(2*a*f - b*(e + (-4*d*f + e^2)^{(1/2)})))^{(1/2)})^{(1/2)}$

Rubi [A] time = 1.75, antiderivative size = 666, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {974, 1032, 724, 206}

$$\frac{2(-cx(-2acf + b^2f - bce + 2c^2d) - bc(cd - 3af) - 2ac^2e + b^2ce + b^3(-f))}{(b^2 - 4ac)\sqrt{a + bx + cx^2}((cd - af)^2 - (bd - ae)(ce - bf))} \frac{f(f(2af - b(\sqrt{e^2 - 4df} + e)) + c)}{\sqrt{2}\sqrt{e^2 - 4df}((cd - af)^2 - (bd - ae)(ce - bf))}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

[Out] $(2*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f) - c*(2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)))*\text{Sqrt}[a + b*x + c*x^2] - (f*(c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]) + f*(2*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f])))*\text{ArcTanh}[(4*a*f - b*(e - \text{Sqrt}[e^2 - 4*d*f]) + 2*(b*f - c*(e - \text{Sqrt}[e^2 - 4*d*f]))*x)/(2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*\text{Sqrt}[e^2 - 4*d*f]])*\text{Sqrt}[a + b*x + c*x^2])]/(\text{Sqrt}[2]*\text{Sqrt}[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*\text{Sqr}$

$$t[c*(e^2 - 2*d*f - e*\sqrt{e^2 - 4*d*f}) + f*(2*a*f - b*(e - \sqrt{e^2 - 4*d*f}))] + (f*(c*(e^2 - 2*d*f - e*\sqrt{e^2 - 4*d*f}) + f*(2*a*f - b*(e - \sqrt{e^2 - 4*d*f}))) * \text{ArcTanh}[(4*a*f - b*(e + \sqrt{e^2 - 4*d*f}) + 2*(b*f - c*(e + \sqrt{e^2 - 4*d*f}))*x] / (2*\sqrt{2}*\sqrt{c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*\sqrt{e^2 - 4*d*f}}*\sqrt{a + b*x + c*x^2}]) / (\sqrt{2}*\sqrt{e^2 - 4*d*f}*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*\sqrt{c*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f}) + f*(2*a*f - b*(e + \sqrt{e^2 - 4*d*f}))})$$

Rule 206

$$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x] / \text{Rt}[a, 2]) / (\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$$

Rule 724

$$\text{Int}[1/(((d_.) + (e_)*(x_))*\sqrt{(a_.) + (b_)*(x_.) + (c_)*(x_.)^2}), x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x) / \sqrt{a + b*x + c*x^2}], x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[2*c*d - b*e, 0]$$

Rule 974

$$\text{Int}[(a_ + (b_)*(x_.) + (c_)*(x_.)^2)^{(p_)}*((d_.) + (e_)*(x_.) + (f_)*(x_.)^2)^{(q_)}, x_Symbol] \rightarrow \text{Simp}[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x]*(a + b*x + c*x^2)^{(p+1)}*(d + e*x + f*x^2)^{(q+1)} / ((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))^{(p+1)}), x] - \text{Dist}[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))^{(p+1)}), \text{Int}[(a + b*x + c*x^2)^{(p+1)}*(d + e*x + f*x^2)^q * \text{Simp}[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))^{(p+1)} - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(a*f*(p+1) - c*d*(p+2)) - e*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))^{(p+q+2)} + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f))^{(p+q+2)} - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*(p+1) - c*e*(2*p+q+4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))^{(2*p+2*q+5)}*x^2, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, q\}, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[e^2 - 4*d*f, 0] \&\& \text{LtQ}[p, -1] \&\& \text{NeQ}[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] \&\& !(!\text{IntegerQ}[p] \&\& \text{ILtQ}[q, -1]) \&\& !\text{IGtQ}[q, 0]$$

Rule 1032

$$\text{Int}[(g_ + (h_)*(x_)) / (((a_ + (b_)*(x_.) + (c_)*(x_.)^2)*\sqrt{(d_.) + (e_)*(x_.) + (f_)*(x_.)^2}), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(2*c*g - h*(b - q))/q, \text{Int}[1/((b - q + 2*c*x)*\sqrt{d + e*x + f*x^2}), x], x] - \text{Dist}[(2*c*g - h*(b + q))/q, \text{Int}[1/((b + q + 2*c*x)*\sqrt{d + e*x + f*x^2}), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h\}, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0]$$

&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{1}{(a + bx + cx^2)^{3/2} (d + ex + fx^2)} dx &= \frac{2(b^2ce - 2ac^2e - b^3f - bc(cd - 3af) - c(2c^2d - bce + b^2f - 2acf)x)}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a + bx + cx^2}} + \\ &= \frac{2(b^2ce - 2ac^2e - b^3f - bc(cd - 3af) - c(2c^2d - bce + b^2f - 2acf)x)}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a + bx + cx^2}} + \\ &= \frac{2(b^2ce - 2ac^2e - b^3f - bc(cd - 3af) - c(2c^2d - bce + b^2f - 2acf)x)}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a + bx + cx^2}} - \\ &= \frac{2(b^2ce - 2ac^2e - b^3f - bc(cd - 3af) - c(2c^2d - bce + b^2f - 2acf)x)}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a + bx + cx^2}} \end{aligned}$$

Mathematica [A] time = 5.04, size = 700, normalized size = 1.05

$$2f \left(\frac{2(-2c(2af + cx(\sqrt{e^2 - 4df} + e)) + 2b^2f - bc(\sqrt{e^2 - 4df} + e - 2fx))}{(b^2 - 4ac)\sqrt{a + x(b + cx)}(4af^2 - 2bf(\sqrt{e^2 - 4df} + e) + c(\sqrt{e^2 - 4df} + e)^2)} + \frac{2c(cx(\sqrt{e^2 - 4df} - e) - 2af) + 2b^2f + bc(\sqrt{e^2 - 4df} - e + 2fx)}{(b^2 - 4ac)\sqrt{a + x(b + cx)}(f(2af + b(\sqrt{e^2 - 4df} - e)) + c(-e\sqrt{e^2 - 4df} - 2df + e^2))} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x]

[Out] (2*f*((2*b^2*f + b*c*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x) + 2*c*(-2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x))/((b^2 - 4*a*c)*(c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f])))*Sqrt[a + x*(b + c*x)]) - (2*(2*b^2*f - b*c*(e + Sqrt[e^2 - 4*d*f] - 2*f*x) - 2*c*(2*a*f + c*(e + Sqrt[e^2 - 4*d*f])*x)))/((b^2 - 4*a*c)*(4*a*f^2 - 2*b*f*(e + Sqrt[e^2 - 4*d*f]) + c*(e + Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + x*(b + c*x)]) + (Sqrt[2]*f^2*ArcTanh

$$\frac{[(4*af - 2*c*(e + \sqrt{e^2 - 4*df}))*x - b*(e + \sqrt{e^2 - 4*df} - 2*f*x)] / (2*\sqrt{2}*\sqrt{c*(e^2 - 2*df + e*\sqrt{e^2 - 4*df}) + f*(2*af - b*(e + \sqrt{e^2 - 4*df}))}) * \sqrt{a + x*(b + c*x)}}{(c*(e^2 - 2*df + e*\sqrt{e^2 - 4*df}) + f*(2*af - b*(e + \sqrt{e^2 - 4*df})))^{3/2} - (\sqrt{2}*f^2*\sqrt{c*(e^2 - 2*df - e*\sqrt{e^2 - 4*df}) + f*(2*af + b*(-e + \sqrt{e^2 - 4*df}))}) * \text{ArcTanh}[(4*af + 2*c*(-e + \sqrt{e^2 - 4*df}))*x + b*(-e + \sqrt{e^2 - 4*df} + 2*f*x)] / (2*\sqrt{2}*\sqrt{c*(e^2 - 2*df - e*\sqrt{e^2 - 4*df}) + f*(2*af + b*(-e + \sqrt{e^2 - 4*df}))}) * \sqrt{a + x*(b + c*x)}}}{(c*(-e^2 + 2*df + e*\sqrt{e^2 - 4*df}) + f*(-2*af + b*(e - \sqrt{e^2 - 4*df})))^2} / \sqrt{e^2 - 4*df}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage2

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] sage2

maple [B] time = 0.02, size = 4099, normalized size = 6.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)

[Out]
$$-2/(-4*df+e^2)^{1/2}/(2*af^2-b*ef-2*c*df+c*e^2-(-4*df+e^2)^{1/2}*bf+(-4*df+e^2)^{1/2}*c*e)*f^2/((x+1/2*(e+(-4*df+e^2)^{1/2}))/f)^2*c+(bf-c*e-(-4*df+e^2)^{1/2}*c)*(x+1/2*(e+(-4*df+e^2)^{1/2}))/f/f+1/2*(2*af^2-b*ef-2*c*df+c*e^2-(-4*df+e^2)^{1/2}*bf+(-4*df+e^2)^{1/2}*c*e)/f^2)^{1/2}-4*af/(2*af^2-b*ef-2*c*df+c*e^2-(-4*df+e^2)^{1/2}*bf+(-4*df+e^2)^{1/2}*c*e)/(4*ac-4*c^2*d/f+c^2*e^2/f^2-(-4*df+e^2)*c^2/f^2-b^2)/((x+1/2*(e+(-4*df+e^2)^{1/2}))/f)^2*c+(bf-c*e-(-4*df+e^2)^{1/2}*c)*(x+1/2*(e+(-4*df+e^2)^{1/2}))/f/f+1/2*(2*af^2-b*ef-2*c*df+c*e^2-(-4*df+e^2)^{1/2}*bf+(-4*df+e^2)^{1/2}*c*e)$$

$$\begin{aligned}
 & a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^{2} \\
 & ^{(1/2)}*x*b*c+4/(-4*d*f+e^2)^{(1/2)}*f/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+ \\
 & e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f \\
 & +e^2)*c^2/f^2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c+(b*f-c*e+(-4*d*f+ \\
 & e^2)^{(1/2)}*c)*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d* \\
 & f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*x*c^2*e-2 \\
 & *f/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c \\
 & *e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2-b^2)/((x-1/2*(-e+(-4* \\
 & d*f+e^2)^{(1/2)})/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^{(1/2)}*c)*(x-1/2*(-e+(-4*d*f+e^ \\
 & 2)^{(1/2)})/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4* \\
 & d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*b*c-2/(-4*d*f+e^2)^{(1/2)}*f^2/(2*a*f^2-b*e*f- \\
 & 2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/(4*a*c-4*c^2*d \\
 & /f+c^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f) \\
 & ^2*c+(b*f-c*e+(-4*d*f+e^2)^{(1/2)}*c)*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)/f+1/2 \\
 & *(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e \\
 &)/f^2)^{(1/2)}*b^2+2/(-4*d*f+e^2)^{(1/2)}*f/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d* \\
 & f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d \\
 & *f+e^2)*c^2/f^2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c+(b*f-c*e+(-4*d* \\
 & f+e^2)^{(1/2)}*c)*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)/f+1/2*(2*a*f^2-b*e*f-2*c* \\
 & d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*b*c*e-2 \\
 & /(-4*d*f+e^2)^{(1/2)}/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4 \\
 & *d*f+e^2)^{(1/2)}*c*e)*f^2*2^{(1/2)}/((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2) \\
 & ^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*\ln(((b*f-c*e+(-4*d*f+e^2)^{(1/ \\
 & 2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4* \\
 & d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(1/2)}*((2*a*f^2-b*e*f- \\
 & 2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4* \\
 & (x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c+4*(b*f-c*e+(-4*d*f+e^2)^{(1/2)}*c)*(x-1 \\
 & /2*(-e+(-4*d*f+e^2)^{(1/2)})/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2) \\
 & ^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)})/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2) \\
 &))/f))
 \end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details)Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(cx^2 + bx + a)^{3/2} (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)

[Out] int(1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d), x)

[Out] Timed out

$$3.125 \quad \int \frac{1}{x(a+bx+cx^2)^{3/2}(d+ex+fx^2)} dx$$

Optimal. Leaf size=816

$$\frac{2(b^2 + cxb - 2ac)}{a(b^2 - 4ac)d\sqrt{cx^2 + bx + a}} - \frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{cx^2+bx+a}}\right)}{a^{3/2}d} + \frac{f((e - \sqrt{e^2 - 4df})(f(be - af) - c(e^2 - df)) - 2(f(be - af) - c(e^2 - df)))}{\sqrt{2}d\sqrt{e^2 - 4df}((cd - af)^2 - (bd - af)^2)}$$

[Out] $-\operatorname{arctanh}\left(\frac{1/2*(b*x+2*a)/a^{1/2}/(c*x^2+b*x+a)^{1/2}}{a^{3/2}/d+2*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/d/(c*x^2+b*x+a)^{1/2}+2*(c*e*(2*a*c*e-b*(a*f+c*d))+(-a*f+b*e)*(2*c^2*d+b^2*f-c*(2*a*f+b*e))+c*(2*c^2*d*e+b*f*(-a*f+b*e)-b*c*(d*f+e^2))*x)/(-4*a*c+b^2)/d/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))/(c*x^2+b*x+a)^{1/2}+1/2*f*\operatorname{arctanh}\left(\frac{1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^{1/2})))-b*(e-(-4*d*f+e^2)^{1/2})}{(c*x^2+b*x+a)^{1/2}/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^{1/2})^{1/2}}\right)+2*c*(-2*d*e*f+e^3)+(f*(-a*f+b*e)-c*(-d*f+e^2))*(e-(-4*d*f+e^2)^{1/2})/d/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))*2^{1/2}/(-4*d*f+e^2)^{1/2}/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^{1/2})^{1/2}-1/2*f*\operatorname{arctanh}\left(\frac{1/4*(4*a*f-b*(e+(-4*d*f+e^2)^{1/2}))+2*x*(b*f-c*(e+(-4*d*f+e^2)^{1/2}))}{(c*x^2+b*x+a)^{1/2}/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^{1/2})^{1/2}}\right)+2*c*(-2*d*e*f+e^3)+(f*(-a*f+b*e)-c*(-d*f+e^2))*(e+(-4*d*f+e^2)^{1/2})/d/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))*2^{1/2}/(-4*d*f+e^2)^{1/2}/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^{1/2})^{1/2}\right)$

Rubi [A] time = 15.92, antiderivative size = 814, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {6728, 740, 12, 724, 206, 1016, 1032}

$$\frac{2(b^2 + cxb - 2ac)}{a(b^2 - 4ac)d\sqrt{cx^2 + bx + a}} - \frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{cx^2+bx+a}}\right)}{a^{3/2}d} - \frac{f(2f(be^2 - afe - bdf) - 2c(e^3 - 2def) - (e - \sqrt{e^2 - 4df}))}{\sqrt{2}d\sqrt{e^2 - 4df}((cd - af)^2 - (bd - af)^2)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}\left[\frac{1}{x*(a + b*x + c*x^2)^{3/2}*(d + e*x + f*x^2)}, x\right]$

[Out] $\frac{2*(b^2 - 2*a*c + b*c*x)}{a*(b^2 - 4*a*c)*d*\operatorname{Sqrt}[a + b*x + c*x^2]} + \frac{2*(c*e*(2*a*c*e - b*(c*d + a*f)) + (b*e - a*f)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(2*c^2*d*e + b*f*(b*e - a*f) - b*c*(e^2 + d*f))*x}{(b^2 - 4*a*c)*d*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*\operatorname{Sqrt}[a + b*x + c*x^2]} - \operatorname{ArcTan}\left[\frac{2a+bx}{2\sqrt{a}\sqrt{cx^2+bx+a}}\right]$

$$\frac{h[(2*a + b*x)/(2*sqrt[a]*sqrt[a + b*x + c*x^2])]/(a^{(3/2)*d}) - (f*(2*f*(b*e^2 - b*d*f - a*e*f) - 2*c*(e^3 - 2*d*e*f) - (e - sqrt[e^2 - 4*d*f])*(f*(b*e - a*f) - c*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e - sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - sqrt[e^2 - 4*d*f]))*x]/(2*sqrt[2]*sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*sqrt[e^2 - 4*d*f]]*sqrt[a + b*x + c*x^2])]/(sqrt[2]*d*sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*sqrt[e^2 - 4*d*f]]) + (f*(2*f*(b*e^2 - b*d*f - a*e*f) - 2*c*(e^3 - 2*d*e*f) - (e + sqrt[e^2 - 4*d*f])*(f*(b*e - a*f) - c*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e + sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + sqrt[e^2 - 4*d*f]))*x]/(2*sqrt[2]*sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*sqrt[e^2 - 4*d*f]]*sqrt[a + b*x + c*x^2])]/(sqrt[2]*d*sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*sqrt[e^2 - 4*d*f]])$$

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1016

```

Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1) * (d + e*x + f*x^2)^(q + 1) * (g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*c*d - 2*a*c*e + a*b*f))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !(IntegerQ[p] && ILtQ[q, -1])

```

Rule 1032

```

Int[((g_.) + (h_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]

```

Rule 6728

```

Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(a+bx+cx^2)^{3/2}(d+ex+fx^2)} dx &= \int \left(\frac{1}{dx(a+bx+cx^2)^{3/2}} + \frac{-e-fx}{d(a+bx+cx^2)^{3/2}(d+ex+fx^2)} \right) dx \\
&= \frac{\int \frac{1}{x(a+bx+cx^2)^{3/2}} dx}{d} + \frac{\int \frac{-e-fx}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)} dx}{d} \\
&= \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)d\sqrt{a+bx+cx^2}} + \frac{2(ce(2ace-b(cd+af))+(be-af)(2(b^2-4ac)d\sqrt{a+bx+cx^2}))}{(b^2-4ac)d\sqrt{a+bx+cx^2}} \\
&= \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)d\sqrt{a+bx+cx^2}} + \frac{2(ce(2ace-b(cd+af))+(be-af)(2(b^2-4ac)d\sqrt{a+bx+cx^2}))}{(b^2-4ac)d\sqrt{a+bx+cx^2}} \\
&= \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)d\sqrt{a+bx+cx^2}} + \frac{2(ce(2ace-b(cd+af))+(be-af)(2(b^2-4ac)d\sqrt{a+bx+cx^2}))}{(b^2-4ac)d\sqrt{a+bx+cx^2}} \\
&= \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)d\sqrt{a+bx+cx^2}} + \frac{2(ce(2ace-b(cd+af))+(be-af)(2(b^2-4ac)d\sqrt{a+bx+cx^2}))}{(b^2-4ac)d\sqrt{a+bx+cx^2}}
\end{aligned}$$

Mathematica [A] time = 6.56, size = 1121, normalized size = 1.37

$$\frac{16\sqrt{2} \left(\frac{ef}{\sqrt{e^2-4df}} + f \right) \sqrt{ce^2 - bfe - c\sqrt{e^2-4df}e + 2af^2 - 2cdf + bf\sqrt{e^2-4df}} (cx^2 + bx + a)^{3/2} \tanh^{-1} \left(\frac{ef}{2\sqrt{2}\sqrt{ce^2 - bfe - c\sqrt{e^2-4df}e + 2af^2 - 2cdf + bf\sqrt{e^2-4df}}} \right)}{d(4af^2 - 2b(e - \sqrt{e^2-4df})f + c(e - \sqrt{e^2-4df})^2) (16af^2 - 8b(e - \sqrt{e^2-4df})f + 4c(e - \sqrt{e^2-4df})^2)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

[Out] (2*(b^2 - 2*a*c + b*c*x)*(a + b*x + c*x^2))/(a*(b^2 - 4*a*c)*d*(a + x*(b + c*x))^(3/2)) - (2*f*(1 + e/Sqrt[e^2 - 4*d*f]))*(2*b^2*f - 4*a*c*f - b*c*(e - Sqrt[e^2 - 4*d*f]) + 2*c*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)*(a + b*x + c*x^2)/((b^2 - 4*a*c)*d*(4*a*f^2 - 2*b*f*(e - Sqrt[e^2 - 4*d*f]) + c*(e - Sqrt[e^2 - 4*d*f])^2)*(a + x*(b + c*x))^(3/2)) - (2*f*(1 - e/Sqrt[e^2 - 4*d*f]))*(2*b^2*f - 4*a*c*f - b*c*(e + Sqrt[e^2 - 4*d*f]) + 2*c*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)*(a + b*x + c*x^2)/((b^2 - 4*a*c)*d*(4*a*f^2 - 2*b*f*(

$$\begin{aligned}
& e + \sqrt{e^2 - 4df}) + c(e + \sqrt{e^2 - 4df})^2(a + x(b + cx))^{3/2} \\
& - ((a + bx + cx^2)^{3/2} \operatorname{ArcTanh}[(2a + bx)/(2\sqrt{a}\sqrt{a + bx + cx^2})]) / (a^{3/2}d(a + x(b + cx))^{3/2}) + (16\sqrt{2}f^2(f + ef) / \sqrt{e^2 - 4df}) \sqrt{ce^2 - 2cdf - b^2ef + 2af^2 - ce\sqrt{e^2 - 4df} + bf\sqrt{e^2 - 4df}} (a + bx + cx^2)^{3/2} \operatorname{ArcTanh}[(4af - b(e - \sqrt{e^2 - 4df}) - (-2bf + 2c(e - \sqrt{e^2 - 4df})))x] / (2\sqrt{2}\sqrt{ce^2 - 2cdf - b^2ef + 2af^2 - ce\sqrt{e^2 - 4df} + bf\sqrt{e^2 - 4df}} \sqrt{a + bx + cx^2}) / (d(4af^2 - 2bf(e - \sqrt{e^2 - 4df}) + c(e - \sqrt{e^2 - 4df})^2)(16af^2 - 8bf(e - \sqrt{e^2 - 4df}) + 4c(e - \sqrt{e^2 - 4df})^2)(a + x(b + cx))^{3/2}) - (16\sqrt{2}f^2(-f + ef) / \sqrt{e^2 - 4df}) \sqrt{ce^2 - 2cdf - b^2ef + 2af^2 + ce\sqrt{e^2 - 4df} - bf\sqrt{e^2 - 4df}} (a + bx + cx^2)^{3/2} \operatorname{ArcTanh}[(4af - b(e + \sqrt{e^2 - 4df}) - (-2bf + 2c(e + \sqrt{e^2 - 4df})))x] / (2\sqrt{2}\sqrt{ce^2 - 2cdf - b^2ef + 2af^2 + ce\sqrt{e^2 - 4df} - bf\sqrt{e^2 - 4df}} \sqrt{a + bx + cx^2}) / (d(4af^2 - 2bf(e + \sqrt{e^2 - 4df}) + c(e + \sqrt{e^2 - 4df})^2)(16af^2 - 8bf(e + \sqrt{e^2 - 4df}) + 4c(e + \sqrt{e^2 - 4df})^2)(a + x(b + cx))^{3/2})
\end{aligned}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage0x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] sage0*x

maple [B] time = 0.04, size = 4594, normalized size = 5.63

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)

[Out]
$$\frac{4f^3/(e+(-4df+e^2)^{1/2})/(-4df+e^2)^{1/2}/(2af^2-bef-2cdf+ce^2-(-4df+e^2)^{1/2}bf+(-4df+e^2)^{1/2}ce)/((x+1/2(e+(-4df+e^2)^{1/2}))/f)^2c+(bf-ce-(-4df+e^2)^{1/2}c)(x+1/2(e+(-4df+e^2)^{1/2}))/f)/f+1/2(2af^2-bef-2cdf+ce^2-(-4df+e^2)^{1/2}bf+(-4df+e^2)^{1/2}ce)/f^2)^{1/2}+8f^2/(e+(-4df+e^2)^{1/2})/(2af^2-bef-2cdf+ce^2-(-4df+e^2)^{1/2}bf+(-4df+e^2)^{1/2}ce)/(4ac-4c^2d/f+c^2e^2/f^2-(-4df+e^2)c^2/f^2-b^2)/((x+1/2(e+(-4df+e^2)^{1/2}))/f)^2c+(bf-ce-(-4df+e^2)^{1/2}c)(x+1/2(e+(-4df+e^2)^{1/2}))/f)/f+1/2(2af^2-bef-2cdf+ce^2-(-4df+e^2)^{1/2}bf+(-4df+e^2)^{1/2}ce)/f^2)^{1/2}xc^2-8f^3/(e+(-4df+e^2)^{1/2})/(-4df+e^2)^{1/2}/(2af^2-bef-2cdf+ce^2-(-4df+e^2)^{1/2}bf+(-4df+e^2)^{1/2}ce)/(4ac-4c^2d/f+c^2e^2/f^2-(-4df+e^2)c^2/f^2-b^2)/((x+1/2(e+(-4df+e^2)^{1/2}))/f)^2c+(bf-ce-(-4df+e^2)^{1/2}c)(x+1/2(e+(-4df+e^2)^{1/2}))/f)/f+1/2(2af^2-bef-2cdf+ce^2-(-4df+e^2)^{1/2}bf+(-4df+e^2)^{1/2}ce)/f^2)^{1/2}xb^2+8f^2/(e+(-4df+e^2)^{1/2})/(-4df+e^2)^{1/2}/(2af^2-bef-2cdf+ce^2-(-4df+e^2)^{1/2}bf+(-4df+e^2)^{1/2}ce)/(4ac-4c^2d/f+c^2e^2/f^2-(-4df+e^2)c^2/f^2-b^2)/((x+1/2(e+(-4df+e^2)^{1/2}))/f)^2c+(bf-ce-(-4df+e^2)^{1/2}c)(x+1/2(e+(-4df+e^2)^{1/2}))/f)/f+1/2(2af^2-bef-2cdf+ce^2-(-4df+e^2)^{1/2}bf+(-4df+e^2)^{1/2}ce)/f^2)^{1/2}b^2+4f^2/(e+(-4df+e^2)^{1/2})/(-4df+e^2)^{1/2}/(2af^2-bef-2cdf+ce^2-(-4df+e^2)^{1/2}bf+(-4df+e^2)^{1/2}ce)/(4ac-4c^2d/f+c^2e^2/f^2-(-4df+e^2)c^2/f^2-b^2)/((x+1/2(e+(-4df+e^2)^{1/2}))/f)^2c+(bf-ce-(-4df+e^2)^{1/2}c)(x+1/2(e+(-4df+e^2)^{1/2}))/f)/f+1/2(2af^2-bef-2cdf+ce^2-(-4df+e^2)^{1/2}bf+(-4df+e^2)^{1/2}ce)/f^2)^{1/2}b^2+4f^3/(e+(-4df+e^2)^{1/2})/(-4df+e^2)^{1/2}/(2af^2-bef-2cdf+ce^2-(-4df+e^2)^{1/2}bf+(-4df+e^2)^{1/2}ce)*2^{1/2}/((2af^2-bef-2cdf+ce^2-(-4df+e^2)^{1/2}bf+(-4df+e^2)^{1/2}ce)/f^2)^{1/2}*\ln(((bf-ce-(-4df+e^2)^{1/2}c)(x+1/2(e+(-4df+e^2)^{1/2}))/f)/f+(2af^2-bef-2cdf+ce^2-(-4df+e^2)^{1/2}bf+(-4df+e^2)^{1/2}ce)/f^2+1/2*2^{1/2}*((2af^2-bef-2cdf+ce^2-(-4df+e^2)^{1/2}bf+(-4df+e^2)^{1/2}ce)/f^2)^{1/2}*(4*(x+1/2(e+(-4df+e^2)^{1/2}))/f)^2c+4*(bf-ce-(-4df+e^2)^{1/2}c)(x+1/2(e+(-4df+e^2)^{1/2}))/f)/f+2*(2af^2-bef-2cdf+ce^2-(-4df+e^2)^{1/2}bf+(-4df+e^2)^{1/2}ce)/f^2)^{1/2})/(x+1/2(e+(-4df+e^2)^{1/2}))/f)+4f^3/(-e+(-4df+e^2)^{1/2})/(-4df+e^2)^{1/2}/(2af^2-bef-2cdf+ce^2+(-4df+e^2)^{1/2}bf-(-4df+e^2)^{1/2}ce)$$

$$\begin{aligned}
& (1/2)*c*e)/((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^{(1/2)} \\
& /2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+ \\
& (-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}-8*f^2/(-e+(-4*d*f \\
& +e^2)^{(1/2)})/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^ \\
& 2)^{(1/2)}*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2-b^2)/((x-1/ \\
& 2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^{(1/2)}*c)*(x-1/2*(-e+ \\
& (-4*d*f+e^2)^{(1/2)}))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)} \\
&)*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*x*c^2-8*f^3/(-e+(-4*d*f+e^2)^{(1/2)} \\
&)/(-4*d*f+e^2)^{(1/2)}/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(- \\
& 4*d*f+e^2)^{(1/2)}*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2-b^2 \\
&)/((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^{(1/2)}*c)*(x- \\
& 1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e \\
& ^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*x*b*c+8*f^2/(-e+(-4*d*f+e^ \\
& 2)^{(1/2)})/(-4*d*f+e^2)^{(1/2)}/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)} \\
&)*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2)*c^2 \\
& /f^2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^{(1/2)} \\
&)*c)*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(- \\
& 4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*x*c^2*e-4*f^2/(-e+ \\
& (-4*d*f+e^2)^{(1/2)})/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4 \\
& *d*f+e^2)^{(1/2)}*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2-b^2) \\
& /((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^{(1/2)}*c)*(x-1 \\
& /2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^ \\
& 2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*b*c-4*f^3/(-e+(-4*d*f+e^2)^ \\
& (1/2))/(-4*d*f+e^2)^{(1/2)}/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b \\
& *f-(-4*d*f+e^2)^{(1/2)}*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^ \\
& 2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^{(1/2)}*c \\
&)*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4* \\
& d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*b^2+4*f^2/(-e+(-4*d*f \\
& +e^2)^{(1/2)})/(-4*d*f+e^2)^{(1/2)}/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(\\
& 1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2)* \\
& c^2/f^2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^{(\\
& 1/2)}*c)*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^ \\
& 2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*b*c*e-4*f^3/(-e \\
& +(-4*d*f+e^2)^{(1/2)})/(-4*d*f+e^2)^{(1/2)}/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d* \\
& f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)*2^{(1/2)}/((2*a*f^2-b*e*f-2*c*d*f+c* \\
& e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*\ln(((b*f-c*e+ \\
& (-4*d*f+e^2)^{(1/2)}*c)*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)/f+(2*a*f^2-b*e*f-2* \\
& c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(1/2)}* \\
& ((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e \\
&)/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c+4*(b*f-c*e+(-4*d*f+e^ \\
& 2)^{(1/2)}*c)*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c* \\
& e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}/(x-1/2*(-e+ \\
& (-4*d*f+e^2)^{(1/2)}))/f)-4*f/(-e+(-4*d*f+e^2)^{(1/2)})/(e+(-4*d*f+e^2)^{(1/2)})/a \\
& /(c*x^2+b*x+a)^{(1/2)}+8*f/(-e+(-4*d*f+e^2)^{(1/2)})/(e+(-4*d*f+e^2)^{(1/2)})*b/a \\
& /(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*c*x+4*f/(-e+(-4*d*f+e^2)^{(1/2)})/(e+(-4*d*f
\end{aligned}$$

$$+e^2)^{1/2}) * b^2/a / (4*a*c - b^2) / (c*x^2 + b*x + a)^{1/2} + 4*f / (-e + (-4*d*f + e^2)^{1/2}) / (e + (-4*d*f + e^2)^{1/2}) / a^{3/2} * \ln((b*x + 2*a + 2*(c*x^2 + b*x + a)^{1/2} * a^{1/2}) / x)$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + bx + a)^{\frac{3}{2}} (fx^2 + ex + d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] integrate(1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 + e*x + d)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x(c x^2 + b x + a)^{3/2} (f x^2 + e x + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)

[Out] int(1/(x*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + bx + cx^2)^{\frac{3}{2}} (d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)

[Out] Integral(1/(x*(a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)

$$3.126 \quad \int \frac{x^4}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx$$

Optimal. Leaf size=140

$$-\frac{1}{4}\sqrt{-x^2-4x-3}x + \frac{5}{2}\sqrt{-x^2-4x-3} + \frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{5}{4}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right) +$$

[Out] 11/2*arcsin(2+x)-5/4*arctanh(x/(-x^2-4*x-3)^(1/2))+1/4*arctan(1/2*(1+(-3-x)/(-x^2-4*x-3)^(1/2))*2^(1/2))*2^(1/2)-1/4*arctan(1/2*(1+(3+x)/(-x^2-4*x-3)^(1/2))*2^(1/2))*2^(1/2)+5/2*(-x^2-4*x-3)^(1/2)-1/4*x*(-x^2-4*x-3)^(1/2)

Rubi [A] time = 0.50, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 14, integrand size = 30, number of rules / integrand size = 0.467, Rules used = {6728, 619, 216, 640, 742, 1028, 986, 12, 1026, 1161, 618, 204, 1027, 206}

$$-\frac{1}{4}\sqrt{-x^2-4x-3}x + \frac{5}{2}\sqrt{-x^2-4x-3} + \frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{5}{4}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right) +$$

Antiderivative was successfully verified.

[In] Int[x^4/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]

[Out] (5*Sqrt[-3 - 4*x - x^2])/2 - (x*Sqrt[-3 - 4*x - x^2])/4 + (11*ArcSin[2 + x])/2 + ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/(2*Sqrt[2]) - ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/(2*Sqrt[2]) - (5*ArcTanh[x/Sqrt[-3 - 4*x - x^2]])/4

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 742

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad raticQ[a, b, c, d, e, m, p, x]

Rule 986

Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -

$$\frac{bfx}{((a + bx + cx^2)\sqrt{d + ex + fx^2})}, x, x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b^2 - 4ac, 0] \&\& \text{NeQ}[e^2 - 4df, 0] \&\& \text{NeQ}[ce - bf, 0] \&\& \text{NegQ}[b^2 - 4ac]$$

Rule 1026

$$\text{Int}[(x_)/(((a_) + (b_)(x_) + (c_)(x_)^2)\sqrt{(d_) + (e_)(x_) + (f_)(x_)^2})], x_Symbol] := \text{Dist}[-2e, \text{Subst}[\text{Int}[(1 - dx^2)/(c_ - bfx - e(2cd - b^2e + 2af)x^2 + d^2(c_ - bfx)x^4)], x], x, (1 + ((e + \sqrt{e^2 - 4df})x)/(2d))/\sqrt{d + ex + fx^2}], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b^2 - 4ac, 0] \&\& \text{NeQ}[e^2 - 4df, 0] \&\& \text{EqQ}[bd - ae, 0]$$

Rule 1027

$$\text{Int}[(g_ + (h_)(x_))/((a_) + (b_)(x_) + (c_)(x_)^2)\sqrt{(d_) + (e_)(x_) + (f_)(x_)^2}], x_Symbol] := \text{Dist}[g, \text{Subst}[\text{Int}[1/(a + (cd - af)x^2)], x], x, x/\sqrt{d + ex + fx^2}], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h\}, x] \&\& \text{NeQ}[b^2 - 4ac, 0] \&\& \text{NeQ}[e^2 - 4df, 0] \&\& \text{EqQ}[bd - ae, 0] \&\& \text{EqQ}[2hd - ge, 0]$$

Rule 1028

$$\text{Int}[(g_ + (h_)(x_))/((a_) + (b_)(x_) + (c_)(x_)^2)\sqrt{(d_) + (e_)(x_) + (f_)(x_)^2}], x_Symbol] := -\text{Dist}[(2hd - ge)/e, \text{Int}[1/(a + bx + cx^2)\sqrt{d + ex + fx^2}], x], x] + \text{Dist}[h/e, \text{Int}[(2d + ex)/(a + bx + cx^2)\sqrt{d + ex + fx^2}], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h\}, x] \&\& \text{NeQ}[b^2 - 4ac, 0] \&\& \text{NeQ}[e^2 - 4df, 0] \&\& \text{EqQ}[bd - ae, 0] \&\& \text{NeQ}[2hd - ge, 0]$$

Rule 1161

$$\text{Int}[(d_ + (e_)(x_)^2)/((a_) + (b_)(x_)^2 + (c_)(x_)^4), x_Symbol] := \text{With}[\{q = \text{Rt}[(2d)/e - b/c, 2]\}, \text{Dist}[e/(2c), \text{Int}[1/\text{Simp}[d/e + qx + x^2], x], x] + \text{Dist}[e/(2c), \text{Int}[1/\text{Simp}[d/e - qx + x^2], x], x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[b^2 - 4ac, 0] \&\& \text{EqQ}[c^2d^2 - ae^2, 0] \&\& (\text{GtQ}[(2d)/e - b/c, 0] || (!\text{LtQ}[(2d)/e - b/c, 0] \&\& \text{EqQ}[d - e\text{Rt}[a/c, 2], 0]))$$

Rule 6728

$$\text{Int}[(u_)/((a_) + (b_)(x_)^{(n_)} + (c_)(x_)^{(2n_)}), x_Symbol] := \text{With}[\{v = \text{RationalFunctionExpand}[u/(a + bx^n + cx^{2n})], x], \text{Int}[v, x] /; \text{SumQ}[v] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{EqQ}[n2, 2n] \&\& \text{IGtQ}[n, 0]$$

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx &= \int \left(\frac{5}{4\sqrt{-3-4x-x^2}} - \frac{x}{\sqrt{-3-4x-x^2}} + \frac{x^2}{2\sqrt{-3-4x-x^2}} - \frac{x^3}{4\sqrt{-3-4x-x^2}} \right) dx \\
&= -\left(\frac{1}{4} \int \frac{15+8x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx \right) + \frac{1}{2} \int \frac{x^2}{\sqrt{-3-4x-x^2}} dx + \frac{5}{4} \int \frac{x^3}{\sqrt{-3-4x-x^2}} dx \\
&= \sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} - \frac{1}{4} \int \frac{3+6x}{\sqrt{-3-4x-x^2}} dx + \frac{1}{2} \int \frac{x^2}{\sqrt{-3-4x-x^2}} dx \\
&= \frac{5}{2}\sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} + \frac{5}{4} \sin^{-1}(2+x) + \frac{1}{8} \int \frac{1}{\sqrt{-3-4x-x^2}} dx \\
&= \frac{5}{2}\sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} + \frac{13}{4} \sin^{-1}(2+x) - \tanh^{-1}\left(\frac{1}{\sqrt{-3-4x-x^2}}\right) \\
&= \frac{5}{2}\sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} + \frac{11}{2} \sin^{-1}(2+x) - \frac{5}{4} \tanh^{-1}\left(\frac{1}{\sqrt{-3-4x-x^2}}\right) \\
&= \frac{5}{2}\sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} + \frac{11}{2} \sin^{-1}(2+x) - \frac{5}{4} \tanh^{-1}\left(\frac{1}{\sqrt{-3-4x-x^2}}\right) \\
&= \frac{5}{2}\sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} + \frac{11}{2} \sin^{-1}(2+x) - \frac{5}{4} \tanh^{-1}\left(\frac{1}{\sqrt{-3-4x-x^2}}\right) \\
&= \frac{5}{2}\sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} + \frac{11}{2} \sin^{-1}(2+x) + \frac{\tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{2\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 0.53, size = 210, normalized size = 1.50

$$\frac{1}{24} \left(-6\sqrt{-x^2-4x-3}x + 60\sqrt{-x^2-4x-3} - \sqrt{1-2i\sqrt{2}} (4\sqrt{2}+7i) \tanh^{-1}\left(\frac{-i\sqrt{2}x+2x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right) - \sqrt{2+4i\sqrt{2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(Sqrt[-3-4*x-x^2]*(3+4*x+2*x^2)),x]

[Out] $(60\sqrt{-3 - 4x - x^2} - 6x\sqrt{-3 - 4x - x^2} + 132\text{ArcSin}[2 + x] - \text{Sqrt}[1 - (2I)\sqrt{2}]) \cdot (7I + 4\sqrt{2}) \cdot \text{ArcTanh}[(2 - (2I)\sqrt{2} + 2x - I\sqrt{2})x / (\sqrt{2 + (4I)\sqrt{2}})\sqrt{-3 - 4x - x^2}]) - \text{Sqrt}[1 + (2I)\sqrt{2}] \cdot (-7I + 4\sqrt{2}) \cdot \text{ArcTanh}[(2 + (2I)\sqrt{2} + (2 + I\sqrt{2})x) / (\sqrt{2 - (4I)\sqrt{2}})\sqrt{-3 - 4x - x^2}]) / 24$

fricas [A] time = 0.99, size = 178, normalized size = 1.27

$$-\frac{1}{4}\sqrt{-x^2 - 4x - 3}(x - 10) + \frac{1}{8}\sqrt{2}\arctan\left(\frac{\sqrt{2}x + 3\sqrt{2}\sqrt{-x^2 - 4x - 3}}{2(2x + 3)}\right) + \frac{1}{8}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x - 3\sqrt{2}\sqrt{-x^2 - 4x - 3}}{2(2x + 3)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")`

[Out] $-1/4\sqrt{-x^2 - 4x - 3}(x - 10) + 1/8\sqrt{2}\arctan(1/2(\sqrt{2}x + 3\sqrt{2}\sqrt{-x^2 - 4x - 3})/(2x + 3)) + 1/8\sqrt{2}\arctan(-1/2(\sqrt{2}x - 3\sqrt{2}\sqrt{-x^2 - 4x - 3})/(2x + 3)) - 11/2\arctan(\sqrt{-x^2 - 4x - 3}(x + 2)/(x^2 + 4x + 3)) + 5/16\log(-(2\sqrt{-x^2 - 4x - 3})x + 4x + 3)/x^2) - 5/16\log((2\sqrt{-x^2 - 4x - 3})x - 4x - 3)/x^2)$

giac [A] time = 0.20, size = 188, normalized size = 1.34

$$-\frac{1}{4}\sqrt{-x^2 - 4x - 3}(x - 10) + \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3(\sqrt{-x^2 - 4x - 3} - 1)}{x + 2} + 1\right)\right) + \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2 - 4x - 3}}{x} - 1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")`

[Out] $-1/4\sqrt{-x^2 - 4x - 3}(x - 10) + 1/4\sqrt{2}\arctan(1/2\sqrt{2}*(3*(\sqrt{-x^2 - 4x - 3} - 1)/(x + 2) + 1)) + 1/4\sqrt{2}\arctan(1/2\sqrt{2}*((\sqrt{-x^2 - 4x - 3} - 1)/(x + 2) + 1)) + 11/2\arcsin(x + 2) - 5/8\log(2*(\sqrt{-x^2 - 4x - 3} - 1)/(x + 2) + 3*(\sqrt{-x^2 - 4x - 3} - 1)^2/(x + 2)^2 + 1) + 5/8\log(2*(\sqrt{-x^2 - 4x - 3} - 1)/(x + 2) + (\sqrt{-x^2 - 4x - 3} - 1)^2/(x + 2)^2 + 3))$

maple [A] time = 0.02, size = 159, normalized size = 1.14

$$-\frac{\sqrt{-x^2-4x-3}x}{4} + \frac{11 \arcsin(x+2)}{2} + \frac{5\sqrt{-x^2-4x-3}}{2} + \frac{\sqrt{3}\sqrt{4}\sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2}-12}}{\left(5 \operatorname{arctanh}\left(\frac{3x}{(-x-\frac{3}{2})\sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2}-12}}\right)\right)} + \frac{24\sqrt{\frac{x^2}{(-x-\frac{3}{2})^2}-4}}{\left(\frac{x}{-x-\frac{3}{2}}+1\right)^2}\left(\frac{x}{-x-\frac{3}{2}}+\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)`

[Out] `-1/4*x*(-x^2-4*x-3)^(1/2)+5/2*(-x^2-4*x-3)^(1/2)+11/2*arcsin(2+x)+1/24*3^(1/2)*4^(1/2)*(3/(-x-3/2)^2*x^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2))+5*arctanh(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(1/2)*x))/(1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^(1/2)/(1/(-x-3/2)*x+1)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(2x^2 + 4x + 3)\sqrt{-x^2 - 4x - 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")`

[Out] `integrate(x^4/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{-x^2 - 4x - 3} (2x^2 + 4x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/((-4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)`

[Out] `int(x^4/((-4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{-(x+1)(x+3)} (2x^2 + 4x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

$$3.127 \quad \int \frac{x^3}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx$$

Optimal. Leaf size=115

$$-\frac{1}{2}\sqrt{-x^2-4x-3} + \frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{2\sqrt{2}} + \tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right) - 2\sin^{-1}(x+2)$$

[Out] -2*arcsin(2+x)+arctanh(x/(-x^2-4*x-3)^(1/2))+1/4*arctan(1/2*(1+(-3-x)/(-x^2-4*x-3)^(1/2))*2^(1/2))*2^(1/2)-1/4*arctan(1/2*(1+(3+x)/(-x^2-4*x-3)^(1/2))*2^(1/2))*2^(1/2)-1/2*(-x^2-4*x-3)^(1/2)

Rubi [A] time = 0.42, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, integrand size = 30, number of rules / integrand size = 0.433, Rules used = {6728, 619, 216, 640, 1028, 986, 12, 1026, 1161, 618, 204, 1027, 206}

$$-\frac{1}{2}\sqrt{-x^2-4x-3} + \frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{2\sqrt{2}} + \tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right) - 2\sin^{-1}(x+2)$$

Antiderivative was successfully verified.

[In] Int[x^3/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]

[Out] -Sqrt[-3 - 4*x - x^2]/2 - 2*ArcSin[2 + x] + ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/(2*Sqrt[2]) - ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/(2*Sqrt[2]) + ArcTanh[x/Sqrt[-3 - 4*x - x^2]]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

$Q[a, 0] \parallel LtQ[b, 0]$

Rule 216

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[(\text{Rt}[-b, 2]*x)/\text{Sqrt}[a]]/\text{Rt}[-b, 2], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ \text{NegQ}[b]$

Rule 618

$\text{Int}[(a_) + (b_)*(x_) + (c_)*(x_)^2]^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 619

$\text{Int}[(a_) + (b_)*(x_) + (c_)*(x_)^2]^{(p_)}, x_Symbol] \rightarrow \text{Dist}[1/(2*c*((-4*c)/(b^2 - 4*a*c))^{p_}), \text{Subst}[\text{Int}[\text{Simp}[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{GtQ}[4*a - b^2/c, 0]$

Rule 640

$\text{Int}[(d_) + (e_)*(x_)]*(a_) + (b_)*(x_) + (c_)*(x_)^2]^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(e*(a + b*x + c*x^2)^{(p+1)})/(2*c*(p+1)), x] + \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[(a + b*x + c*x^2)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, p\}, x] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[p, -1]$

Rule 986

$\text{Int}[1/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*\text{Sqrt}[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)], 2\}, \text{Dist}[1/(2*q), \text{Int}[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)*\text{Sqrt}[d + e*x + f*x^2]), x], x] - \text{Dist}[1/(2*q), \text{Int}[(c*d - a*f - q + (c*e - b*f)*x)/((a + b*x + c*x^2)*\text{Sqrt}[d + e*x + f*x^2]), x], x]] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[e^2 - 4*d*f, 0] \ \&\& \ \text{NeQ}[c*e - b*f, 0] \ \&\& \ \text{NegQ}[b^2 - 4*a*c]$

Rule 1026

$\text{Int}[(x_)/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*\text{Sqrt}[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] \rightarrow \text{Dist}[-2*e, \text{Subst}[\text{Int}[(1 - d*x^2)/(c*e - b*f - e*(2*c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4], x], x, (1 + ((e + \text{Sqrt}[e^2 - 4*d*f])*x)/(2*d))/\text{Sqrt}[d + e*x + f*x^2]], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[e^2 - 4*d*f, 0] \ \&\& \ \text{EqQ}[b*d - a*e, 0]$

Rule 1027

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[g, Subst[Int[1/(a + (c*d - a*f)*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] && EqQ[2*h*d - g*e, 0]
```

Rule 1028

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := -Dist[(2*h*d - g*e)/e, Int[1/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] && NeQ[2*h*d - g*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; SumQ[v] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx &= \int \left(-\frac{1}{\sqrt{-3-4x-x^2}} + \frac{x}{2\sqrt{-3-4x-x^2}} + \frac{6+5x}{2\sqrt{-3-4x-x^2} (3+4x+2x^2)} \right) dx \\
&= \frac{1}{2} \int \frac{x}{\sqrt{-3-4x-x^2}} dx + \frac{1}{2} \int \frac{6+5x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx - \int \frac{1}{\sqrt{-3-4x-x^2}} dx \\
&= -\frac{1}{2} \sqrt{-3-4x-x^2} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}} dx, x, -4-2x \right) - \frac{5}{8} \int \frac{1}{\sqrt{-3-4x-x^2}} dx \\
&= -\frac{1}{2} \sqrt{-3-4x-x^2} - \sin^{-1}(2+x) + \frac{1}{8} \int \frac{-6-4x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx \\
&= -\frac{1}{2} \sqrt{-3-4x-x^2} - 2 \sin^{-1}(2+x) + \frac{5}{4} \tanh^{-1} \left(\frac{x}{\sqrt{-3-4x-x^2}} \right) + \frac{1}{2} \int \frac{1}{\sqrt{-3-4x-x^2}} dx \\
&= -\frac{1}{2} \sqrt{-3-4x-x^2} - 2 \sin^{-1}(2+x) + \tanh^{-1} \left(\frac{x}{\sqrt{-3-4x-x^2}} \right) + 4 \text{Subst} \left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}} dx, x, -4-2x \right) \\
&= -\frac{1}{2} \sqrt{-3-4x-x^2} - 2 \sin^{-1}(2+x) + \tanh^{-1} \left(\frac{x}{\sqrt{-3-4x-x^2}} \right) - \frac{1}{6} \text{Subst} \left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}} dx, x, -4-2x \right) \\
&= -\frac{1}{2} \sqrt{-3-4x-x^2} - 2 \sin^{-1}(2+x) + \tanh^{-1} \left(\frac{x}{\sqrt{-3-4x-x^2}} \right) + \frac{1}{3} \text{Subst} \left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}} dx, x, -4-2x \right) \\
&= -\frac{1}{2} \sqrt{-3-4x-x^2} - 2 \sin^{-1}(2+x) + \frac{\tan^{-1} \left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}} \right)}{2\sqrt{2}} - \frac{\tan^{-1} \left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}} \right)}{2\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 0.44, size = 192, normalized size = 1.67

$$\frac{1}{8} \left(-4 \left(\sqrt{-x^2-4x-3} + 4 \sin^{-1}(x+2) \right) + \frac{(5\sqrt{2}-2i) \tanh^{-1} \left(\frac{i\sqrt{2}x+2x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}} \sqrt{-x^2-4x-3}} \right)}{\sqrt{1-2i\sqrt{2}}} + \frac{(5\sqrt{2}+2i) \tanh^{-1} \left(\frac{(2-i)\sqrt{-x^2-4x-3}}{\sqrt{2+4i\sqrt{2}}} \right)}{\sqrt{1+2i\sqrt{2}}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)), x]

[Out] $(-4*(\text{Sqrt}[-3 - 4*x - x^2] + 4*\text{ArcSin}[2 + x]) + ((-2*I + 5*\text{Sqrt}[2])*\text{ArcTanh}[(2 + (2*I)*\text{Sqrt}[2] + 2*x + I*\text{Sqrt}[2]*x)/(\text{Sqrt}[2 - (4*I)*\text{Sqrt}[2]]*\text{Sqrt}[-3 - 4*x - x^2])])/\text{Sqrt}[1 - (2*I)*\text{Sqrt}[2]] + ((2*I + 5*\text{Sqrt}[2])*\text{ArcTanh}[(2 - (2*I)*\text{Sqrt}[2] + (2 - I*\text{Sqrt}[2])*x)/(\text{Sqrt}[2 + (4*I)*\text{Sqrt}[2]]*\text{Sqrt}[-3 - 4*x - x^2])])/\text{Sqrt}[1 + (2*I)*\text{Sqrt}[2]])/8$

fricas [A] time = 1.50, size = 175, normalized size = 1.52

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{\sqrt{2}x + 3\sqrt{2}\sqrt{-x^2 - 4x - 3}}{2(2x + 3)}\right) + \frac{1}{8}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x - 3\sqrt{2}\sqrt{-x^2 - 4x - 3}}{2(2x + 3)}\right) - \frac{1}{2}\sqrt{-x^2 - 4x - 3} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")`

[Out] $1/8*\text{sqrt}(2)*\arctan(1/2*(\text{sqrt}(2)*x + 3*\text{sqrt}(2)*\text{sqrt}(-x^2 - 4*x - 3))/(2*x + 3)) + 1/8*\text{sqrt}(2)*\arctan(-1/2*(\text{sqrt}(2)*x - 3*\text{sqrt}(2)*\text{sqrt}(-x^2 - 4*x - 3))/(2*x + 3)) - 1/2*\text{sqrt}(-x^2 - 4*x - 3) + 2*\arctan(\text{sqrt}(-x^2 - 4*x - 3)*(x + 2)/(x^2 + 4*x + 3)) - 1/4*\log(-2*\text{sqrt}(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2 + 1/4*\log((2*\text{sqrt}(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)$

giac [A] time = 0.25, size = 185, normalized size = 1.61

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3(\sqrt{-x^2 - 4x - 3} - 1)}{x + 2} + 1\right)\right) + \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2 - 4x - 3} - 1}{x + 2} + 1\right)\right) - \frac{1}{2}\sqrt{-x^2 - 4x - 3} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")`

[Out] $1/4*\text{sqrt}(2)*\arctan(1/2*\text{sqrt}(2)*(3*(\text{sqrt}(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/4*\text{sqrt}(2)*\arctan(1/2*\text{sqrt}(2)*((\text{sqrt}(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) - 1/2*\text{sqrt}(-x^2 - 4*x - 3) - 2*\arcsin(x + 2) + 1/2*\log(2*(\text{sqrt}(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(\text{sqrt}(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) - 1/2*\log(2*(\text{sqrt}(-x^2 - 4*x - 3) - 1)/(x + 2) + (\text{sqrt}(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)$

maple [A] time = 0.02, size = 144, normalized size = 1.25

$$-2 \arcsin(x+2) - \frac{\sqrt{-x^2-4x-3}}{2} + \frac{\sqrt{3} \sqrt{4} \sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2} - 12} \left(-4 \operatorname{arctanh} \left(\frac{3x}{(-x-\frac{3}{2}) \sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2} - 12}} \right) + \sqrt{2} \arctan \left(\sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2} - 12} \right) \right)}{24 \sqrt{\frac{\frac{x^2}{(-x-\frac{3}{2})^2} - 4}{\left(\frac{x}{-x-\frac{3}{2}} + 1\right)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2), x)`

[Out] `-1/2*(-x^2-4*x-3)^(1/2)-2*arcsin(x+2)+1/24*3^(1/2)*4^(1/2)*(3/(-x-3/2))^2*x^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2))-4*arctanh(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(1/2)*x))/((1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^2)^(1/2)/(1/(-x-3/2)*x+1)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(2x^2 + 4x + 3)\sqrt{-x^2 - 4x - 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2), x, algorithm="maxima")`

[Out] `integrate(x^3/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{-x^2 - 4x - 3} (2x^2 + 4x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/((-4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)`

[Out] `int(x^3/((-4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{-(x+1)(x+3)} (2x^2 + 4x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```


$$3.128 \quad \int \frac{x^2}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx$$

Optimal. Leaf size=98

$$-\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{1}{2} \tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right) + \frac{1}{2} \sin^{-1}(x+2)$$

[Out] 1/2*arcsin(2+x)-1/2*arctanh(x/(-x^2-4*x-3)^(1/2))-1/2*arctan(1/2*(1+(-3-x)/(-x^2-4*x-3)^(1/2))*2^(1/2))*2^(1/2)+1/2*arctan(1/2*(1+(3+x)/(-x^2-4*x-3)^(1/2))*2^(1/2))*2^(1/2)

Rubi [A] time = 0.20, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1077, 619, 216, 1028, 986, 12, 1026, 1161, 618, 204, 1027, 206}

$$-\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{1}{2} \tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right) + \frac{1}{2} \sin^{-1}(x+2)$$

Antiderivative was successfully verified.

[In] Int[x^2/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]

[Out] ArcSin[2 + x]/2 - ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2] + ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2] - ArcTanh[x/Sqrt[-3 - 4*x - x^2]]/2

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

$Q[a, 0] \parallel LtQ[b, 0]$

Rule 216

$Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /;$ FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 618

$Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^{-1}, x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /;$ FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 619

$Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^{p_}, x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /;$ FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 986

$Int[1/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e - b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /;$ FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e - b*f, 0] && NegQ[b^2 - 4*a*c]

Rule 1026

$Int[(x_)/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 - 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /;$ FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]

Rule 1027

$Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[g, Subst[Int[1/(a + (c*d - a*f)*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /;$ FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] && EqQ[2*h*d - g*e, 0]

Rule 1028

```
Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_)*(x_)^2]), x_Symbol] := -Dist[(2*h*d - g*e)/e, Int[1/((a + b
*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g
, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
&& NeQ[2*h*d - g*e, 0]
```

Rule 1077

```
Int[((A_) + (C_)*(x_)^2)/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) +
(e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C - b*C*x)/((a + b*x + c*x^2)*Sqrt[d
+ e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 1161

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx &= \frac{1}{2} \int \frac{1}{\sqrt{-3-4x-x^2}} dx + \frac{1}{2} \int \frac{-3-4x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx \\
&= -\left(\frac{1}{4} \text{Subst} \left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}} dx, x, -4-2x \right)\right) + \frac{1}{2} \int \frac{-6-4x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx \\
&= \frac{1}{2} \sin^{-1}(2+x) - \frac{1}{4} \int \frac{-6-4x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx + \frac{1}{4} \int \frac{-6-4x}{\sqrt{-3-4x-x^2}} dx \\
&= \frac{1}{2} \sin^{-1}(2+x) - \tanh^{-1} \left(\frac{x}{\sqrt{-3-4x-x^2}} \right) + \frac{3}{2} \text{Subst} \left(\int \frac{1}{3-3x^2} dx, x, -4-2x \right) \\
&= \frac{1}{2} \sin^{-1}(2+x) - \frac{1}{2} \tanh^{-1} \left(\frac{x}{\sqrt{-3-4x-x^2}} \right) - 8 \text{Subst} \left(\int \frac{1+3x^2}{-4-8x^2-36} dx, x, -4-2x \right) \\
&= \frac{1}{2} \sin^{-1}(2+x) - \frac{1}{2} \tanh^{-1} \left(\frac{x}{\sqrt{-3-4x-x^2}} \right) + \frac{1}{3} \text{Subst} \left(\int \frac{1}{\frac{1}{3} - \frac{2x}{3} + x^2} dx, x, -4-2x \right) \\
&= \frac{1}{2} \sin^{-1}(2+x) - \frac{1}{2} \tanh^{-1} \left(\frac{x}{\sqrt{-3-4x-x^2}} \right) - \frac{2}{3} \text{Subst} \left(\int \frac{1}{-\frac{8}{9} - x^2} dx, x, -4-2x \right) \\
&= \frac{1}{2} \sin^{-1}(2+x) - \frac{\tan^{-1} \left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}} \right)}{\sqrt{2}} + \frac{\tan^{-1} \left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}} \right)}{\sqrt{2}} - \frac{1}{2} \tanh^{-1} \left(\frac{x}{\sqrt{-3-4x-x^2}} \right)
\end{aligned}$$

Mathematica [C] time = 0.19, size = 159, normalized size = 1.62

$$\frac{1}{4} \left(-i\sqrt{1-2i\sqrt{2}} \tanh^{-1} \left(\frac{i\sqrt{2}x+2x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}} \sqrt{-x^2-4x-3}} \right) + i\sqrt{1+2i\sqrt{2}} \tanh^{-1} \left(\frac{(2-i\sqrt{2})x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}} \sqrt{-x^2-4x-3}} \right) + 2 \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(Sqrt[-3-4*x-x^2]*(3+4*x+2*x^2)),x]

[Out] (2*ArcSin[2+x] - I*Sqrt[1-(2*I)*Sqrt[2]]*ArcTanh[(2+(2*I)*Sqrt[2]+2*x+I*Sqrt[2]*x)/(Sqrt[2-(4*I)*Sqrt[2]]*Sqrt[-3-4*x-x^2]]) + I*Sqrt[1+(2*I)*Sqrt[2]]*ArcTanh[(2-(2*I)*Sqrt[2]+(2-I*Sqrt[2])*x)/(Sqrt[2+(4*I)*Sqrt[2]]*Sqrt[-3-4*x-x^2]])/4

fricas [A] time = 0.92, size = 161, normalized size = 1.64

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)-\frac{1}{4}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)-\frac{1}{2}\arctan\left(\frac{\sqrt{-x^2-4x-3}}{x+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")

[Out] -1/4*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) - 1/4*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) - 1/2*arctan(sqrt(-x^2 - 4*x - 3)*(x + 2)/(x^2 + 4*x + 3)) + 1/8*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) - 1/8*log((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)

giac [B] time = 0.23, size = 171, normalized size = 1.74

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3(\sqrt{-x^2-4x-3}-1)}{x+2}+1\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)+\frac{1}{2}\arcsin\left(\frac{x+2}{\sqrt{-x^2-4x-3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")

[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/2*arcsin(x + 2) - 1/4*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) + 1/4*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)

maple [A] time = 0.01, size = 130, normalized size = 1.33

$$\frac{\arcsin(x+2)}{2} \frac{\sqrt{3}\sqrt{4}\sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2}-12}\left(-\operatorname{arctanh}\left(\frac{3x}{(-x-\frac{3}{2})\sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2}-12}}\right)+\sqrt{2}\arctan\left(\frac{\sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2}-12}\sqrt{2}}{6}\right)\right)}{12\sqrt{\frac{\frac{x^2}{(-x-\frac{3}{2})^2}-4}{\left(\frac{x}{-x-\frac{3}{2}}+1\right)^2}}\left(\frac{x}{-x-\frac{3}{2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)

[Out] $\frac{1}{2} \arcsin(x+2) - \frac{1}{12} 3^{(1/2)} 4^{(1/2)} (3/(-x-3/2)^2 x^2 - 12)^{(1/2)} (2^{(1/2)} \arctan(1/6 * (3/(-x-3/2)^2 x^2 - 12)^{(1/2)} * 2^{(1/2)}) - \operatorname{arctanh}(3/(-x-3/2) / (3/(-x-3/2)^2 x^2 - 12)^{(1/2)} * x)) / ((1/(-x-3/2)^2 x^2 - 4) / (1/(-x-3/2) * x + 1)^2)^{(1/2)} / (1/(-x-3/2) * x + 1)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(2x^2 + 4x + 3)\sqrt{-x^2 - 4x - 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")`

[Out] `integrate(x^2/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{-x^2 - 4x - 3} (2x^2 + 4x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/((-4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)`

[Out] `int(x^2/((-4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-(x+1)(x+3)} (2x^2 + 4x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)`

[Out] `Integral(x**2/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)`

$$3.129 \quad \int \frac{x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx$$

Optimal. Leaf size=68

$$\frac{\tan^{-1}\left(\frac{\frac{3\sqrt{-x-1}}{\sqrt{x+3}}+1}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)}{\sqrt{2}}$$

[Out] $-1/2*\arctan(1/2*(1-3*(-1-x)^{(1/2)}/(3+x)^{(1/2)})*2^{(1/2)})*2^{(1/2)}+1/2*\arctan(1/2*(1+3*(-1-x)^{(1/2)}/(3+x)^{(1/2)})*2^{(1/2)})*2^{(1/2)}$

Rubi [A] time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1026, 1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]

[Out] ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2] - ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1026

Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4], x], x, (1 + ((e + Sqrt[e^2 -

$4*d*f))x)/(2*d))/\text{Sqrt}[d + e*x + f*x^2]], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[e^2 - 4*d*f, 0] \&\& \text{EqQ}[b*d - a*e, 0]$

Rule 1161

$\text{Int}[\frac{(d_.) + (e_.)*(x_)^2}{(a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4}, x_Symbol] :$
 $> \text{With}[\{q = \text{Rt}[(2*d)/e - b/c, 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& (\text{GtQ}[(2*d)/e - b/c, 0] || (!\text{LtQ}[(2*d)/e - b/c, 0] \&\& \text{EqQ}[d - e*\text{Rt}[a/c, 2], 0]))$

Rubi steps

$$\begin{aligned} \int \frac{x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx &= 8 \text{Subst} \left(\int \frac{1+3x^2}{-4-8x^2-36x^4} dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}} \right) \\ &= -\left(\frac{1}{3} \text{Subst} \left(\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2} dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}} \right) \right) - \frac{1}{3} \text{Subst} \left(\int \frac{1}{\frac{1}{3}+\frac{2x}{3}} \right) \\ &= \frac{2}{3} \text{Subst} \left(\int \frac{1}{-\frac{8}{9}-x^2} dx, x, \frac{2}{3} \left(-1 + \frac{3+x}{\sqrt{-3-4x-x^2}} \right) \right) + \frac{2}{3} \text{Subst} \left(\int \frac{1}{-\frac{8}{9}} \right) \\ &= \frac{\tan^{-1} \left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}} \right)}{\sqrt{2}} - \frac{\tan^{-1} \left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}} \right)}{\sqrt{2}} \end{aligned}$$

Mathematica [C] time = 0.17, size = 174, normalized size = 2.56

$$\frac{(1-i\sqrt{2})\sqrt{1-2i\sqrt{2}} \tanh^{-1} \left(\frac{(2-i\sqrt{2})x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}} \sqrt{-x^2-4x-3}} \right) + (1+i\sqrt{2})\sqrt{1+2i\sqrt{2}} \tanh^{-1} \left(\frac{(2+i\sqrt{2})x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}} \sqrt{-x^2-4x-3}} \right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)), x]

[Out] $((1 - I*\text{Sqrt}[2])* \text{Sqrt}[1 - (2*I)*\text{Sqrt}[2]]*\text{ArcTanh}[(2 - (2*I)*\text{Sqrt}[2] + (2 - I*\text{Sqrt}[2])*x)/(\text{Sqrt}[2 + (4*I)*\text{Sqrt}[2]]*\text{Sqrt}[-3 - 4*x - x^2])] + (1 + I*\text{Sqrt}[2])* \text{Sqrt}[1 + (2*I)*\text{Sqrt}[2]]*\text{ArcTanh}[(2 + (2*I)*\text{Sqrt}[2] + (2 + I*\text{Sqrt}[2])*x)/(\text{Sqrt}[2 - (4*I)*\text{Sqrt}[2]]*\text{Sqrt}[-3 - 4*x - x^2])])/(6*\text{Sqrt}[2])$

fricas [A] time = 0.88, size = 50, normalized size = 0.74

$$\frac{1}{4} \sqrt{2} \arctan \left(\frac{\sqrt{2} (6x^2 + 20x + 15) \sqrt{-x^2 - 4x - 3}}{4(2x^3 + 11x^2 + 18x + 9)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")

[Out] 1/4*sqrt(2)*arctan(1/4*sqrt(2)*(6*x^2 + 20*x + 15)*sqrt(-x^2 - 4*x - 3)/(2*x^3 + 11*x^2 + 18*x + 9))

giac [A] time = 0.18, size = 68, normalized size = 1.00

$$\frac{1}{2} \sqrt{2} \arctan \left(\frac{1}{2} \sqrt{2} \left(\frac{3(\sqrt{-x^2 - 4x - 3} - 1)}{x + 2} + 1 \right) \right) + \frac{1}{2} \sqrt{2} \arctan \left(\frac{1}{2} \sqrt{2} \left(\frac{\sqrt{-x^2 - 4x - 3} - 1}{x + 2} + 1 \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")

[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))

maple [A] time = 0.01, size = 92, normalized size = 1.35

$$\frac{\sqrt{3} \sqrt{4} \sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2} - 12} \sqrt{2} \arctan \left(\frac{\sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2} - 12} \sqrt{2}}{6} \right)}{12 \sqrt{\frac{\frac{x^2}{(-x-\frac{3}{2})^2} - 4}{\left(\frac{x}{-x-\frac{3}{2}} + 1\right)^2}} \left(\frac{x}{-x-\frac{3}{2}} + 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)

[Out] 1/12*3^(1/2)*4^(1/2)/((1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^2)^(1/2)/(1/(-x-3/2)*x+1)*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2)*arctan(1/6*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(2x^2 + 4x + 3)\sqrt{-x^2 - 4x - 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")

[Out] integrate(x/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{-x^2 - 4x - 3} (2x^2 + 4x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((-4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)

[Out] int(x/((-4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-(x+1)(x+3)} (2x^2 + 4x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)

[Out] Integral(x/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)

$$3.130 \quad \int \frac{1}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx$$

Optimal. Leaf size=95

$$-\frac{1}{3}\sqrt{2} \tan^{-1}\left(\frac{1 - \frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right) + \frac{1}{3}\sqrt{2} \tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}} + 1}{\sqrt{2}}\right) + \frac{1}{3} \tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

[Out] 1/3*arctanh(x/(-x^2-4*x-3)^(1/2))-1/3*arctan(1/2*(1+(-3-x)/(-x^2-4*x-3)^(1/2))*2^(1/2))*2^(1/2)+1/3*arctan(1/2*(1+(3+x)/(-x^2-4*x-3)^(1/2))*2^(1/2))*2^(1/2)

Rubi [A] time = 0.11, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {986, 12, 1026, 1161, 618, 204, 1027, 206}

$$-\frac{1}{3}\sqrt{2} \tan^{-1}\left(\frac{1 - \frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right) + \frac{1}{3}\sqrt{2} \tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}} + 1}{\sqrt{2}}\right) + \frac{1}{3} \tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]

[Out] -(Sqrt[2]*ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/3 + (Sqrt[2]*ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/3 + ArcTanh[x/Sqrt[-3 - 4*x - x^2]]/3

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e - b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e - b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1026

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 - 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1027

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[g, Subst[Int[1/(a + (c*d - a*f)*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] && EqQ[2*h*d - g*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx &= -\left(\frac{1}{6} \int \frac{-6-4x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx\right) + \frac{1}{6} \int -\frac{4x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx \\
&= -\left(\frac{2}{3} \int \frac{x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx\right) + \text{Subst}\left(\int \frac{1}{3-3x^2} dx, x, \frac{x}{\sqrt{-3-4x-x^2}}\right) \\
&= \frac{1}{3} \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - \frac{16}{3} \text{Subst}\left(\int \frac{1+3x^2}{-4-8x^2-36x^4} dx, x, \frac{x}{\sqrt{-3-4x-x^2}}\right) \\
&= \frac{1}{3} \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{2}{9} \text{Subst}\left(\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2} dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right) \\
&= \frac{1}{3} \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - \frac{4}{9} \text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2} dx, x, \frac{2}{3}\left(-1+\frac{x}{\sqrt{-3-4x-x^2}}\right)\right) \\
&= -\frac{1}{3}\sqrt{2} \tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) + \frac{1}{3}\sqrt{2} \tan^{-1}\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) + \frac{1}{3} \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)
\end{aligned}$$

Mathematica [C] time = 0.10, size = 150, normalized size = 1.58

$$\frac{1}{6}i \left(\sqrt{1-2i\sqrt{2}} \tanh^{-1}\left(\frac{(2-i\sqrt{2})x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right) - \sqrt{1+2i\sqrt{2}} \tanh^{-1}\left(\frac{(2+i\sqrt{2})x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)), x]

[Out] (I/6)*(Sqrt[1 - (2*I)*Sqrt[2]]*ArcTanh[(2 - (2*I)*Sqrt[2] + (2 - I*Sqrt[2])*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])] - Sqrt[1 + (2*I)*Sqrt[2]]*ArcTanh[(2 + (2*I)*Sqrt[2] + (2 + I*Sqrt[2])*x)/(Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])])

fricas [A] time = 0.95, size = 132, normalized size = 1.39

$$-\frac{1}{6}\sqrt{2} \arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right) - \frac{1}{6}\sqrt{2} \arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right) - \frac{1}{12} \log\left(-\frac{2\sqrt{-x^2-4x-3}}{2x+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2), x, algorithm="fricas")

[Out] $-\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{-x^2-4x-3}\right)+3\sqrt{2}\sqrt{-x^2-4x-3}\right)/(2x+3) - \frac{1}{6}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{-x^2-4x-3}\right)-3\sqrt{2}\sqrt{-x^2-4x-3}\right)/(2x+3) - \frac{1}{12}\log\left(-\frac{2\sqrt{-x^2-4x-3}x+4x+3}{x^2}\right) + \frac{1}{12}\log\left(\frac{2\sqrt{-x^2-4x-3}x-4x-3}{x^2}\right)$

giac [B] time = 0.19, size = 165, normalized size = 1.74

$$-\frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)-\frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)+\frac{1}{6}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)-\frac{1}{6}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}+1\right)}{x+2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")`

[Out] $-\frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right) - \frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right) + \frac{1}{6}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right) + \frac{3\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2+1} - \frac{1}{6}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\sqrt{-x^2-4x-3}-1\right)^2/(x+2)^2+3$

maple [A] time = 0.01, size = 121, normalized size = 1.27

$$\frac{\sqrt{3}\sqrt{4}\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\left(\operatorname{arctanh}\left(\frac{3x}{\left(-x-\frac{3}{2}\right)\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}}\right)+\sqrt{2}\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\sqrt{2}}{6}\right)\right)}{18\sqrt{\frac{\frac{x^2}{\left(-x-\frac{3}{2}\right)^2}-4}{\left(\frac{x}{-x-\frac{3}{2}}+1\right)^2}}\left(\frac{x}{-x-\frac{3}{2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)`

[Out] $-\frac{1}{18}3^{1/2}4^{1/2}\left(\frac{3}{(-x-3/2)^2x^2-12}\right)^{1/2}\left(2^{1/2}\arctan\left(\frac{1}{6}\left(\frac{3}{(-x-3/2)^2x^2-12}\right)^{1/2}2^{1/2}\right)+\operatorname{arctanh}\left(\frac{3}{(-x-3/2)}\right)/\left(\frac{3}{(-x-3/2)^2x^2-12}\right)^{1/2}\right)x/\left(\frac{1}{(-x-3/2)^2x^2-4}/\left(\frac{1}{(-x-3/2)x+1}\right)^2\right)^{1/2}/\left(\frac{1}{(-x-3/2)x+1}\right)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2x^2+4x+3)\sqrt{-x^2-4x-3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-x^2 - 4x - 3} (2x^2 + 4x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)

[Out] int(1/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-(x+1)(x+3)} (2x^2 + 4x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)

[Out] Integral(1/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)

$$3.131 \quad \int \frac{1}{x\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx$$

Optimal. Leaf size=130

$$-\frac{\tan^{-1}\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right)}{3\sqrt{3}} + \frac{1}{9}\sqrt{2} \tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right) - \frac{1}{9}\sqrt{2} \tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right) - \frac{4}{9} \tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

[Out] -4/9*arctanh(x/(-x^2-4*x-3)^(1/2))+1/9*arctan(1/2*(1+(-3-x)/(-x^2-4*x-3)^(1/2))*2^(1/2))*2^(1/2)-1/9*arctan(1/2*(1+(3+x)/(-x^2-4*x-3)^(1/2))*2^(1/2))*2^(1/2)-1/9*arctan(1/3*(3+2*x)*3^(1/2)/(-x^2-4*x-3)^(1/2))*3^(1/2)

Rubi [A] time = 0.42, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {6728, 724, 204, 1028, 986, 12, 1026, 1161, 618, 1027, 206}

$$-\frac{\tan^{-1}\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right)}{3\sqrt{3}} + \frac{1}{9}\sqrt{2} \tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right) - \frac{1}{9}\sqrt{2} \tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right) - \frac{4}{9} \tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]

[Out] -ArcTan[(3 + 2*x)/(Sqrt[3]*Sqrt[-3 - 4*x - x^2])]/(3*Sqrt[3]) + (Sqrt[2]*ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/9 - (Sqrt[2]*ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/9 - (4*ArcTanh[x/Sqrt[-3 - 4*x - x^2]])/9

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 986

Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e - b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e - b*f, 0] && NegQ[b^2 - 4*a*c]

Rule 1026

Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 - 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]

Rule 1027

Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[g, Subst[Int[1/(a + (c*d - a*f)*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] && EqQ[2*h*d - g*e, 0]

Rule 1028

Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := -Dist[(2*h*d - g*e)/e, Int[1/((a + b

```
*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g
, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
&& NeQ[2*h*d - g*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx &= \int \left(\frac{1}{3x\sqrt{-3-4x-x^2}} - \frac{2(2+x)}{3\sqrt{-3-4x-x^2}(3+4x+2x^2)} \right) dx \\
&= \frac{1}{3} \int \frac{1}{x\sqrt{-3-4x-x^2}} dx - \frac{2}{3} \int \frac{2+x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx \\
&= \frac{1}{6} \int \frac{-6-4x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx - \frac{1}{3} \int \frac{1}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx \\
&= -\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{1}{18} \int \frac{-6-4x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx - \frac{1}{18} \int \frac{1}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx \\
&= -\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} - \frac{1}{3} \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{2}{9} \int \frac{1}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx \\
&= -\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} - \frac{4}{9} \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{16}{9} \text{Subst}\left(\int \frac{1}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx\right) \\
&= -\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} - \frac{4}{9} \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - \frac{2}{27} \text{Subst}\left(\int \frac{1}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx\right) \\
&= -\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} - \frac{4}{9} \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{4}{27} \text{Subst}\left(\int \frac{1}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx\right) \\
&= -\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{1}{9} \sqrt{2} \tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) - \frac{1}{9} \sqrt{2} \tan^{-1}\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)
\end{aligned}$$

Mathematica [C] time = 0.44, size = 200, normalized size = 1.54

$$\frac{1}{54} \left(-6\sqrt{3} \tan^{-1}\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right) - 3\sqrt{1-2i\sqrt{2}} (\sqrt{2}+2i) \tanh^{-1}\left(\frac{(2-i\sqrt{2})x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right) - 3\sqrt{1+2i\sqrt{2}} (\sqrt{2}-2i) \tanh^{-1}\left(\frac{(2+i\sqrt{2})x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)), x]

[Out] (-6*sqrt[3]*ArcTan[(3 + 2*x)/(sqrt[3]*sqrt[-3 - 4*x - x^2]]) - 3*sqrt[1 - (2*I)*sqrt[2]]*(2*I + sqrt[2])*ArcTanh[(2 - (2*I)*sqrt[2] + (2 - I*sqrt[2]))*sqrt[-3 - 4*x - x^2]] - 3*sqrt[1 + (2*I)*sqrt[2]]*(2*I - sqrt[2])*ArcTanh[(2 + (2*I)*sqrt[2] + (2 + I*sqrt[2]))*sqrt[-3 - 4*x - x^2]])/54

$x)/(\text{Sqrt}[2 + (4*I)*\text{Sqrt}[2]]*\text{Sqrt}[-3 - 4*x - x^2]) - 3*\text{Sqrt}[1 + (2*I)*\text{Sqrt}[2]]*(-2*I + \text{Sqrt}[2])*ArcTanh[(2 + (2*I)*\text{Sqrt}[2] + (2 + I*\text{Sqrt}[2])*x)/(\text{Sqrt}[2 - (4*I)*\text{Sqrt}[2]]*\text{Sqrt}[-3 - 4*x - x^2])]/54$

fricas [A] time = 0.67, size = 170, normalized size = 1.31

$$\frac{1}{9} \sqrt{3} \arctan\left(\frac{\sqrt{3} \sqrt{-x^2 - 4x - 3} (2x + 3)}{3(x^2 + 4x + 3)}\right) + \frac{1}{18} \sqrt{2} \arctan\left(\frac{\sqrt{2}x + 3\sqrt{2} \sqrt{-x^2 - 4x - 3}}{2(2x + 3)}\right) + \frac{1}{18} \sqrt{2} \arctan\left(-\frac{\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")

[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(-x^2 - 4*x - 3)*(2*x + 3)/(x^2 + 4*x + 3)) + 1/18*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) + 1/18*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) + 1/9*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) - 1/9*log((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)

giac [A] time = 0.20, size = 199, normalized size = 1.53

$$\frac{1}{9} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} \left(\frac{3(\sqrt{-x^2 - 4x - 3} - 1)}{x + 2} + 1\right)\right) + \frac{2}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} \left(\frac{2(\sqrt{-x^2 - 4x - 3} - 1)}{x + 2} + 1\right)\right) + \frac{1}{9} \sqrt{2} \arctan\left(-\frac{\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")

[Out] 1/9*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/9*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) - 2/9*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) + 2/9*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3))

maple [A] time = 0.02, size = 152, normalized size = 1.17

$$\frac{\sqrt{3} \arctan\left(\frac{(-4x-6)\sqrt{3}}{6\sqrt{-x^2-4x-3}}\right) + \frac{\sqrt{3} \sqrt{4} \sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2} - 12}}{9}}{54 \sqrt{\frac{\frac{x^2}{(-x-\frac{3}{2})^2} - 4}{\left(\frac{x}{-x-\frac{3}{2}} + 1\right)^2}} \left(\frac{x}{-x-\frac{3}{2}} + 1\right)} + \sqrt{2} \arctan\left(\frac{\sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2} - 12} \sqrt{2}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)`

[Out] $\frac{1}{54} \cdot 3^{1/2} \cdot 4^{1/2} \cdot (3/(-x-3/2)^2 \cdot x^2 - 12)^{1/2} \cdot (2^{1/2} \cdot \arctan(1/6 \cdot (3/(-x-3/2)^2 \cdot x^2 - 12)^{1/2} \cdot 2^{1/2})) + 4 \cdot \operatorname{arctanh}(3/(-x-3/2)/(3/(-x-3/2)^2 \cdot x^2 - 12)^{1/2} \cdot x) / ((1/(-x-3/2)^2 \cdot x^2 - 4)/(1/(-x-3/2) \cdot x + 1)^2)^{1/2} / (1/(-x-3/2) \cdot x + 1) + 1/9 \cdot 3^{1/2} \cdot \arctan(1/6 \cdot (-6-4 \cdot x) \cdot 3^{1/2}/(-x^2-4 \cdot x-3)^{1/2})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2x^2 + 4x + 3)\sqrt{-x^2 - 4x - 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)*x), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x \sqrt{-x^2 - 4x - 3} (2x^2 + 4x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(-4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)`

[Out] `int(1/(x*(-4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sqrt{-(x+1)(x+3)} (2x^2 + 4x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)`

[Out] `Integral(1/(x*sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)`

$$3.132 \quad \int \frac{1}{x^2 \sqrt{-3-4x-x^2} (3+4x+2x^2)} dx$$

Optimal. Leaf size=151

$$\frac{\sqrt{-x^2-4x-3}}{9x} + \frac{2 \tan^{-1}\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right)}{3\sqrt{3}} + \frac{2}{27} \sqrt{2} \tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right) - \frac{2}{27} \sqrt{2} \tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right) + \frac{10}{27} \tanh$$

[Out] 10/27*arctanh(x/(-x^2-4*x-3)^(1/2))+2/27*arctan(1/2*(1+(-3-x)/(-x^2-4*x-3)^(1/2))*2^(1/2))*2^(1/2)-2/27*arctan(1/2*(1+(3+x)/(-x^2-4*x-3)^(1/2))*2^(1/2))*2^(1/2)+2/9*arctan(1/3*(3+2*x)*3^(1/2)/(-x^2-4*x-3)^(1/2))*3^(1/2)+1/9*(-x^2-4*x-3)^(1/2)/x

Rubi [A] time = 0.45, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6728, 730, 724, 204, 1028, 986, 12, 1026, 1161, 618, 1027, 206}

$$\frac{\sqrt{-x^2-4x-3}}{9x} + \frac{2 \tan^{-1}\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right)}{3\sqrt{3}} + \frac{2}{27} \sqrt{2} \tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right) - \frac{2}{27} \sqrt{2} \tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right) + \frac{10}{27} \tanh$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]

[Out] Sqrt[-3 - 4*x - x^2]/(9*x) + (2*ArcTan[(3 + 2*x)/(Sqrt[3]*Sqrt[-3 - 4*x - x^2])])/(3*Sqrt[3]) + (2*Sqrt[2]*ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/27 - (2*Sqrt[2]*ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/27 + (10*ArcTanh[x/Sqrt[-3 - 4*x - x^2]])/27

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 730

Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]

Rule 986

Int[1/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e - b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e - b*f, 0] && NegQ[b^2 - 4*a*c]

Rule 1026

Int[(x_)/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 - 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]

Rule 1027

```
Int[((g_) + (h_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[g, Subst[Int[1/(a + (c*d - a*f)*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] && EqQ[2*h*d - g*e, 0]
```

Rule 1028

```
Int[((g_) + (h_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := -Dist[(2*h*d - g*e)/e, Int[1/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] && NeQ[2*h*d - g*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx &= \int \left(\frac{1}{3x^2\sqrt{-3-4x-x^2}} - \frac{4}{9x\sqrt{-3-4x-x^2}} + \frac{2(5+4x)}{9\sqrt{-3-4x-x^2}(3+4x+2x^2)} \right) dx \\
&= \frac{2}{9} \int \frac{5+4x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx + \frac{1}{3} \int \frac{1}{x^2\sqrt{-3-4x-x^2}} dx \\
&= \frac{\sqrt{-3-4x-x^2}}{9x} - \frac{2}{9} \int \frac{1}{x\sqrt{-3-4x-x^2}} dx - \frac{2}{9} \int \frac{1}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx \\
&= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{4 \tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{9\sqrt{3}} + \frac{1}{27} \int \frac{-6-4x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx \\
&= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{2 \tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{4}{9} \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) \\
&= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{2 \tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{10}{27} \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) \\
&= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{2 \tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{10}{27} \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) \\
&= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{2 \tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{10}{27} \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) \\
&= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{2 \tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{10}{27} \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) \\
&= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{2 \tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{2}{27} \sqrt{2} \tan^{-1}\left(\frac{1 - \frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)
\end{aligned}$$

Mathematica [C] time = 0.41, size = 225, normalized size = 1.49

$$3 \left(\sqrt{-x^2-4x-3} + 2\sqrt{3}x \tan^{-1}\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right) \right) + \sqrt{1-2i\sqrt{2}} (2\sqrt{2}+i)x \tanh^{-1}\left(\frac{-i\sqrt{2}x+2x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right) + \sqrt{1-2i\sqrt{2}}$$

$$27x$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*Sqrt[-3-4*x-x^2]*(3+4*x+2*x^2)),x]

[Out] (3*(Sqrt[-3-4*x-x^2]+2*Sqrt[3]*x*ArcTan[(3+2*x)/(Sqrt[3]*Sqrt[-3-4*x-x^2])]) + Sqrt[1-(2*I)*Sqrt[2]]*(1+2*Sqrt[2])*x*ArcTanh[(2-(2*I)

) * Sqrt[2] + 2*x - I*Sqrt[2]*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]) + Sqrt[1 + (2*I)*Sqrt[2]]*(-I + 2*Sqrt[2])*x*ArcTanh[(2 + (2*I)*Sqrt[2] + (2 + I*Sqrt[2])*x)/(Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])]/(27*x)

fricas [A] time = 0.73, size = 194, normalized size = 1.28

$$\frac{12\sqrt{3}x \arctan\left(\frac{\sqrt{3}\sqrt{-x^2-4x-3}(2x+3)}{3(x^2+4x+3)}\right) - 2\sqrt{2}x \arctan\left(\frac{\sqrt{2}x+3\sqrt{-x^2-4x-3}}{2(2x+3)}\right) - 2\sqrt{2}x \arctan\left(-\frac{\sqrt{2}x-3\sqrt{-x^2-4x-3}}{2(2x+3)}\right)}{54x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")

[Out] -1/54*(12*sqrt(3)*x*arctan(1/3*sqrt(3)*sqrt(-x^2 - 4*x - 3)*(2*x + 3)/(x^2 + 4*x + 3)) - 2*sqrt(2)*x*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) - 2*sqrt(2)*x*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) + 5*x*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) - 5*x*log((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2) - 6*sqrt(-x^2 - 4*x - 3))/x

giac [B] time = 0.48, size = 269, normalized size = 1.78

$$\frac{2}{27}\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3(\sqrt{-x^2-4x-3}-1)}{x+2}+1\right)\right) - \frac{4}{9}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2(\sqrt{-x^2-4x-3}-1)}{x+2}+1\right)\right) + \frac{2}{27}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")

[Out] 2/27*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) - 4/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 2/27*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) - 1/18*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 2)/((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) + 5/27*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) - 5/27*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3))

maple [A] time = 0.02, size = 169, normalized size = 1.12

$$\frac{2\sqrt{3} \arctan\left(\frac{(-4x-6)\sqrt{3}}{6\sqrt{-x^2-4x-3}}\right) + \frac{\sqrt{-x^2-4x-3}}{9x} + \frac{\sqrt{3} \sqrt{4} \sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2} - 12}}{\left(-5 \operatorname{arctanh}\left(\frac{3x}{(-x-\frac{3}{2}) \sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2} - 12}}\right) + \sqrt{2}\right)} + \sqrt{2}}{81 \sqrt{\frac{\frac{x^2}{(-x-\frac{3}{2})^2} - 4}{\left(\frac{x}{-x-\frac{3}{2}} + 1\right)^2}} \left(\frac{x}{-x-\frac{3}{2}} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2), x)`

[Out] $\frac{1}{81} 3^{1/2} 4^{1/2} (3/(-x-3/2)^2 x^2 - 12)^{1/2} (2^{1/2} \arctan(1/6 (3/(-x-3/2)^2 x^2 - 12)^{1/2} 2^{1/2}) - 5 \operatorname{arctanh}(3/(-x-3/2) / (3/(-x-3/2)^2 x^2 - 12)^{1/2} x)) / ((1/(-x-3/2)^2 x^2 - 4) / (1/(-x-3/2) x + 1)^2)^{1/2} / (1/(-x-3/2) x + 1) / 9 (-x^2 - 4x - 3)^{1/2} / x - 2/9 3^{1/2} \arctan(1/6 (-4x-6) 3^{1/2} / (-x^2 - 4x - 3)^{1/2})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2x^2 + 4x + 3)\sqrt{-x^2 - 4x - 3} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2), x, algorithm="maxima")`

[Out] `integrate(1/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)*x^2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2 \sqrt{-x^2 - 4x - 3} (2x^2 + 4x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2*(-4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)`

[Out] `int(1/(x^2*(-4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{-(x+1)(x+3)} (2x^2 + 4x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

$$3.133 \quad \int (2+3x)^2 (30 + 31x - 12x^2)^2 \sqrt{6 + 17x + 12x^2} dx$$

Optimal. Leaf size=149

$$-\frac{1}{32}(10-3x)(12x^2 + 17x + 6)^{7/2} - \frac{873(12x^2 + 17x + 6)^{7/2}}{1792} + \frac{25091(24x + 17)(12x^2 + 17x + 6)^{5/2}}{24576} - \frac{125455(24x + 17)(12x^2 + 17x + 6)^{3/2}}{4718592}$$

[Out] -125455/4718592*(17+24*x)*(12*x^2+17*x+6)^(3/2)+25091/24576*(17+24*x)*(12*x^2+17*x+6)^(5/2)-873/1792*(12*x^2+17*x+6)^(7/2)-1/32*(10-3*x)*(12*x^2+17*x+6)^(7/2)-125455/1811939328*arctanh(1/12*(17+24*x)*3^(1/2)/(12*x^2+17*x+6)^(1/2))*3^(1/2)+125455/150994944*(17+24*x)*(12*x^2+17*x+6)^(1/2)

Rubi [A] time = 0.09, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1002, 742, 640, 612, 621, 206}

$$-\frac{1}{32}(10-3x)(12x^2 + 17x + 6)^{7/2} - \frac{873(12x^2 + 17x + 6)^{7/2}}{1792} + \frac{25091(24x + 17)(12x^2 + 17x + 6)^{5/2}}{24576} - \frac{125455(24x + 17)(12x^2 + 17x + 6)^{3/2}}{4718592}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x)^2*(30 + 31*x - 12*x^2)^2*Sqrt[6 + 17*x + 12*x^2], x]

[Out] (125455*(17 + 24*x)*Sqrt[6 + 17*x + 12*x^2])/150994944 - (125455*(17 + 24*x)*(6 + 17*x + 12*x^2)^(3/2))/4718592 + (25091*(17 + 24*x)*(6 + 17*x + 12*x^2)^(5/2))/24576 - (873*(6 + 17*x + 12*x^2)^(7/2))/1792 - ((10 - 3*x)*(6 + 17*x + 12*x^2)^(7/2))/32 - (125455*ArcTanh[(17 + 24*x)/(4*Sqrt[3]*Sqrt[6 + 17*x + 12*x^2])])/(603979776*Sqrt[3])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

```
Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 742

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1002

```
Int[((g_) + (h_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(m_), x_Symbol] := Int[((d*g)/a + (f*h*x)/c)^(m)*(a + b*x + c*x^2)^(m + p), x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x] && EqQ[c*g^2 - b*g*h + a*h^2, 0] && EqQ[c^2*d*g^2 - a*c*e*g*h + a^2*f*h^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int (2 + 3x)^2 (30 + 31x - 12x^2)^2 \sqrt{6 + 17x + 12x^2} dx &= \int (10 - 3x)^2 (6 + 17x + 12x^2)^{5/2} dx \\
&= -\frac{1}{32}(10 - 3x)(6 + 17x + 12x^2)^{7/2} + \frac{1}{96} \int \left(11331 - \frac{785}{2}\right. \\
&\quad \left. - \frac{873(6 + 17x + 12x^2)^{7/2}}{1792} - \frac{1}{32}(10 - 3x)(6 + 17x + 12x^2)^{5/2}\right. \\
&\quad \left. - \frac{25091(17 + 24x)(6 + 17x + 12x^2)^{5/2}}{24576} - \frac{873(6 + 17x + 12x^2)^{3/2}}{1792}\right. \\
&\quad \left. - \frac{125455(17 + 24x)(6 + 17x + 12x^2)^{3/2}}{4718592} + \frac{25091(17 + 24x)}{4718592}\right) dx \\
&= \frac{125455(17 + 24x)\sqrt{6 + 17x + 12x^2}}{150994944} - \frac{125455(17 + 24x)}{4718592} \\
&= \frac{125455(17 + 24x)\sqrt{6 + 17x + 12x^2}}{150994944} - \frac{125455(17 + 24x)}{4718592} \\
&= \frac{125455(17 + 24x)\sqrt{6 + 17x + 12x^2}}{150994944} - \frac{125455(17 + 24x)}{4718592}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 87, normalized size = 0.58

$$\frac{12\sqrt{12x^2 + 17x + 6} (171228266496x^7 - 732816211968x^6 - 1190083166208x^5 + 3438453030912x^4 + 8974844476416x^3 + 3438453030912x^2 - 1190083166208x - 732816211968)}{12683575296}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x)^2*(30 + 31*x - 12*x^2)^2*Sqrt[6 + 17*x + 12*x^2], x]

[Out] (12*Sqrt[6 + 17*x + 12*x^2]*(474999091769 + 3132157281976*x + 7899203409792*x^2 + 8974844476416*x^3 + 3438453030912*x^4 - 1190083166208*x^5 - 732816211968*x^6 + 171228266496*x^7) - 878185*Sqrt[3]*ArcTanh[(17 + 24*x)/(4*Sqrt[18 + 51*x + 36*x^2]])]/12683575296

fricas [A] time = 0.58, size = 88, normalized size = 0.59

$$\frac{1}{1056964608} (171228266496 x^7 - 732816211968 x^6 - 1190083166208 x^5 + 3438453030912 x^4 + 8974844476416 x^3 + 3438453030912 x^2 - 1190083166208 x - 732816211968)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*x)^2*(-12*x^2+31*x+30)^2*(12*x^2+17*x+6)^(1/2),x, algorithm="fricas")

[Out] 1/1056964608*(171228266496*x^7 - 732816211968*x^6 - 1190083166208*x^5 + 3438453030912*x^4 + 8974844476416*x^3 + 7899203409792*x^2 + 3132157281976*x + 474999091769)*sqrt(12*x^2 + 17*x + 6) + 125455/3623878656*sqrt(3)*log(-8*sqrt(3)*sqrt(12*x^2 + 17*x + 6)*(24*x + 17) + 1152*x^2 + 1632*x + 577)

giac [A] time = 0.30, size = 85, normalized size = 0.57

$$\frac{1}{1056964608} (8 (48 (24 (96 (24 (48 (168x - 719)x - 56047)x + 3886417)x + 973832951)x + 20570842213)x + 391519660247)x + 474999091769) \sqrt{12x^2 + 17x + 6} + 125455/1811939328 \sqrt{3} \log(\text{abs}(-4\sqrt{3} * (2\sqrt{3} * x - \sqrt{12x^2 + 17x + 6}) - 17)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*x)^2*(-12*x^2+31*x+30)^2*(12*x^2+17*x+6)^(1/2),x, algorithm="giac")

[Out] 1/1056964608*(8*(48*(24*(96*(24*(48*(168*x - 719)*x - 56047)*x + 3886417)*x + 973832951)*x + 20570842213)*x + 391519660247)*x + 474999091769)*sqrt(12*x^2 + 17*x + 6) + 125455/1811939328*sqrt(3)*log(abs(-4*sqrt(3)*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x + 6)) - 17))

maple [A] time = 0.02, size = 147, normalized size = 0.99

$$\frac{27(12x^2 + 17x + 6)^{\frac{3}{2}}x^5}{2} - \frac{8613(12x^2 + 17x + 6)^{\frac{3}{2}}x^4}{112} + \frac{14991(12x^2 + 17x + 6)^{\frac{3}{2}}x^3}{1792} + \frac{4267751(12x^2 + 17x + 6)^{\frac{3}{2}}x^2}{14336}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2+3*x)^2*(-12*x^2+31*x+30)^2*(12*x^2+17*x+6)^(1/2),x)

[Out] 27/2*x^5*(12*x^2+17*x+6)^(3/2)-8613/112*x^4*(12*x^2+17*x+6)^(3/2)+14991/1792*x^3*(12*x^2+17*x+6)^(3/2)+4267751/14336*x^2*(12*x^2+17*x+6)^(3/2)+129220757/458752*x*(12*x^2+17*x+6)^(3/2)+2473875847/33030144*(12*x^2+17*x+6)^(3/2)-125455/3623878656*ln(1/12*(17/2+12*x)*12^(1/2)+(12*x^2+17*x+6)^(1/2))*12^(1/2)+125455/150994944*(17+24*x)*(12*x^2+17*x+6)^(1/2)

maxima [A] time = 1.00, size = 155, normalized size = 1.04

$$\frac{27}{2} (12x^2 + 17x + 6)^{\frac{3}{2}}x^5 - \frac{8613}{112} (12x^2 + 17x + 6)^{\frac{3}{2}}x^4 + \frac{14991}{1792} (12x^2 + 17x + 6)^{\frac{3}{2}}x^3 + \frac{4267751}{14336} (12x^2 + 17x + 6)^{\frac{3}{2}}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*x)^2*(-12*x^2+31*x+30)^2*(12*x^2+17*x+6)^(1/2),x, algorithm="maxima")


```
[Out] 27/2*(12*x^2 + 17*x + 6)^(3/2)*x^5 - 8613/112*(12*x^2 + 17*x + 6)^(3/2)*x^4
+ 14991/1792*(12*x^2 + 17*x + 6)^(3/2)*x^3 + 4267751/14336*(12*x^2 + 17*x
+ 6)^(3/2)*x^2 + 129220757/458752*(12*x^2 + 17*x + 6)^(3/2)*x + 2473875847/
33030144*(12*x^2 + 17*x + 6)^(3/2) + 125455/6291456*sqrt(12*x^2 + 17*x + 6)
*x - 125455/1811939328*sqrt(3)*log(4*sqrt(3)*sqrt(12*x^2 + 17*x + 6) + 24*x
+ 17) + 2132735/150994944*sqrt(12*x^2 + 17*x + 6)
```

mupad [B] time = 5.08, size = 187, normalized size = 1.26

$$\frac{4267751 x^2 (12 x^2 + 17 x + 6)^{3/2}}{14336} + \frac{14991 x^3 (12 x^2 + 17 x + 6)^{3/2}}{1792} - \frac{8613 x^4 (12 x^2 + 17 x + 6)^{3/2}}{112} + \frac{27 x^5 (12 x^2 + 17 x + 6)^{3/2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 2)^2*(17*x + 12*x^2 + 6)^(1/2)*(31*x - 12*x^2 + 30)^2,x)
```

```
[Out] (4267751*x^2*(17*x + 12*x^2 + 6)^(3/2))/14336 + (14991*x^3*(17*x + 12*x^2 +
6)^(3/2))/1792 - (8613*x^4*(17*x + 12*x^2 + 6)^(3/2))/112 + (27*x^5*(17*x
+ 12*x^2 + 6)^(3/2))/2 - (146030443*12^(1/2)*log((17*x + 12*x^2 + 6)^(1/2)
+ (12^(1/2)*(12*x + 17/2))/12))/88080384 + (438091329*(x/2 + 17/48)*(17*x +
12*x^2 + 6)^(1/2))/229376 + (2473875847*(17*x + 12*x^2 + 6)^(1/2)*(408*x +
1152*x^2 - 291))/3170893824 + (129220757*x*(17*x + 12*x^2 + 6)^(3/2))/4587
52 + (42055889399*12^(1/2)*log(2*(17*x + 12*x^2 + 6)^(1/2) + (12^(1/2)*(24*
x + 17))/12))/25367150592
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{(3x+2)(4x+3)} (3x-10)^2 (3x+2)^2 (4x+3)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**2*(-12*x**2+31*x+30)**2*(12*x**2+17*x+6)**(1/2),x)
```

```
[Out] Integral(sqrt((3*x + 2)*(4*x + 3))*(3*x - 10)**2*(3*x + 2)**2*(4*x + 3)**2,
x)
```

$$3.134 \quad \int (2+3x) (30 + 31x - 12x^2) \sqrt{6 + 17x + 12x^2} dx$$

Optimal. Leaf size=103

$$-\frac{1}{20} (12x^2 + 17x + 6)^{5/2} + \frac{97}{768} (24x+17) (12x^2 + 17x + 6)^{3/2} - \frac{97(24x + 17)\sqrt{12x^2 + 17x + 6}}{24576} + \frac{97 \tanh^{-1}\left(\frac{24x}{4\sqrt{3}\sqrt{12x^2 + 17x + 6}}\right)}{98304\sqrt{3}}$$

[Out] 97/768*(17+24*x)*(12*x^2+17*x+6)^(3/2)-1/20*(12*x^2+17*x+6)^(5/2)+97/294912*arctanh(1/12*(17+24*x)*3^(1/2)/(12*x^2+17*x+6)^(1/2))*3^(1/2)-97/24576*(17+24*x)*(12*x^2+17*x+6)^(1/2)

Rubi [A] time = 0.04, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1002, 640, 612, 621, 206}

$$-\frac{1}{20} (12x^2 + 17x + 6)^{5/2} + \frac{97}{768} (24x+17) (12x^2 + 17x + 6)^{3/2} - \frac{97(24x + 17)\sqrt{12x^2 + 17x + 6}}{24576} + \frac{97 \tanh^{-1}\left(\frac{24x}{4\sqrt{3}\sqrt{12x^2 + 17x + 6}}\right)}{98304\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x)*(30 + 31*x - 12*x^2)*Sqrt[6 + 17*x + 12*x^2], x]

[Out] (-97*(17 + 24*x)*Sqrt[6 + 17*x + 12*x^2])/24576 + (97*(17 + 24*x)*(6 + 17*x + 12*x^2)^(3/2))/768 - (6 + 17*x + 12*x^2)^(5/2)/20 + (97*ArcTanh[(17 + 24*x)/(4*Sqrt[3]*Sqrt[6 + 17*x + 12*x^2])])/(98304*Sqrt[3])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,

b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1002

Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(m_.), x_Symbol] :> Int[((d*g)/a + (f*h*x)/c)^(m*(a + b*x + c*x^2)^(m + p)), x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x] && EqQ[c*g^2 - b*g*h + a*h^2, 0] && EqQ[c^2*d*g^2 - a*c*e*g*h + a^2*f*h^2, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned}
 \int (2 + 3x)(30 + 31x - 12x^2) \sqrt{6 + 17x + 12x^2} \, dx &= \int (10 - 3x)(6 + 17x + 12x^2)^{3/2} \, dx \\
 &= -\frac{1}{20}(6 + 17x + 12x^2)^{5/2} + \frac{97}{8} \int (6 + 17x + 12x^2)^{3/2} \, dx \\
 &= \frac{97}{768}(17 + 24x)(6 + 17x + 12x^2)^{3/2} - \frac{1}{20}(6 + 17x + 12x^2)^{5/2} \\
 &= -\frac{97(17 + 24x)\sqrt{6 + 17x + 12x^2}}{24576} + \frac{97}{768}(17 + 24x)(6 + 17x + 12x^2)^{3/2} \\
 &= -\frac{97(17 + 24x)\sqrt{6 + 17x + 12x^2}}{24576} + \frac{97}{768}(17 + 24x)(6 + 17x + 12x^2)^{3/2} \\
 &= -\frac{97(17 + 24x)\sqrt{6 + 17x + 12x^2}}{24576} + \frac{97}{768}(17 + 24x)(6 + 17x + 12x^2)^{3/2}
 \end{aligned}$$

Mathematica [A] time = 0.03, size = 72, normalized size = 0.70

$$\frac{485\sqrt{3} \tanh^{-1}\left(\frac{24x+17}{4\sqrt{36x^2+51x+18}}\right) + 12\sqrt{12x^2+17x+6} \left(-884736x^4 + 1963008x^3 + 6837888x^2 + 5455144x + 13\right)}{1474560}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x)*(30 + 31*x - 12*x^2)*Sqrt[6 + 17*x + 12*x^2], x]

[Out] (12*Sqrt[6 + 17*x + 12*x^2]*(1353611 + 5455144*x + 6837888*x^2 + 1963008*x^3 - 884736*x^4) + 485*Sqrt[3]*ArcTanh[(17 + 24*x)/(4*Sqrt[18 + 51*x + 36*x^2])))/1474560

fricas [A] time = 0.59, size = 73, normalized size = 0.71

$$-\frac{1}{122880} (884736x^4 - 1963008x^3 - 6837888x^2 - 5455144x - 1353611) \sqrt{12x^2 + 17x + 6} + \frac{97}{589824} \sqrt{3} \log(8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*x)*(-12*x^2+31*x+30)*(12*x^2+17*x+6)^(1/2), x, algorithm="fricas")

[Out] -1/122880*(884736*x^4 - 1963008*x^3 - 6837888*x^2 - 5455144*x - 1353611)*sqrt(12*x^2 + 17*x + 6) + 97/589824*sqrt(3)*log(8*sqrt(3)*sqrt(12*x^2 + 17*x + 6)*(24*x + 17) + 1152*x^2 + 1632*x + 577)

giac [A] time = 0.20, size = 70, normalized size = 0.68

$$-\frac{1}{122880} (8(48(72(32x - 71)x - 17807)x - 681893)x - 1353611) \sqrt{12x^2 + 17x + 6} - \frac{97}{294912} \sqrt{3} \log\left(\left| -4\sqrt{3} \left(8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*x)*(-12*x^2+31*x+30)*(12*x^2+17*x+6)^(1/2), x, algorithm="giac")

[Out] -1/122880*(8*(48*(72*(32*x - 71)*x - 17807)*x - 681893)*x - 1353611)*sqrt(12*x^2 + 17*x + 6) - 97/294912*sqrt(3)*log(abs(-4*sqrt(3)*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x + 6)) - 17))

maple [A] time = 0.01, size = 96, normalized size = 0.93

$$-\frac{3(12x^2 + 17x + 6)^{\frac{3}{2}} x^2}{5} + \frac{349(12x^2 + 17x + 6)^{\frac{3}{2}} x}{160} + \frac{97\sqrt{12} \ln\left(\frac{\left(12x + \frac{17}{2}\right)\sqrt{12}}{12} + \sqrt{12x^2 + 17x + 6}\right)}{589824} + \frac{7093(12x^2 + 17x + 6)^{\frac{3}{2}}}{3840}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2+3*x)*(-12*x^2+31*x+30)*(12*x^2+17*x+6)^(1/2), x)

[Out] -3/5*(12*x^2+17*x+6)^(3/2)*x^2+349/160*(12*x^2+17*x+6)^(3/2)*x+7093/3840*(12*x^2+17*x+6)^(3/2)-97/24576*(24*x+17)*(12*x^2+17*x+6)^(1/2)+97/589824*12^(1/2)*ln(1/12*(12*x+17/2)*12^(1/2)+(12*x^2+17*x+6)^(1/2))

maxima [A] time = 0.98, size = 104, normalized size = 1.01

$$-\frac{3}{5}(12x^2 + 17x + 6)^{\frac{3}{2}}x^2 + \frac{349}{160}(12x^2 + 17x + 6)^{\frac{3}{2}}x + \frac{7093}{3840}(12x^2 + 17x + 6)^{\frac{3}{2}} - \frac{97}{1024}\sqrt{12x^2 + 17x + 6}x + \frac{97}{2949120}\sqrt{12x^2 + 17x + 6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*x)*(-12*x^2+31*x+30)*(12*x^2+17*x+6)^(1/2),x, algorithm="maxima")

[Out] -3/5*(12*x^2 + 17*x + 6)^(3/2)*x^2 + 349/160*(12*x^2 + 17*x + 6)^(3/2)*x + 7093/3840*(12*x^2 + 17*x + 6)^(3/2) - 97/1024*sqrt(12*x^2 + 17*x + 6)*x + 97/2949120*sqrt(12*x^2 + 17*x + 6) + 24*x + 17) - 1649/24576*sqrt(12*x^2 + 17*x + 6)

mupad [B] time = 4.69, size = 136, normalized size = 1.32

$$\frac{3753\left(\frac{x}{2} + \frac{17}{48}\right)\sqrt{12x^2 + 17x + 6}}{80} - \frac{417\sqrt{12}\ln\left(\sqrt{12x^2 + 17x + 6} + \frac{\sqrt{12}\left(\frac{12x + 17}{2}\right)}{12}\right)}{10240} - \frac{3x^2(12x^2 + 17x + 6)^{3/2}}{5} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x + 2)*(17*x + 12*x^2 + 6)^(1/2)*(31*x - 12*x^2 + 30),x)

[Out] (3753*(x/2 + 17/48)*(17*x + 12*x^2 + 6)^(1/2))/80 - (417*12^(1/2)*log((17*x + 12*x^2 + 6)^(1/2) + (12^(1/2)*(12*x + 17/2))/12))/10240 - (3*x^2*(17*x + 12*x^2 + 6)^(3/2))/5 + (7093*(17*x + 12*x^2 + 6)^(1/2)*(408*x + 1152*x^2 - 291))/368640 + (349*x*(17*x + 12*x^2 + 6)^(3/2))/160 + (120581*12^(1/2)*log(2*(17*x + 12*x^2 + 6)^(1/2) + (12^(1/2)*(24*x + 17))/12))/2949120

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-152x\sqrt{12x^2 + 17x + 6}\right)dx - \int\left(-69x^2\sqrt{12x^2 + 17x + 6}\right)dx - \int 36x^3\sqrt{12x^2 + 17x + 6}dx - \int\left(-60\sqrt{12x^2 + 17x + 6}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*x)*(-12*x**2+31*x+30)*(12*x**2+17*x+6)**(1/2),x)

[Out] -Integral(-152*x*sqrt(12*x**2 + 17*x + 6), x) - Integral(-69*x**2*sqrt(12*x**2 + 17*x + 6), x) - Integral(36*x**3*sqrt(12*x**2 + 17*x + 6), x) - Integral(-60*sqrt(12*x**2 + 17*x + 6), x)

$$3.135 \quad \int \frac{\sqrt{6+17x+12x^2}}{(2+3x)(30+31x-12x^2)} dx$$

Optimal. Leaf size=28

$$\frac{1}{42} \tanh^{-1} \left(\frac{291x + 206}{84\sqrt{12x^2 + 17x + 6}} \right)$$

[Out] 1/42*arctanh(1/84*(206+291*x)/(12*x^2+17*x+6)^(1/2))

Rubi [A] time = 0.05, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {1002, 724, 206}

$$\frac{1}{42} \tanh^{-1} \left(\frac{291x + 206}{84\sqrt{12x^2 + 17x + 6}} \right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)*(30 + 31*x - 12*x^2)), x]

[Out] ArcTanh[(206 + 291*x)/(84*Sqrt[6 + 17*x + 12*x^2])]/42

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1002

Int[((g_) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(m_.), x_Symbol] := Int[((d*g)/a + (f*h*x)/c)^(m*(a + b*x + c*x^2)^(m + p)), x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x] && EqQ[c*g^2 - b*g*h + a*h^2, 0] && EqQ[c^2*d*g^2 - a*c*e*g*h + a^2*f*h^2, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{6+17x+12x^2}}{(2+3x)(30+31x-12x^2)} dx &= \int \frac{1}{(10-3x)\sqrt{6+17x+12x^2}} dx \\ &= -\left(2 \operatorname{Subst}\left(\int \frac{1}{7056-x^2} dx, x, \frac{-206-291x}{\sqrt{6+17x+12x^2}}\right)\right) \\ &= \frac{1}{42} \tanh^{-1}\left(\frac{206+291x}{84\sqrt{6+17x+12x^2}}\right) \end{aligned}$$

Mathematica [A] time = 0.10, size = 37, normalized size = 1.32

$$\frac{1}{42} \log\left(84\sqrt{12x^2+17x+6}+291x+206\right) - \frac{1}{42} \log(10-3x)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)*(30 + 31*x - 12*x^2)),x]

[Out] -1/42*Log[10 - 3*x] + Log[206 + 291*x + 84*Sqrt[6 + 17*x + 12*x^2]]/42

fricas [B] time = 0.70, size = 53, normalized size = 1.89

$$\frac{1}{84} \log\left(\frac{291x + 84\sqrt{12x^2+17x+6}+206}{x}\right) - \frac{1}{84} \log\left(\frac{291x - 84\sqrt{12x^2+17x+6}+206}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)/(-12*x^2+31*x+30),x, algorithm="fricas")

[Out] 1/84*log((291*x + 84*sqrt(12*x^2 + 17*x + 6) + 206)/x) - 1/84*log((291*x - 84*sqrt(12*x^2 + 17*x + 6) + 206)/x)

giac [B] time = 0.27, size = 63, normalized size = 2.25

$$\frac{1}{42} \log\left(\left|-6\sqrt{3}x + 20\sqrt{3} + 3\sqrt{12x^2+17x+6} + 42\right|\right) - \frac{1}{42} \log\left(\left|-6\sqrt{3}x + 20\sqrt{3} + 3\sqrt{12x^2+17x+6} - 42\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)/(-12*x^2+31*x+30),x, algorithm="giac")

[Out] 1/42*log(abs(-6*sqrt(3)*x + 20*sqrt(3) + 3*sqrt(12*x^2 + 17*x + 6) + 42)) - 1/42*log(abs(-6*sqrt(3)*x + 20*sqrt(3) + 3*sqrt(12*x^2 + 17*x + 6) - 42))

maple [B] time = 0.02, size = 163, normalized size = 5.82

$$\frac{\operatorname{arctanh}\left(\frac{97x + \frac{206}{3}}{28\sqrt{97x + 12\left(x - \frac{10}{3}\right)^2 - \frac{382}{3}}}\right)}{42} - \frac{97\sqrt{12} \ln\left(\frac{\left(12x + \frac{17}{2}\right)\sqrt{12}}{12} + \sqrt{97x + 12\left(x - \frac{10}{3}\right)^2 - \frac{382}{3}}\right)}{14112} + \frac{\sqrt{12} \ln\left(\frac{\left(12x + \frac{17}{2}\right)\sqrt{12}}{12} + \dots\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((12*x^2+17*x+6)^(1/2)/(2+3*x)/(-12*x^2+31*x+30), x)`

[Out] $-4/49*(12*(x+3/4)^2-x-3/4)^{(1/2)}+1/294*\ln(1/12*(12*x+17/2)*12^{(1/2)}+(12*(x+3/4)^2-x-3/4)^{(1/2)})*12^{(1/2)}+1/12*(12*(x+2/3)^2+x+2/3)^{(1/2)}+1/288*\ln(1/12*(12*x+17/2)*12^{(1/2)}+(12*(x+2/3)^2+x+2/3)^{(1/2)})*12^{(1/2)}-1/588*(12*(x-10/3)^2+97*x-382/3)^{(1/2)}-97/14112*\ln(1/12*(12*x+17/2)*12^{(1/2)}+(12*(x-10/3)^2+97*x-382/3)^{(1/2)})*12^{(1/2)}+1/42*\operatorname{arctanh}(1/28*(206/3+97*x)/(12*(x-10/3)^2+97*x-382/3)^{(1/2)})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{12x^2 + 17x + 6}}{(12x^2 - 31x - 30)(3x + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)/(-12*x^2+31*x+30), x, algorithm="maxima")`

[Out] `-integrate(sqrt(12*x^2 + 17*x + 6)/((12*x^2 - 31*x - 30)*(3*x + 2)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{12x^2 + 17x + 6}}{(3x + 2)(-12x^2 + 31x + 30)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((17*x + 12*x^2 + 6)^(1/2)/((3*x + 2)*(31*x - 12*x^2 + 30)), x)`

[Out] `int((17*x + 12*x^2 + 6)^(1/2)/((3*x + 2)*(31*x - 12*x^2 + 30)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{12x^2 + 17x + 6}}{36x^3 - 69x^2 - 152x - 60} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x**2+17*x+6)**(1/2)/(2+3*x)/(-12*x**2+31*x+30),x)
```

```
[Out] -Integral(sqrt(12*x**2 + 17*x + 6)/(36*x**3 - 69*x**2 - 152*x - 60), x)
```

$$3.136 \quad \int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^2(30+31x-12x^2)^2} dx$$

Optimal. Leaf size=84

$$-\frac{388x+275}{98(10-3x)\sqrt{12x^2+17x+6}} + \frac{3137\sqrt{12x^2+17x+6}}{38416(10-3x)} + \frac{97 \tanh^{-1}\left(\frac{291x+206}{84\sqrt{12x^2+17x+6}}\right)}{3226944}$$

[Out] 97/3226944*arctanh(1/84*(206+291*x)/(12*x^2+17*x+6)^(1/2))+1/98*(-275-388*x)/(10-3*x)/(12*x^2+17*x+6)^(1/2)+3137/38416*(12*x^2+17*x+6)^(1/2)/(10-3*x)

Rubi [A] time = 0.08, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {1002, 740, 806, 724, 206}

$$-\frac{388x+275}{98(10-3x)\sqrt{12x^2+17x+6}} + \frac{3137\sqrt{12x^2+17x+6}}{38416(10-3x)} + \frac{97 \tanh^{-1}\left(\frac{291x+206}{84\sqrt{12x^2+17x+6}}\right)}{3226944}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)^2*(30 + 31*x - 12*x^2)^2), x]

[Out] -(275 + 388*x)/(98*(10 - 3*x)*Sqrt[6 + 17*x + 12*x^2]) + (3137*Sqrt[6 + 17*x + 12*x^2])/(38416*(10 - 3*x)) + (97*ArcTanh[(206 + 291*x)/(84*Sqrt[6 + 17*x + 12*x^2])])/3226944

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 740

Int[((d_.) + (e_.)*(x_)^(m_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e

```
) * x) * (a + b * x + c * x^2)^(p + 1) / ((p + 1) * (b^2 - 4 * a * c) * (c * d^2 - b * d * e + a * e^2)), x] + Dist[1 / ((p + 1) * (b^2 - 4 * a * c) * (c * d^2 - b * d * e + a * e^2)), Int[(d + e * x)^m * Simp[b * c * d * e * (2 * p - m + 2) + b^2 * e^2 * (m + p + 2) - 2 * c^2 * d^2 * (2 * p + 3) - 2 * a * c * e^2 * (m + 2 * p + 3) - c * e * (2 * c * d - b * e) * (m + 2 * p + 4) * x, x] * (a + b * x + c * x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4 * a * c, 0] && NeQ[c * d^2 - b * d * e + a * e^2, 0] && NeQ[2 * c * d - b * e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.) * (x_))^(m_) * ((f_.) + (g_.) * (x_)) * ((a_.) + (b_.) * (x_) + (c_.) * (x_)^2)^(p_), x_Symbol] :> -Simp[((e * f - d * g) * (d + e * x)^(m + 1) * (a + b * x + c * x^2)^(p + 1)) / (2 * (p + 1) * (c * d^2 - b * d * e + a * e^2)), x] - Dist[(b * (e * f + d * g) - 2 * (c * d * f + a * e * g)) / (2 * (c * d^2 - b * d * e + a * e^2)), Int[(d + e * x)^(m + 1) * (a + b * x + c * x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4 * a * c, 0] && NeQ[c * d^2 - b * d * e + a * e^2, 0] && EqQ[Simplify[m + 2 * p + 3], 0]
```

Rule 1002

```
Int[((g_.) + (h_.) * (x_))^(m_) * ((a_.) + (b_.) * (x_) + (c_.) * (x_)^2)^(p_) * ((d_.) + (e_.) * (x_) + (f_.) * (x_)^2)^(m_), x_Symbol] :> Int[((d * g) / a + (f * h * x) / c)^(m) * (a + b * x + c * x^2)^(m + p), x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x] && EqQ[c * g^2 - b * g * h + a * h^2, 0] && EqQ[c^2 * d * g^2 - a * c * e * g * h + a^2 * f * h^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^2(30+31x-12x^2)^2} dx &= \int \frac{1}{(10-3x)^2(6+17x+12x^2)^{3/2}} dx \\
&= -\frac{275+388x}{98(10-3x)\sqrt{6+17x+12x^2}} - \frac{1}{882} \int \frac{-\frac{14859}{2} - 10476x}{(10-3x)^2\sqrt{6+17x+12x^2}} dx \\
&= -\frac{275+388x}{98(10-3x)\sqrt{6+17x+12x^2}} + \frac{3137\sqrt{6+17x+12x^2}}{38416(10-3x)} + \frac{97 \int \frac{1}{(10-3x)\sqrt{6+17x+12x^2}} dx}{76832} \\
&= -\frac{275+388x}{98(10-3x)\sqrt{6+17x+12x^2}} + \frac{3137\sqrt{6+17x+12x^2}}{38416(10-3x)} - \frac{97 \operatorname{Subst}\left(\int \frac{1}{705}\right)}{705} \\
&= -\frac{275+388x}{98(10-3x)\sqrt{6+17x+12x^2}} + \frac{3137\sqrt{6+17x+12x^2}}{38416(10-3x)} + \frac{97 \tanh^{-1}\left(\frac{2}{84\sqrt{\dots}}\right)}{32269}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 114, normalized size = 1.36

$$\frac{\sqrt{12x^2+17x+6} \left(97(36x^3-69x^2-152x-60) \tanh^{-1}\left(\frac{7\sqrt{3x+2}}{6\sqrt{4x+3}}\right) - 42\sqrt{3x+2}\sqrt{4x+3}(37644x^2-98767x-88978) + 37644x^2 \right) + 97(-60-152x-69x^2+36x^3) \operatorname{ArcTanh}\left[\frac{7\sqrt{3x+2}}{6\sqrt{4x+3}}\right]}{1613472(3x-10)(3x+2)^{3/2}(4x+3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)^2*(30 + 31*x - 12*x^2)^2), x]
[Out] (Sqrt[6 + 17*x + 12*x^2]*(-42*Sqrt[2 + 3*x]*Sqrt[3 + 4*x]*(-88978 - 98767*x + 37644*x^2) + 97*(-60 - 152*x - 69*x^2 + 36*x^3)*ArcTanh[(7*Sqrt[2 + 3*x])/(6*Sqrt[3 + 4*x])]))/(1613472*(-10 + 3*x)*(2 + 3*x)^(3/2)*(3 + 4*x)^(3/2))
```

fricas [A] time = 0.72, size = 126, normalized size = 1.50

$$\frac{97(36x^3-69x^2-152x-60) \log\left(\frac{291x+84\sqrt{12x^2+17x+6}+206}{x}\right) - 97(36x^3-69x^2-152x-60) \log\left(\frac{291x-84\sqrt{12x^2+17x+6}+206}{x}\right)}{6453888(36x^3-69x^2-152x-60)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^2/(-12*x^2+31*x+30)^2,x, algorithm="fricas")
```

[Out] $\frac{1}{9680832} \sqrt{3} \left(\sqrt{3} \left(175672 \sqrt{3} + 97 \log \left(\frac{7\sqrt{3} - 12}{7\sqrt{3} + 12} \right) \right) \operatorname{sgn} \left(\frac{1}{3x+2} \right) - \left(97 \sqrt{3} \log \left(\frac{\left| -28\sqrt{3} + 24\sqrt{\frac{1}{3x+2} + 4} \right|}{4 \left(7\sqrt{3} + 6\sqrt{\frac{1}{3x+2} + 4} \right)} \right) \right)$

giac [B] time = 0.27, size = 159, normalized size = 1.89

$$\frac{1}{9680832} \sqrt{3} \left(\sqrt{3} \left(175672 \sqrt{3} + 97 \log \left(\frac{7\sqrt{3} - 12}{7\sqrt{3} + 12} \right) \right) \operatorname{sgn} \left(\frac{1}{3x+2} \right) - \left(97 \sqrt{3} \log \left(\frac{\left| -28\sqrt{3} + 24\sqrt{\frac{1}{3x+2} + 4} \right|}{4 \left(7\sqrt{3} + 6\sqrt{\frac{1}{3x+2} + 4} \right)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^2/(-12*x^2+31*x+30)^2,x, algorithm="giac")`

[Out] $\frac{1}{9680832} \sqrt{3} \left(\sqrt{3} \left(175672 \sqrt{3} + 97 \log \left(\frac{7\sqrt{3} - 12}{7\sqrt{3} + 12} \right) \right) \operatorname{sgn} \left(\frac{1}{3x+2} \right) - \left(97 \sqrt{3} \log \left(\frac{\left| -28\sqrt{3} + 24\sqrt{\frac{1}{3x+2} + 4} \right|}{4 \left(7\sqrt{3} + 6\sqrt{\frac{1}{3x+2} + 4} \right)} \right) \right)$

maple [B] time = 0.02, size = 245, normalized size = 2.92

$$\frac{97 \operatorname{arctanh} \left(\frac{97x + \frac{206}{3}}{28\sqrt{97x + 12\left(x - \frac{10}{3}\right)^2 - \frac{382}{3}}} \right)}{3226944} - \frac{7057\sqrt{12} \ln \left(\frac{\left(\frac{12x+17}{2}\right)\sqrt{12}}{12} + \sqrt{97x + 12\left(x - \frac{10}{3}\right)^2 - \frac{382}{3}} \right)}{813189888} + \frac{\sqrt{12} \ln \left(\frac{\left(\frac{12x+17}{2}\right)\sqrt{12}}{12} + \sqrt{97x + 12\left(x - \frac{10}{3}\right)^2 - \frac{382}{3}} \right)}{813189888}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((12*x^2+17*x+6)^(1/2)/(2+3*x)^2/(-12*x^2+31*x+30)^2,x)`

[Out] $\frac{384}{117649} (-x + 12(x + 3/4)^{-2-3/4})^{1/2} - \frac{16}{117649} 12^{1/2} \ln \left(\frac{1}{12} (12x + 17/2) \cdot 12^{1/2} + (-x + 12(x + 3/4)^{-2-3/4})^{1/2} \right) + \frac{1}{288} (x + 12(x + 2/3)^{-2+2/3})^{1/2} + \frac{1}{6912} 12^{1/2} \ln \left(\frac{1}{12} (12x + 17/2) \cdot 12^{1/2} + (x + 12(x + 2/3)^{-2+2/3})^{1/2} \right) - \frac{97}{45177216} (97x + 12(x - 10/3)^{-2-382/3})^{1/2} - \frac{7057}{813189888} 12^{1/2} \ln \left(\frac{1}{12} (12x + 17/2) \cdot 12^{1/2} + (97x + 12(x - 10/3)^{-2-382/3})^{1/2} \right) + \frac{97}{3226944} \operatorname{arctanh} \left(\frac{1}{28} (97x + 206/3) / (97x + 12(x - 10/3)^{-2-382/3})^{1/2} \right) - \frac{1}{72} (x + 2/3)^{-2} (x + 12(x + 2/3)^{-2+2/3})^{3/2} + \frac{32}{2401} (x + 3/4)^{-2} (-x + 12(x + 3/4)^{-2-3/4})^{3/2} - \frac{1}{67765824} (x - 10/3) (97x + 12(x - 10/3)^{-2-382/3})^{3/2} + \frac{1}{135531648} (24x + 17) (97x + 12(x - 10/3)^{-2-382/3})^{1/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{12x^2 + 17x + 6}}{(12x^2 - 31x - 30)^2 (3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^2/(-12*x^2+31*x+30)^2,x, algorithm="maxima")

[Out] integrate(sqrt(12*x^2 + 17*x + 6)/((12*x^2 - 31*x - 30)^2*(3*x + 2)^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{12x^2 + 17x + 6}}{(3x + 2)^2 (-12x^2 + 31x + 30)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((17*x + 12*x^2 + 6)^(1/2)/((3*x + 2)^2*(31*x - 12*x^2 + 30)^2), x)

[Out] int((17*x + 12*x^2 + 6)^(1/2)/((3*x + 2)^2*(31*x - 12*x^2 + 30)^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(3x + 2)(4x + 3)}}{(3x - 10)^2 (3x + 2)^2 (4x + 3)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((12*x**2+17*x+6)**(1/2)/(2+3*x)**2/(-12*x**2+31*x+30)**2,x)

[Out] Integral(sqrt((3*x + 2)*(4*x + 3))/((3*x - 10)**2*(3*x + 2)**2*(4*x + 3)**2), x)

$$3.137 \quad \int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^3(30+31x-12x^2)^3} dx$$

Optimal. Leaf size=139

$$\frac{388x + 275}{294(10 - 3x)^2 (12x^2 + 17x + 6)^{3/2}} - \frac{1634466587\sqrt{12x^2 + 17x + 6}}{7589772288(10 - 3x)} - \frac{50555899\sqrt{12x^2 + 17x + 6}}{19361664(10 - 3x)^2} + \frac{104256}{8232(10 - 3x)}$$

[Out] 1/294*(-275-388*x)/(10-3*x)^2/(12*x^2+17*x+6)^(3/2)+40325/637540872192*arctanh(1/84*(206+291*x)/(12*x^2+17*x+6)^(1/2))+1/8232*(738029+1042556*x)/(10-3*x)^2/(12*x^2+17*x+6)^(1/2)-50555899/19361664*(12*x^2+17*x+6)^(1/2)/(10-3*x)^2-1634466587/7589772288*(12*x^2+17*x+6)^(1/2)/(10-3*x)

Rubi [A] time = 0.12, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {1002, 740, 822, 834, 806, 724, 206}

$$\frac{388x + 275}{294(10 - 3x)^2 (12x^2 + 17x + 6)^{3/2}} - \frac{1634466587\sqrt{12x^2 + 17x + 6}}{7589772288(10 - 3x)} - \frac{50555899\sqrt{12x^2 + 17x + 6}}{19361664(10 - 3x)^2} + \frac{104256}{8232(10 - 3x)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)^3*(30 + 31*x - 12*x^2)^3), x]

[Out] -(275 + 388*x)/(294*(10 - 3*x)^2*(6 + 17*x + 12*x^2)^(3/2)) + (738029 + 1042556*x)/(8232*(10 - 3*x)^2*Sqrt[6 + 17*x + 12*x^2]) - (50555899*Sqrt[6 + 17*x + 12*x^2])/(19361664*(10 - 3*x)^2) - (1634466587*Sqrt[6 + 17*x + 12*x^2])/(7589772288*(10 - 3*x)) + (40325*ArcTanh[(206 + 291*x)/(84*Sqrt[6 + 17*x + 12*x^2]])/637540872192

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m + 2*p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
```


IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 1002

Int[((g_) + (h_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_ + (e_)*(x_) + (f_)*(x_)^2)^(m_)), x_Symbol] :> Int[((d*g)/a + (f*h*x)/c)^(m*(a + b*x + c*x^2)^(m + p), x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x] && EqQ[c*g^2 - b*g*h + a*h^2, 0] && EqQ[c^2*d*g^2 - a*c*e*g*h + a^2*f*h^2, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^3(30+31x-12x^2)^3} dx &= \int \frac{1}{(10-3x)^3(6+17x+12x^2)^{5/2}} dx \\
 &= -\frac{275+388x}{294(10-3x)^2(6+17x+12x^2)^{3/2}} - \frac{\int \frac{\frac{109953}{2}-41904x}{(10-3x)^3(6+17x+12x^2)^{3/2}} dx}{2646} \\
 &= -\frac{275+388x}{294(10-3x)^2(6+17x+12x^2)^{3/2}} + \frac{738029+1042556x}{8232(10-3x)^2\sqrt{6+17x+12x^2}} + \dots \\
 &= -\frac{275+388x}{294(10-3x)^2(6+17x+12x^2)^{3/2}} + \frac{738029+1042556x}{8232(10-3x)^2\sqrt{6+17x+12x^2}} - \dots \\
 &= -\frac{275+388x}{294(10-3x)^2(6+17x+12x^2)^{3/2}} + \frac{738029+1042556x}{8232(10-3x)^2\sqrt{6+17x+12x^2}} - \dots \\
 &= -\frac{275+388x}{294(10-3x)^2(6+17x+12x^2)^{3/2}} + \frac{738029+1042556x}{8232(10-3x)^2\sqrt{6+17x+12x^2}} - \dots \\
 &= -\frac{275+388x}{294(10-3x)^2(6+17x+12x^2)^{3/2}} + \frac{738029+1042556x}{8232(10-3x)^2\sqrt{6+17x+12x^2}} - \dots
 \end{aligned}$$

Mathematica [A] time = 0.37, size = 131, normalized size = 0.94

$$\frac{\sqrt{12x^2 + 17x + 6} \left(40325 (-36x^3 + 69x^2 + 152x + 60)^2 \tanh^{-1} \left(\frac{7\sqrt{3x+2}}{6\sqrt{4x+3}} \right) + 42\sqrt{3x+2}\sqrt{4x+3} (706089565584x^4 + 10124325497244x^5 + 9848047480070x^6 - 1096520427663x^7 - 3206824169544x^8 + 706089565584x^9) + 40325(60 + 152x + 69x^2 - 36x^3)^2 \operatorname{ArcTanh} \left[\frac{7\sqrt{3x+2}}{6\sqrt{4x+3}} \right] \right)}{318770436096(10 - 3x)^2(2 + 3x)^{5/2}(3 + 4x)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)^3*(30 + 31*x - 12*x^2)^3), x]

[Out] (Sqrt[6 + 17*x + 12*x^2]*(42*Sqrt[2 + 3*x]*Sqrt[3 + 4*x]*(2773753482408 + 10124325497244*x + 9848047480070*x^2 - 1096520427663*x^3 - 3206824169544*x^4 + 706089565584*x^5) + 40325*(60 + 152*x + 69*x^2 - 36*x^3)^2*ArcTanh[(7*Sqrt[2 + 3*x])/(6*Sqrt[3 + 4*x])]))/(318770436096*(10 - 3*x)^2*(2 + 3*x)^(5/2)*(3 + 4*x)^(5/2))

fricas [A] time = 0.64, size = 186, normalized size = 1.34

$$40325 (1296x^6 - 4968x^5 - 6183x^4 + 16656x^3 + 31384x^2 + 18240x + 3600) \log \left(\frac{291x + 84\sqrt{12x^2 + 17x + 6} + 206}{x} \right) - 40325 (1296x^6 - 4968x^5 - 6183x^4 + 16656x^3 + 31384x^2 + 18240x + 3600) \log \left(\frac{291x - 84\sqrt{12x^2 + 17x + 6} + 206}{x} \right) + 168(706089565584x^5 - 3206824169544x^4 - 1096520427663x^3 + 9848047480070x^2 + 10124325497244x + 2773753482408) \sqrt{12x^2 + 17x + 6} / (1296x^6 - 4968x^5 - 6183x^4 + 16656x^3 + 31384x^2 + 18240x + 3600)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^3/(-12*x^2+31*x+30)^3,x, algorithm="fricas")

[Out] 1/1275081744384*(40325*(1296*x^6 - 4968*x^5 - 6183*x^4 + 16656*x^3 + 31384*x^2 + 18240*x + 3600)*log((291*x + 84*sqrt(12*x^2 + 17*x + 6) + 206)/x) - 40325*(1296*x^6 - 4968*x^5 - 6183*x^4 + 16656*x^3 + 31384*x^2 + 18240*x + 3600)*log((291*x - 84*sqrt(12*x^2 + 17*x + 6) + 206)/x) + 168*(706089565584*x^5 - 3206824169544*x^4 - 1096520427663*x^3 + 9848047480070*x^2 + 10124325497244*x + 2773753482408)*sqrt(12*x^2 + 17*x + 6))/(1296*x^6 - 4968*x^5 - 6183*x^4 + 16656*x^3 + 31384*x^2 + 18240*x + 3600)

giac [A] time = 0.26, size = 232, normalized size = 1.67

$$\frac{\sqrt{3} \left(282273 \sqrt{3} (2\sqrt{3}x - \sqrt{12x^2 + 17x + 6})^3 - 11460924 (2\sqrt{3}x - \sqrt{12x^2 + 17x + 6})^2 - 37551180 \sqrt{3} (2\sqrt{3}x - \sqrt{12x^2 + 17x + 6}) \right)}{159385218048 \left(3 (2\sqrt{3}x - \sqrt{12x^2 + 17x + 6})^2 - 40 \sqrt{3} (2\sqrt{3}x - \sqrt{12x^2 + 17x + 6}) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^3/(-12*x^2+31*x+30)^3,x, algorithm="giac")

```
[Out] 1/159385218048*sqrt(3)*(282273*sqrt(3)*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x + 6))^3 - 11460924*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x + 6))^2 - 37551180*sqrt(3)*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x + 6)) - 83365264)/(3*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x + 6))^2 - 40*sqrt(3)*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x + 6)) - 188)^2 + 1/2213683584*((8*(2860316794*x + 6078171227)*x + 34383350229)*x + 8090114146)/(12*x^2 + 17*x + 6)^(3/2) + 40325/637540872192*log(abs(-6*sqrt(3)*x + 20*sqrt(3) + 3*sqrt(12*x^2 + 17*x + 6) + 42)) - 40325/637540872192*log(abs(-6*sqrt(3)*x + 20*sqrt(3) + 3*sqrt(12*x^2 + 17*x + 6) - 42))
```

maple [B] time = 0.02, size = 306, normalized size = 2.20

$$\frac{40325 \operatorname{arctanh}\left(\frac{97x + \frac{206}{3}}{28\sqrt{97x + 12\left(x - \frac{10}{3}\right)^2 - \frac{382}{3}}}\right) + 570457\sqrt{12} \ln\left(\frac{\left(12x + \frac{17}{2}\right)\sqrt{12}}{12} + \sqrt{97x + 12\left(x - \frac{10}{3}\right)^2 - \frac{382}{3}}\right) + 23\sqrt{12} \ln\left(\frac{\left(12x + \frac{17}{2}\right)\sqrt{12}}{12} - \sqrt{97x + 12\left(x - \frac{10}{3}\right)^2 - \frac{382}{3}}\right)}{637540872192} - \frac{570457\sqrt{12} \ln\left(\frac{\left(12x + \frac{17}{2}\right)\sqrt{12}}{12} + \sqrt{97x + 12\left(x - \frac{10}{3}\right)^2 - \frac{382}{3}}\right) + 23\sqrt{12} \ln\left(\frac{\left(12x + \frac{17}{2}\right)\sqrt{12}}{12} - \sqrt{97x + 12\left(x - \frac{10}{3}\right)^2 - \frac{382}{3}}\right)}{31239502737408}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((12*x^2+17*x+6)^(1/2)/(2+3*x)^3/(-12*x^2+31*x+30)^3,x)
```

```
[Out] -1/2592/(x+2/3)^3*(x+12*(x+2/3)^2+2/3)^(3/2)-1261/31239502737408/(x-10/3)*(97*x+12*(x-10/3)^2-382/3)^(3/2)+1261/62479005474816*(24*x+17)*(97*x+12*(x-10/3)^2-382/3)^(1/2)+1/79692609024/(x-10/3)^2*(97*x+12*(x-10/3)^2-382/3)^(3/2)-230400/5764801/(x+3/4)^2*(-x+12*(x+3/4)^2-3/4)^(3/2)-128/352947/(x+3/4)^3*(-x+12*(x+3/4)^2-3/4)^(3/2)-570457/31239502737408*12^(1/2)*ln(1/12*(12*x+17/2)*12^(1/2)+(97*x+12*(x-10/3)^2-382/3)^(1/2))+47/1152/(x+2/3)^2*(x+12*(x+2/3)^2+2/3)^(3/2)-23/110592*12^(1/2)*ln(1/12*(12*x+17/2)*12^(1/2)+(x+12*(x+2/3)^2+2/3)^(1/2))+58752/282475249*12^(1/2)*ln(1/12*(12*x+17/2)*12^(1/2)+(-x+12*(x+3/4)^2-3/4)^(1/2))-40325/8925572210688*(97*x+12*(x-10/3)^2-382/3)^(1/2)+40325/637540872192*arctanh(1/28*(97*x+206/3)/(97*x+12*(x-10/3)^2-382/3)^(1/2))-23/4608*(x+12*(x+2/3)^2+2/3)^(1/2)-1410048/282475249*(-x+12*(x+3/4)^2-3/4)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{12x^2 + 17x + 6}}{(12x^2 - 31x - 30)^3 (3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^3/(-12*x^2+31*x+30)^3,x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(12*x^2 + 17*x + 6)/((12*x^2 - 31*x - 30)^3*(3*x + 2)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{12x^2 + 17x + 6}}{(3x + 2)^3 (-12x^2 + 31x + 30)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((17*x + 12*x^2 + 6)^(1/2)/((3*x + 2)^3*(31*x - 12*x^2 + 30)^3), x)

[Out] int((17*x + 12*x^2 + 6)^(1/2)/((3*x + 2)^3*(31*x - 12*x^2 + 30)^3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{12x^2 + 17x + 6}}{46656x^9 - 268272x^8 - 76788x^7 + 1703619x^6 + 1218456x^5 - 3669588x^4 - 6898688x^3 - 4903920x^2 - 1641600x - 216000} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((12*x**2+17*x+6)**(1/2)/(2+3*x)**3/(-12*x**2+31*x+30)**3, x)

[Out] -Integral(sqrt(12*x**2 + 17*x + 6)/(46656*x**9 - 268272*x**8 - 76788*x**7 + 1703619*x**6 + 1218456*x**5 - 3669588*x**4 - 6898688*x**3 - 4903920*x**2 - 1641600*x - 216000), x)

$$3.138 \quad \int (-3 + 2x) (-3x + x^2)^{2/3} dx$$

Optimal. Leaf size=15

$$\frac{3}{5} (x^2 - 3x)^{5/3}$$

[Out] 3/5*(x^2-3*x)^(5/3)

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {629}

$$\frac{3}{5} (x^2 - 3x)^{5/3}$$

Antiderivative was successfully verified.

[In] Int[(-3 + 2*x)*(-3*x + x^2)^(2/3), x]

[Out] (3*(-3*x + x^2)^(5/3))/5

Rule 629

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rubi steps

$$\int (-3 + 2x) (-3x + x^2)^{2/3} dx = \frac{3}{5} (-3x + x^2)^{5/3}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 0.87

$$\frac{3}{5} ((x - 3)x)^{5/3}$$

Antiderivative was successfully verified.

[In] Integrate[(-3 + 2*x)*(-3*x + x^2)^(2/3), x]

[Out] (3*((-3 + x)*x)^(5/3))/5

fricas [A] time = 0.61, size = 11, normalized size = 0.73

$$\frac{3}{5} (x^2 - 3x)^{5/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3+2*x)*(x^2-3*x)^(2/3),x, algorithm="fricas")

[Out] 3/5*(x^2 - 3*x)^(5/3)

giac [A] time = 0.15, size = 11, normalized size = 0.73

$$\frac{3}{5} (x^2 - 3x)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3+2*x)*(x^2-3*x)^(2/3),x, algorithm="giac")

[Out] 3/5*(x^2 - 3*x)^(5/3)

maple [A] time = 0.00, size = 16, normalized size = 1.07

$$\frac{3(x-3)(x^2-3x)^{\frac{2}{3}}x}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-3+2*x)*(x^2-3*x)^(2/3),x)

[Out] 3/5*(x-3)*x*(x^2-3*x)^(2/3)

maxima [A] time = 0.43, size = 11, normalized size = 0.73

$$\frac{3}{5} (x^2 - 3x)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3+2*x)*(x^2-3*x)^(2/3),x, algorithm="maxima")

[Out] 3/5*(x^2 - 3*x)^(5/3)

mupad [B] time = 3.72, size = 15, normalized size = 1.00

$$\frac{3x(x^2-3x)^{\frac{2}{3}}(x-3)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x - 3)*(x^2 - 3*x)^(2/3),x)

[Out] $(3*x*(x^2 - 3*x)^{(2/3)}*(x - 3))/5$

sympy [B] time = 0.31, size = 31, normalized size = 2.07

$$\frac{3x^2(x^2 - 3x)^{\frac{2}{3}}}{5} - \frac{9x(x^2 - 3x)^{\frac{2}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-3+2*x)*(x**2-3*x)**(2/3),x)`

[Out] $3*x**2*(x**2 - 3*x)**(2/3)/5 - 9*x*(x**2 - 3*x)**(2/3)/5$

$$3.139 \quad \int((-3+x)x)^{2/3}(-3+2x) dx$$

Optimal. Leaf size=16

$$\frac{3}{5}(-((3-x)x))^{5/3}$$

[Out] 3/5*(-(3-x)*x)^(5/3)

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1588}

$$\frac{3}{5}(-((3-x)x))^{5/3}$$

Antiderivative was successfully verified.

[In] Int[((-3 + x)*x)^(2/3)*(-3 + 2*x), x]

[Out] (3*(-((3 - x)*x))^(5/3))/5

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int((-3+x)x)^{2/3}(-3+2x) dx = \frac{3}{5}(-((3-x)x))^{5/3}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{3}{5}((x-3)x)^{5/3}$$

Antiderivative was successfully verified.

[In] Integrate[((-3 + x)*x)^(2/3)*(-3 + 2*x), x]

[Out] (3*((-3 + x)*x)^(5/3))/5

fricas [A] time = 0.56, size = 11, normalized size = 0.69

$$\frac{3}{5} (x^2 - 3x)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((−3+x)*x)^(2/3)*(−3+2*x),x, algorithm="fricas")

[Out] 3/5*(x^2 - 3*x)^(5/3)

giac [A] time = 0.15, size = 11, normalized size = 0.69

$$\frac{3}{5} (x^2 - 3x)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((−3+x)*x)^(2/3)*(−3+2*x),x, algorithm="giac")

[Out] 3/5*(x^2 - 3*x)^(5/3)

maple [A] time = 0.00, size = 14, normalized size = 0.88

$$\frac{3(x-3)((x-3)x)^{\frac{2}{3}}x}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x-3)*x)^(2/3)*(−3+2*x),x)

[Out] 3/5*(x-3)*x*((x-3)*x)^(2/3)

maxima [A] time = 0.43, size = 9, normalized size = 0.56

$$\frac{3}{5} ((x-3)x)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((−3+x)*x)^(2/3)*(−3+2*x),x, algorithm="maxima")

[Out] 3/5*((x - 3)*x)^(5/3)

mupad [B] time = 3.66, size = 13, normalized size = 0.81

$$\frac{3x(x(x-3))^{2/3}(x-3)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x - 3)*(x*(x - 3))^(2/3), x)`

[Out] `(3*x*(x*(x - 3))^(2/3)*(x - 3))/5`

sympy [A] time = 4.25, size = 10, normalized size = 0.62

$$\frac{3(x(x-3))^{\frac{5}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((-3+x)*x)**(2/3)*(-3+2*x), x)`

[Out] `3*(x*(x - 3))**(5/3)/5`

$$3.140 \quad \int \frac{x(9-9x+2x^2)}{\sqrt[3]{-3x+x^2}} dx$$

Optimal. Leaf size=15

$$\frac{3}{5} (x^2 - 3x)^{5/3}$$

[Out] 3/5*(x^2-3*x)^(5/3)

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1631, 629}

$$\frac{3}{5} (x^2 - 3x)^{5/3}$$

Antiderivative was successfully verified.

[In] Int[(x*(9 - 9*x + 2*x^2))/(-3*x + x^2)^(1/3), x]

[Out] (3*(-3*x + x^2)^(5/3))/5

Rule 629

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1631

Int[(Pq_)*((e_)*(x_))^(m_)*((b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Dist[e, Int[(e*x)^(m - 1)*PolynomialQuotient[Pq, b + c*x, x]*(b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{b, c, e, m, p}, x] && PolyQ[Pq, x] && EqQ[PolynomialRemainder[Pq, b + c*x, x], 0]

Rubi steps

$$\begin{aligned} \int \frac{x(9-9x+2x^2)}{\sqrt[3]{-3x+x^2}} dx &= \int (-3+2x)(-3x+x^2)^{2/3} dx \\ &= \frac{3}{5} (-3x+x^2)^{5/3} \end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 0.87

$$\frac{3}{5} ((x-3)x)^{5/3}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(9 - 9*x + 2*x^2))/(-3*x + x^2)^(1/3),x]

[Out] (3*((-3 + x)*x)^(5/3))/5

fricas [A] time = 0.41, size = 11, normalized size = 0.73

$$\frac{3}{5} (x^2 - 3x)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*x^2-9*x+9)/(x^2-3*x)^(1/3),x, algorithm="fricas")

[Out] 3/5*(x^2 - 3*x)^(5/3)

giac [A] time = 0.16, size = 11, normalized size = 0.73

$$\frac{3}{5} (x^2 - 3x)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*x^2-9*x+9)/(x^2-3*x)^(1/3),x, algorithm="giac")

[Out] 3/5*(x^2 - 3*x)^(5/3)

maple [A] time = 0.00, size = 20, normalized size = 1.33

$$\frac{3(x-3)^2 x^2}{5(x^2-3x)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(2*x^2-9*x+9)/(x^2-3*x)^(1/3),x)

[Out] 3/5*(x-3)^2*x^2/(x^2-3*x)^(1/3)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - 9x + 9)x}{(x^2 - 3x)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*x^2-9*x+9)/(x^2-3*x)^(1/3),x, algorithm="maxima")

[Out] integrate((2*x^2 - 9*x + 9)*x/(x^2 - 3*x)^(1/3), x)

mupad [B] time = 3.68, size = 15, normalized size = 1.00

$$\frac{3x(x^2 - 3x)^{2/3}(x - 3)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(2*x^2 - 9*x + 9))/(x^2 - 3*x)^(1/3),x)

[Out] (3*x*(x^2 - 3*x)^(2/3)*(x - 3))/5

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(x-3)(2x-3)}{\sqrt[3]{x(x-3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*x**2-9*x+9)/(x**2-3*x)**(1/3),x)

[Out] Integral(x*(x - 3)*(2*x - 3)/(x*(x - 3))**(1/3), x)

$$3.141 \quad \int \frac{x(9-9x+2x^2)}{\sqrt[3]{(-3+x)x}} dx$$

Optimal. Leaf size=15

$$\frac{3}{5}(x^2 - 3x)^{5/3}$$

[Out] 3/5*(x^2-3*x)^(5/3)

Rubi [A] time = 0.06, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1985, 1631, 629}

$$\frac{3}{5}(x^2 - 3x)^{5/3}$$

Antiderivative was successfully verified.

[In] Int[(x*(9 - 9*x + 2*x^2))/((-3 + x)*x)^(1/3),x]

[Out] (3*(-3*x + x^2)^(5/3))/5

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
  := Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1631

```
Int[(Pq_)*((e_.)*(x_)^(m_.))*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
  > Dist[e, Int[(e*x)^(m - 1)*PolynomialQuotient[Pq, b + c*x, x]*(b*x + c*x^2)
  ]^(p + 1), x] /; FreeQ[{b, c, e, m, p}, x] && PolyQ[Pq, x] && EqQ[Polyn
  omialRemainder[Pq, b + c*x, x], 0]
```

Rule 1985

```
Int[(u_)^(p_.)*(v_)^(q_.)*(z_)^(m_.), x_Symbol] := Int[ExpandToSum[z, x]^m*
  ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{m, p, q}, x] && Linea
  rQ[z, x] && QuadraticQ[{u, v}, x] && !(LinearMatchQ[z, x] && QuadraticMatc
  hQ[{u, v}, x])
```

Rubi steps

$$\begin{aligned} \int \frac{x(9-9x+2x^2)}{\sqrt[3]{(-3+x)x}} dx &= \int \frac{x(9-9x+2x^2)}{\sqrt[3]{-3x+x^2}} dx \\ &= \int (-3+2x)(-3x+x^2)^{2/3} dx \\ &= \frac{3}{5}(-3x+x^2)^{5/3} \end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 0.87

$$\frac{3}{5}((x-3)x)^{5/3}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(9 - 9*x + 2*x^2))/((-3 + x)*x)^(1/3), x]

[Out] (3*((-3 + x)*x)^(5/3))/5

fricas [A] time = 0.42, size = 11, normalized size = 0.73

$$\frac{3}{5}(x^2 - 3x)^{5/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*x^2-9*x+9)/((-3+x)*x)^(1/3), x, algorithm="fricas")

[Out] 3/5*(x^2 - 3*x)^(5/3)

giac [A] time = 0.20, size = 11, normalized size = 0.73

$$\frac{3}{5}(x^2 - 3x)^{5/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*x^2-9*x+9)/((-3+x)*x)^(1/3), x, algorithm="giac")

[Out] 3/5*(x^2 - 3*x)^(5/3)

maple [A] time = 0.00, size = 18, normalized size = 1.20

$$\frac{3(x-3)^2 x^2}{5((x-3)x)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(2*x^2-9*x+9)/((x-3)*x)^(1/3),x)`

[Out] `3/5*(x-3)^2*x^2/((x-3)*x)^(1/3)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - 9x + 9)x}{((x-3)x)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(2*x^2-9*x+9)/((-3+x)*x)^(1/3),x, algorithm="maxima")`

[Out] `integrate((2*x^2 - 9*x + 9)*x/((x - 3)*x)^(1/3), x)`

mupad [B] time = 3.61, size = 13, normalized size = 0.87

$$\frac{3x(x(x-3))^{2/3}(x-3)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*(2*x^2 - 9*x + 9))/(x*(x - 3))^(1/3),x)`

[Out] `(3*x*(x*(x - 3))^(2/3)*(x - 3))/5`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(x-3)(2x-3)}{\sqrt[3]{x(x-3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(2*x**2-9*x+9)/((-3+x)*x)**(1/3),x)`

[Out] `Integral(x*(x - 3)*(2*x - 3)/(x*(x - 3))**(1/3), x)`

$$3.142 \quad \int \frac{g+hx}{\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2} (g^2+3h^2x^2)} dx$$

Optimal. Leaf size=242

$$\frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}} \log(g^2+3h^2x^2)}{6 \cdot 2^{2/3} h \sqrt[3]{9cx^2-\frac{cg^2}{h^2}}} - \frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}} \log\left(\left(1-\frac{3hx}{g}\right)^{2/3} + \sqrt[3]{2} \sqrt[3]{\frac{3hx}{g}+1}\right)}{2 \cdot 2^{2/3} h \sqrt[3]{9cx^2-\frac{cg^2}{h^2}}} + \frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}} \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2^{2/3}\left(1-\frac{3hx}{g}\right)}{\sqrt{3} \sqrt[3]{\frac{3hx}{g}+1}}\right)}{2^{2/3} \sqrt{3} h \sqrt[3]{9cx^2-\frac{cg^2}{h^2}}}$$

[Out] $1/12*(1-9*h^2*x^2/g^2)^(1/3)*\ln(3*h^2*x^2+g^2)*2^(1/3)/h/(-c*g^2/h^2+9*c*x^2)^(1/3)-1/4*(1-9*h^2*x^2/g^2)^(1/3)*\ln\left(\left(1-\frac{3hx}{g}\right)^{2/3}+\sqrt[3]{2}\sqrt[3]{\frac{3hx}{g}+1}\right)*2^(1/3)/h/(-c*g^2/h^2+9*c*x^2)^(1/3)-1/6*(1-9*h^2*x^2/g^2)^(1/3)*\arctan\left(\frac{-1/3*3^(1/2)+1/3*2^(2/3)*(1-3*h*x/g)^(2/3)}{(1+3*h*x/g)^(1/3)*3^(1/2)}\right)*2^(1/3)/h/(-c*g^2/h^2+9*c*x^2)^(1/3)*3^(1/2)$

Rubi [A] time = 0.09, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1009, 1008}

$$\frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}} \log(g^2+3h^2x^2)}{6 \cdot 2^{2/3} h \sqrt[3]{9cx^2-\frac{cg^2}{h^2}}} - \frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}} \log\left(\left(1-\frac{3hx}{g}\right)^{2/3} + \sqrt[3]{2} \sqrt[3]{\frac{3hx}{g}+1}\right)}{2 \cdot 2^{2/3} h \sqrt[3]{9cx^2-\frac{cg^2}{h^2}}} + \frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}} \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2^{2/3}\left(1-\frac{3hx}{g}\right)}{\sqrt{3} \sqrt[3]{\frac{3hx}{g}+1}}\right)}{2^{2/3} \sqrt{3} h \sqrt[3]{9cx^2-\frac{cg^2}{h^2}}}$$

Antiderivative was successfully verified.

[In] Int[(g + h*x)/((-((c*g^2)/h^2) + 9*c*x^2)^(1/3)*(g^2 + 3*h^2*x^2)), x]

[Out] $\left(\left(1-\frac{9h^2x^2}{g^2}\right)^{1/3} \text{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2^{2/3}\left(1-\frac{3hx}{g}\right)}{\sqrt{3} \sqrt[3]{\frac{3hx}{g}+1}}\right]\right) / \left(2^{2/3} \sqrt{3} h \left(-\frac{c g^2}{h^2} + 9 c x^2\right)^{1/3}\right) + \left(\left(1-\frac{9h^2x^2}{g^2}\right)^{1/3} \text{Log}\left[g^2+3h^2x^2\right]\right) / \left(6 \cdot 2^{2/3} h \left(-\frac{c g^2}{h^2} + 9 c x^2\right)^{1/3}\right) - \left(\left(1-\frac{9h^2x^2}{g^2}\right)^{1/3} \text{Log}\left[\left(1-\frac{3hx}{g}\right)^{2/3} + 2^{1/3} \left(1+\frac{3hx}{g}\right)^{1/3}\right]\right) / \left(2 \cdot 2^{2/3} h \left(-\frac{c g^2}{h^2} + 9 c x^2\right)^{1/3}\right)$

Rule 1008

Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)^(1/3)*((d_) + (f_.)*(x_)^2)), x_Symbol] :> Simp[(Sqrt[3]*h*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - (3*h*x)/g)^(2/3))]/(Sqrt[3]*(1 + (3*h*x)/g)^(1/3)))]/(2^(2/3)*a^(1/3)*f), x] + (-Simp[(3*h*Log[(1 - (3*h*x)/g)^(2/3) + 2^(1/3)*(1 + (3*h*x)/g)^(1/3))]/(2^(5/3)*a^(1/3)*f))

$(1/3)*f), x] + \text{Simp}[(h*\text{Log}[d + f*x^2])/(2^(5/3)*a^(1/3)*f), x] /; \text{FreeQ}[\{a, c, d, f, g, h\}, x] \&\& \text{EqQ}[c*d + 3*a*f, 0] \&\& \text{EqQ}[c*g^2 + 9*a*h^2, 0] \&\& \text{GtQ}[a, 0]$

Rule 1009

$\text{Int}[(g_ + (h_)*(x_))/((a_ + (c_)*(x_)^2)^(1/3)*((d_ + (f_)*(x_)^2)), x_Symbol] := \text{Dist}[(1 + (c*x^2)/a)^(1/3)/(a + c*x^2)^(1/3), \text{Int}[(g + h*x)/((1 + (c*x^2)/a)^(1/3)*(d + f*x^2)), x], x] /; \text{FreeQ}[\{a, c, d, f, g, h\}, x] \&\& \text{EqQ}[c*d + 3*a*f, 0] \&\& \text{EqQ}[c*g^2 + 9*a*h^2, 0] \&\& !\text{GtQ}[a, 0]$

Rubi steps

$$\int \frac{g + hx}{\sqrt[3]{-\frac{cg^2}{h^2} + 9cx^2} (g^2 + 3h^2x^2)} dx = \frac{\sqrt[3]{1 - \frac{9h^2x^2}{g^2}} \int \frac{g+hx}{(g^2+3h^2x^2) \sqrt[3]{1 - \frac{9h^2x^2}{g^2}}} dx}{\sqrt[3]{-\frac{cg^2}{h^2} + 9cx^2}}$$

$$= \frac{\sqrt[3]{1 - \frac{9h^2x^2}{g^2}} \tan^{-1} \left(\frac{1}{\sqrt{3}} - \frac{2^{2/3} \left(1 - \frac{3hx}{g}\right)^{2/3}}{\sqrt{3} \sqrt[3]{1 + \frac{3hx}{g}}} \right)}{2^{2/3} \sqrt{3} h \sqrt[3]{-\frac{cg^2}{h^2} + 9cx^2}} + \frac{\sqrt[3]{1 - \frac{9h^2x^2}{g^2}} \log(g^2 + 3h^2x^2)}{6 \cdot 2^{2/3} h \sqrt[3]{-\frac{cg^2}{h^2} + 9cx^2}} - \frac{\sqrt[3]{1 - \frac{9h^2x^2}{g^2}}}{\sqrt[3]{-\frac{cg^2}{h^2} + 9cx^2}}$$

Mathematica [C] time = 0.56, size = 268, normalized size = 1.11

$$\frac{h^2x \left(-hx \sqrt[3]{1 - \frac{9h^2x^2}{g^2}} F_1 \left(1; \frac{1}{3}, 1; 2; \frac{9h^2x^2}{g^2}, -\frac{3h^2x^2}{g^2} \right) - \frac{2g^5 F_1 \left(\frac{1}{2}; \frac{1}{3}, 1; \frac{3}{2}; \frac{9h^2x^2}{g^2}, -\frac{3h^2x^2}{g^2} \right)}{(g^2+3h^2x^2) \left(g^2 F_1 \left(\frac{1}{2}; \frac{1}{3}, 1; \frac{3}{2}; \frac{9h^2x^2}{g^2}, -\frac{3h^2x^2}{g^2} \right) + 2h^2x^2 \left(F_1 \left(\frac{3}{2}; \frac{4}{3}, 1; \frac{5}{2}; \frac{9h^2x^2}{g^2}, -\frac{3h^2x^2}{g^2} \right) - F_1 \left(\frac{3}{2}; \frac{3}{2}, 1; \frac{5}{2}; \frac{9h^2x^2}{g^2}, -\frac{3h^2x^2}{g^2} \right) \right)} \right)}{2cg^2 (g^2 - 9h^2x^2)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(g + h*x)/((-((c*g^2)/h^2) + 9*c*x^2)^(1/3)*(g^2 + 3*h^2*x^2)),x]

[Out] $(h^2*x*(c*(-(g^2/h^2) + 9*x^2))^(2/3)*(-(h*x*(1 - (9*h^2*x^2)/g^2)^(1/3)*\text{AppellF1}[1, 1/3, 1, 2, (9*h^2*x^2)/g^2, (-3*h^2*x^2)/g^2]) - (2*g^5*\text{AppellF1}[1/2, 1/3, 1, 3/2, (9*h^2*x^2)/g^2, (-3*h^2*x^2)/g^2])/(g^2 + 3*h^2*x^2)*(g^2*\text{AppellF1}[1/2, 1/3, 1, 3/2, (9*h^2*x^2)/g^2, (-3*h^2*x^2)/g^2] + 2*h^2*x^2*(-\text{AppellF1}[3/2, 1/3, 2, 5/2, (9*h^2*x^2)/g^2, (-3*h^2*x^2)/g^2] + \text{AppellF1}[3/2, 4/3, 1, 5/2, (9*h^2*x^2)/g^2, (-3*h^2*x^2)/g^2]))))/(2*c*g^2*(g^2 - 9*h^2*x^2))$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{hx + g}{(3h^2x^2 + g^2)\left(9cx^2 - \frac{cg^2}{h^2}\right)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2),x, algorithm="giac")

[Out] integrate((h*x + g)/((3*h^2*x^2 + g^2)*(9*c*x^2 - c*g^2/h^2)^(1/3)), x)

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{hx + g}{\left(9cx^2 - \frac{cg^2}{h^2}\right)^{\frac{1}{3}} (3h^2x^2 + g^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2),x)

[Out] int((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{hx + g}{(3h^2x^2 + g^2)\left(9cx^2 - \frac{cg^2}{h^2}\right)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2),x, algorithm="maxima")

[Out] integrate((h*x + g)/((3*h^2*x^2 + g^2)*(9*c*x^2 - c*g^2/h^2)^(1/3)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{g + hx}{(g^2 + 3h^2x^2) \left(9cx^2 - \frac{cg^2}{h^2}\right)^{1/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g + h*x)/((g^2 + 3*h^2*x^2)*(9*c*x^2 - (c*g^2)/h^2)^(1/3)), x)

[Out] int((g + h*x)/((g^2 + 3*h^2*x^2)*(9*c*x^2 - (c*g^2)/h^2)^(1/3)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{g + hx}{\sqrt[3]{c \left(-\frac{g}{h} + 3x\right) \left(\frac{g}{h} + 3x\right) (g^2 + 3h^2x^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)/(-c*g**2/h**2+9*c*x**2)**(1/3)/(3*h**2*x**2+g**2), x)

[Out] Integral((g + h*x)/((c*(-g/h + 3*x)*(g/h + 3*x))**(1/3)*(g**2 + 3*h**2*x**2)), x)

$$3.143 \quad \int \frac{g+hx}{\sqrt[3]{\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2} \left(\frac{f\left(b^2-\frac{-c^2g^2+bcgh+2b^2h^2}{3h^2}\right)}{c^2} + \frac{bfx}{c} + fx^2 \right)} dx$$

Optimal. Leaf size=488

$$\frac{3^{2/3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}} \log\left(\frac{f\left(b^2h^2-bcgh+c^2g^2\right)}{3c^2h^2} + \frac{bfx}{c} + fx^2\right)}{2f\sqrt[3]{-\frac{(cg-2bh)(bh+cg)}{ch^2}+9bx+9cx^2}} - \frac{3^{2/3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}} \log\left(\left(1 - \frac{(cg-2bh)(bh+cg)}{ch^2} + 9bx + 9cx^2\right)\right)}{2f\sqrt[3]{-\frac{(cg-2bh)(bh+cg)}{ch^2}+9bx+9cx^2}}$$

[Out] $-3^{1/6}h(c^2h^2(-2bh+cg)(bh+cg)/c/h^2-9bx-9cx^2)/(-bh+2c^2g)^{1/3} \arctan\left(\frac{-1/3^{1/2}+1/3^{2/3}(1-3h(2cx+b)/(-bh+2c^2g))^{2/3}}{(1+3h(2cx+b)/(-bh+2c^2g))^{1/3}}\right) \sqrt[3]{\frac{f(b^2h^2-bcgh+c^2g^2)}{3c^2h^2} + \frac{bfx}{c} + fx^2} / (-2bh+cg)(bh+cg)/c/h^2+9bx+9cx^2)^{1/3} + 1/2 \sqrt[3]{\frac{f(b^2h^2-bcgh+c^2g^2)}{3c^2h^2} + \frac{bfx}{c} + fx^2} \ln\left(\frac{1/3f(b^2h^2-bcgh+c^2g^2)/c^2/h^2+bfx/c+fx^2}{(-2bh+cg)(bh+cg)/c/h^2+9bx+9cx^2}\right)^{1/3} - 3/2 \sqrt[3]{\frac{f(b^2h^2-bcgh+c^2g^2)}{3c^2h^2} + \frac{bfx}{c} + fx^2} \ln\left(\frac{1-3h(2cx+b)/(-bh+2c^2g)^{2/3}+2^{1/3}(1+3h(2cx+b)/(-bh+2c^2g))^{1/3}}{(-2bh+cg)(bh+cg)/c/h^2+9bx+9cx^2}\right)^{1/3}$

Rubi [A] time = 0.36, antiderivative size = 488, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 104, $\frac{\text{number of rules}}{\text{integrand size}} = 0.019$, Rules used = {1041, 1040}

$$\frac{3^{2/3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}} \log\left(\frac{f\left(b^2h^2-bcgh+c^2g^2\right)}{3c^2h^2} + \frac{bfx}{c} + fx^2\right)}{2f\sqrt[3]{-\frac{(cg-2bh)(bh+cg)}{ch^2}+9bx+9cx^2}} - \frac{3^{2/3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}} \log\left(\left(1 - \frac{(cg-2bh)(bh+cg)}{ch^2} + 9bx + 9cx^2\right)\right)}{2f\sqrt[3]{-\frac{(cg-2bh)(bh+cg)}{ch^2}+9bx+9cx^2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}\left[\frac{(g+hx)/\left(\left(-c^2g^2+bcgh+2b^2h^2\right)/\left(9c^2h^2+bx+cx^2\right)\right)^{1/3} \left(\frac{f\left(b^2-\left(-c^2g^2+bcgh+2b^2h^2\right)/\left(3h^2\right)\right)}{c^2} + \frac{bfx}{c} + fx^2\right)}{c+fx^2}\right], x$

[Out] $\left(\frac{3^{1/6}h\left(\frac{c^2h^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}\right)^{1/3} \text{ArcTan}\left[\frac{1/\sqrt[3]{\frac{f\left(b^2-\left(-c^2g^2+bcgh+2b^2h^2\right)/\left(3h^2\right)\right)}{c^2} + \frac{bfx}{c} + fx^2}}{\left(1-\frac{(cg-2bh)(bh+cg)}{ch^2}+9bx+9cx^2\right)^{1/3}}\right]}{\left(2c^2g-bh\right)^{2/3}}\right) / \left(\sqrt[3]{\frac{f\left(b^2-\left(-c^2g^2+bcgh+2b^2h^2\right)/\left(3h^2\right)\right)}{c^2} + \frac{bfx}{c} + fx^2}\right) \left(1+\frac{3h\left(b+2cx\right)}{2c^2g-bh}\right)^{1/3}$

$$\left. \right) \left. \right) / (f * (-((c * g - 2 * b * h) * (c * g + b * h)) / (c * h^2)) + 9 * b * x + 9 * c * x^2)^{(1/3)} + (3^{(2/3)} * h * ((c * h^2 * ((c * g - 2 * b * h) * (c * g + b * h)) / (c * h^2) - 9 * b * x - 9 * c * x^2)) / (2 * c * g - b * h)^{(2/3)}) * \text{Log}[(f * (c^2 * g^2 - b * c * g * h + b^2 * h^2)) / (3 * c^2 * h^2 + (b * f * x) / c + f * x^2)] / (2 * f * (-((c * g - 2 * b * h) * (c * g + b * h)) / (c * h^2)) + 9 * b * x + 9 * c * x^2)^{(1/3)} - (3 * 3^{(2/3)} * h * ((c * h^2 * ((c * g - 2 * b * h) * (c * g + b * h)) / (c * h^2) - 9 * b * x - 9 * c * x^2)) / (2 * c * g - b * h)^{(2/3)}) * \text{Log}[(1 - (3 * h * (b + 2 * c * x)) / (2 * c * g - b * h))^{(2/3)} + 2^{(1/3)} * (1 + (3 * h * (b + 2 * c * x)) / (2 * c * g - b * h))^{(1/3)}] / (2 * f * (-((c * g - 2 * b * h) * (c * g + b * h)) / (c * h^2)) + 9 * b * x + 9 * c * x^2)^{(1/3)}$$

Rule 1040

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] := With[{q = ((-9*c*h^2)/(2*c*g - b*h)^2)^(1/3)}, Simp[(Sqrt[3]*h*q*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - (3*h*(b + 2*c*x))/(2*c*g - b*h))^(2/3))]/(Sqrt[3]*(1 + (3*h*(b + 2*c*x))/(2*c*g - b*h))^(1/3))]/f, x] + (-Simp[(3*h*q*Log[(1 - (3*h*(b + 2*c*x))/(2*c*g - b*h))^(2/3) + 2^(1/3)*(1 + (3*h*(b + 2*c*x))/(2*c*g - b*h))^(1/3)]]/(2*f), x] + Simp[(h*q*Log[d + e*x + f*x^2])/(2*f), x]) /; FreeQ[{a, b, c, d, e, f, g, h}, x] && EqQ[c*e - b*f, 0] && EqQ[c^2*d - f*(b^2 - 3*a*c), 0] && EqQ[c^2*g^2 - b*c*g*h - 2*b^2*h^2 + 9*a*c*h^2, 0] && GtQ[(-9*c*h^2)/(2*c*g - b*h)^2, 0]
```

Rule 1041

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] := With[{q = -(c/(b^2 - 4*a*c))}, Dist[(q*(a + b*x + c*x^2)^(1/3)/(a + b*x + c*x^2)^(1/3), Int[(g + h*x)/((q*a + b*q*x + c*q*x^2)^(1/3)*(d + e*x + f*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && EqQ[c*e - b*f, 0] && EqQ[c^2*d - f*(b^2 - 3*a*c), 0] && EqQ[c^2*g^2 - b*c*g*h - 2*b^2*h^2 + 9*a*c*h^2, 0] && !GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \frac{g + hx}{\sqrt[3]{\frac{-c^2g^2 + bcgh + 2b^2h^2}{9ch^2} + bx + cx^2} \left(\frac{f\left(\frac{b^2 - \frac{-c^2g^2 + bcgh + 2b^2h^2}{3h^2}}{c^2}\right) + \frac{bfx}{c} + fx^2}{c^2} \right)} dx = \frac{\sqrt[3]{\frac{c\left(\frac{-c^2g^2 + bcgh + 2b^2h^2}{9ch^2} + bx + cx^2\right)}{b^2 - \frac{4(-c^2g^2 + bcgh + 2b^2h^2)}{9h^2}}} \int \frac{f\left(\frac{b^2 - \frac{-c^2g^2}{3h^2}}{c^2}\right)}{\left(\frac{b^2 - \frac{-c^2g^2}{3h^2}}{c^2}\right)} dx}{3\sqrt[6]{3} h \sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(cg+bh)}{ch^2} - 9bx - 9cx^2\right)}{(2cg-bh)^2}} \tan^{-1} \frac{f\sqrt[3]{-\frac{(cg-2bh)(cg+bh)}{ch^2} + 9bx}}{f\sqrt[3]{-\frac{(cg-2bh)(cg+bh)}{ch^2} + 9bx}}}$$

Mathematica [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{g + hx}{\sqrt[3]{\frac{-c^2g^2 + bcgh + 2b^2h^2}{9ch^2} + bx + cx^2} \left(\frac{f\left(\frac{b^2 - \frac{-c^2g^2 + bcgh + 2b^2h^2}{3h^2}}{c^2}\right) + \frac{bfx}{c} + fx^2}{c^2} \right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(g + h*x)/((((-c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(9*c*h^2) + b*x + c*x^2)^(1/3)*((f*(b^2 - (-c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(3*h^2)))/c^2 + (b*f*x)/c + f*x^2)),x]

[Out] Integrate[(g + h*x)/((((-c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(9*c*h^2) + b*x + c*x^2)^(1/3)*((f*(b^2 - (-c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(3*h^2)))/c^2 + (b*f*x)/c + f*x^2)), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/3)/(f*(b^2+1/3*(-2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^2),x, algorithm m="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3(hx + g)}{\left(cx^2 + bx - \frac{c^2g^2 - bcgh - 2b^2h^2}{9ch^2}\right)^{\frac{1}{3}} \left(3fx^2 + \frac{3bfx}{c} + \frac{\left(3b^2 + \frac{c^2g^2 - bcgh - 2b^2h^2}{h^2}\right)f}{c^2}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/3)/(f*(b^2+1/3*(-2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^2),x, algorithm m="giac")

[Out] integrate(3*(h*x + g)/((c*x^2 + b*x - 1/9*(c^2*g^2 - b*c*g*h - 2*b^2*h^2)/c*h^2))^(1/3)*(3*f*x^2 + 3*b*f*x/c + (3*b^2 + (c^2*g^2 - b*c*g*h - 2*b^2*h^2)/h^2)*f/c^2), x)

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{hx + g}{\left(cx^2 + bx + \frac{2b^2h^2 + bcgh - c^2g^2}{9ch^2}\right)^{\frac{1}{3}} \left(fx^2 + \frac{bfx}{c} + \frac{\left(b^2 + \frac{-2b^2h^2 - bcgh + c^2g^2}{3h^2}\right)f}{c^2}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/3)/(f*(b^2+1/3*(-2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^2),x)

[Out] int((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/3)/(f*(b^2+1/3*(-2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$3 \int \frac{hx + g}{\left(cx^2 + bx - \frac{c^2g^2 - bcgh - 2b^2h^2}{9ch^2}\right)^{\frac{1}{3}} \left(3fx^2 + \frac{3bfx}{c} + \frac{\left(3b^2 + \frac{c^2g^2 - bcgh - 2b^2h^2}{h^2}\right)f}{c^2}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/3)/(f*(b^2+1/3*(-2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^2),x, algorithm m="maxima")


```
[Out] 3*integrate((h*x + g)/((c*x^2 + b*x - 1/9*(c^2*g^2 - b*c*g*h - 2*b^2*h^2)/(c*h^2))^(1/3)*(3*f*x^2 + 3*b*f*x/c + (3*b^2 + (c^2*g^2 - b*c*g*h - 2*b^2*h^2)/h^2)*f/c^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{g + hx}{\left(bx + cx^2 + \frac{2b^2h^2 + bcgh - c^2g^2}{9ch^2} \right)^{1/3} \left(fx^2 - \frac{f \left(\frac{2b^2h^2 + bcgh - c^2g^2}{3h^2} - b^2 \right)}{c^2} + \frac{bf x}{c} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)/((b*x + c*x^2 + ((2*b^2*h^2)/9 - (c^2*g^2)/9 + (b*c*g*h)/9)/(c*h^2))^(1/3)*(f*x^2 - (f*((2*b^2*h^2)/3 - (c^2*g^2)/3 + (b*c*g*h)/3)/h^2 - b^2))/c^2 + (b*f*x)/c), x)
```

```
[Out] int((g + h*x)/((b*x + c*x^2 + ((2*b^2*h^2)/9 - (c^2*g^2)/9 + (b*c*g*h)/9)/(c*h^2))^(1/3)*(f*x^2 - (f*((2*b^2*h^2)/3 - (c^2*g^2)/3 + (b*c*g*h)/3)/h^2 - b^2))/c^2 + (b*f*x)/c), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$3 \cdot 3^{\frac{2}{3}} c^2 h^2 \int \frac{g}{b^2 h^2 \sqrt[3]{\frac{2b^2}{c} + \frac{bg}{h} + 9bx - \frac{cg^2}{h^2} + 9cx^2 - bcgh} \sqrt[3]{\frac{2b^2}{c} + \frac{bg}{h} + 9bx - \frac{cg^2}{h^2} + 9cx^2 + 3bch^2 x} \sqrt[3]{\frac{2b^2}{c} + \frac{bg}{h} + 9bx - \frac{cg^2}{h^2} + 9cx^2 + c^2 g^2} \sqrt[3]{\frac{2b^2}{c} + \frac{bg}{h} + 9bx - \frac{cg^2}{h^2} + 9cx^2 + c^2 g^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(1/9*(2*b**2*h**2+b*c*g*h-c**2*g**2)/c/h**2+b*x+c*x**2)**(1/3)/(f*(b**2+1/3*(-2*b**2*h**2-b*c*g*h+c**2*g**2)/h**2)/c**2+b*f*x/c+f*x**2), x)
```

```
[Out] 3*3**(2/3)*c**2*h**2*(Integral(g/(b**2*h**2*(2*b**2/c + b*g/h + 9*b*x - c*g**2/h**2 + 9*c*x**2)**(1/3) - b*c*g*h*(2*b**2/c + b*g/h + 9*b*x - c*g**2/h**2 + 9*c*x**2)**(1/3) + 3*b*c*h**2*x*(2*b**2/c + b*g/h + 9*b*x - c*g**2/h**2 + 9*c*x**2)**(1/3) + c**2*g**2*(2*b**2/c + b*g/h + 9*b*x - c*g**2/h**2 + 9*c*x**2)**(1/3) + 3*c**2*h**2*x**2*(2*b**2/c + b*g/h + 9*b*x - c*g**2/h**2 + 9*c*x**2)**(1/3)), x) + Integral(h*x/(b**2*h**2*(2*b**2/c + b*g/h + 9*b*x - c*g**2/h**2 + 9*c*x**2)**(1/3) - b*c*g*h*(2*b**2/c + b*g/h + 9*b*x - c*g**2/h**2 + 9*c*x**2)**(1/3) + 3*b*c*h**2*x*(2*b**2/c + b*g/h + 9*b*x - c*g**2/h**2 + 9*c*x**2)**(1/3) + c**2*g**2*(2*b**2/c + b*g/h + 9*b*x - c*g**2/h**2 + 9*c*x**2)**(1/3) + 3*c**2*h**2*x**2*(2*b**2/c + b*g/h + 9*b*x - c*g**2/h**2 + 9*c*x**2)**(1/3)), x))/f
```


Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```

If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
  If[LeafCount[result]<=2*LeafCount[optimal],
    "A",
    "B"],
  "C"],
If[FreeQ[result,Integrate] && FreeQ[result,Int],
  "C",
"F"]]

```

```
(* ::Text:: *)
```

```
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
```

```
(*1 = rational function*)
```

```
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
```

```
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
```

```
(*6 = appell function*)
```

```
(*7 = rootsum function*)
```

```
(*8 = integrate function*)
```

```
(*9 = unknown function*)
```

```
ExpnType[expn_] :=
```

```
  If[AtomQ[expn],
```

```
    1,
```

```
  If[ListQ[expn],
```

```
    Max[Map[ExpnType,expn]],
```

```
  If[Head[expn]===Power,
```

```
    If[IntegerQ[expn[[2]],
```

```
      ExpnType[expn[[1]],
```

```
    If[Head[expn[[2]]===Rational,
```

```
      If[IntegerQ[expn[[1]] || Head[expn[[1]]===Rational,
```

```
        1,
```

```
        Max[ExpnType[expn[[1]],2]],
```

```
      Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
```

```
    If[Head[expn]===Plus || Head[expn]===Times,
```

```
      Max[ExpnType[First[expn],ExpnType[Rest[expn]]],
```

```
    If[ElementaryFunctionQ[Head[expn]],
```

```
      Max[3,ExpnType[expn[[1]]],
```

```
    If[SpecialFunctionQ[Head[expn]],
```

```
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
```

```
    If[HypergeometricFunctionQ[Head[expn]],
```

```
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
```

```
    If[AppellFunctionQ[Head[expn]],
```

```
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```

If[Head[expn]===RootSum,
  Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
If[Head[expn]===Integrate || Head[expn]===Int,
  Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
9]]]]]]]]]]

ElementaryFunctionQ[func_] :=
MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]

HypergeometricFunctionQ[func_] :=
MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]

AppellFunctionQ[func_] :=
MemberQ[{AppellF1},func]

```

4.0.2 Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000

```

```

#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
# see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

    leaf_count_result:=leafcount(result);
    #do NOT call ExpnType() if leaf size is too large. Recursion problem
    if leaf_count_result > 500000 then
        return "B";
    fi;

    leaf_count_optimal:=leafcount(optimal);

    ExpnType_result:=ExpnType(result);
    ExpnType_optimal:=ExpnType(optimal);

    if debug then
        print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
            ExpnType_optimal);
    fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;

```

```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do not
as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false

```

```

#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  end if
end proc:

```



```

elif HypergeometricFunctionQ(op(0,expn)) then
  max(5,apply(max,map(ExpnType,[op(expn)])))
elif AppellFunctionQ(op(0,expn)) then
  max(6,apply(max,map(ExpnType,[op(expn)])))
elif op(0,expn)='int' then
  max(8,apply(max,map(ExpnType,[op(expn)]))) else
9
end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
    MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
        asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
        asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
        fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
        gamma,loggamma,digamma,zeta,polylog,LambertW,
        elliptic_f,elliptic_e,elliptic_pi,exp_polar
        ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'`^`')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
    ))
    else:
        return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
    ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
    (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))

```

```

elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
    return max(7,m1)
elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```

4.0.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fracas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:

```

```

        print ("func ", func , " is elementary_function")
    else:
        print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:

```

```

        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(), "base_ring") and hasattr(expn.parent(), "gens"):
        return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
    return False

except AttributeError as error:
    return False

def expnType(expn):

    if debug:
        print(">>>>Enter expnType, expn=", expn)
        print(">>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #instance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(instance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #instance(expn,Pow)
        if type(expn.operands()[1])==Integer: #instance(expn.args[1],Integer)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #instance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #instance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
instance(expn,Add) or instance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))

```

```

    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(4,m1) #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(5,m1) #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(6,m1) #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
#is checked before calling the grading function that is passed.
#but kept it here.
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

```

```
#main function
```

```
def grade_antiderivative(result,optimal):
```

```

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

```

```

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

```

```

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

```

```

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex

```



```
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
    else: #result contains complex but optimal is not
        return "C"
else: # result do not contain complex, this assumes optimal do not as
well
    if leaf_count_result <= 2*leaf_count_optimal:
        return "A"
    else:
        return "B"
else:
    return "C"
```